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Optical Phase Conjugation

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Download this document: http://moodle.univ-metz.fr/
Useful reading... [YY84, Yar97, San99]


...and many others.
Contents

1. Principle and application of phase conjugation
   - Experiment
   - Distortion correction theorem

2. Generation of phase conjugate waves
   - Non Linear Polarization Development
   - Four wave mixing coupled mode formulation
   - Conjugate wave amplitude

3. Self Pumped Phase Conjugation and Holography
   - Four Wave Mixing from a holographic point of view
   - Phase Conjugation without pumping
A peculiar phenomenon
Discovered in the early 70\textsuperscript{ies}

Use Non Linear Material
- Third order : non zero $\chi^3$
  - Photorefractivity
  - Stimulated Brillouin Scattering
  - Stimulated Raman Scattering
- Four wave mixing

Wavefront correction
- Distorted incident beam
- Reflected back \textit{as is}
- Distortion corrected

Figure: Phase conjugation principle.
Beams are reflected back *as if time was reversed*

Images source: http://sharp.bu.edu/~slehar/PhaseConjugate/PhaseConjugate.html

Incident wavefronts...

- Are reflected back **exactly**
- Back and forth wavefronts are **identical**
Attractive applications
All based on wavefront distortion correction

Phase conjugation applications
- All optical image transmission through fibers
- Distortion correction in high power lasers
- Dynamic wave front correction for optical sensors
- Dynamic Holography
- ...
A phase conjugate wave travels time the wrong way
Phase conjugation is also known as *time reversal*

Take some input monochromatic wave

\[ E_1 = \Re \{ \psi(r) \exp(i(\omega t - kz)) \} = \Re \{ \psi(r) \} \cos(\omega t - kz) \]

Take the phase conjugate over space only

- \[ E_2 = \Re \{ \psi(r) \exp(i(-kz))e^{i\omega t} \} \]
- \[ E_2 = \Re \{ \overline{\psi(r)} \exp(i(\omega t + kz)) \} \]
- \[ E_2 = \Re \{ \overline{\psi(r)} \} \cos(\omega t + kz) \]
- \[ E_2 = \Re \{ \psi(r) \} \cos(-\omega t - kz) \]
Principle and application of phase conjugation

Generation of phase conjugate waves

Self Pumping and Holography

Distortion correction theorem

The distortion correction theorem

If a backward traveling wave is phase conjugate somewhere then it is everywhere

Take a paraxial forward propagating wave

- Expressed as: \( E_1 (r, t) = \psi_1 (r) e^{i(\omega t - k z)} \)
- Obeys the wave equation: \( \Delta E_1 + \omega^2 \mu_0 \varepsilon (r) E_1 = 0 \)
- In the paraxial limit: \( \Delta \psi_1 + \left[ \omega^2 \mu_0 \varepsilon (r) - k^2 \right] \psi_1 - 2ik \frac{\partial \psi_1}{\partial z} = 0 \)
- Conjugate equation: \( \Delta \overline{\psi_1} + \left[ \omega^2 \mu_0 \varepsilon (r) - k^2 \right] \overline{\psi_1} + 2ik \frac{\partial \overline{\psi_1}}{\partial z} = 0 \)

Had we taken a backward propagating wave

- Expressed as: \( E_2 (r, t) = \psi_2 (r) e^{i(\omega t + k z)} \)
- Paraxial equation \((z \rightarrow -z)\): \( \Delta \psi_2 + \left[ \omega^2 \mu_0 \varepsilon (r) - k^2 \right] \psi_2 + 2ik \frac{\partial \psi_2}{\partial z} = 0 \)

Same second order linear differential equations for loss-less media

\( \varepsilon (r) \in \mathbb{R} \Rightarrow \left[ \psi_2 (z = 0) = a.\overline{\psi_1} (z = 0) \iff \forall z, \psi_2 (z) = a.\overline{\psi_1} (z) \right] \)
Flashback: Second Order

- Relies on $\chi_2: P_{NL} \propto E^2$
- Two waves mix to generate a third one
  - $\omega_1 \pm \omega_2 \rightarrow \omega_3$
- Second Harmonic Generation, Optical Parametric Amplification, Optical Parametric Oscillation...

Third Order

- Relies on $\chi_3: P_{NL} \propto E^3$
- Three waves mix to generate a fourth one
  - $\omega_1 \pm \omega_2 \pm \omega_3 \rightarrow \omega_4$
- Phase conjugation for $\omega_4 = \omega_1 + \omega_2 - \omega_3$? Let’s see...
Principle and application of phase conjugation

Generation of phase conjugate waves

Self Pumping and Holography

Non Linear Polarization Development

Non Linear Polarization \( P_{NL} \)

**General polarization development**

\[
[P]_i = \varepsilon_0 [\chi]_{ij} [E]_j + 2 [d]_{ijk} [E]_j [E]_k + 4 [\chi]_{ijkl} [E]_j [E]_k [E]_l
\]

**Third order non linear development** around \( \omega_4 = \omega_1 + \omega_2 - \omega_3 \)

\[
[P_{NL}]_i (\omega_4) = 6 [\chi]_{ijkl} [E]_j (\omega_1) [E]_k (\omega_2) [E]_l (\omega_3) e^{i(\omega_4 t + k_4 r)}
\]

**Degenerate Four wave mixing** : \( \omega = \omega + \omega - \omega \)

\[
[P_{NL}]_i (\omega) = 6 [\chi]_{ijkl} [E]_j (\omega) [E]_k (\omega) [E]_l (\omega) e^{i(\omega t + kr)}
\]

---

\(^{1}\)As an exercise, you can multiply, sum-up and keep only \( \omega_4 \) related terms... and find \( k_4 \)
Degenerate Four Wave mixing configuration

- $A_1 = \overline{A}_2$ intense plane pumps
- $A_3$ is the signal
- We seek $A_4$
Principle and application of phase conjugation

Generation of phase conjugate waves

Self Pumping and Holography

Four wave mixing coupled mode formulation

Signal wave equation

Let us start with the standard non linear wave equation

\[ \Delta E - \mu_0 \varepsilon \frac{\partial^2 E}{\partial t^2} = \mu_0 \frac{\partial^2 P_{NL}}{\partial t^2} \]

Signal \( A_3 \) propagation

- Each wave has its own direction and polarization
- They can be treated separately
- With \( \Delta E_3 = \Re \left[ \left( -k^2 A_3 - 2ik \frac{\partial A_3}{\partial z} + \frac{\partial^2 A_3}{\partial z^2} \right) e^{i(\omega t - kz)} \right] \)
- \( \Re \left[ \left( -2ik \frac{\partial A_3}{\partial z} + \frac{\partial^2 A_3}{\partial z^2} \right) e^{i(\omega t - kz)} \right] = \mu_0 \frac{\partial^2 P_{NL}}{\partial t^2} \)

Evaluation of the non linear polarization \( P_{NL} \)

Let us take a look at the terms which involve \( e^{i(\omega t \pm kz)} \)
Principle and application of phase conjugation

Generation of phase conjugate waves

Self Pumping and Holography

Four wave mixing coupled mode formulation

Stripping the non linear polarization to useful terms
Keeping only the relevant terms which contain $e^{i(\omega t \pm kz)}$

Expansion of third order non linear polarization

$$[P_{NL}]_i = \Re e \left[ 6 \left( \begin{array}{c} [\chi]_{ijkl} [A_1]_j [A_2]_k [A_4]_l \\
+ [\chi]_{ijji} [A_1]_j \overline{[A_1]_j} [A_3]_l \\
+ [\chi]_{ikkj} [A_2]_k \overline{[A_2]_k} [A_3]_j \end{array} \right) e^{i(\omega t - kz)} \right]$$

Simplifying assumptions

- Intense pump beam terms are dominant
- Polarizations are
  - either all the same, only $[\chi]_{iii}$ involved
  - or $(A_1/A_2) \perp (A_3/A_4)$, only $[\chi]_{ijji}, i \neq j$ involved

$$[P_{NL}]_i = \chi^{(3)} \left[ \left( \|A_1\|^2 + \|A_2\|^2 \right) A_3 + A_1 A_2 \overline{A_4} \right] e^{i(\omega t - kz)}$$

$$\chi^{(3)} = 6[\chi]_{iii} \text{ or } \chi^{(3)} = 6[\chi]_{ijji}$$
Resulting coupled wave propagation equation

Coupled wave equation resulting of $P_{NL}$

$$\frac{\partial A_3}{\partial z} = -i \frac{\omega}{2} \sqrt{\frac{\mu_0}{\varepsilon}} \chi^{(3)} \left[ (\|A_1\|^2 + \|A_2\|^2) A_3 + A_1 A_2 \overline{A_4} \right]$$

Further simplification

- Homogeneous refraction index modulation: Kerr effect
  - Simple phase factor change
  - Remove it from equation $A_i' = A_i e^{-i \frac{\omega}{2} \sqrt{\frac{\mu_0}{\varepsilon}} \chi^{(3)} (\|A_1\|^2 + \|A_2\|^2) z}$

- Set $\kappa = \frac{\omega}{2} \sqrt{\frac{\mu_0}{\varepsilon}} \chi^{(3)} A_1 A_2$

Simplified coupling equations

$$\frac{\partial A_3'}{\partial z} = i \kappa A_4'$$
$$\frac{\partial A_4'}{\partial z} = i \kappa A_3'$$

obtained through the same kind of derivation
## Conjugate wave amplitude

### General solution

Boundary conditions at $z = 0$ and $z = L$

\[
A_3 \text{ is forward propagating} \quad A_4 \text{ is backward propagating}
\]

- \[
A_3' (z) = -i \frac{|\kappa| \sin (|k| z)}{\kappa \cos (|k| L)} A_4'(L) + \frac{\cos (|k| (z - L))}{\cos (|k| L)} A_3'(0)
\]

- \[
A_4' (z) = \cos (|k| z) \cos (|k| L) A_4'(L) + i \frac{|\kappa| \sin (|k| (z - L))}{|k| \cos (|k| L)} A_3'(0)
\]

### One beam experiment

\[
A_4' (L) = 0
\]

- \[
A_3' (L) = \frac{A_3'(0)}{\cos(|k| L)} \quad \text{Coherent amplifier}
\]

- \[
A_4' (0) = -i \frac{|\kappa|}{|\kappa|} \tan (|\kappa| L) A_3'(0) \quad \text{Reflectivity can exceed 1}
\]
One Beam experiment and phase factor

One beam experiment

- \( A'_3 (L) = \frac{A'_3(0)}{\cos(|k|L)} \)
- \( A'_4 (0) = -i \frac{\kappa}{|\kappa|} \tan (|\kappa| L) \overline{A'_3 (0)} \)

What if \( \cos (|k|L) = 0 \)?
- Infinite gain
- \( A_3 \) and \( A_4 \) start from noise
- Spontaneous oscillations

Coherent amplifier
Reflectivity can exceed 1

Graph: I/I_0 vs. KZ

Values:
0.25 0.5 0.75 1 1.25
Four Wave Mixing is Real Time Holography

Write Hologram

\[ T \propto \| A_1 + A_3 \|^2 = \| A_1 \|^2 + \| A_3 \|^2 + A_1 A_3 + A_3 A_1 \]

Read Hologram with \( A_2 = \overline{A_1} \)

\[ A_4 \propto T A_2 = (\| A_1 \|^2 + \| A_3 \|^2) A_2 + A_2 A_1 \overline{A_3} + A_2 A_3 \overline{A_1} \]
CAT Conjugator using Beam Fanning

Total internal reflection

Figure: Beam fanning in photorefractive Baryum Titanate