

Second Harmonic Generation and related second order Nonlinear Optics

Nicolas Fressengeas

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Second Harmonic Generation and related second order Nonlinear Optics

N. Fressengeas

Laboratoire Matériaux Optiques, Photonique et Systèmes Unité de Recherche commune à l'Université de Lorraine et à Supélec

November 12, 2012



Useful reading... [YY84, DGN91, LKW99]

V. G. Dmitriev, G. G. Gurzadyan, and D. N. Nikogosyan.

Handbook of Nonlinear Optical Crystals, volume 64 of Springer Series in Optical Sciences.

Springer Verlag, Heidelberg, Germany, 1991.

- W. Lauterborn, T. Kurz, and M. Wiesenfeldt.

 Coherent Optics: Fundamentals and Applications.

 Springer-Verlag, New York, 1999.
 - A. Yariv and P. Yeh.

 Optical waves in crystals. Propagation and control of laser radiation.

 Wiley series in pure and applied optics. Wiley-Interscience, Stanford University, 1984.

Contents

- Three-wave interaction
 - Assumptions framework
 - Three Wave propagation equation
 - Sum frequency generation
 - Scalar approximation
- Non Linear Optics Application
 - Second Harmonic Generation
 - Optical Parametric Amplifier
 - Optical Parametric Oscillator
- Phase matching
 - Phase matching conditions
 - Phase matching in uni-axial crystals
 - Quasi-phase matching





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With standard assumptions: no charge, no current, no magnet and no conductivity

Maxwell Model

- $\operatorname{div}(D) = 0$
- div(B) = 0
- $\operatorname{curl}(E) = -\frac{\partial B}{\partial t}$
- $\operatorname{curl}(H) = \frac{\partial D}{\partial t}$

Matter equations

•
$$D = \varepsilon_0 E + P = \varepsilon E$$

•
$$B = \mu_0 H$$

$$P = \varepsilon_0 \chi_L E + P_{NL}$$

Wave equation in isotropic medium

$$\Delta E - \mu_0 \varepsilon \frac{\partial^2 E}{\partial t^2} = \mu_0 \frac{\partial^2 P_{NL}}{\partial t^2}$$

Solutions to be found only in a specific frameworkk

We look here for :

- Quadratic non-linearity
- Three wave interaction



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All waves are transverse plane waves propagating in the z direction

- Transversal E: has x and y components
- Wave equation : $\frac{\partial^2 E}{\partial z^2} \mu_0 \varepsilon \frac{\partial^2 E}{\partial t^2} = \mu_0 \frac{\partial^2 P_{NL}(z)}{\partial t^2}$

Quadratic non-linearity

- \bullet P_{NL} is transversal
- $[P_{NL}]_i = \sum_{\{j,k\} \in \{x,y\}^2} [d]_{ijk} [E]_j [E]_k = [d]_{ijk} [E]_j [E]_k$

Three wave interaction only

- Three waves only are present : ω_1 , ω_2 and ω_3
- Non linear interaction of two waves: sum, difference, doubling, rectification...
- We consider only those for which $\omega_1 + \omega_2 = \omega_3$

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Sum of three waves

•
$$[E]_{x,y}(z,t) = \mathcal{R}e\left(\sum_{\nu=1}^{3} \exp(ik_{\nu}z - i\omega_{\nu}t)\right)$$

- Dispersion law : $k_{\nu}^2 = \mu_0 \varepsilon_{\nu} \omega_{\nu}^2$
- ε dispersion : $\varepsilon_{\nu} = \varepsilon \left(\omega_{\nu} \right)$

How good is this ansatz?

- We have assumed $\omega_1 + \omega_2 = \omega_3$
- Why would ω_3 not be a source as well ?
- OK if wave 3 is small enough

Separate investigation at each frequency

- ullet At $\omega_{
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$$\nu \in \{1, 2, 3\}$$

3 Perturbed wave equations

$$\frac{\partial^2 E^{(\omega_{\nu})}(z,t)}{\partial z^2} - \mu_0 \varepsilon_{\omega_{\nu}} \frac{\partial^2 E^{(\omega_{\nu})}(z,t)}{\partial t^2} = \mu_0 \frac{\partial^2 P^{\omega_{\nu}}_{NL}(z,t)}{\partial t^2}$$

Temporal harmonic notation

- $E^{(\omega_{\nu})}$ is a plane wave
- Non linear polarization is a plane wave
- Considering only...the ω_{ν} part \Leftrightarrow the plane wave part

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$$\frac{\partial^2 E^{(\omega_{\nu})}(z,t)}{\partial z^2} + \mu_0 \varepsilon_{\omega_{\nu}} \omega_{\nu}^2 E^{(\omega_{\nu})}(z,t) = -\mu_0 \omega_{\nu}^2 P_{NL}^{\omega_{\nu}}(z,t)$$





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Reminder

$$[P_{NL}]_i = [d]_{ijk}[E]_j[E]_k$$

Temporal harmonic framework

or ω_1

- Multiply complex fields
- Include Conjugates to take Real Part
- Select only the ω_1 component

$$\left[P_{NL}^{\omega_{1}}\left(z,t\right)\right]_{i}=\mathcal{R}e\left(\left[d\right]_{ijk}\left[E^{\left(\omega_{3}\right)}\left(z\right)\right]_{j}\overline{\left[E^{\left(\omega_{2}\right)}\left(z\right)\right]_{k}}e^{\left(i\left(k_{3}-k_{2}\right)z-i\left(\omega_{3}-\omega_{2}\right)t\right)}\right)$$

Wave propagation equation

$$\frac{\partial^{2} E^{(\omega_{1})}(z,t)}{\partial z^{2}} + \mu_{0} \varepsilon_{\omega_{1}} \omega_{1}^{2} E^{(\omega_{1})}(z,t) = -\mu_{0} \omega_{1}^{2} P_{NL}^{\omega_{1}}(z,t)$$

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N. Fressengeas (LMOPS)

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Closely related to the paraxial approximation

The Slow Varying Approximation

- Beam envelope is assumed to vary slowly in the longitudinal direction
- Equivalent as assuming a narrow beam
- Second derivative with respect z neglected compared to
 - the first one with respect to z
 - the others with respect to x and y

To put it in maths.

•
$$\frac{\partial^{2} E^{(\omega_{1})}(z,t)}{\partial z^{2}} = \frac{\partial^{2}}{\partial z^{2}} \mathcal{R}e\left(E^{(\omega_{1})}(z) \exp\left(i\left(k_{1}z - \omega_{1}t\right)\right)\right)$$
•
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Wave Propagation Equation under SVA approximation

Obtaining an envelope equation, which is simpler

Non SVA wave propagation equation

$$\frac{\partial^{2} E^{(\omega_{1})}\left(z,t\right)}{\partial z^{2}}+\mu_{0} \varepsilon_{\omega_{1}} \omega_{1}^{2} E^{(\omega_{1})}\left(z,t\right)=-\mu_{0} \omega_{1}^{2} P_{NL}^{\omega_{1}}\left(z,t\right)$$

SVA equation

•
$$\operatorname{Re}\left(\left[2ik_1\frac{\partial E^{(\omega_1)}(z)}{\partial z}-k_1^2E^{(\omega_1)}(z)\right]e^{(i(k_1z-\omega_1t))}\right)$$

• +
$$\mathcal{R}e\left(\mu_0\varepsilon_{\omega_1}\omega_1^2E^{(\omega_1)}(z)e^{(i(k_1z-\omega_1t))}\right)$$

$$\bullet = \mathcal{R}e\left(-\mu_0\omega_1^2[d]_{ijk}\left[E^{(\omega_3)}(z)\right]_i\overline{\left[E^{(\omega_2)}(z)\right]_k}e^{(i(k_3-k_2)z-i(\omega_3-\omega_2)t)}\right)$$





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SVA equation

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$$\Re e \left(\left[2ik_1 \frac{\partial E^{(\omega_1)}(z)}{\partial z} - k_1^2 E^{(\omega_1)}(z) \right] e^{(i(k_1 z - \omega_1 t))} \right)$$

• +
$$\Re e \left(\mu_0 \varepsilon_{\omega_1} \omega_1^2 E^{(\omega_1)}(z) e^{(i(k_1 z - \omega_1 t))} \right)$$

$$\bullet = \mathcal{R}e\left(-\mu_0\omega_1^2[d]_{ijk}\left[E^{(\omega_3)}(z)\right]_i\overline{\left[E^{(\omega_2)}(z)\right]_k}e^{(i(k_3-k_2)z-i(\omega_3-\omega_2)t)}\right)$$



November 12, 2012

Wave Propagation Equation under SVA approximation

Obtaining an envelope equation, which is simpler

Non SVA wave propagation equation

$$\frac{\partial^{2} E^{(\omega_{1})}\left(z,t\right)}{\partial z^{2}}+\mu_{0} \varepsilon_{\omega_{1}} \omega_{1}^{2} E^{(\omega_{1})}\left(z,t\right)=-\mu_{0} \omega_{1}^{2} P_{NL}^{\omega_{1}}\left(z,t\right)$$

SVA equation

•
$$\Re e \left(\left[2ik_1 \frac{\partial E^{(\omega_1)}(z)}{\partial z} - k_1^2 E^{(\omega_1)}(z) \right] e^{(i(k_1 z - \omega_1 t))} \right)$$

• +
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$$\bullet = \mathcal{R}e\left(-\mu_0\omega_1^2[d]_{ijk}\big[E^{(\omega_3)}\left(z\right)\big]_j\overline{\big[E^{(\omega_2)}\left(z\right)\big]_k}e^{(i(k_3-k_2)z-i(\omega_3-\omega_2)t)}\right)$$



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《中医《國医》(基医《基医》) 基 ()

Obtaining an envelope equation, which is simpler

Non SVA wave propagation equation

$$\frac{\partial^{2} E^{(\omega_{1})}(z,t)}{\partial z^{2}} + \mu_{0} \varepsilon_{\omega_{1}} \omega_{1}^{2} E^{(\omega_{1})}(z,t) = -\mu_{0} \omega_{1}^{2} P_{NL}^{\omega_{1}}(z,t)$$

•
$$\Re \left(\left[2ik_1\frac{\partial E^{(\omega_1)}(z)}{\partial z}-k_1^2E^{(\omega_1)}(z)\right]e^{(i(k_1z-\omega_1t))}\right)$$

- + $\Re \left(\mu_0 \varepsilon_{\omega_1} \omega_1^2 E^{(\omega_1)}(z) e^{(i(k_1 z \omega_1 t))}\right)$
- $\bullet = \mathcal{R}e\left(-\mu_0\omega_1^2[d]_{ijk}\left[E^{(\omega_3)}\left(z\right)\right]_j\overline{\left[E^{(\omega_2)}\left(z\right)\right]_k}e^{(i(k_3-k_2)z-i(\omega_3-\omega_2)t)}\right)$





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•
$$\operatorname{Re}\left(\left[2ik_1\frac{\partial E^{(\omega_1)}(z)}{\partial z}\right]e^{(i(k_1z-\omega_1t))}\right)$$

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The three waves

Phase mismatch and dispersion relationship

- Phase mismatch : $\Delta k = k_1 + k_2 k_3$
- ullet Recall the dispersion relationship : $k_1^2=\mu_0 arepsilon_{\omega_1} \omega_1^2$
- Wave impedance : $\eta_{
 u} = \sqrt{\frac{\mu_0}{arepsilon_{\omega_{
 u}}}}$

Three wave propagation, rotating i o j o k $(i,j,k) \in \{x,y\}^3$

•
$$\left[\frac{\partial E^{(\omega_2)}}{\partial z}\right]_k = -\frac{i\omega_2}{2}\eta_2[d]_{kij}\left[E^{(\omega_1)}\right]_i\overline{\left[E^{(\omega_3)}\right]_j}\exp\left(-i\Delta kz\right)$$

•
$$\left[\frac{\partial E^{(\omega_3)}}{\partial z}\right]_i = +\frac{i\omega_3}{2}\eta_3[d]_{jki}\left[E^{(\omega_2)}\right]_k\left[E^{(\omega_1)}\right]_i \exp\left(i\Delta kz\right)$$





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Three wave propagation, rotating
$$i \rightarrow j \rightarrow k$$

$$(i,j,k) \in \{x,y\}^3$$

•
$$\left[\frac{\partial E^{(\omega_1)}}{\partial z}\right]_i = +\frac{i\omega_1}{2}\eta_1[d]_{ijk}\left[E^{(\omega_3)}\right]_j\overline{\left[E^{(\omega_2)}\right]_k}\exp\left(-i\Delta kz\right)$$

•
$$\overline{\left[\frac{\partial E^{(\omega_2)}}{\partial z}\right]_k} = -\frac{i\omega_2}{2}\eta_2[d]_{kij}[E^{(\omega_1)}]_i\overline{\left[E^{(\omega_3)}\right]_j}\exp(-i\Delta kz)$$
•
$$\left[\frac{\partial E^{(\omega_3)}}{\partial z}\right]_i = +\frac{i\omega_3}{2}\eta_3[d]_{jki}[E^{(\omega_2)}]_k[E^{(\omega_1)}]_i\exp(i\Delta kz)$$

•
$$\left[\frac{\partial E^{(\omega_3)}}{\partial z}\right]_i = +\frac{i\omega_3}{2}\eta_3[d]_{jki}[E^{(\omega_2)}]_k[E^{(\omega_1)}]_i \exp(i\Delta kz)$$





6 equations for various quadratic phenomena

All in one for: frequency sum and difference, second harmonic generation and optical rectification, parametric amplifier...

Six equations

$$(i,j,k) \in \{x,y\}^3$$

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$$\left[\frac{\partial E^{(\omega_1)}}{\partial z}\right]_i = +\frac{i\omega_1}{2}\eta_1[d]_{ijk}\left[E^{(\omega_3)}\right]_j\overline{\left[E^{(\omega_2)}\right]_k}\exp\left(-i\Delta kz\right)$$

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•
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Why all those names ?

They differ by:

- The input frequencies and the generated ones
- The one that is the smallest and those which are large
-

- Three-wave interaction
 - Assumptions framework
 - Three Wave propagation equation
 - Sum frequency generation
 - Scalar approximation
- Non Linear Optics Application
 - Second Harmonic Generation
 - Optical Parametric Amplifier
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- Phase matching
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 - Phase matching in uni-axial crystals
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Example: sum frequency generation

Input beams assumed constant

Undepleted pump approximation

Assumptions

- $\omega_1 + \omega_2 = \omega_3$
- Generated beam null at z=0: $\left[E^{(\omega_3)}(z=0)\right]_i=0$

$$\bullet \ \frac{\partial \left[E^{(\omega_1)} \right]_i}{\partial z} = \overline{\frac{\partial \left[E^{(\omega_2)} \right]_k}{\partial z}} = 0$$

$$\left[\frac{\partial E^{(\omega_3)}}{\partial z}\right]_j = +\frac{i\omega_3}{2}\eta_3[d]_{jki}\left[E^{(\omega_2)}\right]_k\left[E^{(\omega_1)}\right]_i \exp\left(-i\Delta kz\right)$$





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One equation remains

$$\left[\frac{\partial E^{(\omega_3)}}{\partial z}\right]_i = +\frac{i\omega_3}{2}\eta_3[d]_{jki}\left[E^{(\omega_2)}\right]_k\left[E^{(\omega_1)}\right]_i \exp\left(-i\Delta kz\right)$$



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Equation to solve

$$\left[\frac{\partial E^{(\omega_3)}}{\partial z}\right]_j = +\frac{i\omega_3}{2}\eta_3[d]_{jki}\left[E^{(\omega_2)}\right]_k\left[E^{(\omega_1)}\right]_i \exp\left(-i\Delta kz\right)$$

$$\Delta k \neq 0$$

 $y' = ae^{(ibx)} \Rightarrow y = \frac{ia}{b} (1 - e^{(ibx)})$

Wave solution

$$\left[E^{(\omega_3)}\right]_j$$

$$\frac{i\omega_3}{2}\eta_3[d]_{jki}\big[E^{(\omega_2)}\big]_k\big[E^{(\omega_1)}\big]_i\frac{e^{(i\Delta kz)}-1}{i\Delta k}$$

Intensity
$$\propto [E^{(\omega_3)}]_j \overline{[E^{(\omega_3)}]}_j$$

$$\omega_3^2 \eta_3^2 \left[d^2 \right]_{jki} \left| E^{(\omega_2)} \right|_k^2 \left| E^{(\omega_1)} \right|_i^2 \frac{\sin^2\left(\frac{\Delta k_z}{2}\right)}{\Delta k^2}$$





Equation to solve

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ye solution
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$$y' = a \Rightarrow y = ax$$

$$\frac{i\omega_3}{2}\eta_3[d]_{jki}[E^{(\omega_2)}]_k[E^{(\omega_1)}]_iz$$

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 $[E^{(\omega_3)}]_i$

Equation to solve

$$\left[\frac{\partial E^{(\omega_3)}}{\partial z}\right]_j = +\frac{i\omega_3}{2}\eta_3[d]_{jki}\left[E^{(\omega_2)}\right]_k\left[E^{(\omega_1)}\right]_i \exp\left(-i\Delta kz\right)$$

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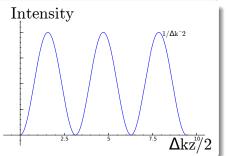
Phase match or not phase match

Phase matching is a key issue to sum frequency generation

Phase mismatch

 $\Delta k \neq 0$

- Oscillating intensity
- Max intensity $\propto \frac{1}{\Delta L^2}$







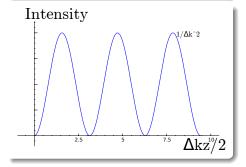
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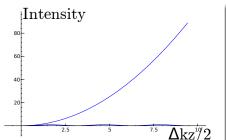
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Phase match

 $\Delta k = 0$

- Intensity quadratic increase
- Approximations do not hold long



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Further approximations to remove vectors

Simplifying notations

- ullet Set indexes equal for polarization and frequency: $A_
 u = ig[E^{(\omega_
 u)}ig]_
 u$
- Consider $\varepsilon_{\nu} = n_{\nu}^2 \varepsilon_0$
- Abbreviate $C = \sqrt{\frac{\mu_0}{\varepsilon_0}} \sqrt{\frac{\omega_1 \omega_2 \omega_3}{n_1 n_2 n_3}}$
- For lossless media, d is isotropic
- Assume d is frequency independent
- Let K = dC/2

- $\frac{\partial A_1}{\partial z} = +iK\overline{A_2}A_3 \exp(-i\Delta kz)$
- $\frac{\partial A_2}{\partial z} = -iKA_1\overline{A_3}\exp(+i\Delta kz)$
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Further approximations to remove vectors

Simplifying notations

- ullet Set indexes equal for polarization and frequency: $A_
 u = ig[E^{(\omega_
 u)} ig]_
 u$
- Consider $\varepsilon_{\nu} = n_{\nu}^2 \varepsilon_0$
- Abbreviate $C = \sqrt{\frac{\mu_0}{\varepsilon_0}} \sqrt{\frac{\omega_1 \omega_2 \omega_3}{n_1 n_2 n_3}}$
- For lossless media, d is isotropic.
- Assume *d* is frequency independent
- Let K = dC/2

- $\frac{\partial A_1}{\partial z} = +iK\overline{A_2}A_3\exp\left(-i\Delta kz\right)$
- $\bullet \ \ \overline{\frac{\partial A_2}{\partial z}} = -iKA_1\overline{A_3}\exp\left(+i\Delta kz\right)$
- $\frac{\partial A_3}{\partial z} = +iKA_2A_1\exp(+i\Delta kz)$

Non Linear Optics Application

- - Assumptions framework
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SHG

Sum frequency generation of two equal frequencies from the same source

One input beam counts for two

•
$$\omega_1 = \omega_2$$
, $A_1 = A_2$, $k_1 = k_2$, $\omega_3 = 2\omega_1$

$$\Delta k = 2k_1 - k_3$$

•
$$\frac{\partial A_1}{\partial z} = +iK\overline{A_1}A_3 \exp\left(-i\Delta kz\right)$$

•
$$\frac{\partial A_3}{\partial z} = +iKA_1^2 \exp\left(+i\Delta kz\right)$$

$$\bullet$$
 $\frac{\partial A_1}{\partial x} = +iK\overline{A_1}A_2$

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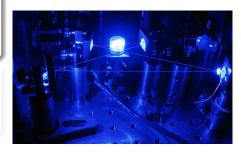


Figure: Closeup of a BBO crystal inside a resonant build-up ring cavity for frequency doubling 461 nm blue light into the ultraviolet. (source flicky

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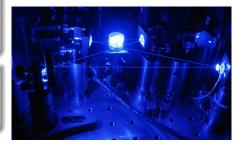
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《中》《圖》《意》《意》

 $\Delta k = 0$

Remember...

member...
$$\Delta k = 0$$

•
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• $\frac{\partial A_3}{\partial z} = +iKA_1^2$

$$A_3 \in i\mathbb{R}$$

•
$$\frac{\partial A_1}{\partial z} = -K\overline{A_1}\tilde{A}_3$$

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$$\frac{\partial \tilde{A}_3}{\partial z} = KA_1^2$$

$$\left(A_1^2(z) + \tilde{A}_3^2(z)\right) = A_1^2(0)$$

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Real equations

•
$$\frac{\partial \tilde{A}_3}{\partial z} = KA_1^2$$

Multiply by A_1 and $ilde{A}_3$

 $\frac{\partial \left(A_1^2 + \tilde{A}_3^2\right)}{\partial z} = 0$

This is Energy Conservation

Start with no harmonic

$$\left(A_1^2(z) + \tilde{A}_3^2(z)\right) = A_1^2(0)$$

\tilde{A}_3 equation

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 $ilde{A}_3$ expression

$$\tilde{A}_3(z) = A_1(0) \tanh(KA_1(0)z)$$

 I_3 expression

$$I_3(z) = I_1(0) \tanh^2(KA_1(0)z)$$

$$I_1(z) = I_1(0) - I_3(z)$$

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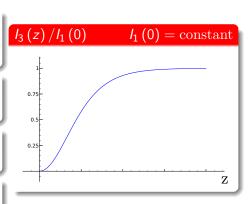
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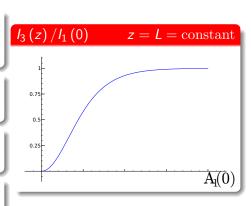
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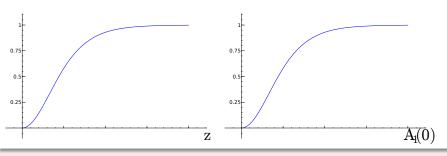
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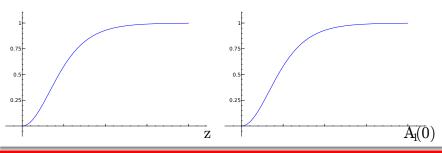
Second Harmonic Beam Evolution



Suprinsingly.

- It is possible to convert 100% of a beam, with large interaction length or intensity
- The process has no threshold and does not need noise to start
- We have retrieved Energy Conservation in spite of drastic approximations

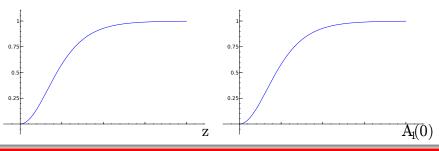
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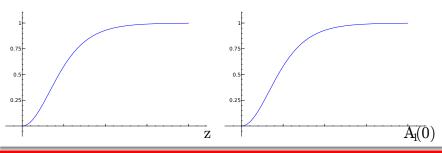
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Optical Parametric Amplifier

OPA

Optical Amplification of a weak signal beam thanks to a powerful pump beam

Signal beam amplification

- ω_1 : weak signal to be amplified
- ullet ω_3 : intense pump beam
- $\omega_2 = \omega_3 \omega_1$: difference frequency generation (idler)

Undepleted pump approximation

$$A_3(z) = A_3(0) = K_p/K$$

Phase matched equations

- $\bullet \ \frac{\partial A_1}{\partial z} = +iK_p\overline{A_2}$
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Figure: White light continuum seeded optical parametric amplifier (OPA) able to generate extremely short pulses. (source Freie Universität Berlin)

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Initial conditions

- A weak signal : $A_1(0) \neq 0$
- No idler : $A_2(0) = 0$

Amplitude solution

Amplified signal :

$$A_1(z) = A_1(0) \cosh(K_p z)$$

• Idler :

$$\overline{A_2(z)} = -iA_1(0)\sinh(K_p z)$$





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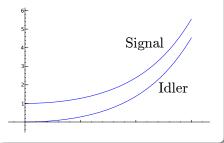
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Intensities

- Amplified signal : $I_1(z) = I_1(0) \cosh^2(K_p z)$
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Amplification



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Optical Parametric Oscillator

OPO

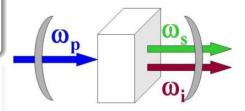
Use Optical Parametric Amplification to make a tunable laser

OPA pumped with ω_3

- Amplifier for ω_1 and ω_2
- With $\omega_1 + \omega_2 = \omega_3$
- Phase matching: $k_1 + k_2 = k_3$
- ω_1 and ω_2 initiated from noise

Frequency tunable laser

- Get Non Linear Medium
- Adjust Cavity for ω_1 and ω_2
- Pump with ω_3
- You got it!





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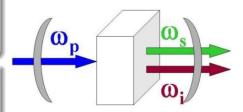


Figure: Optical Parametric Oscillato (source Cristal Laser)

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Colinear (scalar) phase matching

Phase matching for co-propagation waves

- $k_1 + k_2 = k_3 \Rightarrow \omega_1 n_1 + \omega_2 n_2 = \omega_3 n_3$
- for SHG : $2k_1 = k_3 \Rightarrow n_1 = n_3$
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One and only solution

Use birefringent crystals and different polarizations





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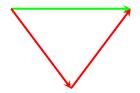
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Non colinear phase matching



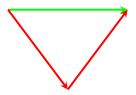




Non colinear phase matching

Use clever geometries

With reflections



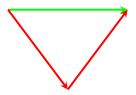




Non colinear phase matching

Use clever geometries

With reflections
Or even more clever...





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 - Phase matching conditions
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 - Quasi-phase matching





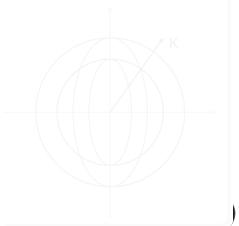
Waves polarization

- 1 incident wave counts for 2
- They share the same polarization
- Second Harmonic polarization is orthogonal

Type I phase matching

- One refraction index for Fundamental
- The other for Second Harmonic
- They must be equal
- Propagate in the right direction

 n_0 and n_e function of propagation direction: index ellipsoid cross-section





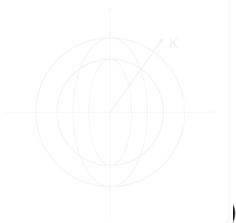
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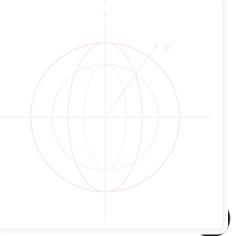
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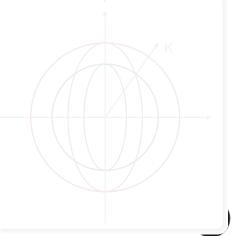
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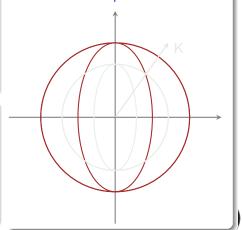
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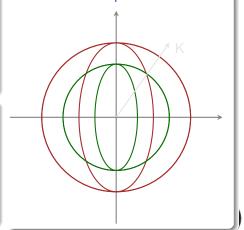
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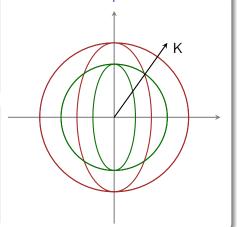
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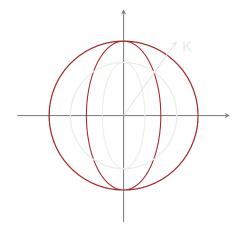
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Fundamental index ellipsoïd section

$$\frac{1}{n_{\rm e}^2(\theta)} = \frac{\cos^2(\theta)}{n_{\rm o}^2} + \frac{\sin^2(\theta)}{n_{\rm e}^2}$$

Harmonic index ellipsoïd section

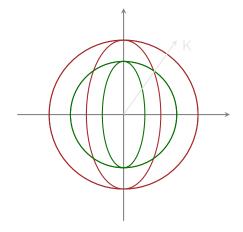
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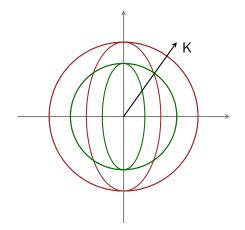
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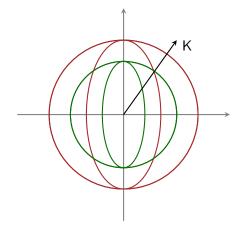
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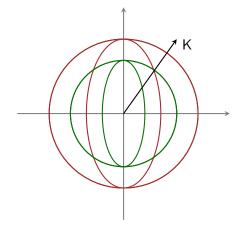
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- Both input beams ω_1 and ω_2 share the same polarization
- The generated beam ω_3 polarization is orthogonal

Another solution: Type II

- Input beams polarization are orthogonal
- Generated beam share one of them
- Not possible for SHG
- How is the angle calculated ?



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Phase matching in bi-axial crystals

A hard task

- Phase matching is seldom colinear
- Vector phase matching in a complex index ellipsoïd
- I will let you think on it

Paper by Boeuf can help



N. Boeuf, D. Branning, I. Chaperot, E. Dauler, S. Guerin, G. Jaeger, A. Muller, and A. Migdall.

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- Three-wave interaction
 - Assumptions framework
 - Three Wave propagation equation
 - Sum frequency generation
 - Scalar approximation
- Non Linear Optics Application
 - Second Harmonic Generation
 - Optical Parametric Amplifier
 - Optical Parametric Oscillator
- Phase matching
 - Phase matching conditions
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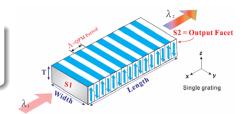


Phase matching

Quasi phase matching in layered media

Periodically Poled Lithium Niobate

- Periodic Domain Reversal
- d sign reversal



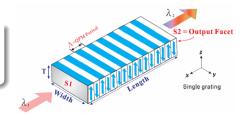
$$\left[\frac{\partial E^{(\omega_3)}}{\partial z}\right]$$

$$\frac{i\omega_3}{2}\eta_3[d]_{jki} |E^{(\omega_1)}|_i^2 \frac{e^{(i\Delta kA)}-1}{\Delta k} \sum_{n=1}^N (-1)^n e^{(i\Delta kA)}$$



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Wave solution

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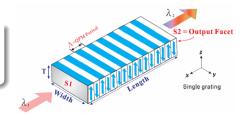
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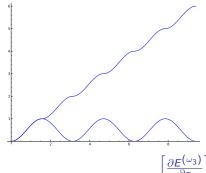
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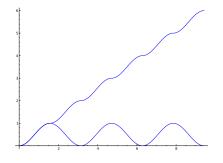
Wave solution

$$\left[\frac{\partial E^{(\omega_3)}}{\partial z}\right]_j$$

$$\frac{i\omega_3}{2}\eta_3[d]_{jki}|E^{(\omega_1)}|_i^2\frac{e^{(i\Delta k\Lambda)}-1}{\Delta k}\sum_{n=1}^N(-1)^ne^{(i\Delta k\Lambda)}$$

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Intsensity Solution

$$\left|\frac{i\omega_3}{2}\eta_3[d]_{jki}\left|E^{(\omega_1)}\right|_i^2\right|^2$$
 4 Λ^2 -



