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Optical Phase Conjugation

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Download this document : <http://moodle.univ-metz.fr/>

Usefull reading. . .

[YY84, Yar97, San99]



F. Sanchez.

Optique non-linéaire - Cours et problèmes résolus.

Ellipses, 1999.



A. Yariv.

Optical Electronics in Modern Communications.

Oxford Series in Electrical and Computer Engineering. Oxford University Press, 1997.



A. Yariv and P. Yeh.

Optical waves in crystals. Propagation and control of laser radiation.

Wiley series in pure and applied optics. Wiley-Interscience, Stanford University, 1984.

... and many others

Contents

- 1 Principle and application of phase conjugation
 - Experiment
 - Distortion correction theorem

- 2 Generation of phase conjugate waves
 - Non Linear Polarization Development
 - Four wave mixing coupled mode formulation
 - Conjugate wave amplitude

- 3 Self Pumped Phase Conjugation and Holography
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A peculiar phenomenon

Discovered in the early 70^{ies}

Use Non Linear Material

- Third order : non zero χ_3
 - Photorefractivity
 - Stimulated Brillouin Scattering
 - Stimulated Raman Scattering
- Four wave mixing

Wavefront correction

- Distorted incident beam
- Reflected back **as is**
- Distortion corrected

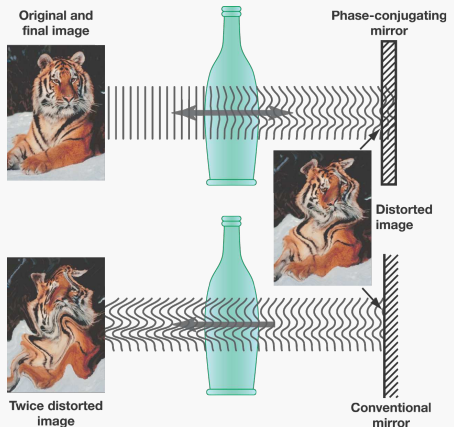


Figure: Phase conjugation principle.

Source : Wikipedia

Experiment

Beams are reflected back *as if time was reversed*

Images source :

<http://sharp.bu.edu/~slehar/PhaseConjugate/PhaseConjugate.html>

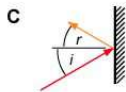
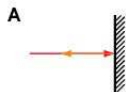
Incident wavefronts. . .

- Are reflected back **exactly**
- Back and forth wavefronts are **identical**

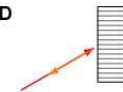
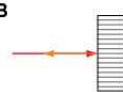
regular mirror

distorting
glass

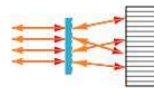
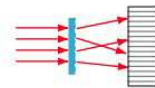
regular mirror



phase conjugate mirror



phase conjugate mirror



Attractive applications

All based on wavefront distortion correction

Phase conjugation applications

- All optical image transmission through fibers
- Distortion correction in high power lasers
- Dynamic wave front correction for optical sensors
- Dynamic Holography
- ...

A phase conjugate wave travels time the wrong way

Phase conjugation is also known as *time reversal*

Take some input monochromatic wave

$$E_1 = \mathcal{R}e [\psi(r) \exp(i(\omega t - kz))] = \mathcal{R}e [\psi(r)] \cos(\omega t - kz)$$

Take the phase conjugate *over space only*

- $E_2 = \mathcal{R}e [\overline{\psi(r) \exp(i(-kz))} e^{i\omega t}]$
- $E_2 = \mathcal{R}e [\overline{\psi(r)} \exp(i(\omega t + kz))]$
- $E_2 = \mathcal{R}e [\overline{\psi(r)}] \cos(\omega t + kz)$
- $E_2 = \mathcal{R}e [\psi(r)] \cos(-\omega t - kz)$

The distortion correction theorem

If a backward traveling wave is phase conjugate *somewhere* then it is *everywhere*

Take a paraxial forward propagating wave

- Expressed as: $E_1(r, t) = \psi_1(r) e^{i(\omega t - kz)}$
- Obeys the wave equation: $\Delta E_1 + \omega^2 \mu_0 \varepsilon(r) E_1 = 0$
- In the paraxial limit: $\Delta \psi_1 + [\omega^2 \mu_0 \varepsilon(r) - k^2] \psi_1 - 2ik \frac{\partial \psi_1}{\partial z} = 0$
- Conjugate equation: $\Delta \overline{\psi_1} + [\omega^2 \mu_0 \overline{\varepsilon(r)} - k^2] \overline{\psi_1} + 2ik \frac{\partial \overline{\psi_1}}{\partial z} = 0$

Had we taken a backward propagating wave

- Expressed as: $E_2(r, t) = \psi_2(r) e^{i(\omega t + kz)}$
- Paraxial equation ($z \rightarrow -z$):

$$\Delta \psi_2 + [\omega^2 \mu_0 \varepsilon(r) - k^2] \psi_2 + 2ik \frac{\partial \psi_2}{\partial z} = 0$$

Same second order **linear** differential equations for loss-less media

$$\varepsilon(r) \in \mathbb{R} \Rightarrow \left[\psi_2(z=0) = a \cdot \overline{\psi_1(z=0)} \Leftrightarrow \forall z, \psi_2(z) = a \cdot \overline{\psi_1(z)} \right]$$

Four wave mixing

Third Order Non Linear Optics

Flashback : Second Order

- Relies on χ_2 : $P_{NL} \propto E^2$
- Two waves mix to generate a third one
- $\omega_1 \pm \omega_2 \rightarrow \omega_3$
- Second Harmonic Generation, Optical Parametric Amplification, Optical Parametric Oscillation...

Third order

- Relies on χ_3 : $P_{NL} \propto E^3$
- **Three** waves mix to generate a **fourth** one
- $\omega_1 \pm \omega_2 \pm \omega_3 \rightarrow \omega_4$
- Phase conjugation for $\omega_4 = \omega_1 + \omega_2 - \omega_3$? Let's see...

Non Linear Polarization P_{NL}

General polarization development

$$[P]_i = \varepsilon_0[\chi]_{ij} + 2[d]_{ijk}[E]_j[E]_k + 4[\chi]_{ijkl}[E]_j[E]_k[E]_l$$

Third order non linear development¹ around $\omega_4 = \omega_1 + \omega_2 - \omega_3$

$$[P_{NL}]_i(\omega_4) = 6[\chi]_{ijkl}[E]_j(\omega_1)[E]_k(\omega_2)\overline{[E]_l(\omega_3)}e^{i(\omega_4 t + k_4 r)}$$

Degenerate Four wave mixing : $\omega = \omega + \omega - \omega$

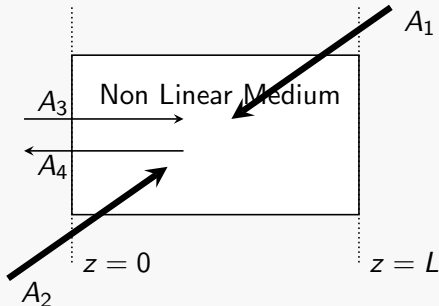
$$[P_{NL}]_i(\omega) = 6[\chi]_{ijkl}[E]_j(\omega)[E]_k(\omega)\overline{[E]_l(\omega)}e^{i(\omega t + k r)}$$

¹As an exercise, you can multiply, sum-up and keep only ω_4 related terms... and find k_4

Degenerate Four Wave mixing configuration

Degenerate configuration

- $A_1 = \overline{A_2}$ intense plane pumps
- A_3 is the signal
- We seek A_4



Signal wave equation

Let us start with the standard non linear wave equation

$$\Delta E - \mu_0 \varepsilon \frac{\partial^2 E}{\partial t^2} = \mu_0 \frac{\partial^2 P_{NL}}{\partial t^2}$$

Signal A_3 propagation

- Each wave has its own direction and polarization
- They can be treated separately

- With $\Delta E_3 = \mathcal{R}e \left[\left(-k^2 A_3 - 2ik \frac{\partial A_3}{\partial z} + \frac{\partial^2 A_3}{\partial z^2} \right) e^{i(\omega t - kz)} \right]$

- $\mathcal{R}e \left[\left(-2ik \frac{\partial A_3}{\partial z} + \frac{\partial^2 A_3}{\partial z^2} \right) e^{i(\omega t - kz)} \right] = \mu_0 \frac{\partial^2 P_{NL}}{\partial t^2}$

Evaluation of the non linear polarization P_{NL}

Let us take a look at the terms which involve $e^{i(\omega t \pm kz)}$

Four wave mixing coupled mode formulation

Stripping the non linear polarization to useful terms

Keeping only the relevant terms which contain $e^{i(\omega t \pm kz)}$

Expansion of third order non linear polarization

$$[P_{NL}]_i = \mathcal{R}e \left[6 \begin{pmatrix} [\chi]_{ijkl} & [A_1]_j & [A_2]_k & \overline{[A_4]}_l \\ + & [\chi]_{ijji} & [A_1]_j & \overline{[A_1]}_j & [A_3]_i \\ + & [\chi]_{ikki} & [A_2]_k & \overline{[A_2]}_k & [A_3]_i \end{pmatrix} e^{i(\omega t - kz)} \right]$$

Simplifying assumptions

- Intense pump beam terms are dominant
- Polarizations are
 - either all the same, only $[\chi]_{iiii}$ involved
 - or $(A_1/A_2) \perp (A_3/A_4)$, only $[\chi]_{ijji}$, $i \neq j$ involved
 - $\chi^{(3)} = 6[\chi]_{iiii}$ or $\chi^{(3)} = 6[\chi]_{ijji}$

Resulting coupled wave propagation equation

Coupled wave equation resulting of P_{NL}

$$\frac{\partial A_3}{\partial z} = -i\frac{\omega}{2}\sqrt{\frac{\mu_0}{\epsilon}}\chi^{(3)} \left[(\|A_1\|^2 + \|A_2\|^2) A_3 + A_1 A_2 \overline{A_4} \right]$$

Further simplification

- Homogeneous refraction index modulation : Kerr effect
 - Simple phase factor change
 - Remove it from equation $A'_i = A_i e^{-i\frac{\omega}{2}\sqrt{\frac{\mu_0}{\epsilon}}\chi^{(3)}(\|A_1\|^2 + \|A_2\|^2)z}$
- Set $\overline{\kappa} = \frac{\omega}{2}\sqrt{\frac{\mu_0}{\epsilon}}\chi^{(3)}A_1A_2$

Simplified coupling equations

- $\frac{\partial \overline{A'_3}}{\partial z} = i\overline{\kappa}A'_4$
- $\frac{\partial A'_4}{\partial z} = i\overline{\kappa}\overline{A'_3}$ obtained through the same kind of derivation

Conjugate wave amplitude

General solution

Boundary conditions at $z = 0$ and $z = L$ A_3 is forward propagating A_4 is backward propagating

- $A'_3(z) = -i \frac{|\kappa| \sin(|k|z)}{\kappa \cos(|k|L)} \overline{A'_4(L)} + \frac{\cos(|k|(z-L))}{\cos(|k|L)} A'_3(0)$
- $A'_4(z) = \frac{\cos(|k|z)}{\cos(|k|L)} A'_4(L) + i \frac{\bar{\kappa} \sin(|k|(z-L))}{|k| \cos(|k|L)} \overline{A'_3(0)}$

One beam experiment

 $A'_4(L) = 0$

- $A'_3(L) = \frac{A'_3(0)}{\cos(|k|L)}$ Coherent amplifier
- $A'_4(0) = -i \frac{\bar{\kappa}}{|\kappa|} \tan(|\kappa|L) \overline{A'_3(0)}$ Reflectivity can exceed 1

One Beam experiment and phase factor

One beam experiment

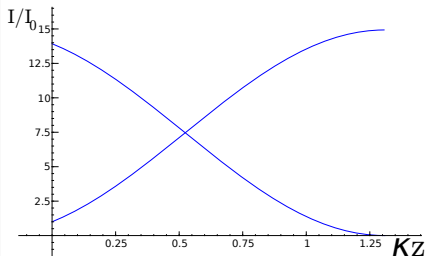
$$A'_4(L) = 0$$

- $A'_3(L) = \frac{A'_3(0)}{\cos(|k|L)}$

Coherent amplifier

- $A'_4(0) = -i \frac{\bar{\kappa}}{|\kappa|} \tan(|\kappa|L) \overline{A'_3(0)}$

Reflectivity can exceed 1



What if $\cos(|k|L) = 0$?

- Infinite gain
- A_3 and A_4 start from noise
- **Spontaneous oscillations**

Four Wave Mixing from a holographic point of view

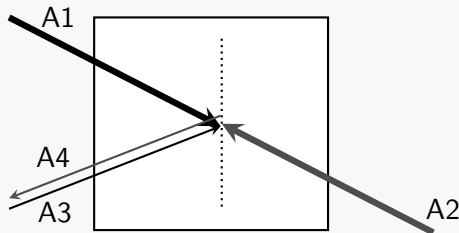
Four Wave Mixing is Real Time Holography

Write Hologram

$$T \propto \|A_1 + A_3\|^2 = \|A_1\|^2 + \|A_3\|^2 + A_1\overline{A_3} + A_3\overline{A_1}$$

Read Hologram with $A_2 = \overline{A_1}$

$$A_4 \propto TA_2 = (\|A_1\|^2 + \|A_3\|^2) A_2 + A_2A_1\overline{A_3} + A_2A_3\overline{A_1}$$



CAT Conjugator using Beam Fanning

Total internal reflection

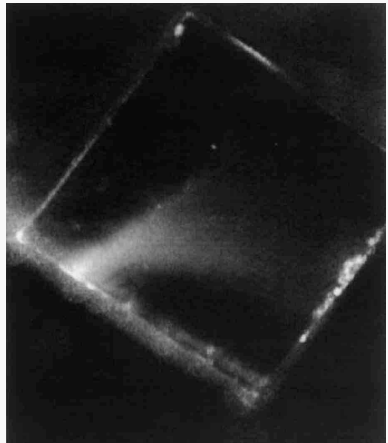
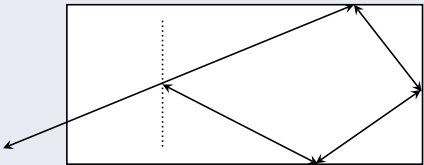


Figure: Beam fanning in photorefractive Baryum Titanate