

UE SPM-PHO-S09-112

Optical Phase Conjugation

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Download this document : <http://moodle.univ-metz.fr/>

Usefull reading. . .

[YY84, Yar97, San99]



F. Sanchez.

Optique non-linéaire - Cours et problèmes résolus.

Ellipses, 1999.



A. Yariv.

Optical Electronics in Modern Communications.

Oxford Series in Electrical and Computer Engineering. Oxford University Press, 1997.



A. Yariv and P. Yeh.

Optical waves in crystals. Propagation and control of laser radiation.

Wiley series in pure and applied optics. Wiley-Interscience, Stanford University, 1984.

... and many others

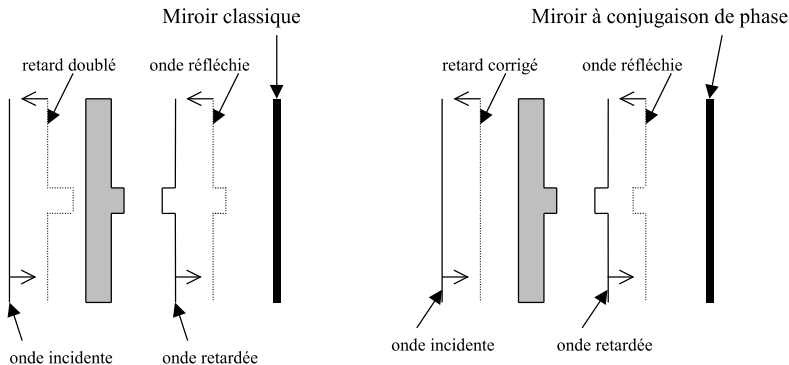
Contents

- 1 Principle and application of phase conjugation
 - Experiment
 - Distortion correction theorem
- 2 Generation of phase conjugate waves
 - Non Linear Polarization Development
 - Four wave mixing coupled mode formulation
 - Conjugate wave amplitude
- 3 Self Pumped Phase Conjugation and Holography
 - Four Wave Mixing from a holographic point of view
 - Phase Conjugation without pumping

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A peculiar phenomenon

Discovered in the early 70^{ies}



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Use Non Linear Material

- Third order : non zero χ_3
 - Photorefractivity
 - Stimulated Brillouin Scattering
 - Stimulated Raman Scattering
- Four wave mixing

Wavefront correction

- Distorted incident beam
- Reflected back as is
- Distortion corrected

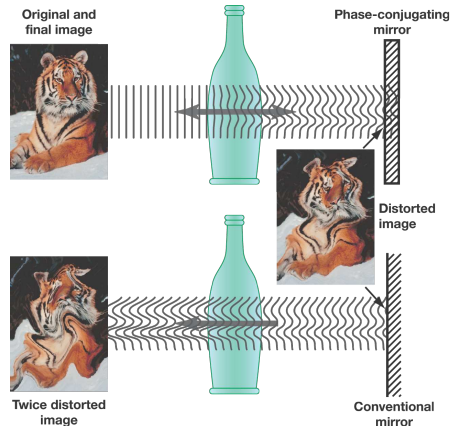


Figure: Phase conjugation principle.

Source : Wikipedia

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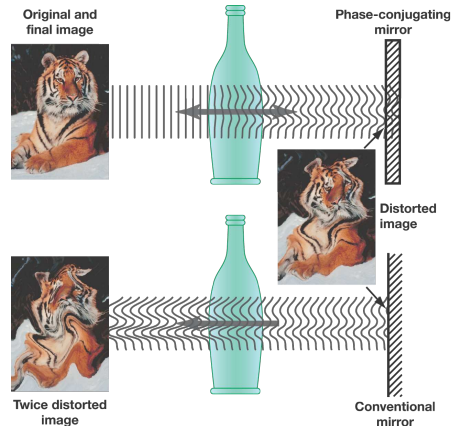


Figure: Phase conjugation principle.

Source : Wikipedia

<http://sharp.bu.edu/~slehar/PhaseConjugate/PhaseConjugate.html>

- Are reflected back **exactly**
- Back and forth wavefronts are **identical**

A diagram showing four parallel red arrows representing light rays entering from the left. They pass through a vertical blue rectangle labeled "glass". After exiting the glass, the rays converge and strike a vertical black line with diagonal hatching, representing a mirror. The rays then reflect back to the left.

A





Diagram B shows a horizontal line with a red arrow pointing right and a blue arrow pointing left, both originating from a vertical bar on the right.

Diagram C shows a ray of light incident on a vertical surface at an angle i and reflecting at an angle r .

D



A diagram showing four parallel red light rays entering a blue converging lens from the left. The rays converge and intersect at a single point on the right side of the lens, which is marked with a vertical line. To the right of this focal point, the rays are shown as parallel again, having crossed each other.

Experiment

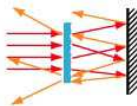
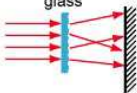
Beams are reflected back *as if time was reversed*

Images source :

<http://sharp.bu.edu/~slehar/PhaseConjugate/PhaseConjugate.html>

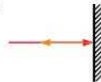
Incident wavefronts...

- Are reflected back **exactly**
- Back and forth wavefronts are **identical**

regular mirror
distorting
glass

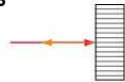
regular mirror

A



phase conjugate mirror

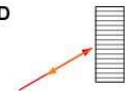
B



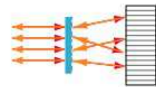
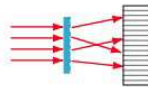
C



D



phase conjugate mirror



Attractive applications

All based on wavefront distortion correction

Phase conjugation applications

- All optical image transmission through fibers
- Distortion correction in high power lasers
- Dynamic wave front correction for optical sensors
- Dynamic Holography
- . . .

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A phase conjugate wave travels time the wrong way

Phase conjugation is also known as *time reversal*

Take some input monochromatic wave

$$E_1 = \mathcal{Re} [\psi(r) \exp(i(\omega t - kz))] = \mathcal{Re} [\psi(r)] \cos(\omega t - kz)$$

Take the phase conjugate over space only

- $E_2 = \mathcal{Re} [\overline{\psi(r)} \exp(i(-kz)) e^{i\omega t}]$
- $E_2 = \mathcal{Re} [\overline{\psi(r)} \exp(i(\omega t + kz))]$
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The distortion correction theorem

If a backward traveling wave is phase conjugate *somewhere* then it is *everywhere*

Take a paraxial forward propagating wave

- Expressed as: $E_1(r, t) = \psi_1(r) e^{i(\omega t - kz)}$
- Obeys the wave equation: $\Delta E_1 + \omega^2 \mu_0 \varepsilon(r) E_1 = 0$
- In the paraxial limit: $\Delta \psi_1 + [\omega^2 \mu_0 \varepsilon(r) - k^2] \psi_1 - 2ik \frac{\partial \psi_1}{\partial z} = 0$
- Conjugate equation: $\Delta \overline{\psi_1} + [\omega^2 \mu_0 \overline{\varepsilon(r)} - k^2] \overline{\psi_1} + 2ik \frac{\partial \overline{\psi_1}}{\partial z} = 0$

Had we taken a backward propagating wave

- Expressed as: $E_2(r, t) = \psi_2(r) e^{i(\omega t + kz)}$
- Paraxial equation ($z \rightarrow -z$):

$$\Delta \psi_2 + [\omega^2 \mu_0 \varepsilon(r) - k^2] \psi_2 + 2ik \frac{\partial \psi_2}{\partial z} = 0$$

Same second order *linear* differential equations for loss-less media

$$\varepsilon(r) \in \mathbb{R} \Rightarrow \left[\psi_2(z=0) = a \cdot \overline{\psi_1(z=0)} \Leftrightarrow \forall z, \psi_2(z) = a \cdot \overline{\psi_1(z)} \right]$$

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Four wave mixing

Third Order Non Linear Optics

Flashback : Second Order

- Relies on χ_2 : $P_{NL} \propto E^2$
- Two waves mix to generate a third one
- $\omega_1 \pm \omega_2 \rightarrow \omega_3$
- Second Harmonic Generation, Optical Parametric Amplification, Optical Parametric Oscillation...

Third order

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- $\omega_1 \pm \omega_2 \pm \omega_3 \rightarrow \omega_4$
- Phase conjugation for $\omega_4 = \omega_1 + \omega_2 - \omega_3$? Let's see...

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Non Linear Polarization P_{NL}

General polarization development

$$[P]_i = \varepsilon_0[\chi]_{ij} + 2[d]_{ijk}[E]_j[E]_k + 4[\chi]_{ijkl}[E]_j[E]_k[E]_l$$

Third order non linear development¹ around $\omega_4 = \omega_1 + \omega_2 - \omega_3$

$$[P_{NL}]_i(\omega_4) = 6[\chi]_{ijkl}[E]_j(\omega_1)[E]_k(\omega_2)\overline{[E]_l(\omega_3)}e^{i(\omega_4 t + k_4 r)}$$

Degenerate Four wave mixing : $\omega = \omega + \omega - \omega$

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¹As an exercise, you can multiply, sum-up and keep only ω_4 related terms... and find k_4

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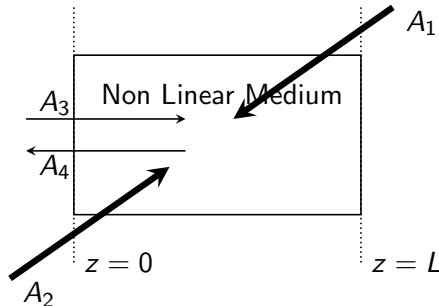
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Degenerate Four Wave mixing configuration

Degenerate configuration

- $A_1 = \overline{A_2}$ intense plane pumps
- A_3 is the signal
- We seek A_4



Signal wave equation

Let us start with the standard non linear wave equation

$$\Delta E - \mu_0 \varepsilon \frac{\partial^2 E}{\partial t^2} = \mu_0 \frac{\partial^2 P_{NL}}{\partial t^2}$$

Signal A_3 propagation

- Each wave has its own direction and polarization
- They can be treated separately

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- Each wave has its own direction and polarization
- They can be treated separately
- With $\Delta E_3 = \mathcal{R}e \left[\left(-k^2 A_3 - 2ik \frac{\partial A_3}{\partial z} + \frac{\partial^2 A_3}{\partial z^2} \right) e^{i(\omega t - kz)} \right]$
- $\mathcal{R}e \left[\left((\omega^2 \mu_0 \varepsilon - k^2) A_3 - 2ik \frac{\partial A_3}{\partial z} + \frac{\partial^2 A_3}{\partial z^2} \right) e^{i(\omega t - kz)} \right] = \mu_0 \frac{\partial^2 P_{NL}}{\partial t^2}$

Signal wave equation

Let us start with the standard non linear wave equation

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Dispersion equation

$$\omega^2 \mu_0 \varepsilon = k^2$$

Signal wave equation

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Assumption

Slow varying assumed : $\|k \frac{\partial A_4}{\partial z}\| \gg \|\frac{\partial^2 A_4}{\partial z^2}\|$

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Evaluation of the non linear polarization P_{NL}

Let us take a look at the terms which involve $e^{i(\omega t \pm kz)}$

Stripping the non linear polarization to useful terms

Keeping only the relevant terms which contain $e^{i(\omega t \pm kz)}$

Expansion of third order non linear polarization

$$[P_{NL}]_i = \mathcal{R}e \left[6 \left(\begin{array}{ccccc} [\chi]_{ijkl} & [A_1]_j & [A_2]_k & \overline{[A_4]}_l \\ + & [\chi]_{ijji} & [A_1]_j & \overline{[A_1]}_j & \overline{[A_3]}_i \\ + & [\chi]_{ikki} & [A_2]_k & \overline{[A_2]}_k & [A_3]_i \\ + & [\chi]_{iiii} & [A_3]_i & \overline{[A_3]}_i & [A_3]_i \\ + & [\chi]_{illi} & [A_4]_l & \overline{[A_4]}_l & [A_3]_i \end{array} \right) e^{i(\omega t - kz)} \right]$$

Simplifying assumptions

- Intense pump beam terms are dominant
- Polarizations are
 - either all the same, only $[\chi]_{ijkl}$ involved
 - or $(A_1/A_2) \neq (A_3/A_4)$, only $[\chi]_{ikki} \neq 0$ involved

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 - $\chi^{(2)} = 0$ or $\chi^{(3)} = 0$

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$$\frac{\partial A_3}{\partial z} = -i\frac{\omega}{2}\sqrt{\frac{\mu_0}{\varepsilon}}\chi^{(3)}\left[\left(\|A1\|^2 + \|A2\|^2\right)A_3 + A_1A_2\overline{A_4}\right]$$

Resulting coupled wave propagation equation

Coupled wave equation resulting of P_{NL}

$$\frac{\partial A_3}{\partial z} = -i\frac{\omega}{2}\sqrt{\frac{\mu_0}{\epsilon}}\chi^{(3)} \left[(\|A_1\|^2 + \|A_2\|^2) A_3 + A_1 A_2 \overline{A_4} \right]$$

Further simplification

- Homogeneous refraction index modulation : Kerr effect
 - Simple phase factor change
 - Remove it from equation $A'_i = A_i e^{-i\frac{\omega}{2}\sqrt{\frac{\mu_0}{\epsilon}}\chi^{(3)}(\|A_1\|^2 + \|A_2\|^2)z}$
- Set $\bar{\kappa} = \frac{\omega}{2}\sqrt{\frac{\mu_0}{\epsilon}}\chi^{(3)}A_1A_2$

Simplified coupling equations

$$\frac{\partial A_1}{\partial z} = i\kappa A_2$$

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Simplified coupling equations

- $\frac{\partial A_1}{\partial z} = i\bar{\kappa}A_2$
- $\frac{\partial A_2}{\partial z} = i\bar{\kappa}A_1$ obtained through the same kind of derivation

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Simplified coupling equations

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- $\frac{\partial A'_4}{\partial z} = i\kappa \overline{A'_3}$

obtained through the same kind of derivation

Resulting coupled wave propagation equation

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General solution

Boundary conditions at $z = 0$ and $z = L$

A_3 is forward propagating

A_4 is backward propagating

- $A'_3(z) = -i \frac{|\kappa| \sin(|k|z)}{\kappa \cos(|k|L)} \overline{A'_4(L)} + \frac{\cos(|k|(z-L))}{\cos(|k|L)} A'_3(0)$

- $A'_4(z) = \frac{\cos(|k|z)}{\cos(|k|L)} A'_4(L) + i \frac{\bar{\kappa} \sin(|k|(z-L))}{|k| \cos(|k|L)} \overline{A'_3(0)}$

One beam experiment

$$A'_4(L) = 0$$

- $A'_3(L) = \frac{A'_3(0)}{\cos(|k|L)}$

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One Beam experiment and phase factor

One beam experiment

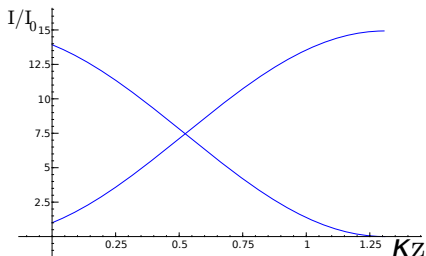
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Reflectivity can exceed 1



What if $\cos(|k|L) = 0$?

- Infinite gain
- A_3 and A_4 start from noise
- Spontaneous oscillations

One Beam experiment and phase factor

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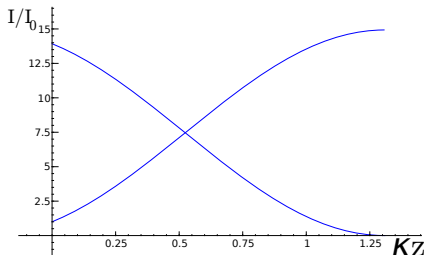
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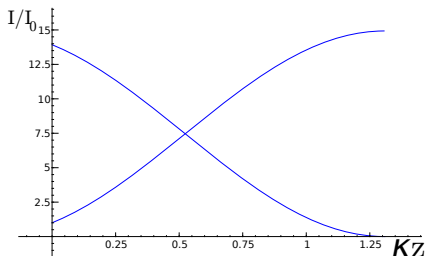
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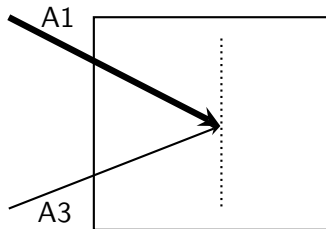
Four Wave Mixing is Real Time Holography

Write Hologram

$$T \propto \|A_1 + A_3\|^2 = \|A_1\|^2 + \|A_3\|^2 + A_1 \overline{A_3} + A_3 \overline{A_1}$$

Read Hologram with $A_2 = \overline{A_1}$

$$A_4 \propto T A_2 = (\|A_1\|^2 + \|A_3\|^2) A_2 + A_2 A_1 \overline{A_3} + A_2 A_3 \overline{A_1}$$



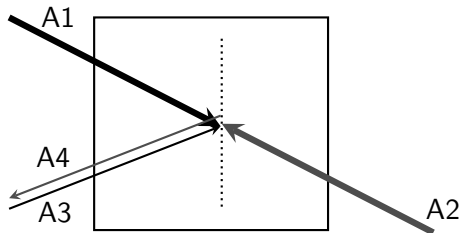
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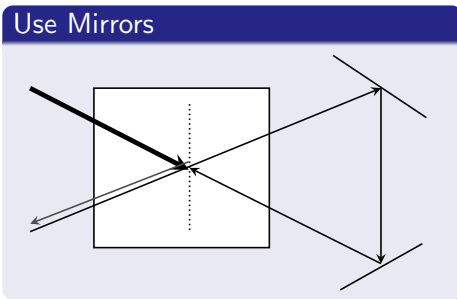
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Phase Conjugation without pumping

CAT Conjugator using Beam Fanning



CAT Conjugator using Beam Fanning

Beam fanning for pump

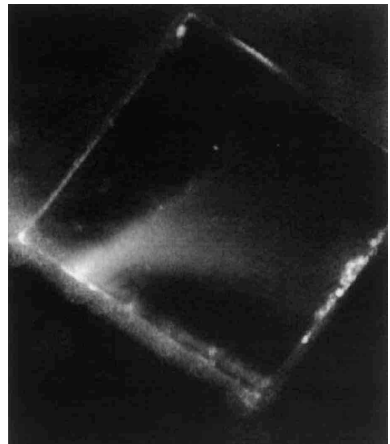
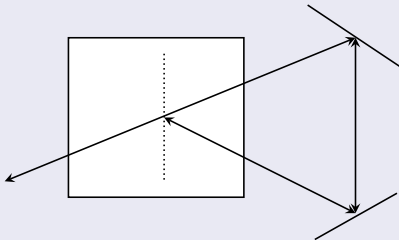


Figure: Beam fanning in photorefractive Baryum Titanate

CAT Conjugator using Beam Fanning

Total internal reflection

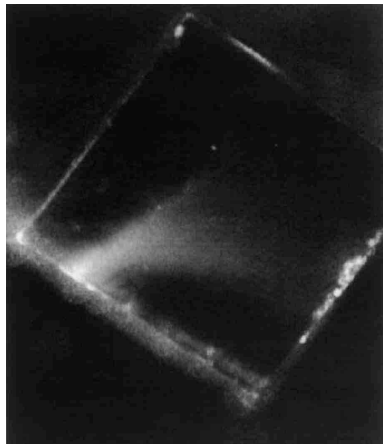
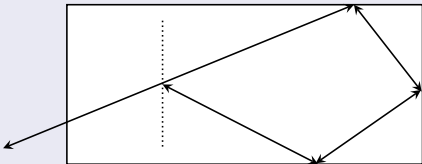


Figure: Beam fanning in photorefractive Baryum Titanate