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## Optical Phase Conjugation

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Download this document : <http://moodle.univ-metz.fr/>

## Useful reading. . .

[YY84, Yar97, San99]



F. Sanchez.

*Optique non-linéaire - Cours et problèmes résolus.*

Ellipses, 1999.



A. Yariv.

*Optical Electronics in Modern Communications.*

Oxford Series in Electrical and Computer Engineering. Oxford University Press, 1997.



A. Yariv and P. Yeh.

*Optical waves in crystals. Propagation and control of laser radiation.*

Wiley series in pure and applied optics. Wiley-Interscience, Stanford University, 1984.

... and many others

# Contents

- 1 Principle and application of phase conjugation
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# A peculiar phenomenon

Discovered in the early 70<sup>ies</sup>

## Use Non Linear Material

- Third order : non zero  $\chi_3$ 
  - Photorefractivity
  - Stimulated Brillouin Scattering
  - Stimulated Raman Scattering
- Four wave mixing

## Wavefront correction

- Distorted incident beam
- Reflected back **as is**
- Distortion corrected

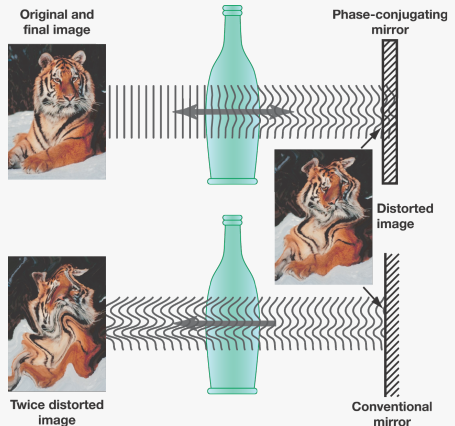


Figure: Phase conjugation principle.

Source : Wikipedia

## Experiment

Beams are reflected back *as if time was reversed*

Images source :

<http://sharp.bu.edu/~slehar/PhaseConjugate/PhaseConjugate.html>

## Incident wavefronts. . .

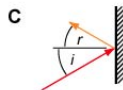
- Are reflected back **exactly**
- Back and forth wavefronts are **identical**

regular mirror  
distorting  
glass

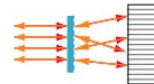
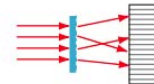
regular mirror



phase conjugate mirror



phase conjugate mirror



# Attractive applications

All based on wavefront distortion correction

## Phase conjugation applications

- All optical image transmission through fibers
- Distortion correction in high power lasers
- Dynamic wave front correction for optical sensors
- Dynamic Holography
- ...

# A phase conjugate wave travels time the wrong way

Phase conjugation is also known as *time reversal*

Take some input monochromatic wave

$$E_1 = \mathcal{R}e [\psi(r) \exp(i(\omega t - kz))] = \mathcal{R}e [\psi(r)] \cos(\omega t - kz)$$

Take the phase conjugate *over space only*

- $E_2 = \mathcal{R}e [\overline{\psi(r) \exp(i(-kz))} e^{i\omega t}]$
- $E_2 = \mathcal{R}e [\overline{\psi(r)} \exp(i(\omega t + kz))]$
- $E_2 = \mathcal{R}e [\overline{\psi(r)}] \cos(\omega t + kz)$
- $E_2 = \mathcal{R}e [\psi(r)] \cos(-\omega t - kz)$



# The distortion correction theorem

If a backward traveling wave is phase conjugate *somewhere* then it is *everywhere*

## Take a paraxial forward propagating wave

- Expressed as:  $E_1(r, t) = \psi_1(r) e^{i(\omega t - kz)}$
- Obeys the wave equation:  $\Delta E_1 + \omega^2 \mu_0 \varepsilon(r) E_1 = 0$
- In the paraxial limit:  $\Delta \psi_1 + [\omega^2 \mu_0 \varepsilon(r) - k^2] \psi_1 - 2ik \frac{\partial \psi_1}{\partial z} = 0$
- Conjugate equation:  $\Delta \overline{\psi_1} + [\omega^2 \mu_0 \overline{\varepsilon(r)} - k^2] \overline{\psi_1} + 2ik \frac{\partial \overline{\psi_1}}{\partial z} = 0$

## Had we taken a backward propagating wave

- Expressed as:  $E_2(r, t) = \psi_2(r) e^{i(\omega t + kz)}$
- Paraxial equation ( $z \rightarrow -z$ ):  

$$\Delta \psi_2 + [\omega^2 \mu_0 \varepsilon(r) - k^2] \psi_2 + 2ik \frac{\partial \psi_2}{\partial z} = 0$$

## Same second order **linear** differential equations for loss-less media

$$\varepsilon(r) \in \mathbb{R} \Rightarrow \left[ \psi_2(z=0) = a \cdot \overline{\psi_1(z=0)} \Leftrightarrow \forall z, \psi_2(z) = a \cdot \overline{\psi_1(z)} \right]$$

# Four wave mixing

## Third Order Non Linear Optics

### Flashback : Second Order

- Relies on  $\chi_2$  :  $P_{NL} \propto E^2$
- Two waves mix to generate a third one
- $\omega_1 \pm \omega_2 \rightarrow \omega_3$
- Second Harmonic Generation, Optical Parametric Amplification, Optical Parametric Oscillation. . .

### Third order

- Relies on  $\chi_3$  :  $P_{NL} \propto E^3$
- **Three** waves mix to generate a **fourth** one
- $\omega_1 \pm \omega_2 \pm \omega_3 \rightarrow \omega_4$
- Phase conjugation for  $\omega_4 = \omega_1 + \omega_2 - \omega_3$ ? Let's see. . .

# Non Linear Polarization $P_{NL}$

General polarization development

$$[P]_i = \varepsilon_0[\chi]_{ij}[E]_j + 2[d]_{ijk}[E]_j[E]_k + 4[\chi]_{ijkl}[E]_j[E]_k[E]_l$$

Third order non linear development<sup>1</sup> around  $\omega_4 = \omega_1 + \omega_2 - \omega_3$

$$[P_{NL}]_i(\omega_4) = 6[\chi]_{ijkl}[E]_j(\omega_1)[E]_k(\omega_2)\overline{[E]_l(\omega_3)}e^{i(\omega_4 t + k_4 r)}$$

Degenerate Four wave mixing :  $\omega = \omega + \omega - \omega$

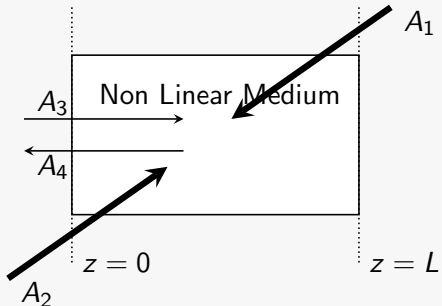
$$[P_{NL}]_i(\omega) = 6[\chi]_{ijkl}[E]_j(\omega)[E]_k(\omega)\overline{[E]_l(\omega)}e^{i(\omega t + k r)}$$

<sup>1</sup>As an exercise, you can multiply, sum-up and keep only  $\omega_4$  related terms. . . and find  $k_4$

# Degenerate Four Wave mixing configuration

## Degenerate configuration

- $A_1 = \overline{A_2}$  intense plane pumps
- $A_3$  is the signal
- We seek  $A_4$



# Signal wave equation

Let us start with the standard non linear wave equation

$$\Delta E - \mu_0 \varepsilon \frac{\partial^2 E}{\partial t^2} = \mu_0 \frac{\partial^2 P_{NL}}{\partial t^2}$$

Signal  $A_3$  propagation

- Each wave has its own direction and polarization
- They can be treated separately
- With  $\Delta E_3 = \mathcal{R}e \left[ \left( -k^2 A_3 - 2ik \frac{\partial A_3}{\partial z} + \frac{\partial^2 A_3}{\partial z^2} \right) e^{i(\omega t - kz)} \right]$
- $\mathcal{R}e \left[ \left( -2ik \frac{\partial A_3}{\partial z} + \frac{\partial^2 A_3}{\partial z^2} \right) e^{i(\omega t - kz)} \right] = \mu_0 \frac{\partial^2 P_{NL}}{\partial t^2}$

Evaluation of the non linear polarization  $P_{NL}$

Let us take a look at the terms which involve  $e^{i(\omega t \pm kz)}$

Four wave mixing coupled mode formulation

## Stripping the non linear polarization to useful terms

Keeping only the relevant terms which contain  $e^{i(\omega t \pm kz)}$ 

## Expansion of third order non linear polarization

$$[P_{NL}]_i = \mathcal{R}e \left[ 6 \left( \begin{array}{ccccc} [\chi]_{ijkl} & [A_1]_j & [A_2]_k & \overline{[A_4]}_l \\ + & [\chi]_{ijji} & [A_1]_j & [A_1]_j & [A_3]_i \\ + & [\chi]_{ikki} & [A_2]_k & [A_2]_k & [A_3]_i \end{array} \right) e^{i(\omega t - kz)} \right]$$

## Simplifying assumptions

- Intense pump beam terms are dominant
- Polarizations are
  - either all the same, only  $[\chi]_{iiii}$  involved
  - or  $(A_1/A_2) \perp (A_3/A_4)$ , only  $[\chi]_{ijji}, i \neq j$  involved
  - $[P_{NL}]_i = \chi^{(3)} [(\|A_1\|^2 + \|A_2\|^2) A_3 + A_1 A_2 \overline{A_4}] e^{i(\omega t - kz)}$   
 $\chi^{(3)} = 6[\chi]_{iiii}$  or  $\chi^{(3)} = 6[\chi]_{ijji}$

# Resulting coupled wave propagation equation

## Coupled wave equation resulting of $P_{NL}$

$$\frac{\partial A_3}{\partial z} = -i\frac{\omega}{2}\sqrt{\frac{\mu_0}{\epsilon}}\chi^{(3)} [ (\|A_1\|^2 + \|A_2\|^2) A_3 + A_1 A_2 \overline{A_4} ]$$

## Further simplification

- Homogeneous refraction index modulation : Kerr effect
  - Simple phase factor change
  - Remove it from equation  $A'_i = A_i e^{-i\frac{\omega}{2}\sqrt{\frac{\mu_0}{\epsilon}}\chi^{(3)}(\|A_1\|^2 + \|A_2\|^2)z}$
- Set  $\overline{\kappa} = \frac{\omega}{2}\sqrt{\frac{\mu_0}{\epsilon}}\chi^{(3)}A_1A_2$

## Simplified coupling equations

- $\frac{\partial \overline{A'_3}}{\partial z} = i\overline{\kappa}A'_4$
- $\frac{\partial A'_4}{\partial z} = i\overline{\kappa}\overline{A'_3}$  obtained through the same kind of derivation

Conjugate wave amplitude

# General solution

Boundary conditions at  $z = 0$  and  $z = L$  $A_3$  is forward propagating $A_4$  is backward propagating

- $A'_3(z) = -i \frac{|\kappa| \sin(|k|z)}{\kappa \cos(|k|L)} \overline{A'_4(L)} + \frac{\cos(|k|(z-L))}{\cos(|k|L)} A'_3(0)$
- $A'_4(z) = \frac{\cos(|k|z)}{\cos(|k|L)} A'_4(L) + i \frac{\bar{\kappa} \sin(|k|(z-L))}{|k| \cos(|k|L)} \overline{A'_3(0)}$

One beam experiment

 $A'_4(L) = 0$ 

- $A'_3(L) = \frac{A'_3(0)}{\cos(|k|L)}$  Coherent amplifier
- $A'_4(0) = -i \frac{\bar{\kappa}}{|\kappa|} \tan(|\kappa|L) \overline{A'_3(0)}$  Reflectivity can exceed 1



# One Beam experiment and phase factor

## One beam experiment

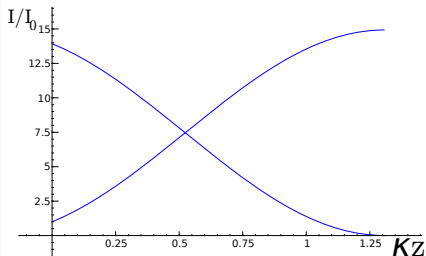
$$A'_4(L) = 0$$

- $A'_3(L) = \frac{A'_3(0)}{\cos(|k|L)}$

Coherent amplifier

- $A'_4(0) = -i \frac{\bar{\kappa}}{|\kappa|} \tan(|\kappa|L) \overline{A'_3(0)}$

Reflectivity can exceed 1



## What if $\cos(|k|L) = 0$ ?

- Infinite gain
- $A_3$  and  $A_4$  start from noise
- **Spontaneous oscillations**

Four Wave Mixing from a holographic point of view

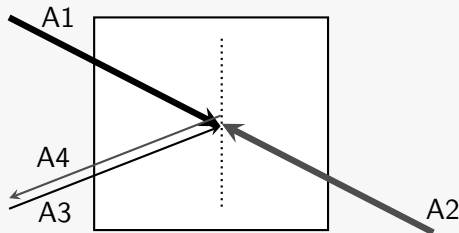
# Four Wave Mixing is Real Time Holography

## Write Hologram

$$T \propto \|A_1 + A_3\|^2 = \|A_1\|^2 + \|A_3\|^2 + A_1\overline{A_3} + A_3\overline{A_1}$$

## Read Hologram with $A_2 = \overline{A_1}$

$$A_4 \propto TA_2 = (\|A_1\|^2 + \|A_3\|^2) A_2 + A_2A_1\overline{A_3} + A_2A_3\overline{A_1}$$



Phase Conjugation without pumping

# CAT Conjugator using Beam Fanning

Total internal reflection

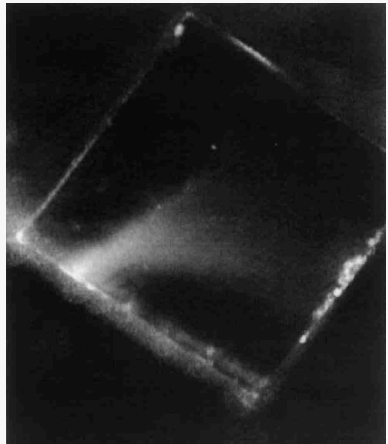
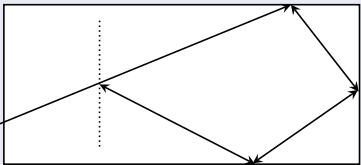


Figure: Beam fanning in photorefractive Baryum Titanate