

# UE SPM-PHO-S09-112 Photorefractivity

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Photorefractivity Version 1.2 frame 1

Band Transport

Harmonic illumination

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History began in 1966 as Optical Damage



# Optical Damage in ${\rm LiNbO_3}$

- Shine a light on  $LiNbO_3$
- Remove it
- Shine another : damaged crysta

#### Semi-permanent effect

- Leave it in the dark : still damaged
- Leave it under uniform light : sometimes repaired

#### Today

- Photorefractivity can prove useful
- Some people still call it *optical damage*:
  - Bad for linear optics (electro-optic modulators...)
  - Bad for *instant* Non Linear Optics (SHG,OPA...)

Photorefractivity Version 1.2 frame 2

sand Transport

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# Photorefractivity is attractive

Non linear optics at low optical power



# Non Linear Optics

- Dynamic Holography
- Phase conjugation
- All optical computing
- **.**..
- At milliwatts and below power levels

#### Observed in many non linear crystals

- Sillenites :  $Bi_{12}SiO_{20}$ ,  $Bi_{12}TiO_{20}$ ,  $Bi_{12}GeO_{20}$
- Tungsten-Bronze :  $Sr_xBa_{1-x}Nb_2O_6$
- Ferroelectrics : LiNbO<sub>3</sub>, BaTiO<sub>3</sub>
- Semiconductors : InP:Fe, AsGa

#### Photorefractivity Version 1.2 frame 3

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and Transport

Harmonic Illumination

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# Usefull reading... [Yeh93, GH88, GH89]



P. Günter and J. P. Huignard. Photorefractive materials and their applications I, volume 61 of Topics in Applied Physics. Springer Verlag, Berlin, 1988.



P. Günter and J. P. Huignard. Photorefractive materials and their applications II, volume 62 of Topics in Applied Physics. Springer Verlag, Berlin, 1989.



P. Yeh.

Introduction to photorefractive nonlinear optics. Wiley Interscience, New York, 1993.



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  - Electro-optic effect
- 2 Harmonic illumination
  - Harmonic framework
  - Uniform background: order 0
  - Periodic modulation : order 1
  - Implications, Simplifications, Diffusion and Saturation
- 3 Two Wave Mixing
  - Gratings graphical view
  - Coupled waves
  - Two Beam Coupling

Photorefractivity Version 1.2 frame 5

Band Transport

Harmonic Ilumination





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Photorefractivity Version 1.2 frame 6

#### Band Transport

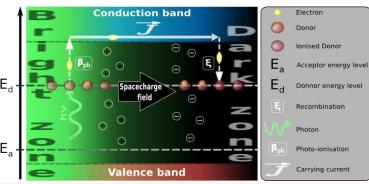
Schematics

Charge Transport

Harmonic Iluminatio

# Photorefractive charge transport and trapping





Linear Index Modulation

Space charge electric field generates refractive index variation through electro-optic effect

Photorefractivity Version 1.2 frame 6

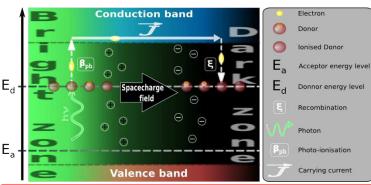
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Photorefractivity Version 1.2 frame 6

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Band Transport

Schematics

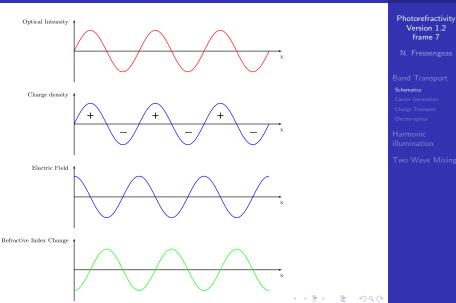
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# Periodic illumination of a photorefractive material



frame 7





## 1 Band Transport Model

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Photorefractivity Version 1.2 frame 8

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#### Band Transport

Carrier Generation
Charge Transport

Harmonic



Carriers are generated by donors: no donors, no carriers



# Nominally pure crystals

- No in-band-gap level
  - No donor nor acceptor
  - No photorefractive effect
- Structural defects often present
- As well as pollutants
- They create in-band-gap levels
- Photorefractivity can arise from them

#### More efficient: doping

- Introduce in-band-gap species
- LiNbO<sub>3</sub>:Fe, InP:Fe...

#### Photorefractivity Version 1.2 frame 8

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sand Transport

Carrier Generation
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Charge Transport Electro-optics

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Band Transport

Carrier Generation Charge Transport

llumination



### Introduce Donors of electrons

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- Energy level close to conduction band
- They easily give electrons to conduction band

### Introducing Acceptors

 $N_A \ll N_D$ 

- Photorefractivity needs traps
- lonized donors are traps
- Introduce Acceptors close to the valence band
- They catch Donors electrons
- Donors are partially ionized  $N_D^+ = N_A$

Photorefractivity Version 1.2 frame 9

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Band Transpo

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Carrier Generation Charge Transport

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We assume here that the only carriers are electrons...what if not?

### Comes from temperature induced Brownian motion

- Temperature induced
- Electrons are kicked into conduction band

Rate proportional to donors-left-to-ionize density

$$\frac{\partial n_{\rm e}}{\partial t} = \beta \left( N_D - N_D^+ \right)$$

One generated electron leaves one ionized donor

$$\frac{\partial n_{\rm e}^*}{\partial t} = \frac{\partial N_D^+}{\partial t} = \beta \left( N_D - N_D^+ \right)$$

Photorefractivity Version 1.2 frame 10

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Sand Transport

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The photoelectric effect at work



#### Photoelectric effect

- Photon energy sufficient to reach conduction band
- lacksquare Rate proportional to light intensity  ${\mathcal I}$
- And to left-to-ionize donors

#### Photo-excitation rate

$$\frac{\partial n_{\rm e}^*}{\partial t} = \frac{\partial N_D^+}{\partial t} = \sigma \mathcal{I} \left( N_D - N_D^+ \right)$$

#### The photo-ionization cross section

- Has the dimensions of a surface
- $\blacksquare$  If  ${\mathcal I}$  is given as a number of photon per surface units and time
- Sometimes the case, sometimes not...

#### Photorefractivity Version 1.2 frame 11

#### and Transpor

Carrier Generati

Charge Transport Electro-optics

# llumination

The photoelectric effect at work



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and Transport

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Electro-optics

Two Ways Mixing



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#### Photorefractivity Version 1.2 frame 11

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# Carrier recombination



#### Photorefractivity Version 1.2 frame 12

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#### Schematics

Carrier Generation
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 $N_D^+$ 

Two Wave Mixing

# Recombination needs luck, electrons and empty traps

- A luck factor
- Carriers density
- Empty trap density
  - $\frac{\partial n_{\rm e}}{\partial t} = \frac{\partial N_D^+}{\partial t} = -\xi n_{\rm e} N_D^+$

# Carrier recombination



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 $N_D^+$ 

# Carriers rate equation

A combination of generation and recombination



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Two Wave Mixing

# A combination of generation and recombination

$$\frac{\partial N_D^+}{\partial t} = (\beta + \sigma \mathcal{I}) \left( N_D - N_D^+ \right) - \xi n_{\hat{e}} N_D^+$$



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#### Band Transport

Carrier Generation
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#### Diffusion

- Due to Temperature and Brownian motion
- Think of it as sugar in water (or coffee)
- Depends on concentration variations

#### Drift under electric-field

- Needs electric-field
- Externally applied or diffusion generated
- Depends on electric field and mobility

#### Photovoltaic effect

- Sometimes called *photo-galvanic*
- Non-isotropic effect
- Think of solar cells: light generates current
- Depends on light intensity

Photorefractivity Version 1.2 frame 14

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Carrier Generation
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Harmonic

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Diffusion current from Fick's first law linked to Einstein relation



#### Fick's first law

Particle flow

$$\overrightarrow{J_p} = -D\operatorname{grad}(p)$$

#### Einstein relation

- Links diffusion, absolute temperature T and Brownian motion through mobility
- lacksquare Mobility  $\mu$  is the velocity to electric field ratio

$$D = \mu_{\rm e} k_B T/e$$

Diffusion Current

$$\overrightarrow{\mathcal{J}} = -e\overrightarrow{J_{\mathring{e}}}$$

Photorefractivity Version 1.2 frame 15

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Photorefractivity Version 1.2 frame 15

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Band Transport

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Harmonic

## Drift

#### A charged particle in an electric field...



#### Electric Field

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- Externally applied
- Due to charged carrier diffusion

#### Drift curren

- Electrons velocity:  $\overrightarrow{V} = -\mu_{\mathring{e}} \overrightarrow{E}$
- Drift Current:  $\overrightarrow{\mathcal{J}} = -e \times \overrightarrow{v}$

$$\overrightarrow{\mathcal{J}} = e n_{\mathring{e}} \mu_{\mathring{e}} \overrightarrow{E}$$

#### Photorefractivity Version 1.2 frame 16

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Schematics
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Harmonic Ilumination

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A charged particle in an electric field...



#### Electric Field

Externally applied

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■ Drift Current:  $\overrightarrow{\mathcal{J}} = -e \times \overrightarrow{v}$ 

 ${\cal J}={\it en}_{
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Schematics
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#### Electric Field

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$$\overrightarrow{\mathcal{J}} = \mathit{en}_{\mbox{\scriptsize e}}\mu_{\mbox{\scriptsize e}}\overrightarrow{E}$$

#### Photorefractivity Version 1.2 frame 16

Band Transport

Charge Transport

Harmonic

An non isotropic effect stemming from crystal asymmetry



## Origins

- Non centro-symmetric crystal
- Anisotropic photo-electric effect
- Depends on light polarization

#### Photovoltaic tenso

- Rank 2
- Main component along polar axis
- Often reduced to a scala

#### Photovoltaic Current

$$\begin{bmatrix} \overrightarrow{\mathcal{J}} \end{bmatrix}_{i} = (N_{D} - N_{D}^{+}) \sum_{j,k} [\beta^{ph}]_{j,k} [\overrightarrow{E}]_{j} [\overrightarrow{E}]_{k} \overrightarrow{u_{i}}$$
$$\overrightarrow{\mathcal{J}} \approx \beta^{ph} \mathcal{I} (N_{D} - N_{D}^{+}) \overrightarrow{c}$$

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Band Transport

Carrier Generation Charge Transport

Harmonic

Two Wave Mixing

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#### lonized donors rate equation

$$\frac{\partial N_D^+}{\partial t} = (\beta + \sigma \mathcal{I}) \left( N_D - N_D^+ \right) - \xi n_{\rm e} N_D^+$$

#### Current density expression

$$\overrightarrow{\mathcal{J}} = \mu_{\begin{subarray}{c} E \end{subarray}} T_{\begin{subarray}{c} E \end{subarray}} + 
ho_{\begin{subarray}{c} E \end{subarray}} + eta^{ph} \mathcal{I} \left( N_D - N_D^+ 
ight) \overrightarrow{c}$$

#### Quasi-static Maxwell mode

- Continuity :  $\operatorname{div}\left(\overrightarrow{\mathcal{J}}\right) + \frac{\partial \rho}{\partial t} = 0$
- lacksquare Charge :  $ho=e\left(N_D^+-N_A^--n_{
  m e}
  ight)$
- lacksquare Maxwell-Gauss :  $\operatorname{div}\left(\overrightarrow{D}\right)=
  ho$ , with  $\overrightarrow{D}=\widehat{\varepsilon}\,\overrightarrow{E}$

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#### Quasi-static Maxwell model

■ Continuity : 
$$\operatorname{div}\left(\overrightarrow{\mathcal{J}}\right) + \frac{\partial \rho}{\partial t} = 0$$

$$lacksquare$$
 Charge :  $ho = e \left( N_D^+ - N_A^- - n_{
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ight)$ 

■ Maxwell-Gauss : 
$$\operatorname{div}\left(\overrightarrow{D}\right) = \rho$$
, with  $\overrightarrow{D} = \widehat{\varepsilon}\overrightarrow{E}$ 

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#### 1 Band Transport Model

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- Carrier Generation
- Charge Transport
- Electro-optic effect
- 2 Harmonic illumination
  - Harmonic framework
  - Uniform background: order 0
  - Periodic modulation : order 1
  - Implications, Simplifications, Diffusion and Saturation
- 3 Two Wave Mixing
  - Gratings graphical view
  - Coupled waves
  - Two Beam Coupling

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#### Band Transport

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# Refractive index modulation through electro-optics

Space-charge electric field induces refractive index variations



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Two Wave Mixing

## This is not an electro-optics lesson

Please refer to the electro-optics lesson

#### Anyhow

- $lue{}$  Light generated electric field: the space charge field  $\it{E}$
- In electro-optic materials : creates index modulation

$$\left[\Delta \frac{1}{n^2}\right]_{i,j} = \sum_{k} \left[\hat{r}\right]_{ijk} \left[\overrightarrow{E}\right]_{k}$$

- Local modulation of refractive index
- Local modification of refractive index ellipsoïd

# Refractive index modulation through electro-optics

Space-charge electric field induces refractive index variations



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Two Wave Mixing

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Electro-optics
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Two Wave Mixing

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# Periodic illumination from plane waves interference



## Two plane waves interfering

- Same wavelength and coherent
- non collinear wave vectors  $\overrightarrow{k_1}$  and  $\overrightarrow{k_2}$ :  $\overrightarrow{K} = \overrightarrow{k_2} \overrightarrow{k_1}$
- Interference pattern :  $\mathcal{I}_{(0)}\left(1+m\cos\left(\overrightarrow{K}\cdot\overrightarrow{r}\right)\right)$ 

  - $m = 2 \frac{\sqrt{\mathcal{I}_1 \mathcal{I}_2}}{\mathcal{I}_1 + \mathcal{I}_2}$

#### Harmonic assumptions

- lacksquare  $m\ll 1$ : intensities are very different
- All unknowns are sum of
  - A large uniform background : order 0
  - A small harmonic modulation : order 1
- Linearity : orders can be uncoupled
  - Uniform intensity analysis
  - Followed by small signal analysis

Photorefractivity Version 1.2 frame 20

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# Periodic illumination from plane waves interference



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Sand Transport

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Harmonic framework Order 0

Order 1 Simplification and

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# Periodic illumination from plane waves interference



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#### One dimension problem

1D

- Plane waves interference
- All phenomena are collinear to  $\overrightarrow{K}$

Drift-diffusion transport only assumed

Photovoltaic effect assumed negligible

Photo-generation only assumed

Large intensities: thermal generation can be neglected

Steady state study

All temporal derivatives assumed zero

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## Carrier generation-recombination equilibrium



#### Steady state equilibrium

$$\frac{\partial N_D^+}{\partial t} = \sigma \mathcal{I} \left( N_D - N_D^+ \right) - \xi n_{\mathring{e}} N_D^+$$

#### Uniform electric field

 $D_{(0)}$  is homogeneous

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## Carrier generation-recombination equilibrium



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#### Uniform electric field

 $= \operatorname{div}(\overrightarrow{D}_{(0)}) = \operatorname{homogeneous}$ 

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### Steady state equilibrium

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#### Uniform electric field

- $\overrightarrow{D}_{(0)}$  is homogeneous
- $\operatorname{div}\left(\overrightarrow{D}_{(0)}\right) = \rho_{(0)} = 0$

Small illumination

- $\blacksquare$   $n_{\rm e}\ll N_{\rm A}$
- $\sigma \mathcal{I} \ll \xi N$

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Two Wave Mixing

$$N_{D(0)}^+ = N_A + n_{\hat{e}(0)}$$

$$n_{e(0)} = \frac{N_D - N_A}{\xi N_A} \sigma \mathcal{I}_{(0)}$$



### Steady state equilibrium

$$\sigma \mathcal{I} \left( N_D - N_D^+ \right) = \xi n_{\mathring{e}} N_D^+$$

#### Uniform electric field

- $\overrightarrow{D}_{(0)}$  is homogeneous
- $\bullet \operatorname{div}\left(\overrightarrow{D}_{(0)}\right) = \rho_{(0)} = 0$
- $N_{D(0)}^{\frac{\rho(0)}{e}} = N_{D(0)}^{+} N_{A}^{-} n_{e(0)}^{*} = 0$

### Small illumination

- $n_{\rm e}\ll N_A$
- $\sigma \mathcal{I} \ll \xi N_A$

#### Photorefractivity Version 1.2 frame 22

Dand Transport

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Two Wave Mixing

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Two Wave Mixing

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Two Wave Mixing

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wo Wave Mixing

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- 1 Band Transport Model
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### Order 1 framework

Basic multi-scale modeling



Photorefractivity Version 1.2 frame 23

### All quantities are assumed periodic

- . . . .

#### Order 0 assumed known

Order 1 assumed small

- $\forall X, X_{(1)} \ll X_{(0)}$ 
  - $(X \times Y)_{(1)} = X_{(0)}Y_{(0)} + X_{(0)}Y_{(1)} + X_{(1)}Y_{(0)} + X_{(1)}Y_{(1)}$
  - Order 0 is independently found

mination

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### Steady state equilibrium

$$\sigma \mathcal{I}_{(1)} \left( N_D - N_{D(0)}^+ \right) + \sigma \mathcal{I}_{(0)} \left( -N_{D(1)}^+ \right) = \xi n_{\hat{e}(0)} N_{D(1)}^+ + \xi n_{\hat{e}(1)} N_{D(0)}^+$$

### Harmonic Current density

$$\overrightarrow{\mathcal{J}}_{(1)} = \mu_{\mathring{e}} k_B T \operatorname{grad} (n_{\mathring{e}})_{(1)} + e \mu_{\mathring{e}} n_{\mathring{e}(1)} \overrightarrow{E}_{(1)}$$

Harmonic Current density divergence is null  $i \overrightarrow{K} \cdot \overrightarrow{\mathcal{J}} =$ 

$$i\overrightarrow{K}\cdot\left(\mu_{\tilde{e}}k_{B}Tin_{\tilde{e}(1)}\overrightarrow{K}+e\mu_{\tilde{e}}n_{\tilde{e}(1)}\overrightarrow{E}_{(1)}\right)=0$$

### Harmonic Poisson

$$\operatorname{div}\left(\overrightarrow{D}_{(1)}\right) = \rho_{(1)}$$

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### Steady state equilibrium

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larmonic framework

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Simplification and Consequences



### Steady state equilibrium

$$\sigma \mathcal{I}_{(1)} \left( N_D - N_{D(0)}^+ \right) + \sigma \mathcal{I}_{(0)} \left( -N_{D(1)}^+ \right) = \\ \xi n_{\hat{e}(0)} N_{D(1)}^+ + \xi n_{\hat{e}(1)} N_{D(0)}^+$$

### Harmonic Current density

$$\overrightarrow{\mathcal{J}}_{(1)} = \mu_{\mathring{\mathrm{e}}} k_{B} Ti n_{\mathring{\mathrm{e}}(1)} \overrightarrow{K} + e \mu_{\mathring{\mathrm{e}}} n_{\mathring{\mathrm{e}}(1)} \overrightarrow{E}_{(1)}$$

Harmonic Current density divergence is null

$$i\overrightarrow{K}\cdot\left(\mu_{\mathring{\mathrm{e}}}k_{B}Tin_{\mathring{\mathrm{e}}(1)}\overrightarrow{K}+e\mu_{\mathring{\mathrm{e}}}n_{\mathring{\mathrm{e}}(1)}\overrightarrow{E}_{(1)}
ight)=0$$

Harmonic Poisson

$$\operatorname{div}\left(\overrightarrow{D}_{(1)}\right) = \rho_{(1)}$$

Photorefractivity Version 1.2 frame 24

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armonic framework

Order 1

 $i\overrightarrow{K}\cdot\overrightarrow{\mathcal{J}}=0$ 

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### Steady state equilibrium

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and Transport

Harmonic Ilumination

Order 0

 $i\overrightarrow{K}\cdot\overrightarrow{\overrightarrow{\mathcal{J}}}=0$ 

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### Steady state equilibrium

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#### Harmonic Poisson

$$i\overrightarrow{K}\cdot\left(\widehat{\varepsilon}\cdot\overrightarrow{E}_{(1)}\right)=e\left(N_{D(1)}^{+}-n_{\mathring{e}(1)}\right)$$

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Order 0 Order 1

 $i\overrightarrow{K}\cdot\overrightarrow{\mathcal{J}}=0$ 

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### Order One Space Charge Field



#### General Expression

$$\overrightarrow{E}_{(1)} = \frac{i\overrightarrow{K}\frac{k_BT}{e} - \frac{\overrightarrow{K}\cdot\mu E_{(0)}}{\overrightarrow{K}<\mu>}}{1 + \frac{\|\overrightarrow{K}\|^2}{k_D^2} + i\frac{e}{k_BT}\frac{\overrightarrow{K}\cdot\mu E_{(0)}}{k_D^2<\mu>}} \frac{\mathcal{I}_{(1)}}{\mathcal{I}_{(0)}}$$

#### Effective permittivity

$$<\varepsilon>=\frac{\overrightarrow{K}\cdot\widehat{\varepsilon}\overrightarrow{K}}{\|\overrightarrow{K}\|^2}$$

### Effective permeability

$$<\mu> = \frac{\overrightarrow{K} \cdot \mu \overrightarrow{K}}{\|\overrightarrow{K}\|^2}$$

### Debye vector

$$k_D^2 = \frac{e}{\langle \varepsilon \rangle} \frac{e}{k_B T} \frac{N_D}{N_A} (N_D - N_A)$$

Photorefractivity Version 1.2 frame 25

Band Transport

illumination

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Simplification and



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Photorefractivity Version 1.2 frame 26

Band Transport

Harmonic illumination

Order 0

Order 1

Simplification and Consequences

### Let's simplify this complex expression



#### General Expression

$$\overrightarrow{E}_{(1)} = \frac{i\overrightarrow{K}\frac{k_BT}{e} - \frac{\overrightarrow{K}\cdot\mu E_{(0)}}{\overrightarrow{K}<\mu>}}{1 + \frac{\|\overrightarrow{K}\|^2}{k_D^2} + i\frac{e}{k_BT}\frac{\overrightarrow{K}\cdot\mu E_{(0)}}{k_D^2<\mu>}} \frac{\mathcal{I}_{(1)}}{\mathcal{I}_{(0)}}$$

#### Simplifying assumptions

- lacktriangle Very often  $\overrightarrow{E}_{(1)} \parallel \overrightarrow{K}$
- When no field is applied :  $\overrightarrow{E}_{(0)} = 0$
- Quarter period phase shift between Intensity and Space-Charge Field gratings

Photorefractivity Version 1.2 frame 26

Band Transport

Harmonic illumination

Order 0

Order 1

Simplification and Consequences

### Let's simplify this complex expression



#### General Expression

$$\overrightarrow{E}_{(1)} = \frac{i\overrightarrow{K}\frac{k_BT}{e} - \frac{\overrightarrow{K} \cdot E_{(0)}}{\overrightarrow{K}}}{1 + \frac{\|\overrightarrow{K}\|^2}{k_D^2} + i\frac{e}{k_BT}\frac{\overrightarrow{K} \cdot E_{(0)}}{k_D^2}} \frac{\mathcal{I}_{(1)}}{\mathcal{I}_{(0)}}$$

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Simplification and Consequences

# Space charge field *vs.* grating spacing $\Lambda = 2\pi/\|\overrightarrow{K}\|$



### Large grating spacing

- Small  $\overrightarrow{K}$
- $\overrightarrow{E}_{(1)} = i \overrightarrow{K} \frac{k_B T}{e} \frac{\mathcal{I}_{(1)}}{\mathcal{I}_{(0)}}$
- Diffusion field :  $\overrightarrow{E_d} = \overrightarrow{K} \frac{k_B T}{e}$

#### Small grating spacing

- Large K
- $\overrightarrow{E}_{(1)} = i\overrightarrow{K} \frac{k_B T}{e} \frac{k_D^2}{\|\overrightarrow{K}\|^2} \frac{\mathcal{I}_{(1)}}{\mathcal{I}_{(0)}}$
- Saturation Field :  $\overrightarrow{E_q} = \overrightarrow{K} \frac{k_B T}{e} \frac{k_D^2}{\|\overrightarrow{K}\|^2}$

Photorefractivity Version 1.2 frame 27

Band Transport

Harmonic illumination

Harmonic framew Order 0

Order 1

Simplification and Consequences

# Space charge field *vs.* grating spacing $\Lambda = 2\pi/\|\overrightarrow{K}\|$



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Band Transport

Harmonic illumination

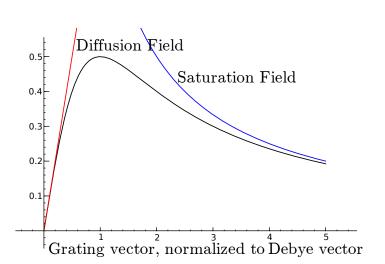
Harmonic frame Order 0

Order 1

Simplification and Consequences

# Space charge field as a function of grating spacing





Photorefractivity Version 1.2 frame 28

Sand Transport

Harmonic illumination

Harmonic fi Order 0

Simplification and Consequences

### Space charge field with externally applied field



### No applied field

$$\overrightarrow{E}_{(1)} = i \frac{\overrightarrow{E_d}}{1 + \frac{E_d}{E_q}} \frac{\mathcal{I}_{(1)}}{\mathcal{I}_{(0)}}$$

Applied field  $\overrightarrow{E}_a$ 

$$\overrightarrow{E}_{(1)} = i \frac{\overrightarrow{E}_d}{1 + \frac{E_d}{E_q}} \left[ \frac{1 + i \frac{E_a}{E_d}}{1 + i \frac{E_a}{E_d + E_q}} \right] \frac{\mathcal{I}_{(1)}}{\mathcal{I}_{(0)}}$$

Photorefractivity Version 1.2 frame 29

Band Transport

llumination

Order 0

Order 1

Simplification and Consequences

### Space charge field with externally applied field



### No applied field

$$\overrightarrow{E}_{(1)} = i \frac{\overrightarrow{E_d}}{1 + \frac{E_d}{E_a}} \frac{\mathcal{I}_{(1)}}{\mathcal{I}_{(0)}}$$

### Applied field $\overrightarrow{E_a}$

$$\overrightarrow{E}_{(1)} = i \frac{\overrightarrow{E_d}}{1 + \frac{E_d}{E_q}} \left[ \frac{1 + i \frac{E_a}{E_d}}{1 + i \frac{E_a}{E_d + E_q}} \right] \frac{\mathcal{I}_{(1)}}{\mathcal{I}_{(0)}}$$

Photorefractivity Version 1.2 frame 29

Band Transport

llumination

Order 0

Simplification and Consequences

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### Applied field effect



### Standard approximations

- lacksquare For most gratings and materials :  $E_d \ll E_q$
- Applied field in the middle :  $E_d \ll E_a \ll E_q$

In phase<sup>1</sup>illumination and space charge gratings

$$\overrightarrow{E}_{(1)} = i \frac{\overrightarrow{E}_d}{1 + \frac{E_d}{E_a}} \left[ i \frac{E_a}{E_d} \right] \frac{\mathcal{I}_{(1)}}{\mathcal{I}_{(0)}}$$

Photorefractivity Version 1.2 frame 30

Band Transport

illumination

Order 0

Order 1

Simplification and

Consequences

¹Actually, they are π phase shifted. A possible negative sign on the electro-optic coefficient renders in-phase index are illumination gratings of

### Applied field effect



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Photorefractivity Version 1.2 frame 30

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Order 1

Simplification and Consequences

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$$\overrightarrow{E}_{(1)} = -\frac{\overrightarrow{E_a}}{1 + \frac{E_d}{E_q}} \frac{\mathcal{I}_{(1)}}{\mathcal{I}_{(0)}}$$

Photorefractivity Version 1.2 frame 30

Dand Transport

llumination

Order 0

Order 1 Simplification and

Simplification and Consequences

 $<sup>^1</sup>$ Actually, they are  $\pi$  phase shifted. A possible negative sign on the electro-optic coefficient renders in-phase index and illumination gratings.



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Two Wave Mixir Gratings graphical view Coupled waves



### In phase intensity and index gratings

Beam interference is destructive, owing to reflection sign reversal



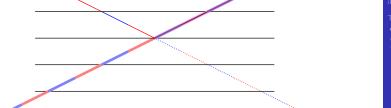


and Transport

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Coupled waves



### Quarter period shifted gratings

Beam interference is constructive



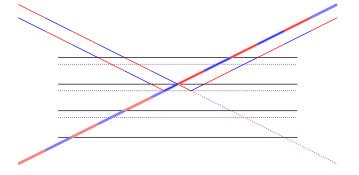
Photorefractivity Version 1.2 frame 32

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Photorefractivity Version 1.2 frame 33

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Coupled waves
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### Two waves and a grating



### Two waves make an intensity grating

- Waves are coherent and same wavelength
- Wave vectors are  $\overrightarrow{k_1}$  and  $\overrightarrow{k_2}$
- Intensity grating vector is  $\overrightarrow{K} = \overrightarrow{k_2} \overrightarrow{k_1}$
- Waves amplitudes are  $A_i = \sqrt{\mathcal{I}_i} e^{-i\psi_i}$

#### Index Grating

- Assume  $E_d \ll E_a \ll E_q$
- Index grating  $\propto \Phi$  shifted illumination grating

$$n = n_{(0)} + \mathcal{R}e \left[ n_{(1)}e^{i\Phi} \frac{\overline{A_1}A_2}{\mathcal{I}_{(0)}}e^{\overrightarrow{K}\cdot\overrightarrow{r}} \right]$$

 $\Phi = \pi/2$  if no applied field and  $\Phi = 0$  if field applied

Photorefractivity Version 1.2 frame 33

Band Transport

Harmonic illumination

Two Wave Mixit
Gratings graphical view
Coupled waves
Two Beam Coupling

## Two waves and a grating



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Two Beam Coupling

## Assumption framework



**Photorefractivity** Version 1.2 frame 34

Paraxial Framework

Coupled waves

Slow Varying Approximation

■ Propagation equation :  $\Delta A + \frac{\omega^2}{c^2} n^2 A = 0$ 

■ SVA:  $\|\frac{\partial^2 A}{\partial z^2}\| \ll \|\beta \frac{\partial A}{\partial z}\|$ 

 $\beta$  such as  $\beta z = \overrightarrow{k} \cdot \overrightarrow{r}$ 



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#### Conventions

- z = 0: entrance in the photorefractive material
- Symmetric coupling :  $\beta_1 = \beta_2 = \|\overrightarrow{k}\| \cos(\theta)$  $\theta$  is the half angle between input beams

$$\frac{\partial A_1}{\partial z} = -\frac{1}{2\mathcal{I}_{(0)}} \Gamma \|A_2\|^2 A_1 - \alpha A_1$$

$$\frac{\partial A_2}{\partial z} = -\frac{1}{2\mathcal{I}_{(0)}} \overline{\Gamma} \|A_1\|^2 A_2 - \alpha A_2$$

$$\Gamma = i \frac{2\pi n_{(1)}}{\lambda \cos(\theta)} e^{-i\Phi}$$

Photorefractivity Version 1.2 frame 35

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#### Conventions

- z = 0: entrance in the photorefractive material
- Symmetric coupling :  $\beta_1 = \beta_2 = \|\overrightarrow{k}\| \cos(\theta)$  $\theta$  is the half angle between input beams

## After Coupled Mode calculations<sup>2</sup>

$$\Gamma = i \frac{2\pi n_{(1)}}{\lambda \cos(\theta)} e^{-i\Phi}$$

lacksquare  $\alpha$  is absorption

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sand Transpor

Harmonic illumination

Gratings graphical view
Coupled waves

Photorefractivity Version 1.2 frame 35



## Separate Diffusion and Drift influences

$$\Gamma = \gamma + 2i\zeta$$

$$\gamma = \frac{2\pi n_{(1)}}{\lambda \cos(\theta)} \sin(\Phi)$$

$$\zeta = \frac{\pi n_{(1)}}{\lambda \cos(\theta)} \cos(\Phi)$$

#### Intensity coupling

$$\frac{\partial L_2}{\partial z} = +\gamma \frac{L_1 L_2}{\mathcal{I}_1 + \mathcal{I}_2} - \alpha \mathcal{I}_2$$

## Phase coupling

#### Energy transfe

- For small absorption  $\alpha$ , energy is transferred from one beam to the other
- $\blacksquare$  Transfer direction is given by sign of  $\gamma$

Photorefractivity Version 1.2 frame 36

Sand Transport

Harmonic illumination

Gratings graphical view

#### Drift induces phase coupling



## Separate Diffusion and Drift influences

$$\Gamma = \gamma + 2i\zeta$$

$$\gamma = \frac{2\pi n_{(1)}}{\lambda \cos(\theta)} \sin(\Phi)$$

$$\zeta = \frac{\pi n_{(1)}}{\lambda \cos(\theta)} \cos(\Phi)$$

## Intensity coupling

$$\frac{\partial \mathcal{I}_1}{\partial z} = -\gamma \frac{\mathcal{I}_1 \mathcal{I}_2}{\mathcal{I}_1 + \mathcal{I}_2} - \alpha \mathcal{I}_1$$

$$\frac{\partial \mathcal{I}_2}{\partial z} = +\gamma \frac{\mathcal{I}_1 \mathcal{I}_2}{\mathcal{I}_1 + \mathcal{I}_2} - \alpha \mathcal{I}_2$$

## Phase coupling

$$\frac{\partial \psi_1}{\partial z} = \zeta \frac{\mathcal{I}_2}{\mathcal{I}_1 + \mathcal{I}_2}$$

#### Energy transfer

- For small absorption  $\alpha$ , energy is transferred from one beam to the other
- Transfer direction is given by sign of  $\gamma$

Photorefractivity Version 1.2 frame 36

Sand Transport

illumination

Gratings graphical view

#### Drift induces phase coupling



## Separate Diffusion and Drift influences

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Gratings graphical view
Coupled waves

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## Phase coupling

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Photorefractivity Version 1.2 frame 36

Band Transport

Harmonic illumination

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Gratings graphical view
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## Photorefractive Two Wave Mixing



#### Photorefractivity Version 1.2 frame 37

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#### Band Transport

Harmonic Ilumination

## Two Wave Mixing Gratings graphical view

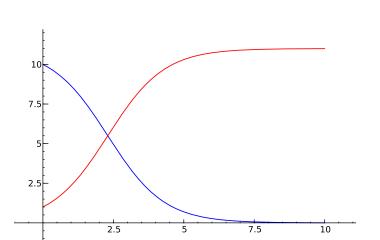
Two Beam Coupling

Coupled Modes Solution

$$\blacksquare \text{ Let } m = \frac{\mathcal{I}_1(0)}{\mathcal{I}_2(0)}$$

## Two Wave Mixing Intensity Coupling





Photorefractivity Version 1.2 frame 38

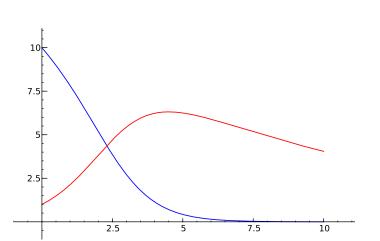
Band Transport

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Two Wave Mixing
Gratings graphical view

# Two Wave Mixing Intensity Coupling with Absorption





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Band Transport

Harmonic Ilumination

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Gratings graphical view