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Photorefractivity

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Photorefractive effect
History began in 1966 as *Optical Damage*

Optical Damage in $LiNbO_3$

- Shine a light on $LiNbO_3$
- Remove it
- Shine another : damaged crystal

Semi-permanent effect

- Leave it in the dark : still damaged
- Leave it under uniform light : sometimes repaired

Today

- Photorefractivity can prove useful
- Some people still call it *optical damage*:
  - Bad for linear optics (electro-optic modulators. . .)
  - Bad for *instant* Non Linear Optics (SHG,OPA. . .)

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Photorefractivity is attractive
Non linear optics at low optical power

Non Linear Optics

- Dynamic Holography
- Phase conjugation
- All optical computing
- ...
- At milliwatts and below power levels

Observed in many non linear crystals

- Sillenites : $Bi_{12}SiO_{20}$, $Bi_{12}TiO_{20}$, $Bi_{12}GeO_{20}$
- Tungsten-Bronze : $Sr_xBa_{1-x}Nb_2O_6$
- Ferroelectrics : $LiNbO_3$, $BaTiO_3$
- Semiconductors : $InP{:}Fe$, $AsGa$

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Usefull reading...
[Yeh93, GH88, GH89]

P. Günter and J. P. Huignard.
Photorefractive materials and their applications I, volume 61 of *Topics in Applied Physics*.
Springer Verlag, Berlin, 1988.

P. Günter and J. P. Huignard.
Photorefractive materials and their applications II, volume 62 of *Topics in Applied Physics*.
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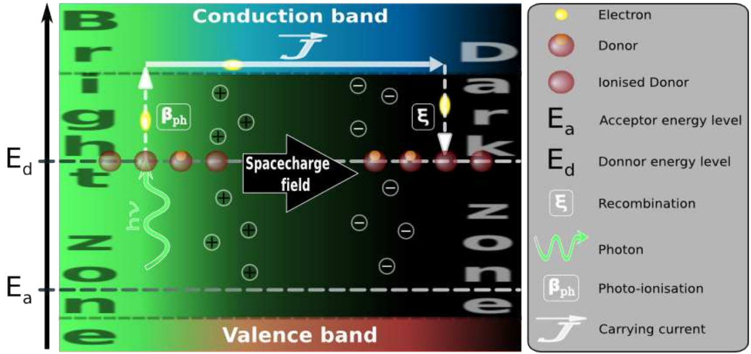
P. Yeh.
Introduction to photorefractive nonlinear optics.
Wiley Interscience, New York, 1993.

Contents

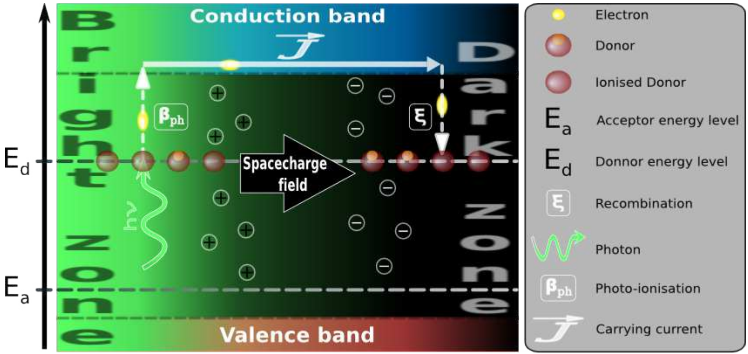
Photorefractive charge transport and trapping



Linear Index Modulation

Space charge electric field generates refractive index variation through electro-optic effect

Photorefractive charge transport and trapping



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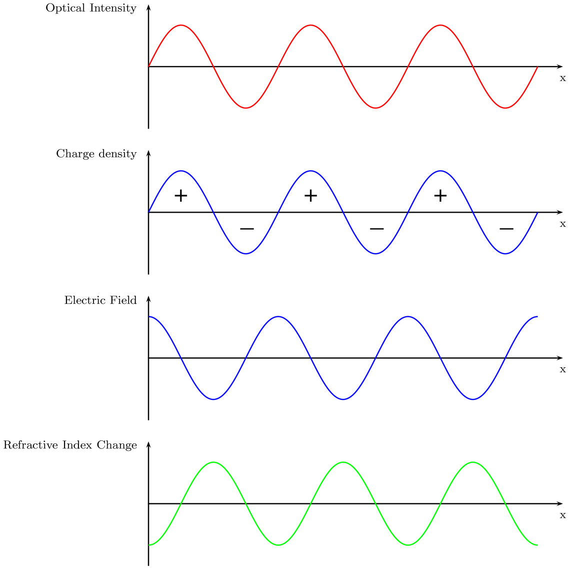
Periodic illumination of a photorefractive material



Donors wanted

Carriers are generated by donors: no donors, no carriers

Nominally pure crystals

- No in-band-gap level
  - No donor nor acceptor
  - No photorefractive effect
- Structural defects often present
- As well as pollutants
- They create in-band-gap levels
- Photorefractivity can arise from them

More efficient: doping

- Introduce in-band-gap species
- $LiNbO_3$:Fe, InP:Fe…

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Introducing Donors and Acceptors

Introduce Donors of electrons N_D

- Energy level close to conduction band
- They easily give electrons to conduction band

Introducing Acceptors $N_A \ll N_D$

- Photorefractivity needs traps
- Ionized donors are traps
- Introduce Acceptors close to the valence band
- They catch Donors electrons
- Donors are partially ionized $N_D^+ = N_A$

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Thermal carrier generation $n_{\mathring{e}}$

We assume here that the only carriers are electrons. . . what if not?

Comes from temperature induced Brownian motion

- Temperature induced
- Electrons are kicked into conduction band

Rate proportional to donors-left-to-ionize density

$$\frac{\partial n_{\mathring{e}}}{\partial t} = \beta\left(N_D - N_D^+\right)$$

One generated electron leaves one ionized donor

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Photo-excitation of carriers
The photoelectric effect at work

Photoelectric effect

- Photon energy sufficient to reach conduction band
- Rate proportional to light intensity $\mathcal{I}$
- And to left-to-ionize donors

Photo-excitation rate

$$\frac{\partial n_e}{\partial t} = \frac{\partial N_D^+}{\partial t} = \sigma \mathcal{I} \left(N_D - N_D^+ \right)$$

The photo-ionization cross section σ

- Has the dimensions of a surface
- If $\mathcal{I}$ is given as a number of photon per surface units and time
- Sometimes the case, sometimes not...

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Carrier recombination

Recombination needs luck, electrons and empty traps

- A luck factor ξ
- Carriers density $n_{\mathring{e}}$
- Empty trap density N_D^+

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Carriers rate equation

A combination of generation and recombination

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Charge transport in the conduction band
An assumption of the Band Transport Model…sometimes untrue

Diffusion
- Due to Temperature and Brownian motion
- Think of it as sugar in water (or coffee)
- Depends on concentration variations

Drift under electric-field
- Needs electric-field
- Externally applied or diffusion generated
- Depends on electric field and mobility

Photovoltaic effect
- Sometimes called *photo-galvanic*
- Non-isotropic effect
- Think of solar cells: light generates current
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Diffusion transport

Diffusion current from Fick's first law linked to Einstein relation

Fick's first law — **Particle flow**

$$\overrightarrow{J_p} = -D\mathrm{grad}\,(p)$$

Einstein relation

- Links diffusion, absolute temperature T and Brownian motion through mobility
- Mobility μ is the velocity to electric field ratio

$$D = \mu_{\mathring{e}} k_B T / e$$

Diffusion Current

$$\overrightarrow{\mathcal{J}} = -e\overrightarrow{J_{\mathring{e}}}$$

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Diffusion Current

$$\overrightarrow{\mathcal{J}} = +\mu_{\mathring{e}} k_B T \mathrm{grad}\,(n_{\mathring{e}})$$

Drift

A charged particle in an electric field. . .

Electric Field $\overrightarrow{E}$

- Externally applied
- Due to charged carrier diffusion

Drift current

- Electrons velocity: $\overrightarrow{v} = -\mu_{\text{e}}\overrightarrow{E}$
- Drift Current: $\overrightarrow{\mathcal{J}} = -e \times \overrightarrow{v}$

$$\overrightarrow{\mathcal{J}} = en_{\text{e}}\mu_{\text{e}}\overrightarrow{E}$$

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Photovoltaic current

An non isotropic effect stemming from crystal asymmetry

Origins

- Non centro-symmetric crystal *e.g.* $\mathrm{LiNbO_3}$
- Anisotropic photo-electric effect
- Depends on light polarization

Photovoltaic tensor

- Rank 2
- Main component along polar axis
- Often reduced to a scalar

Photovoltaic Current

$$\left[\vec{\mathcal{J}}\right]_i = \left(N_D - N_D^+\right) \sum_{j,k} \left[\beta^{ph}\right]_{j,k} \left[\vec{E}\right]_j \left[\vec{E}\right]_k \vec{u_i}$$

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Band Transport Model
Also known as Kukhtarev model

Published in 1979

Ionized donors rate equation

$$\frac{\partial N_D^+}{\partial t} = (\beta + \sigma \mathcal{I}) \left(N_D - N_D^+\right) - \xi n_{\mathring{e}} N_D^+$$

Current density expression

$$\overrightarrow{\mathcal{J}} = \mu_{\mathring{e}} k_B T \operatorname{grad}(n_{\mathring{e}}) + e n_{\mathring{e}} \mu_{\mathring{e}} \overrightarrow{E} + \beta^{ph} \mathcal{I} \left(N_D - N_D^+\right) \overrightarrow{c}$$

Quasi-static Maxwell model

- Continuity : $\operatorname{div}\left(\overrightarrow{\mathcal{J}}\right) + \frac{\partial \rho}{\partial t} = 0$
- Charge : $\rho = e\left(N_D^+ - N_A^- - n_{\mathring{e}}\right)$
- Maxwell-Gauss : $\operatorname{div}\left(\overrightarrow{D}\right) = \rho$, with $\overrightarrow{D} = \hat{\varepsilon} \overrightarrow{E}$

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Refractive index modulation through electro-optics
Space-charge electric field induces refractive index variations

This is not an electro-optics lesson

Please refer to the electro-optics lesson

Anyhow. . .

- Light generated electric field: the space charge field $\overrightarrow{E}$
- In electro-optic materials : creates index modulation

$$\left[\Delta \frac{1}{n^2}\right]_{i,j} = \sum_k \left[\hat{r}\right]_{ijk} \left[\overrightarrow{E}\right]_k$$

- Local modulation of refractive index
- Local modification of refractive index ellipsoïd

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$$\left[\Delta \frac{1}{n^2}\right]_{i,j} = \sum_k [\widehat{r}]_{ijk} \left[\overrightarrow{E}\right]_k$$

- Local modulation of refractive index
- Local modification of refractive index ellipsoïd

Refractive index modulation through electro-optics

Space-charge electric field induces refractive index variations

This is not an electro-optics lesson

Please refer to the electro-optics lesson

Anyhow. . .

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Periodic illumination from plane waves interference

Two plane waves interfering

- Same wavelength and coherent
- non collinear wave vectors $\overrightarrow{k_1}$ and $\overrightarrow{k_2}$: $\overrightarrow{K} = \overrightarrow{k_2} - \overrightarrow{k_1}$
- Interference pattern : $\mathcal{I}_{(0)}\left(1 + m\cos\left(\overrightarrow{K}\cdot\overrightarrow{r}\right)\right)$
  - $\mathcal{I}_{(0)} = \mathcal{I}_1 + \mathcal{I}_2$
  - $m = 2\frac{\sqrt{\mathcal{I}_1\mathcal{I}_2}}{\mathcal{I}_1+\mathcal{I}_2}$

Harmonic assumptions

- $m \ll 1$: intensities are very different
- All unknowns are sum of
  - A large uniform background : order 0
  - A small harmonic modulation : order 1
- Linearity : orders can be uncoupled
  - Uniform intensity analysis
  - Followed by small signal analysis

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Simplifying assumptions

One dimension problem — 1D

- Plane waves interference
- All phenomena are collinear to $\overrightarrow{K}$

Drift-diffusion transport only assumed

Photovoltaic effect assumed negligible

Photo-generation only assumed

Large intensities: thermal generation can be neglected

Steady state study

All temporal derivatives assumed zero

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- Charge Transport
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3 Two Wave Mixing
- Gratings graphical view
- Coupled waves
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Carrier generation–recombination equilibrium

Steady state equilibrium

$$\frac{\partial N_D^+}{\partial t} = \sigma \mathcal{I} \left(N_D - N_D^+ \right) - \xi n_{\mathring{e}} N_D^+$$

Uniform electric field

Carrier generation–recombination equilibrium

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- $\overrightarrow{D}_{(0)}$ is homogeneous
- $\mathrm{div}\left(\overrightarrow{D}_{(0)}\right) = \rho_{(0)} = 0$
- $\frac{\rho_{(0)}}{e} = N_{D\,(0)}^+ - N_A^- - n_{\mathring{e}\,(0)} = 0$

Small illumination

- $n_{\mathring{e}} \ll N_A$
- $\sigma\mathcal{I} \ll \xi N_A$

Equilibrium homogeneous densities

- $N_{D\,(0)}^+ = N_A + n_{\mathring{e}\,(0)}$
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Order 1 framework
Basic multi-scale modeling

All quantities are assumed periodic

- $\mathrm{div}\left(\overrightarrow{X}\right) = i\overrightarrow{K} \cdot \overrightarrow{X}$
- ...

Order 0 assumed known Order 1 assumed small

- $\forall X,\ X_{(1)} \ll X_{(0)}$
- $(X \times Y)_{(1)} = X_{(0)}Y_{(0)} + X_{(0)}Y_{(1)} + X_{(1)}Y_{(0)} + X_{(1)}Y_{(1)}$
- Order 0 is independently found
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Steady state equilibrium

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Harmonic Current density

$$\overrightarrow{\mathcal{J}}_{(1)} = \mu_{\mathring{e}} k_B T \,\mathrm{grad}\left(n_{\mathring{e}}\right)_{(1)} + e\mu_{\mathring{e}} n_{\mathring{e}(1)} \overrightarrow{E}_{(1)}$$

Harmonic Current density divergence is null $\quad i\overrightarrow{K} \cdot \overrightarrow{\mathcal{J}} = 0$

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$$\imath\overrightarrow{K}\cdot\left(\widehat{\varepsilon}\cdot\overrightarrow{E}_{(1)}\right) = e\left(N^+_{D\,(1)} - n_{\mathring{e}(1)}\right)$$

Order One Space Charge Field

General Expression

$$\overrightarrow{E}_{(1)} = \frac{i\overrightarrow{K}\frac{k_B T}{e} - \frac{\overrightarrow{K}\cdot\mu E_{(0)}}{\overrightarrow{K}<\mu>}}{1 + \frac{\|\overrightarrow{K}\|^2}{k_D^2} + i\frac{e}{k_B T}\frac{\overrightarrow{K}\cdot\mu E_{(0)}}{k_D^2<\mu>}} \frac{\mathcal{I}_{(1)}}{\mathcal{I}_{(0)}}$$

Effective permittivity

$$<\varepsilon>= \frac{\overrightarrow{K}\cdot\widehat{\varepsilon}\overrightarrow{K}}{\|\overrightarrow{K}\|^2}$$

Effective permeability

$$<\mu>= \frac{\overrightarrow{K}\cdot\mu\overrightarrow{K}}{\|\overrightarrow{K}\|^2}$$

Debye vector $k_D = \frac{2\pi}{\lambda_D}$

$$k_D^2 = \frac{e}{<\varepsilon>}\frac{e}{k_B T}\frac{N_D}{N_A}(N_D - N_A)$$

Let's simplify this complex expression

General Expression

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Simplifying assumptions

- Very often $\overrightarrow{E}_{(1)} \parallel \overrightarrow{K}$
- When no field is applied : $\overrightarrow{E}_{(0)} = 0$
- Quarter period phase shift between Intensity and Space-Charge Field gratings

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Space charge field *vs.* grating spacing $\Lambda = 2\pi / \| \overrightarrow{K} \|$

Large grating spacing

- Small $\overrightarrow{K}$
- $\overrightarrow{E}_{(1)} = i\overrightarrow{K} \frac{k_B T}{e} \frac{\mathcal{I}_{(1)}}{\mathcal{I}_{(0)}}$
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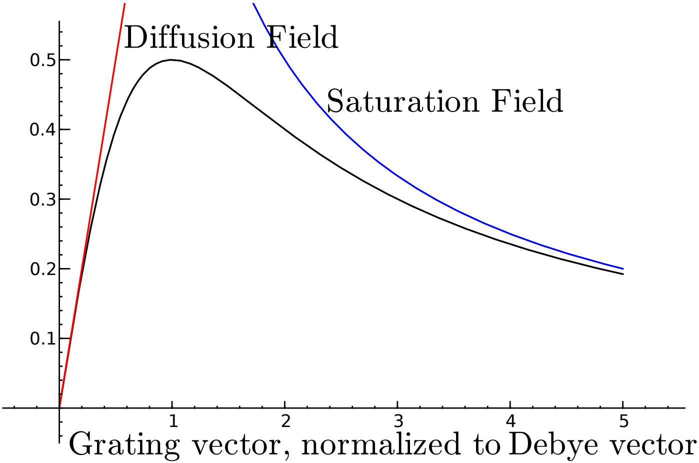
Space charge field as a function of grating spacing



Space charge field with externally applied field

No applied field

$$\overrightarrow{E}_{(1)} = i \frac{\overrightarrow{E_d}}{1 + \frac{E_d}{E_q}} \frac{\mathcal{I}_{(1)}}{\mathcal{I}_{(0)}}$$

Applied field $\overrightarrow{E_a}$

$$\overrightarrow{E}_{(1)} = i \frac{\overrightarrow{E_d}}{1 + \frac{E_d}{E_q}} \left[\frac{1 + i\frac{E_a}{E_d}}{1 + i\frac{E_a}{E_d + E_q}} \right] \frac{\mathcal{I}_{(1)}}{\mathcal{I}_{(0)}}$$

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Applied field effect

Standard approximations

- For most gratings and materials : $E_d \ll E_q$
- Applied field in the middle : $E_d \ll E_a \ll E_q$

In phase[1] illumination and space charge gratings

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[1] Actually, they are π phase shifted. A possible negative sign on the electro-optic coefficient renders in-phase index and illumination gratings.

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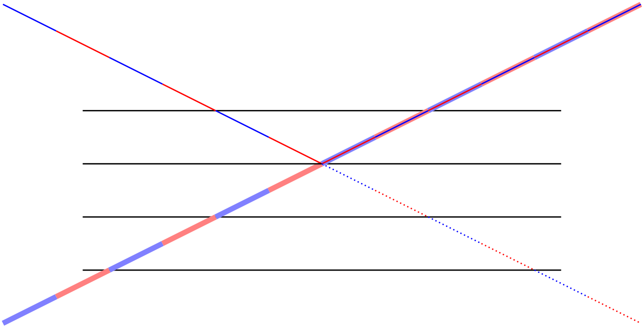
- Electro-optic effect

2 Harmonic illumination
- Harmonic framework
- Uniform background: order 0
- Periodic modulation : order 1
- Implications, Simplifications, Diffusion and Saturation

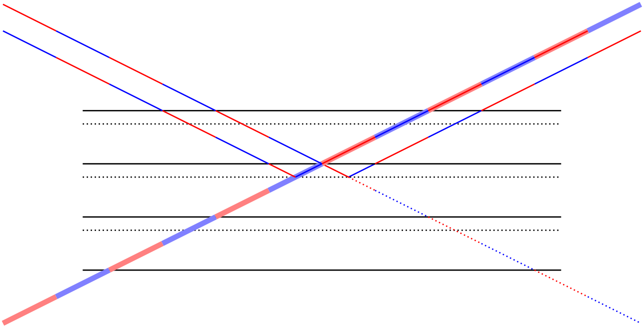
3 Two Wave Mixing
- Gratings graphical view
- Coupled waves
- Two Beam Coupling

In phase intensity and index gratings

Beam interference is destructive, owing to reflection sign reversal



Quarter period shifted gratings
Beam interference is constructive



Two waves and a grating

Two waves make an intensity grating

- Waves are coherent and same wavelength
- Wave vectors are $\overrightarrow{k_1}$ and $\overrightarrow{k_2}$
- Intensity grating vector is $\overrightarrow{K} = \overrightarrow{k_2} - \overrightarrow{k_1}$
- Waves amplitudes are $A_i = \sqrt{\mathcal{I}_i} e^{-i\psi_i}$

Index Grating

- Assume $E_d \ll E_a \ll E_q$
- Index grating $\propto \Phi$ shifted illumination grating

$$n = n_{(0)} + \mathcal{R}e\left[n_{(1)} e^{i\Phi} \frac{\overline{A_1} A_2}{\mathcal{I}_{(0)}} e^{\overrightarrow{K} \cdot \overrightarrow{r}}\right]$$

- $\Phi = \pi/2$ if no applied field and $\Phi = 0$ if field applied

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Assumption framework

Slow Varying Approximation **Paraxial Framework**

- Propagation equation : $\Delta A + \frac{\omega^2}{c^2} n^2 A = 0$
- SVA: $\|\frac{\partial^2 A}{\partial z^2}\| \ll \|\beta \frac{\partial A}{\partial z}\|$
- β such as $\beta z = \overrightarrow{k} \cdot \overrightarrow{r}$

Co-propagative coupling $\beta_1\beta_2 > 0$

Conventions

- $z = 0$: entrance in the photorefractive material
- Symmetric coupling : $\beta_1 = \beta_2 = \|\overrightarrow{k}\| \cos(\theta)$

  θ is the half angle between input beams

After Coupled Mode calculations[2]

- $\frac{\partial A_1}{\partial z} = -\frac{1}{2\mathcal{I}_{(0)}}\Gamma \|A_2\|^2 A_1 - \alpha A_1$
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- $\Gamma = i\frac{2\pi n_{(1)}}{\lambda\cos(\theta)} e^{-i\Phi}$
- α is absorption

[2] See lessons on Second Harmonic Generation and Optical Phase Conjugation for details.

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$$\Gamma = \gamma + 2i\zeta$$

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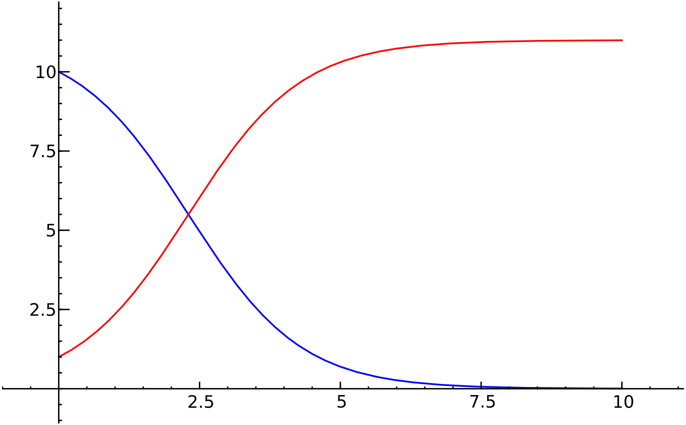
Photorefractive Two Wave Mixing

Coupled Modes Solution

- Let $m = \frac{\mathcal{I}_1(0)}{\mathcal{I}_2(0)}$
- $\mathcal{I}_1(z) = \mathcal{I}_1(0)\frac{1+m^{-1}}{1+m^{-1}e^{\gamma z}}e^{-\alpha z}$
- $\mathcal{I}_2(z) = \mathcal{I}_2(0)\frac{1+m}{1+me^{-\gamma z}}e^{-\alpha z}$

Two Wave Mixing Intensity Coupling



Two Wave Mixing Intensity Coupling with Absorption