

# UE SPM-PHY-S07-101

## Polarization Optics

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## Further reading

[Hua94, K85]



S. Huard.

*Polarisation de la lumière.*

Masson, 1994.



G. P. Können.

*Polarized light in Nature.*

Cambridge University Press, 1985.

# Course Outline

- 1 The physics of polarization optics
  - Polarization states
  - Jones Calculus
  - Stokes parameters and the Poincare Sphere
- 2 Polarized light propagation
  - Jones Matrices
  - Polarizers
  - Linear and Circular Anisotropy
  - Jones Matrices Composition
- 3 Partially polarized light
  - Formalisms used
  - Propagation through optical devices

# The vector nature of light

Optical wave can be polarized, sound waves cannot

## The scalar monochromatic plane wave

The electric field reads:

$$A \cos(\omega t - kz - \varphi)$$

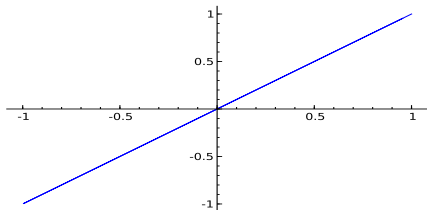
## A **vector** monochromatic plane wave

- Electric field is orthogonal to wave and Poynting vectors
- Lies in the wave vector normal **plane**
- Needs **2 components**
  - $E_x = A_x \cos(\omega t - kz - \varphi_x)$
  - $E_y = A_y \cos(\omega t - kz - \varphi_y)$

## Linear and circular polarization states

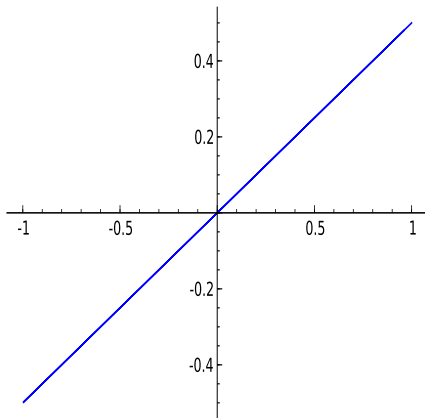
In phase components

$$\varphi_y = \varphi_x$$



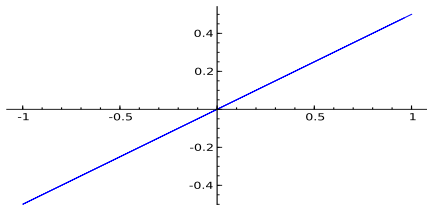
$\pi/2$  shift

$$\varphi_y = \varphi_x \pm \pi/2$$



$\pi$  shift

$$\varphi_y = \varphi_x + \pi$$

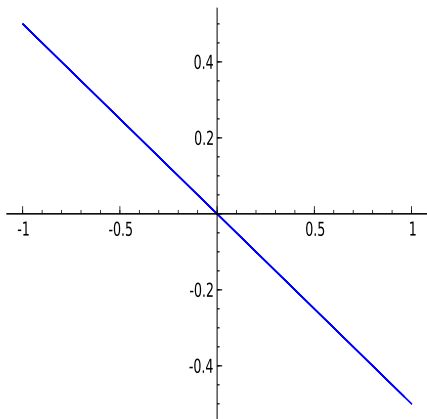


Left or Right

# The elliptic polarization state

The polarization state of ANY monochromatic wave

$$\varphi_y - \varphi_x = \pm\pi/4$$



## Electric field

- $E_x = A_x \cos(\omega t - kz - \varphi_x)$
- $E_y = A_y \cos(\omega t - kz - \varphi_y)$

## 4 real numbers

- $A_x, \varphi_x$
- $A_y, \varphi_y$

## 2 complex numbers

- $A_x \exp(i\varphi_x)$
- $A_y \exp(i\varphi_y)$

# Polarization states are vectors

Monochromatic polarizations belong to a 2D vector space based on the Complex Ring

ANY elliptic polarization state  $\iff$  Two complex numbers

A set of **two** ordered complex numbers is **one** 2D complex vector

## Canonical Basis

$$\left( \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix} \right)$$

## Link with optics ?

- These two vectors represent two polarization states
- We must decide which ones !

## Polarization Basis

Two independent polarizations :

- Crossed Linear
- Reversed circular
- ...
- **YOUR** choice

## Examples : Linear Polarizations

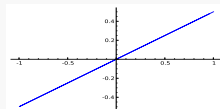
### Canonical Basis Choice

- $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$  : horizontal linear polarization
- $\begin{bmatrix} 0 \\ 1 \end{bmatrix}$  : vertical linear polarization

Tilt

$\theta$

$$\begin{bmatrix} \cos(\theta) \\ \sin(\theta) \end{bmatrix}$$



### Linear polarization Jones vector

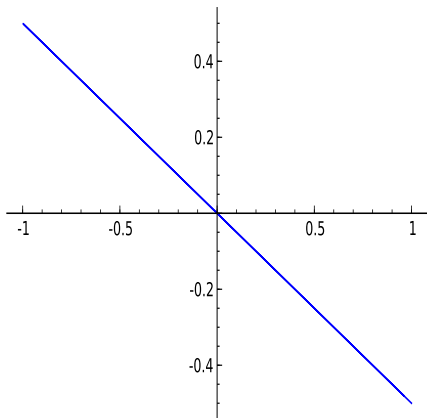
- Linear Polarization : two **in phase** components
  - Two **real** numbers
- In a linear polarization basis



## Examples : Circular Polarizations

In the same canonical basis choice : linear polarizations

$$\varphi_y - \varphi_x = \pm\pi/2$$



### Electric field

- $E_x = A_x \cos(\omega t - kz - \varphi_x)$
- $E_y = A_y \cos(\omega t - kz - \varphi_y)$

### Jones vector

$$\begin{bmatrix} 1 \\ \pm i \end{bmatrix}$$

## About changing basis

A polarization state Jones vector is basis dependent

### Some elementary algebra

- The polarization vector space dimension is 2
- Therefore : two non colinear vectors form a basis
- Any polarization state can be expressed as the sum of two non colinear other states
- Remark : two colinear polarization states are identical

### Homework

Find the transformation matrix between between the two following bases :

- Horizontal and Vertical Linear Polarizations
- Right and Left Circular Polarizations

## Relationship between Jones and Poynting vectors

Jones vectors also provide information about intensity

Choose an orthonormal basis

$(J_1, J_2)$

- Hermitian product is null :  $\overline{J_1} \cdot J_2 = 0$
- Each vector norm is unity :  $\overline{J_1} \cdot J_1 = \overline{J_2} \cdot J_2 = 1$

Hermitian Norm is Intensity

Simple calculations show that :

- If each Jones component is one complex electric field component
- The Hermitian product is proportional to beam intensity

## Polarization as a unique complex number

If the intensity information disappears, polarization is summed up in one complex number

### Rule out the intensity

Norm the Jones vector to **unity**

### Put 1 as first component

- Multiplying Jones vector by a complex number does not change the polarization state
- Norm the first component to 1 :  $\begin{bmatrix} 1 \\ \xi \end{bmatrix}$
- The sole  $\xi$  describes the polarization state

### Choose between the two

Either you norm the vector, or its first component. **Not both !**

# The Stokes parameters

A set of 4 dependent real parameters that can be measured

## Sample Jones Vector

$$\begin{bmatrix} A_x \exp(+i\varphi/2) \\ A_y \exp(-i\varphi/2) \end{bmatrix}$$

### $P_0$ Overall Intensity

$$P_0 = A_x^2 + A_y^2 = I$$

### $P_1$ Intensity Différence

$$P_1 = A_x^2 - A_y^2 = I_x - I_y$$

### $P_2$ $\pi/4$ Tilted Basis

$$J_{\pi/4} = \frac{1}{\sqrt{2}} \begin{bmatrix} A_x e^{-i\varphi/2} + A_y e^{+i\varphi/2} \\ A_x e^{-i\varphi/2} - A_y e^{+i\varphi/2} \end{bmatrix}$$

$$P_2 = I_{\pi/4} - I_{-\pi/4} = 2A_x A_y \cos(\varphi)$$

### $P_3$ Circular Basis

$$J_{\text{cir}} = \frac{1}{\sqrt{2}} \begin{bmatrix} A_x e^{-i\varphi/2} - iA_y e^{+i\varphi/2} \\ A_x e^{-i\varphi/2} + iA_y e^{+i\varphi/2} \end{bmatrix}$$

$$P_3 = I_L - I_R = 2A_x A_y \sin(\varphi)$$

## 4 dependent parameters

$$P_0^2 = P_1^2 + P_2^2 + P_3^2$$

# The Poincare Sphere

Polarization states can be described geometrically on a sphere

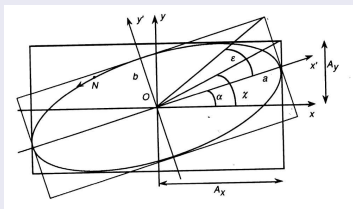
Normalized Stokes parameters

$$S_i = P_i / P_0$$

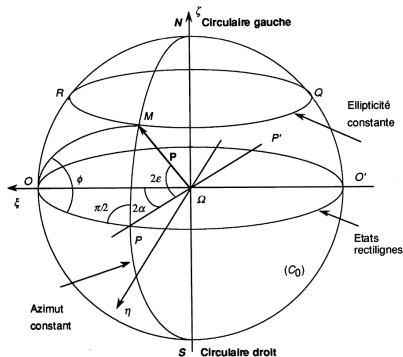
Unit Radius Sphere

$$\sum_{i=1}^3 S_i = 1$$

General Polarisation



$(S_1, S_2, S_3)$  on a unit radius sphere



Figures from [Hua94]

## Eigen Polarization states

Polarization states that do not change after propagation in an anisotropic medium

### Eigen Polarization states

- Do not change
- Except for Intensity

### Orthogonality

2 eigen polarization states are orthonormal

### Linear Anisotropy Eigen Polarizations

- Quarter and half wave plates and Birefringent materials
- Eigen Polarizations are linear along the eigen axes

### Circular Anisotropy

- Also called optical activity
- e.g in Faraday rotators and in gyrotory non linear crystals
- Linear polarization is rotated by an angle proportional to propagation distance
- Eigen polarizations are the circular polarizations

# Jones Matrices

2D Linear Algebra to compute polarization propagation through devices

## Jones matrices in the eigen basis

- Let  $\lambda_1$  and  $\lambda_2$  be the two eigenvalues of a given device
- e.g. for linear anisotropy :  $\lambda_i = e^{n_i k_0 \Delta z}$
- Jones Matrix is  $\begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix}$

## In another basis

use Transformation Matrix

- Let  $\vec{J}_1 = \begin{bmatrix} u \\ v \end{bmatrix}$  and  $\vec{J}_2 = \begin{bmatrix} -\bar{v} \\ \bar{u} \end{bmatrix}$  be the orthonormal eigen vectors
- $\begin{cases} \mathbf{M} \vec{J}_1 = \lambda_1 \vec{J}_1 \\ \mathbf{M} \vec{J}_2 = \lambda_2 \vec{J}_2 \end{cases} \Rightarrow \mathbf{M} = \begin{bmatrix} \lambda_1 u \bar{u} + \lambda_2 v \bar{v} & (\lambda_1 - \lambda_2) u \bar{v} \\ (\lambda_1 - \lambda_2) v \bar{u} & \lambda_2 v \bar{u} + \lambda_1 v \bar{v} \end{bmatrix}$



# The particular case of non absorbing devices

Jones matrix is a unitary operator when  $\lambda_1 = \lambda_2 = 1$

## Non absorbing neither amplifying devices

- $\lambda_1 = \lambda_2 = 1$
- $\text{Det}(\mathbf{M}) = 1$
- $\mathbf{M} \cdot \overline{\mathbf{M}^t} = \overline{\mathbf{M}^t} \cdot \mathbf{M} = I$
- $\mathbf{M}$  is a unitary operator

## Unitary operator properties

- Norm is conserved : Intensity is unchanged after propagation
- Orthogonality is conserved : two initially orthogonal states will remain so after propagation

## Jones Matrix of a polarizer

### In its eigen basis

- A polarizer is designed for :
  - Full transmission of one linear polarization
  - Zero transmission of its orthogonal counterpart
- Eigen basis Jones matrix :  $\mathbf{P}_x = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$  or  $\mathbf{P}_y = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$

### When transmitted polarization is $\theta$ tilted

Change base through  $\theta$  rotation Transformation Matrix

$$\mathbf{R}(-\theta) = \begin{bmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{bmatrix}$$

$$\mathbf{P}(\theta) = \mathbf{R}(\theta) \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \mathbf{R}(-\theta) = \begin{bmatrix} \cos^2(\theta) & \sin(\theta)\cos(\theta) \\ \sin(\theta)\cos(\theta) & \sin^2(\theta) \end{bmatrix}$$

## Intensity transmitted through a polarizer

From natural or non polarized light

Half the intensity is transmitted

From linearly polarized light

- Transmitted Jones vector in polarizer eigen basis:

$$\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \cos(\theta) \\ \sin(\theta) \end{bmatrix} = \begin{bmatrix} \cos(\theta) \\ 0 \end{bmatrix}$$

- Transmitted Intensity :  $\cos^2(\theta)$  **MALUS** law

From circularly polarized light

Show that whatever the polarizer orientation, the transmitted intensity is half the incident intensity.

## Linear anisotropy eigen polarization vectors

- Two orthogonal polarization directions
- Two different refraction indexes  $n_1$  and  $n_2$
- Two linear eigen modes along the eigen directions

## Jones Matrix in the eigen basis

Express phase delay only

$$\begin{bmatrix} e^{in_1k\Delta z} & 0 \\ 0 & e^{in_2k\Delta z} \end{bmatrix} = e^{i\psi} \begin{bmatrix} e^{i\phi/2} & 0 \\ 0 & e^{-i\phi/2} \end{bmatrix} \approx \begin{bmatrix} e^{i\phi/2} & 0 \\ 0 & e^{-i\phi/2} \end{bmatrix}$$

## Quarter and Half wave plates

Homework

- Find the Jones Matrices of Quarter and Half wave plates
- Find their action on tilted linear polarization (special case dor  $\pi/4$  tilt)
- Find their action on circular polarization

## What is circular anisotropy ?

- Two orthogonal circular eigen polarization states
- Two different refraction indexes  $n_L$  and  $n_R$

## Jones Matrix in the circular eigen basis      Express phase delay only

$$\begin{bmatrix} e^{in_L k \Delta z} & 0 \\ 0 & e^{in_R k \Delta z} \end{bmatrix} = e^{i\psi} \begin{bmatrix} e^{i\phi/2} & 0 \\ 0 & e^{-i\phi/2} \end{bmatrix} \approx \begin{bmatrix} e^{i\phi/2} & 0 \\ 0 & e^{-i\phi/2} \end{bmatrix}$$

## Jones Matrix in a linear polarization basis      Rotation matrix

- use  $\mathbf{P}_{\text{Cir}} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ i & -i \end{bmatrix}$  transformation matrix
- $\mathbf{P}_{\text{Cir}} \mathbf{M} \mathbf{P}_{\text{Cir}}^{-1} = e^{i\psi} \begin{bmatrix} \cos(\phi/2) & \sin(\phi/2) \\ -\sin(\phi/2) & \cos(\phi/2) \end{bmatrix}$

## Jones Matrices Composition

The Jones matrices of cascaded optical elements can be composed through Matrix multiplication

### Matrix composition

- If a  $\vec{J}_0$  incident light passes through  $\mathbf{M}_1$  and  $\mathbf{M}_2$  in that order
- First transmission:  $\mathbf{M}_1 \vec{J}_0$
- Second transmission:  $\mathbf{M}_2 \mathbf{M}_1 \vec{J}_0$
- Composed Jones Matrix :  $\mathbf{M}_2 \mathbf{M}_1$  Reversed order

### Beware of non commutativity

- Matrix product does not commute in general
- Think of the case of a linear anisotropy followed by optical activity
  - in that order
  - in the reverse order

## Stokes parameters for partially polarized light

Generalize the coherent definition using the statistical average intensity

### Stokes Vector

$$\vec{S} = \begin{bmatrix} P_0 \\ P_1 \\ P_2 \\ P_3 \end{bmatrix} = \begin{bmatrix} \langle I_x + I_y \rangle \\ \langle I_x - I_y \rangle \\ \langle I_{\pi/4} - I_{-\pi/4} \rangle \\ \langle I_L - I_R \rangle \end{bmatrix}$$

### Polarization degree

$$0 \leq p \leq 1$$

$$p = \frac{\sqrt{P_1^2 + P_2^2 + P_3^2}}{P_0}$$

### Stokes decomposition

### Polarized and depolarized sum

$$\vec{S} = \begin{bmatrix} P_0 \\ P_1 \\ P_2 \\ P_3 \end{bmatrix} = \begin{bmatrix} pP_0 \\ P_1 \\ P_2 \\ P_3 \end{bmatrix} + \begin{bmatrix} (1-p)P_0 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \vec{S}_P + \vec{S}_{NP}$$

# The Jones Coherence Matrix

## Jones Vectors are out

- They describe phase differences
- Meaningless when not monochromatic

## Jones Coherence Matrix

- If  $\vec{J} = \begin{bmatrix} A_x(t) e^{i\varphi_x(t)} \\ A_y(t) e^{i\varphi_y(t)} \end{bmatrix}$
- $\Gamma_{ij} = \langle \vec{J}_i(t) \overline{\vec{J}_j(t)} \rangle$
- $\Gamma = \langle \vec{J}(t) \otimes \overline{\vec{J}(t)} \rangle$

## Coherence Matrix: explicit formulation

$$\Gamma = \begin{bmatrix} \langle |A_x(t)|^2 \rangle & \langle A_x(t) \overline{A_y(t)} e^{i(\varphi_x - \varphi_y)} \rangle \\ \langle \overline{A_x(t)} A_y(t) e^{-i(\varphi_x - \varphi_y)} \rangle & \langle |A_y(t)|^2 \rangle \end{bmatrix}$$



## Jones Coherence Matrix: properties

Trace is Intensity

$$\text{Tr}(\Gamma) = I$$

Base change

Transformation  $\mathbf{P}$

$$\mathbf{P}^{-1}\Gamma\mathbf{P}$$

Relationship with Stokes parameters

from definition

$$\begin{bmatrix} P_0 \\ P_1 \\ P_2 \\ P_3 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & i & -i \end{bmatrix} \begin{bmatrix} \Gamma_{xx} \\ \Gamma_{yy} \\ \Gamma_{xy} \\ \Gamma_{yx} \end{bmatrix}$$

Inverse relationship

$$\begin{bmatrix} \Gamma_{xx} \\ \Gamma_{yy} \\ \Gamma_{xy} \\ \Gamma_{yx} \end{bmatrix} = \frac{1}{2} \begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & -i \\ 0 & 0 & -1 & i \end{bmatrix} \begin{bmatrix} P_0 \\ P_1 \\ P_2 \\ P_3 \end{bmatrix}$$

## Coherence Matrix: further properties

### Polarization degree

$$p = \sqrt{\frac{P_1^2 + P_2^2 + P_3^2}{P_0^2}} = \sqrt{1 - \frac{4(\Gamma_{xx}\Gamma_{yy} - \Gamma_{xy}\Gamma_{yx})}{(\Gamma_{xx} + \Gamma_{yy})^2}} = \sqrt{1 - \frac{4\text{Det}(\Gamma)}{\text{Tr}(\Gamma)^2}}$$

### $\Gamma$ Decomposition in polarized and depolarized components

- $\Gamma = \Gamma_P + \Gamma_{NP}$
- Find  $\Gamma_P$  and  $\Gamma_{NP}$  using the relationship with the Stokes parameters

# Propagation of the Coherence Matrix

## Jones Calculus

- If incoming polarization is  $\overrightarrow{J}(t)$
- Output one is  $\overrightarrow{J}'(t) = \mathbf{M}\overrightarrow{J}(t)$

## Coherence Matrix if $\mathbf{M}$ is unitary

- $\mathbf{M}$  unitary means : linear and/or circular anisotropy only
- $\Gamma' = \langle \overrightarrow{J}'(t) \otimes \overrightarrow{J}'(t) \rangle$
- $\Gamma' = \mathbf{M} \langle \overrightarrow{J}(t) \otimes \overrightarrow{J}(t) \rangle \mathbf{M}^{-1}$

## Polarization degree

- Unaltered for unitary operators      Tr and Det are unaltered
- Not the case if a polarizer is present :  $\rho$  becomes **1**

# Mueller Calculus

Propagating the Jones coherence matrix is difficult if the operator is not unitary

## Jones Calculus raises some difficulties

- Coherence matrix OK for partially polarized light
- Propagation through unitary optical devices  
(linear or circular anisotropy only)
- Hard Times if Polarizers are present

## The Stokes parameters may be an alternative

- Describing intensity, they can be readily measured
- We will show they can be propagated using  $4 \times 4$  **real** matrices
- They are the **Mueller** matrices



# The projection on a polarization state

Matrix of the polarizer with axis parallel to  $\vec{V}$

## Projection on $\vec{V}$ in Jones Basis

$P_V$

- Orthogonal Linear Polarizations Basis:  $\vec{X}$  and  $\vec{Y}$
- Normed Projection Base Vector :
  - $\vec{V} = A_x e^{-i\frac{\varphi}{2}} \vec{X} + A_y e^{i\frac{\varphi}{2}} \vec{Y}$
  - $\vec{V}^t \vec{V} = 1$
- $P_V = \vec{V} \vec{V}^t$ <sup>a</sup>

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<sup>a</sup>Easy to check in the projection eigen basis :  $(\vec{V}, \vec{V}^t)$

# The Pauli Matrices

A base for the 4D  $2 \times 2$  matrix vector space

$$\sigma_1 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \sigma_2 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}, \sigma_3 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \sigma_4 = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}$$

$\mathbf{P}_V$  decomposition

$$\mathbf{P}_V = \frac{1}{2} (p_0 \sigma_0 + p_1 \sigma_1 + p_2 \sigma_2 + p_3 \sigma_3)$$

## $P_V$ composition and Trace property

### Projection property

$$\overrightarrow{V}^t \cdot \sigma_j \overrightarrow{V} = \left( \overrightarrow{V}^t \overrightarrow{V} \right) \overrightarrow{V}^t \cdot \sigma_j \overrightarrow{V} = \overrightarrow{V}^t \left( \overrightarrow{V} \overrightarrow{V}^t \right) \sigma_j \overrightarrow{V} = \overrightarrow{V}^t \cdot \mathbf{P}_V \sigma_j \overrightarrow{V}$$

### Projection Trace in its eigen basis

$$\left( V, \overrightarrow{V}^t \right)$$

- $\mathbf{P}_V$  eigenvalues : 0 & 1  $\text{Tr}(P_V) = 1$
- $\mathbf{P}_V \sigma_j$  eigenvalues : 0 &  $\alpha$   $\alpha \leq 1$   $\text{Tr}(P_V \sigma_j) = \alpha$
- $\overrightarrow{V}^t \cdot \mathbf{P}_V \sigma_j \overrightarrow{V} = \alpha = \text{Tr}(P_V \sigma_j)$

## $\mathbf{P}_V$ Pauli composition

$$\overrightarrow{V}^t \cdot \sigma_j \overrightarrow{V} = \text{Tr}(\mathbf{P}_V \sigma_j) \qquad \text{Tr}(\sigma_i \sigma_j) = 2\delta_{ij}$$

$$\overrightarrow{V}^t \cdot \sigma_j \overrightarrow{V} = \text{Tr}(\mathbf{P}_V \sigma_j) = \frac{1}{2} \sum_i \text{Tr}(\sigma_i \sigma_j) p_i = \frac{1}{2} \sum_i 2\delta_{ij} p_i = p_j$$

### Projection $\mathbf{P}_V$ of the polarization base vectors

- Using  $\mathbf{P}_V = \overrightarrow{V} \overrightarrow{V}^t$ 
  - $\mathbf{P}_V \overrightarrow{X} = A_x^2 \overrightarrow{X} + A_x A_y e^{i\varphi} \overrightarrow{Y}$
  - $\mathbf{P}_V \overrightarrow{Y} = A_y^2 \overrightarrow{Y} + A_x A_y e^{-i\varphi} \overrightarrow{X}$
- Using the Pauli Basis
  - $\mathbf{P}_V \overrightarrow{X} = \frac{1}{2} (p_0 + p_1) \overrightarrow{X} + \frac{1}{2} (p_2 + ip_3) \overrightarrow{Y}$
  - $\mathbf{P}_V \overrightarrow{Y} = \frac{1}{2} (p_0 - p_1) \overrightarrow{Y} + \frac{1}{2} (p_2 - ip_3) \overrightarrow{X}$



## $P_V$ Pauli composition and Stokes parameters

### Stokes parameters as $P_V$ decomposition on the Pauli base

- $p_0 = P_0 = A_x^2 - a_y^2 = I_x - I_y$
- $p_1 = P_1 = A_x^2 - a_y^2 = I_x - I_y$
- $p_2 = P_2 = 2A_x A_y \cos(\varphi) = I_{\pi/4} - I_{-\pi/4}$
- $p_3 = P_3 = 2A_x A_y \sin(\varphi) = I_L - I_R$

## Propagating through devices: Mueller matrices

$$\vec{V}' = \mathbf{M}_J \vec{V}$$

Projection on  $\vec{V}'$

$$\mathbf{P}_{V'} = \vec{V}' \vec{V}'^t = \mathbf{M}_J \vec{V} \vec{V}^t \mathbf{M}_J^t = \mathbf{M}_J \mathbf{P}_V \mathbf{M}_J^t$$

Trace relationship

$$P'_i = \text{Tr}(\mathbf{P}_{V'} \sigma_i) = \text{Tr}(\mathbf{M}_J \mathbf{P}_V \mathbf{M}_J^t \sigma_i) = \\ \frac{1}{2} \sum_{j=0}^3 \text{Tr}(\mathbf{M}_J \sigma_j \mathbf{M}_J^t \sigma_i) P_j$$

Mueller matrix

$$\vec{S}' = \mathbf{M}_M \vec{S}$$

$$(M_M)_{ij} = \frac{1}{2} \text{Tr}(\mathbf{M}_J \sigma_j \mathbf{M}_J^t \sigma_i)$$

# Mueller matrices and partially polarized light

Time average of the previous study

Mueller matrices are time independent

$$\langle \vec{S}' \rangle = \mathbf{M}_M \langle \vec{S} \rangle$$

Mueller calculus can be extended to . . .

- Partially coherent light
- Cascaded optical devices

Final homework

Find the Mueller matrix of each :

- Polarizers along eigen axis or  $\theta$  tilted
- half and quarter wave plates
- linearly and circularly birefringent crystal