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Polarization Optics

Nicolas Fressengeas

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| Nicolas Fressengeas. Polarization Optics. DEA. Université Paul Verlaine Metz, 2010. cel-00521501v1

HAL Id: cel-00521501

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Submitted on 27 Sep 2010 (v1), last revised 6 Feb 2014 (v4)

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UE SPM-PHY-S07-101

Polarization Optics

N. Fressengeas

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Unité de Recherche commune à l'Université Paul Verlaine Metz et à Supélec

Document à télécharger sur <http://moodle.univ-metz.fr/>

Further reading

[Hua94, K85]



S. Huard.

Polarisation de la lumière.

Masson, 1994.



G. P. Können.

Polarized light in Nature.

Cambridge University Press, 1985.

Course Outline

- 1 The physics of polarization optics
 - Polarization states
 - Jones Calculus
 - Stokes parameters and the Poincare Sphere
- 2 Polarized light propagation
 - Jones Matrices
 - Polarizers
 - Linear and Circular Anisotropy
 - Jones Matrices Composition
- 3 Partially polarized light
 - Formalisms used
 - Propagation through optical devices

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The vector nature of light

Optical wave can be polarized, sound waves cannot

The scalar monochromatic plane wave

The electric field reads:

$$A \cos(\omega t - kz - \varphi)$$

A vector monochromatic plane wave

- Electric field is orthogonal to wave and Poynting vectors
- Lies in the wave vector normal plane

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- Electric field is orthogonal to wave and Poynting vectors
- Lies in the wave vector normal plane
- Needs 2 components
 - $E_x = A_x \cos(\omega t - kz - \varphi_x)$
 - $E_y = A_y \cos(\omega t - kz - \varphi_y)$

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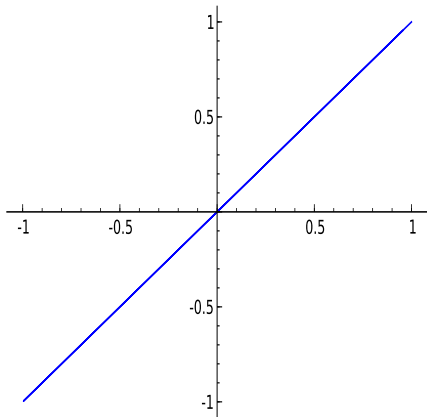
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Linear and circular polarization states

In phase components

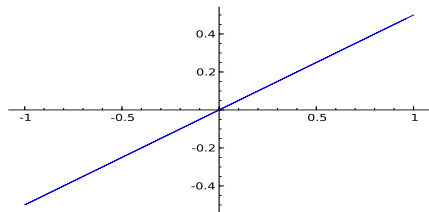
$$\varphi_y = \varphi_x$$



Linear and circular polarization states

In phase components

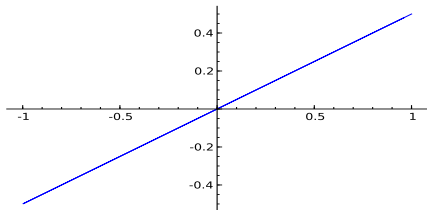
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Linear and circular polarization states

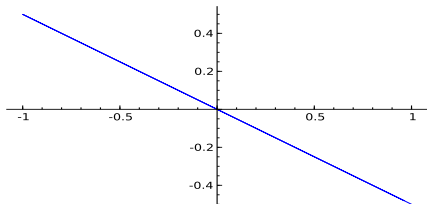
In phase components

$$\varphi_y = \varphi_x$$



π shift

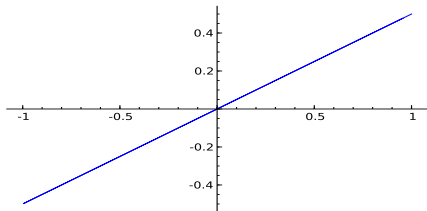
$$\varphi_y = \varphi_x + \pi$$



Linear and circular polarization states

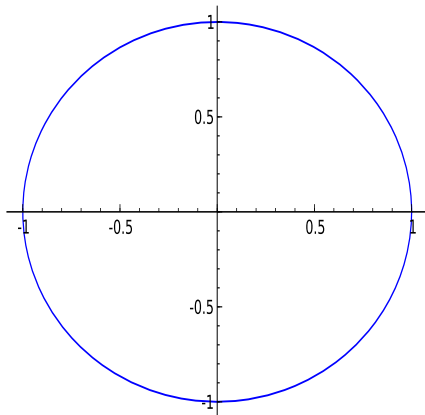
In phase components

$$\varphi_y = \varphi_x$$



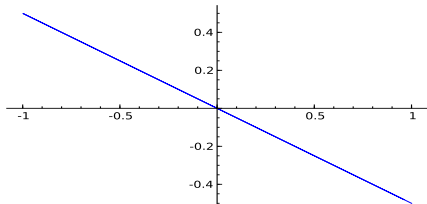
$\pi/2$ shift

$$\varphi_y = \varphi_x \pm \pi/2$$



π shift

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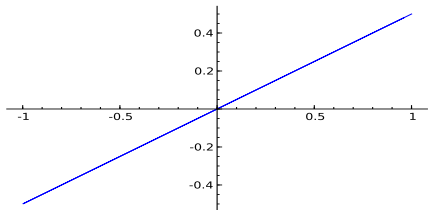


Left or Right

Linear and circular polarization states

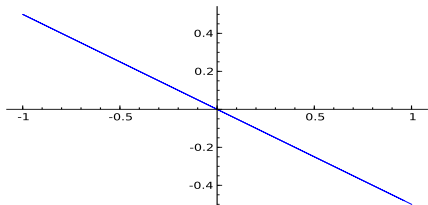
In phase components

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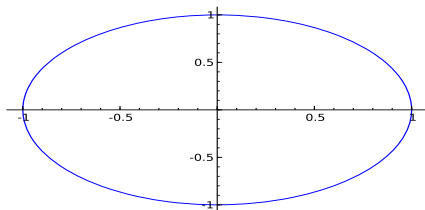
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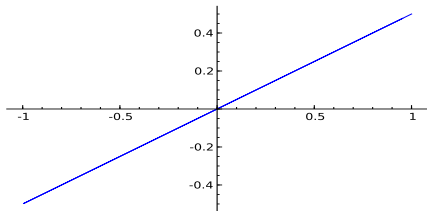


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Linear and circular polarization states

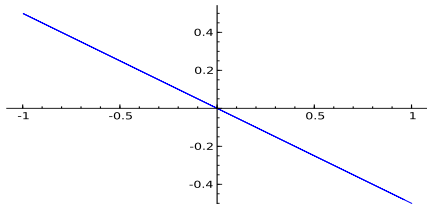
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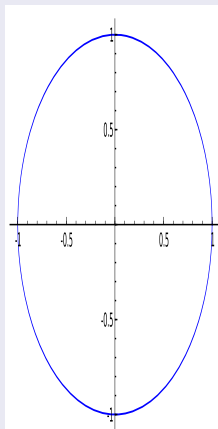
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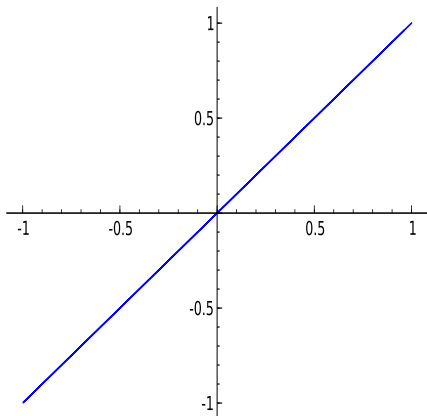


Left or Right

The elliptic polarization state

The polarization state of ANY monochromatic wave

$$\varphi_y - \varphi_x = \pm 0$$



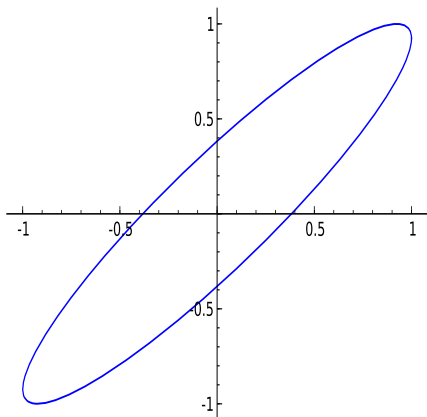
Electric field

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- $E_y = A_y \cos(\omega t - kz - \varphi_y)$

The elliptic polarization state

The polarization state of ANY monochromatic wave

$$\varphi_y - \varphi_x = \pm\pi/8$$



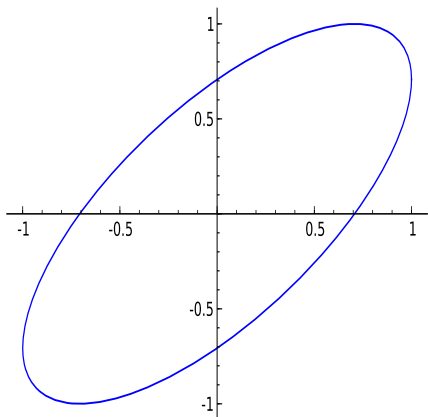
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$$\varphi_y - \varphi_x = \pm\pi/4$$



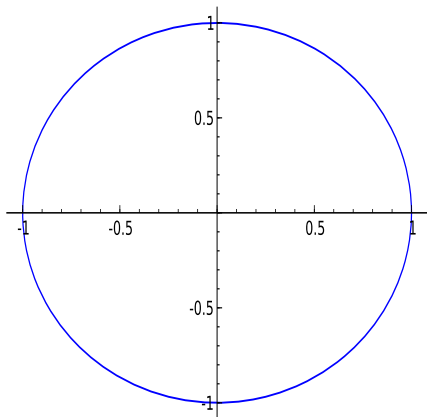
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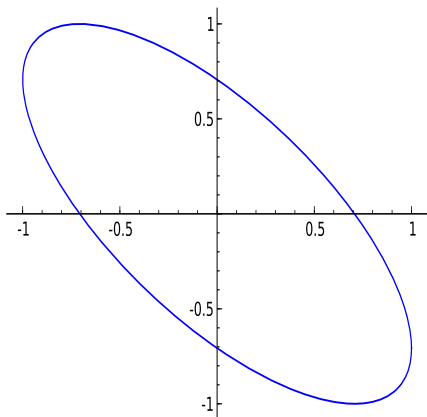
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$$\varphi_y - \varphi_x = \pm 3\pi/4$$



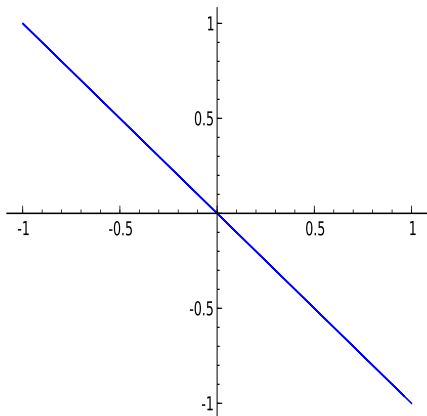
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$$\varphi_y - \varphi_x = \pm\pi$$



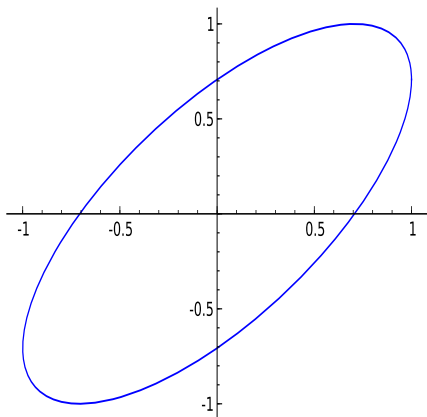
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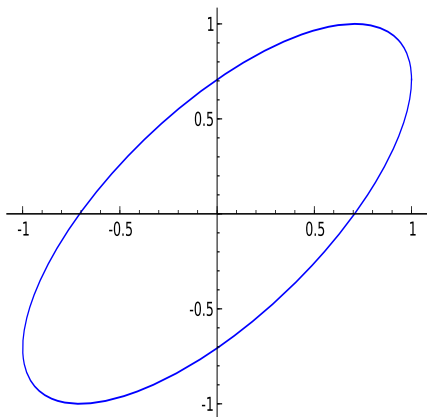
4 real numbers

- A_x, φ_x
- A_y, φ_y

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4 real numbers

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2 complex numbers

- $A_x \exp(i\varphi_x)$
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Polarization states are vectors

Monochromatic polarizations belong to a 2D vector space based on the Complex Ring

ANY elliptic polarization state \iff Two complex numbers

A set of **two** ordered complex numbers is **one** 2D complex vector

Canonical Basis

$$\left(\begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix} \right)$$

Link with optics ?

- These two vectors represent two polarization states
- We must decide which ones !

Polarization Basis

Two independent polarizations :

- Crossed Linear
- Reversed circular
- ...
- **YOUR** choice

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Examples : Linear Polarizations

Canonical Basis Choice

- $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$: horizontal linear polarization
- $\begin{bmatrix} 0 \\ 1 \end{bmatrix}$: vertical linear polarization

Examples : Linear Polarizations

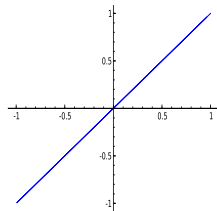
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Tilt

$$\theta = \pi/4$$

$$\frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$



Examples : Linear Polarizations

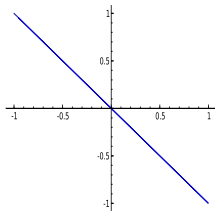
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Tilt

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$$\frac{1}{\sqrt{2}} \begin{bmatrix} -1 \\ 1 \end{bmatrix}$$



Examples : Linear Polarizations

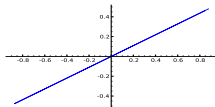
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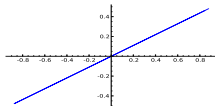
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Linear polarization Jones vector

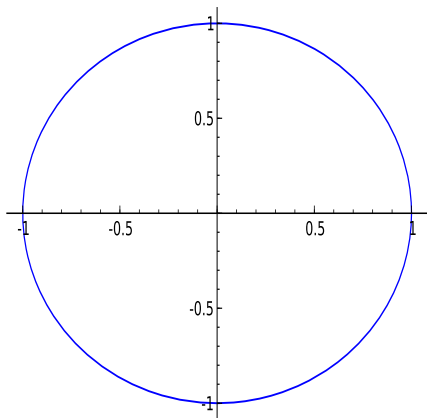
- Linear Polarization : two **in phase** components
- Two **real** numbers

In a linear polarization basis

Examples : Circular Polarizations

In the same canonical basis choice : linear polarizations

$$\varphi_y - \varphi_x = \pm\pi/2$$



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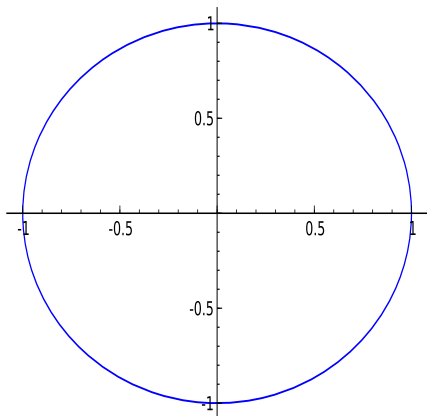
Jones vector

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About changing basis

A polarization state Jones vector is basis dependent

Some elementary algebra

- The polarization vector space dimension is 2
- Therefore : two non colinear vectors form a basis
- Any polarization state can be expressed as the sum of two non colinear other states
- Remark : two colinear polarization states are identical

Homework

Find the transformation matrix between between the two following bases :

- Horizontal and Vertical Linear Polarizations
- Right and Left Circular Polarizations

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Relationship between Jones and Poynting vectors

Jones vectors also provide information about intensity

Choose an orthonormal basis

(J_1, J_2)

- Hermitian product is null : $\overline{J_1} \cdot J_2 = 0$
- Each vector norm is unity : $\overline{J_1} \cdot J_1 = \overline{J_2} \cdot J_2 = 1$

Hermitian Norm is Intensity

Simple calculations show that :

- If each Jones component is one complex electric field component
- The Hermitian product is proportional to beam intensity

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Polarization as a unique complex number

If the intensity information disappears, polarization is summed up in one complex number

Rule out the intensity

Norm the Jones vector to **unity**

Put 1 as first component

- Multiplying Jones vector by a complex number does not change the polarization state
- Norm the first component to 1 : $\begin{bmatrix} 1 \\ \xi \end{bmatrix}$
- The sole ξ describes the polarization state

Choose between the two

Either you norm the vector, or its first component. **Not both !**

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The Stokes parameters

A set of 4 dependent real parameters that can be measured

Sample Jones Vector

$$\begin{bmatrix} A_x \exp(+i\varphi/2) \\ A_y \exp(-i\varphi/2) \end{bmatrix}$$

P_0 Overall Intensity

$$P_0 = A_x^2 + A_y^2 = I$$

P_1 Intensity Difference

$$P_1 = A_x^2 - A_y^2 = I_x - I_y$$

P_2 $\pi/4$ Tilted Basis

$$J_{\pi/4} = \frac{1}{\sqrt{2}} \begin{bmatrix} A_x e^{-i\varphi/2} + A_y e^{+i\varphi/2} \\ A_x e^{-i\varphi/2} - A_y e^{+i\varphi/2} \end{bmatrix}$$
$$P_2 = I_{\pi/4} - I_{-\pi/4} = 2A_x A_y \cos(\varphi)$$

P_3 Circular Basis

$$J_{\text{cir}} = \frac{1}{\sqrt{2}} \begin{bmatrix} A_x e^{-i\varphi/2} - iA_y e^{+i\varphi/2} \\ A_x e^{-i\varphi/2} + iA_y e^{+i\varphi/2} \end{bmatrix}$$
$$P_3 = I_L - I_R = 2A_x A_y \sin(\varphi)$$

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P_2 $\pi/4$ Tilted Basis

$$J_{\pi/4} = \frac{1}{\sqrt{2}} \begin{bmatrix} A_x e^{-i\varphi/2} + A_y e^{+i\varphi/2} \\ A_x e^{-i\varphi/2} - A_y e^{+i\varphi/2} \end{bmatrix}$$
$$P_2 = I_{\pi/4} - I_{-\pi/4} = 2A_x A_y \cos(\varphi)$$

P_3 Circular Basis

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A set of 4 dependent real parameters that can be measured

Sample Jones Vector

$$\begin{bmatrix} A_x \exp(+i\varphi/2) \\ A_y \exp(-i\varphi/2) \end{bmatrix}$$

P_0 Overall Intensity

$$P_0 = A_x^2 + A_y^2 = I$$

P_1 Intensity Différence

$$P_1 = A_x^2 - A_y^2 = I_x - I_y$$

P_2 $\pi/4$ Tilted Basis

$$J_{\pi/4} = \frac{1}{\sqrt{2}} \begin{bmatrix} A_x e^{-i\varphi/2} + A_y e^{+i\varphi/2} \\ A_x e^{-i\varphi/2} - A_y e^{+i\varphi/2} \end{bmatrix}$$
$$P_2 = I_{\pi/4} - I_{-\pi/4} = 2A_x A_y \cos(\varphi)$$

P_3 Circular Basis

$$J_{\text{cir}} = \frac{1}{\sqrt{2}} \begin{bmatrix} A_x e^{-i\varphi/2} - iA_y e^{+i\varphi/2} \\ A_x e^{-i\varphi/2} + iA_y e^{+i\varphi/2} \end{bmatrix}$$
$$P_3 = I_L - I_R = 2A_x A_y \sin(\varphi)$$

The Stokes parameters

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4 dependent parameters

$$P_0^2 = P_1^2 + P_2^2 + P_3^2$$

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Homework

Find the reverse relationship : φ, A_x, A_y from the Stokes parameters

The Poincare Sphere

Polarization states can be described geometrically on a sphere

Recall the Stokes parameters

$$P_0^2 = P_1^2 + P_2^2 + P_3^2$$

Normalized Stokes parameters

$$S_i = P_i/P_0$$

Unit Radius Sphere

$$\sum_{i=1}^3 S_i = 1$$

The Poincare Sphere

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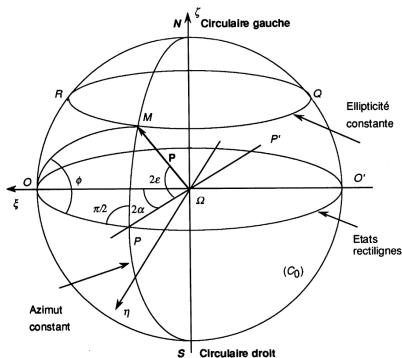
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(S_1, S_2, S_3) on a unit radius sphere



Figures from [Hua94]

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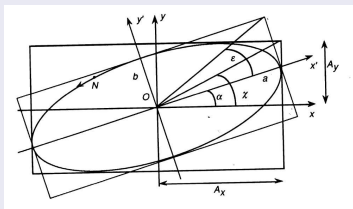
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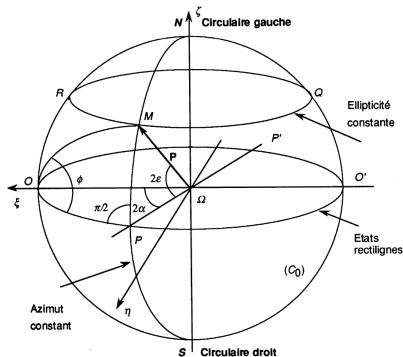
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General Polarisation



(S_1, S_2, S_3) on a unit radius sphere



Figures from [Hua94]

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Eigen Polarization states

Polarization states that do not change after propagation in an anisotropic medium

Eigen Polarization states

- Do not change
- Except for Intensity

Linear Anisotropy Eigen Polarizations

- Quarter and half wave plates and Birefringent materials
- Eigen Polarizations are linear along the eigen axes

Circular Anisotropy

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Orthogonality

2 eigen polarization states are orthonormal

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Jones Matrices

2D Linear Algebra to compute polarization propagation through devices

Jones matrices in the eigen basis

- Let λ_1 and λ_2 be the two eigenvalues of a given device
- e.g. for linear anisotropy : $\lambda_i = e^{n_i k_0 \Delta z}$
- Jones Matrix is $\begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix}$

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Homework

Find half and quarter wave plates Jones Matrices

Jones Matrices

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In another basis

- Let $\vec{J}_1 = \begin{bmatrix} u \\ v \end{bmatrix}$ and $\vec{J}_2 = \begin{bmatrix} -\bar{v} \\ \bar{u} \end{bmatrix}$ be the orthonormal eigen vectors
- $\begin{cases} \mathbf{M} \vec{J}_1 = \lambda_1 \vec{J}_1 \\ \mathbf{M} \vec{J}_2 = \lambda_2 \vec{J}_2 \end{cases} \Rightarrow \mathbf{M} = \begin{bmatrix} \lambda_1 u \bar{u} + \lambda_2 v \bar{v} & (\lambda_1 - \lambda_2) u \bar{v} \\ (\lambda_1 - \lambda_2) v \bar{u} & \lambda_2 v \bar{u} + \lambda_1 v \bar{v} \end{bmatrix}$

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The particular case of non absorbing devices

Jones matrix is a unitary operator when $\lambda_1 = \lambda_2 = 1$

Non absorbing neither amplifying devices

- $\lambda_1 = \lambda_2 = 1$
- $\text{Det}(\mathbf{M}) = 1$
- $\mathbf{M} \cdot \overline{\mathbf{M}^t} = \overline{\mathbf{M}^t} \cdot \mathbf{M} = I$
- \mathbf{M} is a unitary operator

Unitary operator properties

- Norm is conserved : Intensity is unchanged after propagation
- Orthogonality is conserved : two initially orthogonal states will remain so after propagation

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Jones Matrix of a polarizer

In its eigen basis

- A polarizer is designed for :
 - Full transmission of one linear polarization
 - Zero transmission of its orthogonal counterpart

- Eigen basis Jones matrix : $\mathbf{P}_x = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$ or $\mathbf{P}_y = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$

When transmitted polarization is θ tilted

Change base through θ rotation Transformation Matrix

$$\mathbf{R}(-\theta) = \begin{bmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{bmatrix}$$

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Homework

Find again the θ tilted polarizer Jones Matrix by using physics arguments only

Intensity transmitted through a polarizer

From natural or non polarized light

Half the intensity is transmitted

From linearly polarized light

- Transmitted Jones vector in polarizer eigen basis:

$$\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \cos(\theta) \\ \sin(\theta) \end{bmatrix} = \begin{bmatrix} \cos(\theta) \\ 0 \end{bmatrix}$$

- Transmitted Intensity : $\cos^2(\theta)$ MALUS law

From circularly polarized light

Show that whatever the polarizer orientation, the transmitted intensity is half the incident intensity.

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Linear anisotropy eigen polarization vectors

- Two orthogonal polarization directions
- Two different refraction indexes n_1 and n_2
- Two linear eigen modes along the eigen directions

Jones Matrix in the eigen basis

Express phase delay only

$$\begin{bmatrix} e^{im_1k\Delta z} & 0 \\ 0 & e^{im_2k\Delta z} \end{bmatrix} = e^{i\psi} \begin{bmatrix} e^{i\phi/2} & 0 \\ 0 & e^{-i\phi/2} \end{bmatrix} \approx \begin{bmatrix} e^{i\phi/2} & 0 \\ 0 & e^{-i\phi/2} \end{bmatrix}$$

Quarter and Half wave plates

Homework

- Find the Jones Matrices of Quarter and Half wave plates
- Find their action on tilted linear polarization (special case dor $\pi/4$ tilt)
- Find their action on circular polarization

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What is circular anisotropy ?

- Two orthogonal circular eigen polarization states
- Two different refraction indexes n_L and n_R

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Jones Matrix in a linear polarization basis Rotation matrix

- use $\mathbf{P}_{\text{Cir}} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ i & -i \end{bmatrix}$ transformation matrix
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 - Linear and Circular Anisotropy
 - **Jones Matrices Composition**
- 3 Partially polarized light
 - Formalisms used
 - Propagation through optical devices

Jones Matrices Composition

The Jones matrices of cascaded optical elements can be composed through Matrix multiplication

Matrix composition

- If a \vec{J}_0 incident light passes through \mathbf{M}_1 and \mathbf{M}_2 in that order
- First transmission: $\mathbf{M}_1 \vec{J}_0$
- Second transmission: $\mathbf{M}_2 \mathbf{M}_1 \vec{J}_0$
- Composed Jones Matrix : $\mathbf{M}_2 \mathbf{M}_1$ Reversed order

Beware of non commutativity

- Matrix product does not commute in general
- Think of the case of a linear anisotropy followed by optical activity
 - in that order
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Stokes parameters for partially polarized light

Generalize the coherent definition using the statistical average intensity

Stokes Vector

$$\vec{S} = \begin{bmatrix} P_0 \\ P_1 \\ P_2 \\ P_3 \end{bmatrix} = \begin{bmatrix} \langle I_x + I_y \rangle \\ \langle I_x - I_y \rangle \\ \langle I_{\pi/4} - I_{-\pi/4} \rangle \\ \langle I_L - I_R \rangle \end{bmatrix}$$

Polarization degree $0 \leq p \leq 1$

$$p = \frac{\sqrt{P_1^2 + P_2^2 + P_3^2}}{P_0}$$

Stokes decomposition

$$\vec{S} = \begin{bmatrix} P_0 \\ P_1 \\ P_2 \\ P_3 \end{bmatrix} = \begin{bmatrix} pP_0 \\ P_1 \\ P_2 \\ P_3 \end{bmatrix} + \begin{bmatrix} (1-p)P_0 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \vec{S}_P + \vec{S}_{NP}$$

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The Jones Coherence Matrix

Jones Vectors are out

- They describe phase differences
- Meaningless when not monochromatic

Jones Coherence Matrix

- If $\vec{J} = \begin{bmatrix} A_x(t) e^{i\varphi_x(t)} \\ A_y(t) e^{i\varphi_y(t)} \end{bmatrix}$
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Coherence Matrix: explicit formulation

$$\Gamma = \begin{bmatrix} \Gamma_{xx} & \Gamma_{xy} \\ \Gamma_{yx} & \Gamma_{yy} \end{bmatrix}$$

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Trace is Intensity

$$\text{Tr}(\Gamma) = I$$

Base change

Transformation P

$$P^{-1} \Gamma P$$

Relationship with Stokes parameters

from definition

$$\begin{bmatrix} P_0 \\ P_1 \\ P_2 \\ P_3 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & i & -i \end{bmatrix} \begin{bmatrix} \Gamma_{xx} \\ \Gamma_{yy} \\ \Gamma_{xy} \\ \Gamma_{yx} \end{bmatrix}$$

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Inverse relationship

$$\begin{bmatrix} \Gamma_{xx} \\ \Gamma_{yy} \\ \Gamma_{xy} \\ \Gamma_{yx} \end{bmatrix} = \frac{1}{2} \begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & -i \\ 0 & 0 & -1 & i \end{bmatrix} \begin{bmatrix} P_0 \\ P_1 \\ P_2 \\ P_3 \end{bmatrix}$$

Coherence Matrix: further properties

Polarization degree

$$p = \sqrt{\frac{P_1^2 + P_2^2 + P_3^2}{P_0^2}} = \sqrt{1 - \frac{4(\Gamma_{xx}\Gamma_{yy} - \Gamma_{xy}\Gamma_{yx})}{(\Gamma_{xx} + \Gamma_{yy})^2}} = \sqrt{1 - \frac{4\text{Det}(\Gamma)}{\text{Tr}(\Gamma)^2}}$$

Γ Decomposition in polarized and depolarized components

- $\Gamma = \Gamma_P + \Gamma_{NP}$
- Find Γ_P and Γ_{NP} using the relationship with the Stokes parameters

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Propagation of the Coherence Matrix

Jones Calculus

- If incoming polarization is $\overrightarrow{J}(t)$
- Output one is $\overrightarrow{J}'(t) = \mathbf{M}\overrightarrow{J}(t)$

Coherence Matrix if M is unitary

- M unitary means : linear and/or circular anisotropy only

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Mueller Calculus

Propagating the Jones coherence matrix is difficult if the operator is not unitary

Jones Calculus raises some difficulties

- Coherence matrix OK for partially polarized light
- Propagation through unitary optical devices
(linear or circular anisotropy only)
- Hard Times if Polarizers are present

The Stokes parameters may be an alternative

- Describing intensity, they can be readily measured

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The projection on a polarization state

Matrix of the polarizer with axis parallel to \vec{V}

Projection on \vec{V} in Jones Basis

P_V

- Orthogonal Linear Polarizations Basis: \vec{X} and \vec{Y}
- Normed Projection Base Vector :
 - $\vec{V} = A_x e^{-i\frac{\varphi}{2}} \vec{X} + A_y e^{i\frac{\varphi}{2}} \vec{Y}$
 - $\vec{V}^t \vec{V} = 1$
- $P_V = \vec{V} \vec{V}^t$

^aEasy to check in the projection eigen basis : (\vec{V}, \vec{V}^t)



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The Pauli Matrices

A base for the 4D 2×2 matrix vector space

$$\sigma_1 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \sigma_2 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}, \sigma_3 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \sigma_4 = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}$$

\mathbf{P}_V decomposition

$$\mathbf{P}_V = \frac{1}{2} (p_0 \sigma_0 + p_1 \sigma_1 + p_2 \sigma_2 + p_3 \sigma_3)$$

P_V composition and Trace property

Projection property

$$\vec{V}^t \cdot \sigma_j \vec{V} = \left(\vec{V}^t \vec{V} \right) \vec{V}^t \cdot \sigma_j \vec{V}$$

Projection Trace in its eigen basis

- P_V eigenvalues : 0 & 1

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- $P_{V\alpha}$ eigenvalues : 0 & α

$$\alpha \leq 1 \Rightarrow \text{Tr}(P_{V\alpha}) = \alpha$$

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Projection P_V of the polarization base vectors

- Using $P_V = \vec{V} \vec{V}^{\dagger}$
 - $P_V \vec{X} = A_x^2 \vec{X} + A_x A_y e^{i\varphi} \vec{Y}$
 - $P_V \vec{Y} = A_y^2 \vec{Y} + A_x A_y e^{-i\varphi} \vec{X}$

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Using the Pauli Basis

$$\begin{aligned}
 P_V \overrightarrow{X} &= \frac{1}{2} (\rho_0 + \rho_1) \overrightarrow{X} + \frac{1}{2} (\rho_2 + i\rho_3) \overrightarrow{Y} \\
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Projection \mathbf{P}_V of the polarization base vectors

- Using $\mathbf{P}_V = \overrightarrow{V} \overrightarrow{V}^t$
 - $\mathbf{P}_V \overrightarrow{X} = A_x^2 \overrightarrow{X} + A_x A_y e^{i\varphi} \overrightarrow{Y}$
 - $\mathbf{P}_V \overrightarrow{Y} = A_y^2 \overrightarrow{Y} + A_x A_y e^{-i\varphi} \overrightarrow{X}$
- Using the Pauli Basis
 - $\mathbf{P}_V \overrightarrow{X} = \frac{1}{2} (p_0 + p_1) \overrightarrow{X} + \frac{1}{2} (p_2 + ip_3) \overrightarrow{Y}$
 - $\mathbf{P}_V \overrightarrow{Y} = \frac{1}{2} (p_0 - p_1) \overrightarrow{Y} + \frac{1}{2} (p_2 - ip_3) \overrightarrow{X}$

\mathbf{P}_V Pauli composition

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P_V Pauli composition and Stokes parameters

Stokes parameters as P_V decomposition on the Pauli base

- $p_0 = P_0 = A_x^2 - a_y^2 = I_x - I_y$
- $p_1 = P_1 = A_x^2 - a_y^2 = I_x - I_y$
- $p_2 = P_2 = 2A_x A_y \cos(\varphi) = I_{\pi/4} - I_{-\pi/4}$
- $p_3 = P_3 = 2A_x A_y \sin(\varphi) = I_L - I_R$

Propagating through devices: Mueller matrices

$$\vec{V}' = \mathbf{M}_J \vec{V}$$

Projection on \vec{V}'

$$\mathbf{P}_{V'} = \vec{V}' \vec{V}'^t$$

Trace relationship

$$P'_i = \text{Tr}(\mathbf{P}_{V'} \sigma_i)$$

Mueller matrix

$$\vec{S}' = \mathbf{M}_M \vec{S}$$

$$(M_M)_{ij} = \frac{1}{2} \text{Tr}(\mathbf{M}_J \sigma_j \overline{\mathbf{M}_J^t} \sigma_i)$$

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Mueller matrices and partially polarized light

Time average of the previous study

Mueller matrices are time independent

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Mueller calculus can be extended to ...

- Partially coherent light
- Cascaded optical devices

Final homework

Find the Mueller matrix of each :

- Polarizers along eigen axis or θ tilted
- half and quarter wave plates
- linearly and circularly birefringent crystal

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