



Photorefractivity

Nicolas Fressengeas

► To cite this version:

| Nicolas Fressengeas. Photorefractivity. DEA. Université Paul Verlaine Metz, 2010. cel-00520586

HAL Id: cel-00520586

<https://cel.hal.science/cel-00520586>

Submitted on 23 Sep 2010

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

UE SPM-PHO-S09-112

Photorefractivity

N. Fressengeas

Laboratoire Matériaux Optiques, Photonique et Systèmes
Unité de Recherche commune à l'Université Paul Verlaine Metz et à
Supélec

Download this document :
<http://moodle.univ-metz.fr/>

Photorefractive effect

History began in 1966 as *Optical Damage*



Optical Damage in LiNbO_3

- Shine a light on LiNbO_3
- Remove it
- Shine another : **damaged crystal**

Semi-permanent effect

- Leave it in the dark : still damaged
- Leave it under uniform light : sometimes repaired

Today

- Photorefractivity can prove useful
- Some people still call it *optical damage*:
 - Bad for linear optics (electro-optic modulators...)
 - Bad for *instant* Non Linear Optics (SHG, OPA...)

Photorefractivity
Version 1.2
frame 2

N. Fressengeas

Band Transport

Harmonic
illumination

Two Wave Mixing

Photorefractivity is attractive

Non linear optics at low optical power



Non Linear Optics

- Dynamic Holography
- Phase conjugation
- All optical computing
- ...
- At milliwatts and below power levels

Observed in many non linear crystals

- Sillenites : $\text{Bi}_{12}\text{SiO}_{20}$, $\text{Bi}_{12}\text{TiO}_{20}$, $\text{Bi}_{12}\text{GeO}_{20}$
- Tungsten-Bronze : $\text{Sr}_x\text{Ba}_{1-x}\text{Nb}_2\text{O}_6$
- Ferroelectrics : LiNbO_3 , BaTiO_3
- Semiconductors : InP:Fe , AsGa

Photorefractivity
Version 1.2
frame 3

N. Fressengeas

Band Transport

Harmonic
illumination

Two Wave Mixing

Usefull reading. . .

[Yeh93, GH88, GH89]



Photorefractivity
Version 1.2
frame 4

N. Fressengeas

Band Transport

Harmonic
illumination

Two Wave Mixing



P. Günter and J. P. Huignard.

Photorefractive materials and their applications I,
volume 61 of *Topics in Applied Physics*.
Springer Verlag, Berlin, 1988.



P. Günter and J. P. Huignard.

Photorefractive materials and their applications II,
volume 62 of *Topics in Applied Physics*.
Springer Verlag, Berlin, 1989.



P. Yeh.

Introduction to photorefractive nonlinear optics.
Wiley Interscience, New York, 1993.

1 Band Transport Model

- Schematics
- Carrier Generation
- Charge Transport
- Electro-optic effect

2 Harmonic illumination

- Harmonic framework
- Uniform background: order 0
- Periodic modulation : order 1
- Implications, Simplifications, Diffusion and Saturation

3 Two Wave Mixing

- Gratings graphical view
- Coupled waves
- Two Beam Coupling

Photorefractivity
Version 1.2
frame 5

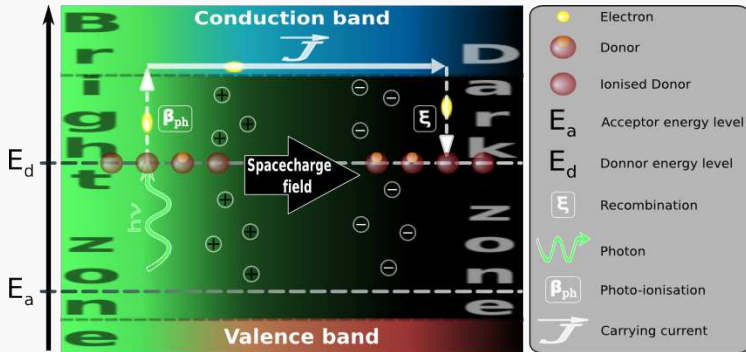
N. Fressengeas

Band Transport

Harmonic
illumination

Two Wave Mixing

Photorefractive charge transport and trapping



Linear Index Modulation

Space charge **electric field** generates **refractive index variation** through **electro-optic effect**

Photorefractivity
Version 1.2
frame 6

N. Fressegeas

Band Transport

Schematics
Carrier Generation
Charge Transport
Electro-optics

Harmonic
illumination

Two Wave Mixing

Periodic illumination of a photorefractive material

Photorefractivity
Version 1.2
frame 7

N. Fressengeas

Band Transport

Schematics

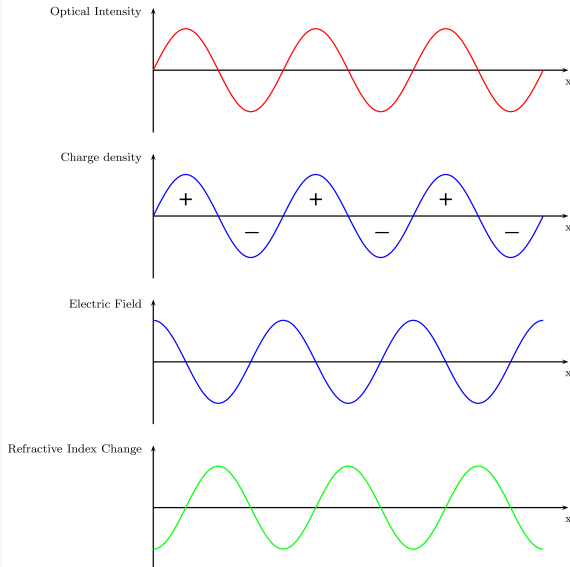
Carrier Generation

Charge Transport

Electro-optics

Harmonic
illumination

Two Wave Mixing



Donors wanted

Carriers are generated by donors: no donors, no carriers

Nominally pure crystals

- No in-band-gap level
 - No donor nor acceptor
 - No photorefractive effect
- Structural defects often present
- As well as pollutants
- They create in-band-gap levels
- Photorefractivity can arise from them

More efficient: doping

- Introduce in-band-gap species
- $\text{LiNbO}_3\text{:Fe}$, InP:Fe ...

Photorefractivity
Version 1.2
frame 8

N. Fressengeas

Band Transport

Schematics

Carrier Generation

Charge Transport

Electro-optics

Harmonic
illumination

Two Wave Mixing

Introducing Donors and Acceptors

Introduce Donors of electrons

N_D

- Energy level close to conduction band
- They easily **give electrons** to conduction band

Introducing Acceptors

$N_A \ll N_D$

- Photorefractivity needs traps
- Ionized donors are traps
- Introduce Acceptors close to the valence band
- They **catch Donors electrons**
- Donors are partially ionized $N_D^+ = N_A$

Photorefractivity
Version 1.2
frame 9

N. Fressengeas

Band Transport

Schematics

Carrier Generation

Charge Transport

Electro-optics

Harmonic
illumination

Two Wave Mixing

Thermal carrier generation

We assume here that the only carriers are electrons. . . what if not?

n_e



Comes from temperature induced Brownian motion

- Temperature induced
- Electrons are **kicked** into conduction band

Rate proportional to donors-left-to-ionize density

$$\frac{\partial n_e}{\partial t} = \beta (N_D - N_D^+)$$

One generated electron leaves one ionized donor

$$\frac{\partial n_e}{\partial t} = \frac{\partial N_D^+}{\partial t} = \beta (N_D - N_D^+)$$

Photorefractivity
Version 1.2
frame 10

N. Fressengeas

Band Transport

Schematics

Carrier Generation

Charge Transport

Electro-optics

Harmonic
illumination

Two Wave Mixing

Photo-excitation of carriers

The photoelectric effect at work



Photoelectric effect

- Photon **energy sufficient** to reach conduction band
- Rate proportional to light intensity \mathcal{I}
- And to left-to-ionize donors

Photo-excitation rate

$$\frac{\partial n_e}{\partial t} = \frac{\partial N_D^+}{\partial t} = \sigma \mathcal{I} (N_D - N_D^+)$$

The photo-ionization cross section

σ

- Has the dimensions of a surface
- If \mathcal{I} is given as a number of photon per surface units and time
- Sometimes the case, sometimes not...

Photorefractivity
Version 1.2
frame 11

N. Fressengeas

Band Transport

Schematics

Carrier Generation

Charge Transport

Electro-optics

Harmonic
illumination

Two Wave Mixing

Recombination needs luck, electrons and empty traps

- A luck factor

ξ

- Carriers density

n_e

- Empty trap density

N_D^+

$$\frac{\partial n_e}{\partial t} = \frac{\partial N_D^+}{\partial t} = -\xi n_e N_D^+$$

Band Transport

Schematics

Carrier Generation

Charge Transport

Electro-optics

Harmonic
illumination

Two Wave Mixing

Carriers rate equation

A combination of generation and recombination

Photorefractivity
Version 1.2
frame 13

N. Fressengeas

Band Transport

Schematics

Carrier Generation

Charge Transport

Electro-optics

Harmonic
illumination

Two Wave Mixing

A combination of generation and recombination

$$\frac{\partial N_D^+}{\partial t} = (\beta + \sigma \mathcal{I}) (N_D - N_D^+) - \xi n_e N_D^+$$

Charge transport in the conduction band

An assumption of the Band Transport Model... sometimes untrue



Diffusion

- Due to Temperature and Brownian motion
- Think of it as **sugar in water** (or coffee)
- Depends on concentration **variations**

Drift under electric-field

- Needs electric-field
- Externally applied or diffusion generated
- Depends on **electric field** and **mobility**

Photovoltaic effect

- Sometimes called *photo-galvanic*
- Non-isotropic effect
- Think of solar cells: **light generates current**
- Depends on light intensity

Photorefractivity
Version 1.2
frame 14

N. Fressengeas

Band Transport

Schematics

Carrier Generation

Charge Transport

Electro-optics

Harmonic
illumination

Two Wave Mixing

Diffusion transport

Diffusion current from Fick's first law linked to Einstein relation



Fick's first law

Particle flow

$$\vec{J}_p = -D \text{grad}(p)$$

Einstein relation

- Links diffusion, absolute temperature T and Brownian motion through mobility
- Mobility μ is the velocity to electric field ratio

$$D = \mu_{\tilde{e}} k_B T / e$$

Diffusion Current

$$\vec{J} = -e \vec{J}_{\tilde{e}} = +\mu_{\tilde{e}} k_B T \text{grad}(n_{\tilde{e}})$$

Photorefractivity
Version 1.2
frame 15

N. Fressengeas

Band Transport

Schematics

Carrier Generation

Charge Transport

Electro-optics

Harmonic
illumination

Two Wave Mixing

Drift

A charged particle in an electric field...

Electric Field



- Externally applied
- Due to charged carrier diffusion

Drift current

- Electrons velocity: $\vec{v} = -\mu_e \vec{E}$
- Drift Current: $\vec{J} = -e \times \vec{v}$
$$\vec{J} = en_e \mu_e \vec{E}$$

Photovoltaic current

An non isotropic effect stemming from crystal asymmetry



Origins

- Non centro-symmetric crystal
- Anisotropic photo-electric effect
- Depends on light polarization

e.g. LiNbO_3

Photovoltaic tensor

- Rank 2
- Main component along polar axis
- Often reduced to a scalar

Photovoltaic Current

$$\begin{aligned} [\vec{\mathcal{J}}]_i &= (N_D - N_D^+) \sum_{j,k} [\beta^{ph}]_{j,k} [\vec{E}]_j [\vec{E}]_k \vec{u}_i \\ \vec{\mathcal{J}} &\approx \beta^{ph} \mathcal{I} (N_D - N_D^+) \vec{c} \end{aligned}$$

Photorefractivity
Version 1.2
frame 17

N. Fressengeas

Band Transport

Schematics

Carrier Generation

Charge Transport

Electro-optics

Harmonic
illumination

Two Wave Mixing

Band Transport Model

Also known as Kukhtarev model

Published in 1979



Ionized donors rate equation

$$\frac{\partial N_D^+}{\partial t} = (\beta + \sigma \mathcal{I}) (N_D - N_D^+) - \xi n_{\bar{e}} N_D^+$$

Current density expression

$$\vec{\mathcal{J}} = \mu_{\bar{e}} k_B T \text{grad} (n_{\bar{e}}) + e n_{\bar{e}} \mu_{\bar{e}} \vec{E} + \beta^{ph} \mathcal{I} (N_D - N_D^+) \vec{c}$$

Quasi-static Maxwell model

- Continuity : $\text{div}(\vec{\mathcal{J}}) + \frac{\partial \rho}{\partial t} = 0$
- Charge : $\rho = e (N_D^+ - N_A^- - n_{\bar{e}})$
- Maxwell-Gauss : $\text{div}(\vec{D}) = \rho$, with $\vec{D} = \hat{\epsilon} \vec{E}$

Photorefractivity
Version 1.2
frame 18

N. Fressengeas

Band Transport

Schematics

Carrier Generation

Charge Transport

Electro-optics

Harmonic
illumination

Two Wave Mixing

Refractive index modulation through electro-optics

Space-charge electric field induces refractive index variations



Photorefractivity
Version 1.2
frame 19

N. Fressengeas

Band Transport

Schematics

Carrier Generation

Charge Transport

Electro-optics

Harmonic
illumination

Two Wave Mixing

This is not an electro-optics lesson

Please refer to the electro-optics lesson

Anyhow. . .

- Light generated electric field: the **space charge field** \vec{E}
- In electro-optic materials : creates index modulation

$$\left[\Delta \frac{1}{n^2}\right]_{i,j} = \sum_k [\hat{r}]_{ijk} \left[\vec{E}\right]_k$$

- Local modulation of refractive index
- Local modification of refractive index ellipsoid

Periodic illumination from plane waves interference

Two plane waves interfering

- Same wavelength and coherent
- non collinear wave vectors \vec{k}_1 and \vec{k}_2 : $\vec{K} = \vec{k}_2 - \vec{k}_1$
- Interference pattern : $\mathcal{I}_{(0)} \left(1 + m \cos \left(\vec{K} \cdot \vec{r} \right) \right)$
 - $\mathcal{I}_{(0)} = \mathcal{I}_1 + \mathcal{I}_2$
 - $m = 2 \frac{\sqrt{\mathcal{I}_1 \mathcal{I}_2}}{\mathcal{I}_1 + \mathcal{I}_2}$

Harmonic assumptions

- $m \ll 1$: intensities are very different
- All unknowns are sum of
 - A large uniform background : order 0
 - A small harmonic modulation : order 1
- Linearity : orders can be uncoupled
 - Uniform intensity analysis
 - Followed by **small signal** analysis

Simplifying assumptions

One dimension problem

1D

- Plane waves interference
- All phenomena are collinear to \vec{k}

Drift-diffusion transport only assumed

Photovoltaic effect assumed negligible

Photo-generation only assumed

Large intensities: thermal generation can be neglected

Steady state study

All temporal derivatives assumed zero

Photorefractivity
Version 1.2
frame 21

N. Fressengeas

Band Transport

Harmonic
illumination

Harmonic framework

Order 0

Order 1

Simplification and
Consequences

Two Wave Mixing

Carrier generation–recombination equilibrium

Steady state equilibrium

$$\sigma \mathcal{I} (N_D - N_D^+) = \xi n_{\tilde{e}} N_D^+$$

Uniform electric field

- $\vec{D}_{(0)}$ is homogeneous
- $\text{div}(\vec{D}_{(0)}) = \rho_{(0)} = 0$
- $N_{D(0)}^+ - N_A^- - n_{\tilde{e}(0)} = 0$

Small illumination

- $n_{\tilde{e}} \ll N_A$
- $\sigma \mathcal{I} \ll \xi N_A$

Equilibrium homogeneous densities

- $N_{D(0)}^+ = N_A + n_{\tilde{e}(0)}$
- $n_{\tilde{e}(0)} = \frac{N_D - N_A}{\xi N_A} \sigma \mathcal{I}_{(0)}$

Order 1 framework

Basic multi-scale modeling



Photorefractivity
Version 1.2
frame 23

N. Fressengeas

Band Transport

Harmonic
illumination

Harmonic framework

Order 0

Order 1

Simplification and
Consequences

Two Wave Mixing

All quantities are assumed periodic

- $\text{div}(\vec{X}) = i\vec{K} \cdot \vec{X}$
- ...

Order 0 assumed known

Order 1 assumed small

- $\forall X, X_{(1)} \ll X_{(0)}$
- $(X \times Y)_{(1)} = X_{(0)} Y_{(1)} + X_{(1)} Y_{(0)}$
- Order 0 is independently found
- Order 1 products is Order 2 : assumed negligible

Order one charge and current equilibrium

Steady state equilibrium

- $\sigma \mathcal{I} (N_D - N_D^+) = \xi n_{\hat{e}} N_D^+$
- $\sigma \mathcal{I}_{(1)} (N_D - N_{D(0)}^+) + \sigma \mathcal{I}_{(0)} (-N_{D(1)}^+) = \xi n_{\hat{e}(0)} N_{D(1)}^+ + \xi n_{\hat{e}(1)} N_{D(0)}^+$

Harmonic Current density

$$\vec{\mathcal{J}}_{(1)} = \mu_{\hat{e}} k_B T i n_{\hat{e}(1)} \vec{K} + e \mu_{\hat{e}} n_{\hat{e}(1)} \vec{E}_{(1)}$$

Harmonic Current density divergence is null $i \vec{K} \cdot \vec{\mathcal{J}} = 0$

$$i \vec{K} \cdot (\mu_{\hat{e}} k_B T i n_{\hat{e}(1)} \vec{K} + e \mu_{\hat{e}} n_{\hat{e}(1)} \vec{E}_{(1)}) = 0$$

Harmonic Poisson

$$i \vec{K} \cdot (\hat{\varepsilon} \cdot \vec{E}_{(1)}) = e (N_{D(1)}^+ - n_{\hat{e}(1)})$$

Photorefractivity
Version 1.2
frame 24

N. Fressengeas

Band Transport

Harmonic
illumination

Harmonic framework

Order 0

Order 1

Simplification and
Consequences

Two Wave Mixing

Order One Space Charge Field

General Expression

$$\vec{E}_{(1)} = \frac{i \vec{K} \frac{k_B T}{e} - \frac{\vec{K} \cdot \mu E_{(0)}}{\vec{K} \cdot \mu} \frac{\mathcal{I}_{(1)}}{\mathcal{I}_{(0)}}}{1 + \frac{\|\vec{K}\|^2}{k_D^2} + i \frac{e}{k_B T} \frac{\vec{K} \cdot \mu E_{(0)}}{k_D^2 \langle \mu \rangle} \frac{\mathcal{I}_{(1)}}{\mathcal{I}_{(0)}}}$$

Effective permittivity

$$\langle \varepsilon \rangle = \frac{\vec{K} \cdot \hat{\varepsilon} \vec{K}}{\|\vec{K}\|^2}$$

Effective permeability

$$\langle \mu \rangle = \frac{\vec{K} \cdot \mu \vec{K}}{\|\vec{K}\|^2}$$

Debye vector

$$k_D = \frac{2\pi}{\lambda_D}$$

$$k_D^2 = \frac{e}{\langle \varepsilon \rangle} \frac{e}{k_B T} \frac{N_D}{N_A} (N_D - N_A)$$

Photorefractivity
Version 1.2
frame 25

N. Fressengeas

Band Transport

Harmonic
illumination

Harmonic framework

Order 0

Order 1

Simplification and
Consequences

Two Wave Mixing

Let's simplify this complex expression

General Expression

$$\vec{E}_{(1)} = \frac{i \vec{K} \frac{k_B T}{e} \mathcal{I}_{(1)}}{1 + \frac{\|\vec{K}\|^2}{k_D^2} \mathcal{I}_{(0)}}$$

Simplifying assumptions

- Very often $\vec{E}_{(1)} \parallel \vec{K}$
- When no field is applied : $\vec{E}_{(0)} = 0$
- Quarter period phase shift between Intensity and Space-Charge Field gratings

Space charge field vs. grating spacing

$$\Lambda = 2\pi / \|\vec{K}\|$$

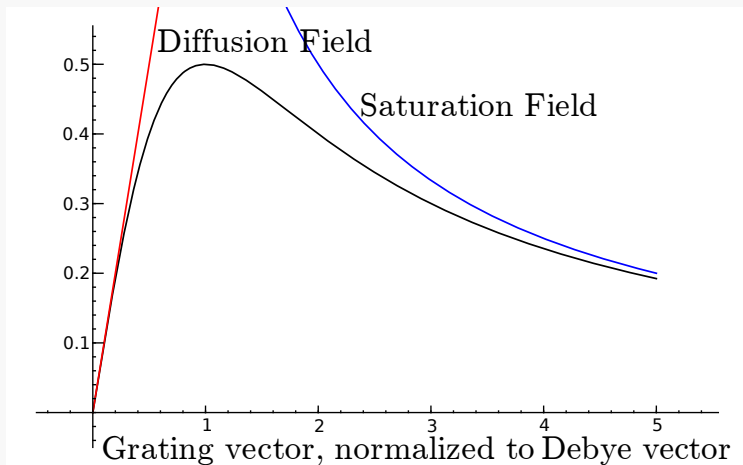
Large grating spacing

- Small \vec{K}
- $\vec{E}_{(1)} = i\vec{K} \frac{k_B T}{e} \frac{\mathcal{I}_{(1)}}{\mathcal{I}_{(0)}}$
- Diffusion field : $\vec{E}_d = \vec{K} \frac{k_B T}{e}$

Small grating spacing

- Large \vec{K}
- $\vec{E}_{(1)} = i\vec{K} \frac{k_B T}{e} \frac{k_D^2}{\|\vec{K}\|^2} \frac{\mathcal{I}_{(1)}}{\mathcal{I}_{(0)}}$
- Saturation Field : $\vec{E}_q = \vec{K} \frac{k_B T}{e} \frac{k_D^2}{\|\vec{K}\|^2}$

Space charge field as a function of grating spacing



Space charge field with externally applied field

No applied field

$$\vec{E}_{(1)} = i \frac{\vec{E}_d}{1 + \frac{E_d}{E_q}} \frac{\mathcal{I}_{(1)}}{\mathcal{I}_{(0)}}$$

Applied field \vec{E}_a

$$\vec{E}_{(1)} = i \frac{\vec{E}_d}{1 + \frac{E_d}{E_q}} \left[\frac{1 + i \frac{E_a}{E_d}}{1 + i \frac{E_a}{E_d + E_q}} \right] \frac{\mathcal{I}_{(1)}}{\mathcal{I}_{(0)}}$$

Standard approximations

- For most gratings and materials : $E_d \ll E_q$
- Applied field in the middle : $E_d \ll E_a \ll E_q$

In phase¹ illumination and space charge gratings

$$\vec{E}_{(1)} = - \frac{\vec{E}_a}{1 + \frac{E_d}{E_q}} \frac{\mathcal{I}_{(1)}}{\mathcal{I}_{(0)}}$$

¹Actually, they are π phase shifted. A possible negative sign on the electro-optic coefficient renders in-phase index and illumination gratings.

In phase intensity and index gratings

Beam interference is destructive, owing to reflection sign reversal



Photorefractivity
Version 1.2
frame 31

N. Fressengeas

Band Transport

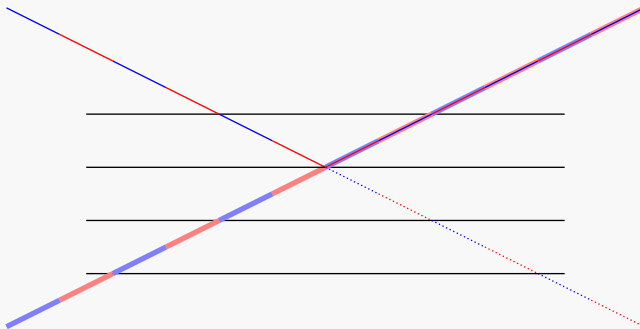
Harmonic
illumination

Two Wave Mixing

Gratings graphical view

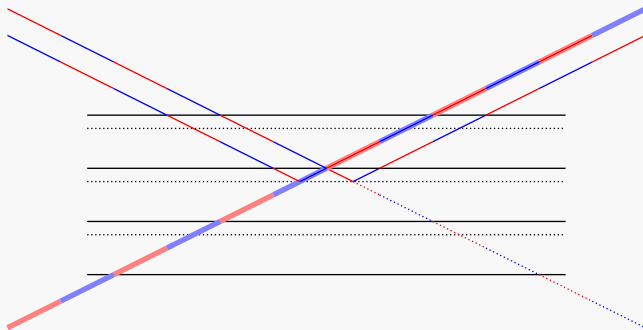
Coupled waves

Two Beam Coupling



Quarter period shifted gratings

Beam interference is constructive



Photorefractivity
Version 1.2
frame 32

N. Fressengeas

Band Transport

Harmonic
illumination

Two Wave Mixing

Gratings graphical view

Coupled waves

Two Beam Coupling

Two waves and a grating

Two waves make an intensity grating

- Waves are coherent and same wavelength
- Wave vectors are \vec{k}_1 and \vec{k}_2
- Intensity grating vector is $\vec{K} = \vec{k}_2 - \vec{k}_1$
- Waves amplitudes are $A_i = \sqrt{I_i} e^{-i\psi_i}$

Index Grating

- Assume $E_d \ll E_a \ll E_q$
- Index grating $\propto \Phi$ shifted illumination grating

$$n = n_{(0)} + \mathcal{R}e \left[n_{(1)} e^{i\Phi} \frac{\overline{A_1} A_2}{I_{(0)}} e^{\vec{K} \cdot \vec{r}} \right]$$

- $\Phi = \pi/2$ if no applied field and $\Phi = 0$ if field applied

Slow Varying Approximation

Paraxial Framework

- Propagation equation : $\Delta A + \frac{\omega^2}{c^2} n^2 A = 0$
- SVA: $\|\frac{\partial^2 A}{\partial z^2}\| \ll \|\beta \frac{\partial A}{\partial z}\|$
- β such as $\beta z = \vec{k} \cdot \vec{r}$

Conventions

- $z = 0$: entrance in the photorefractive material
- Symmetric coupling : $\beta_1 = \beta_2 = \|\vec{k}\| \cos(\theta)$
 θ is the half angle between input beams

After Coupled Mode calculations²

- $\frac{\partial A_1}{\partial z} = -\frac{1}{2I_{(0)}} \Gamma \|A_2\|^2 A_1 - \alpha A_1$
- $\frac{\partial A_2}{\partial z} = -\frac{1}{2I_{(0)}} \bar{\Gamma} \|A_1\|^2 A_2 - \alpha A_2$
- $\Gamma = i \frac{2\pi n_{(1)}}{\lambda \cos(\theta)} e^{-i\Phi}$
- α is absorption

²See lessons on Second Harmonic Generation and Optical Phase Conjugation for details.

Intensity and phase coupling

Diffusion induces intensity coupling

Drift induces phase coupling



Separate Diffusion and Drift influences

$$\Gamma = \gamma + 2i\zeta$$

$$\gamma = \frac{2\pi n_{(1)}}{\lambda \cos(\theta)} \sin(\Phi)$$

$$\zeta = \frac{\pi n_{(1)}}{\lambda \cos(\theta)} \cos(\Phi)$$

Intensity coupling

- $\frac{\partial \mathcal{I}_1}{\partial z} = -\gamma \frac{\mathcal{I}_1 \mathcal{I}_2}{\mathcal{I}_1 + \mathcal{I}_2} - \alpha \mathcal{I}_1$
- $\frac{\partial \mathcal{I}_2}{\partial z} = +\gamma \frac{\mathcal{I}_1 \mathcal{I}_2}{\mathcal{I}_1 + \mathcal{I}_2} - \alpha \mathcal{I}_2$

Phase coupling

- $\frac{\partial \psi_1}{\partial z} = \zeta \frac{\mathcal{I}_2}{\mathcal{I}_1 + \mathcal{I}_2}$
- $\frac{\partial \psi_2}{\partial z} = \zeta \frac{\mathcal{I}_1}{\mathcal{I}_1 + \mathcal{I}_2}$

Energy transfer

- For small absorption α , energy is transferred from one beam to the other
- Transfer direction is given by sign of γ

Photorefractivity
Version 1.2
frame 36

N. Fressengeas

Band Transport

Harmonic
illumination

Two Wave Mixing

Gratings graphical view

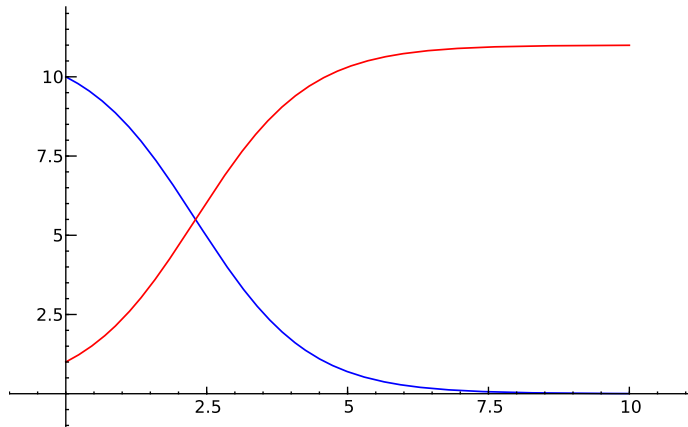
Coupled waves

Two Beam Coupling

Coupled Modes Solution

- Let $m = \frac{\mathcal{I}_1(0)}{\mathcal{I}_2(0)}$
- $\mathcal{I}_1(z) = \mathcal{I}_1(0) \frac{1 + m^{-1}}{1 + m^{-1}e^{\gamma z}} e^{-\alpha z}$
- $\mathcal{I}_2(z) = \mathcal{I}_2(0) \frac{1 + m}{1 + me^{-\gamma z}} e^{-\alpha z}$

Two Wave Mixing Intensity Coupling



Two Wave Mixing Intensity Coupling with Absorption

