

Polarization Optics

Nicolas Fressengeas

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UE SPM-PHY-S07-101 Polarization Optics

N. Fressengeas

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Further reading [Hua94, K85]



S. Huard.

Polarisation de la lumière. Masson, 1994.



G. P. Können.

Polarized light in Nature. Cambridge University Press, 1985.



Course Outline

- The physics of polarization optics
 - Polarization states
 - Jones Calculus
 - Stokes parameters and the Poincare Sphere
- 2 Polarized light propagation
 - Jones Matrices
 - Polarizers
 - Linear and Circular Anisotropy
 - Jones Matrices Composition
- 3 Partially polarized light
 - Formalisms used
 - Propagation through optical devices



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Polarization states Jones Calculus Stokes parameters and the Poincare Sphere

The physics of polarization optics Polarization states

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Polarization states Jones Calculus Stokes parameters and the Poincare Sphere

The vector nature of light Optical wave can be polarized, sound waves cannot

The scalar monochromatic plane wave

The electric field reads:

 $A\cos(\omega t - kz - \varphi)$

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- Electric field is orthogonal to wave and Poynting vectors
- Lies in the wave vector normal plane
- Needs 2 components
 - $\bullet: E_{\rm x} = A_{\rm x} \cos\left(\omega t kz \varphi_{\rm x}\right)$
 - $E_y = A_y \cos(\omega t kz \varphi_y)$



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Polarization states Jones Calculus Stokes parameters and the Poincare Sphere

Linear and circular polarization states





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Polarization states Jones Calculus Stokes parameters and the Poincare Sphere

Linear and circular polarization states





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Polarization states Jones Calculus Stokes parameters and the Poincare Sphere

Linear and circular polarization states





Polarization states Jones Calculus Stokes parameters and the Poincare Sphere

Linear and circular polarization states



Polarization states Jones Calculus Stokes parameters and the Poincare Sphere

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Linear and circular polarization states



Polarization states Jones Calculus Stokes parameters and the Poincare Sphere

The elliptic polarization state

The polarization state of ANY monochromatic wave



Polarization states Jones Calculus Stokes parameters and the Poincare Sphere

The elliptic polarization state The polarization state of ANY monochromatic wave



Electric field

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Polarization states Jones Calculus Stokes parameters and the Poincare Sphere

The elliptic polarization state The polarization state of ANY monochromatic wave



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Polarization states Jones Calculus Stokes parameters and the Poincare Sphere

The elliptic polarization state The polarization state of ANY monochromatic wave



Electric field

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4 real numbers

• A_x, φ_x

•
$$A_y, \varphi_y$$



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Polarization states Jones Calculus Stokes parameters and the Poincare Sphere

The elliptic polarization state

The polarization state of ANY monochromatic wave



Polarization states Jones Calculus Stokes parameters and the Poincare Sphere

The physics of polarization optics

Polarization states

Jones Calculus

- Stokes parameters and the Poincare Sphere
- 2 Polarized light propagation
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 - Linear and Circular Anisotropy
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Polarization states Jones Calculus Stokes parameters and the Poincare Sphere

Polarization states are vectors

Monochromatic polarizations belong to a 2D vector space based on the Complex Ring

ANY elliptic polarization state

Two complex numbers

A set of two ordered complex numbers is one 2D complex vector

Canonical Basis

 $\left(\begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix} \right)$

Link with optics ?

- These two vectors represent two polarization states
- We must decide which ones !

Polarization Basis

Two independent polarizations :

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- Crossed Linear
- Reversed circular
- . . .
- YOUR choice



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Examples : Linear Polarizations

Canonical Basis Choice





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Polarization states Jones Calculus Stokes parameters and the Poincare Sphere

Examples : Linear Polarizations

Canonical Basis Choice







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Examples : Linear Polarizations

Canonical Basis Choice







Polarization states Jones Calculus Stokes parameters and the Poincare Sphere

Examples : Linear Polarizations

Canonical Basis Choice

- $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$: horizontal linear polarization
- $\begin{bmatrix} 0\\1 \end{bmatrix}$: vertical linear polarization





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Polarization states Jones Calculus Stokes parameters and the Poincare Sphere

Examples : Linear Polarizations



Polarization states Jones Calculus Stokes parameters and the Poincare Sphere

Examples : Circular Polarizations

In the same canonical basis choice : linear polarizations



Electric field

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$$E_x = A_x \cos(\omega t - kz - \varphi_x)$$

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Jones vector



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Polarization states Jones Calculus Stokes parameters and the Poincare Sphere

Examples : Circular Polarizations

In the same canonical basis choice : linear polarizations



Electric field

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Jones vector

$$\frac{1}{\sqrt{2}}\begin{bmatrix}1\\\pm i\end{bmatrix}$$

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About changing basis A polarization state Jones vector is basis dependent

Some elementary algebra

- The polarization vector space dimension is 2
- Therefore : two non colinear vectors form a basis
- Any polarization state can be expressed as the sum of two non colinear other states
- Remark : two colinear polarization states are identical

Homework

Find the transformation matrix between between the two following bases :

- Horizontal and Vertical Linear Polarizations
- Right and Left Circular Polarizations



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Polarization states Jones Calculus Stokes parameters and the Poincare Sphere

Relationship between Jones and Poynting vectors

Jones vectors also provide information about intensity

Choose an orthonormal basis

- Hermitian product is null : $\overline{J_1} \cdot J_2 = 0$
- Each vector norm is unity : $\overline{J_1} \cdot J_1 = \overline{J_2} \cdot J_2 = 1$

Hermitian Norm is Intensity

Simple calculations show that :

- If each Jones component is one complex electric field component
- The Hermitian norm is proportional to beam intensity



 (J_1, J_2)

Polarization states Jones Calculus Stokes parameters and the Poincare Sphere

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Simple calculations show that :

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 (J_1, J_2)







If the intensity information disappears, polarization is summed up in one complex number

Rule out the intensity Norm the Jones vector to unity Put 1 as first component • Multiplying Jones vector by a complex number does not

- change the polarization state
- Norm the first component to 1 : $\begin{bmatrix} 1 \\ \xi \end{bmatrix}$
- The sole ξ describes the polarization state

Choose between the two

Either you norm the vector, or its first component. Not both !





Polarization states Jones Calculus Stokes parameters and the Poincare Sphere

The physics of polarization optics

- Polarization states
- Jones Calculus

• Stokes parameters and the Poincare Sphere

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The Stokes parameters

A set of 4 dependent real parameters that can be measured





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The Stokes parameters





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Polarization states Jones Calculus Stokes parameters and the Poincare Sphere

The Poincare Sphere Polarization states can be described geometrically on a sphere

Recall the Stokes parameters

 $P_0^2 = P_1^2 + P_2^2 + P_3^2$

Normalized Stokes parameters

 $S_i = P_i/P_0$

Unit Radius Sphere

$$\sum_{i=1}^{3} S_i = 1$$



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The Poincare Sphere

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Jones Matrices

Polarizers Linear and Circular Anisotropy Jones Matrices Composition

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Jones Matrices Polarizers Linear and Circular Anisotropy Jones Matrices Composition

Eigen Polarization states

Polarization states that do not change after propagation in an anisotropic medium

Eigen Polarization states

- Do not change
- Except for Intensity

Linear Anisotropy Eigen Polarizations

- Quarter and half wave plates and Birefringent materials
- Eigen Polarizations are linear along the eigen axes

- Also called optical activity
- e e.g. in Faraday rotators and in gyratory non-linear crystals
- Linear polarization is notated by an angle proportional in propagation distance



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Hermitian operator

2 eigen polarization states are orthonormal

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Jones Matrices 2D Linear Algebra to compute polarization propagation through devices

Jones matrices in the eigen basis

- Let λ_1 and λ_2 be the two eigenvalues of a given device
- e.g. for linear anisotropy : $\lambda_i = e^{n_i k_0 \Delta z}$

• Jones Matrix is



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Homework

Find half and quarter wave plates Jones Matrices



Jones Matrices

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In another basis

• Let
$$\overrightarrow{J_1} = \begin{bmatrix} u \\ v \end{bmatrix}$$
 and $\overrightarrow{J_2} = \begin{bmatrix} -\overline{v} \\ \overline{u} \end{bmatrix}$ be the orthonormal eigen
vectors
• $\begin{cases} \mathbf{M} \overrightarrow{J_1} = \lambda_1 \overrightarrow{J_1} \\ \mathbf{M} \overrightarrow{J_2} = \lambda_2 \overrightarrow{J_2} \end{cases} \Rightarrow \mathbf{M} = \begin{bmatrix} \lambda_1 u \overline{u} + \lambda_2 v \overline{v} & (\lambda_1 - \lambda_2) u \overline{v} \\ (\lambda_1 - \lambda_2) v \overline{u} & \lambda_2 v \overline{u} + \lambda_1 v \overline{v} \end{bmatrix}$

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In another basis

use Transformation Matrix

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Jones Matrices Polarizers Linear and Circular Anisotropy Jones Matrices Composition

The particular case of non absorbing devices Jones matrix is a unitary operator when $\lambda_1 = \lambda_2 = 1$

Nor absorbing neither amplifying devices

- $|\lambda_1| = |\lambda_2| = 1$
- $\mathbf{M} \cdot \overline{\mathbf{M}^{t}} = \overline{\mathbf{M}^{t}} \cdot \mathbf{M} = I$
- M is a unitary operator

Unitary operator properties

- Norm is conserved : Intensity is unchanged after propagation
- Orthogonality is conserved : two initially orthogonal states will remain so after propagation



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Jones Matrices Polarizers Linear and Circular Anisotropy Jones Matrices Composition

Jones Matrix of a polarizer

In its eigen basis

- A polarized is designed for :
 - Full transmission of one linear polarization
 - Zero transmission of its orthogonal counterpart

• Eigen basis Jones matrix : P

When transmitted polarization is heta tilted

Change base through θ rotation Transformation Matrix

$$\mathbf{R}(\theta) = \begin{bmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{bmatrix}$$



Jones Matrices Polarizers Linear and Circular Anisotropy Jones Matrices Composition

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$$\mathbf{P}_{\mathbf{x}} = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$$
 or $\mathbf{P}_{\mathbf{y}} = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$

When transmitted polarization is heta tilted

Change base through θ rotation Transformation Matrix

$$\mathbf{R}(\theta) = \begin{bmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{bmatrix}$$

Jones Matrices Polarizers Linear and Circular Anisotropy Jones Matrices Composition

Jones Matrix of a polarizer

In its eigen basis

- A polarized is designed for :
 - Full transmission of one linear polarization
 - Zero transmission of its orthogonal counterpart

• Eigen basis Jones matrix :
$$\mathbf{P}_{\mathbf{x}} = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$$
 or $\mathbf{P}_{\mathbf{y}} = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$

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$$\mathbf{P}\left(heta
ight)=\mathbf{R}\left(heta
ight)egin{bmatrix}1&0\0&0\end{bmatrix}\mathbf{R}\left(- heta
ight)$$

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Jones Matrices Polarizers Linear and Circular Anisotropy Jones Matrices Composition

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Homework

Find again the $\boldsymbol{\theta}$ tilted polarizer Jones Matrix by using physics arguments only

Jones Matrices Polarizers Linear and Circular Anisotropy Jones Matrices Composition

Intensity transmitted through a polarizer

From natural or non polarized light

Half the intensity is transmitted

From linearly polarized light

• Transmitted Jones vector in polarizer eigen basis:

$$\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \cos(\theta) \\ \sin(\theta) \end{bmatrix} = \begin{bmatrix} \cos(\theta) \\ 0 \end{bmatrix}$$

• Transmitted Intensity : $\cos^2(\theta)$

MALUS law

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From circularly polarized light

Show that whatever the polarizer orientation, the transmitted intensity is half the incident intensity.



Jones Matrices Polarizers Linear and Circular Anisotropy Jones Matrices Composition

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Jones Matrices Polarizers **Linear and Circular Anisotropy** Jones Matrices Composition

The physics of polarization optics

- Polarization states
- Jones Calculus
- Stokes parameters and the Poincare Sphere

2 Polarized light propagation

- Jones Matrices
- Polarizers

• Linear and Circular Anisotropy

Jones Matrices Composition

3 Partially polarized light

- Formalisms used
- Propagation through optical devices



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Linear anisotropy eigen polarization vectors

- Two orthogonal polarization directions
- Two different refraction indexes n_1 and n_2
- Two linear eigen modes along the eigen directions

Iones Matrix in the eigen basis Express phase delay only $\begin{bmatrix} e^{in_1k\Delta z} & 0\\ 0 & e^{in_2k\Delta z} \end{bmatrix} = e^{i\psi} \begin{bmatrix} e^{i\phi/2} & 0\\ 0 & e^{-i\phi/2} \end{bmatrix} \approx \begin{bmatrix} e^{i\phi/2} & 0\\ 0 & e^{-i\phi/2} \end{bmatrix}$

Quarter and Half wave plates

- Find the Jones Matrices of Quarter and Half wave plates
- Find their action on tilted linear polarization (special case for $\pi/4$ tilt)
- Find their action on circular polarization





Linear anisotropy eigen polarization vectors Two orthogonal polarization directions • Two different refraction indexes n_1 and n_2 Two linear eigen modes along the eigen directions Jones Matrix in the eigen basis Express phase delay only $\begin{bmatrix} e^{in_1k\Delta z} & 0\\ 0 & e^{in_2k\Delta z} \end{bmatrix} = e^{i\psi} \begin{bmatrix} e^{i\phi/2} & 0\\ 0 & e^{-i\phi/2} \end{bmatrix} \approx \begin{bmatrix} e^{i\phi/2} & 0\\ 0 & e^{-i\phi/2} \end{bmatrix}$ Homework Quarter and Half wave plates • Find the Jones Matrices of Quarter and Half wave plates Find their action on tilted linear polarization (special case for $\pi/4$ tilt) Find their action on circular polarization < ロ > < 同 > < 回 > < 回 > N. Fressengeas UE SPM-PHY-S07-109, version 1.1, frame 20

What is circular anisotropy ?

- Two orthogonal circular eigen polarization states
- Two different refraction indexes n_L and n_R

Jones Matrix in the circular eigen basis — Express phase delay only

$$egin{bmatrix} e^{in_Lk\Delta z} & 0 \ 0 & e^{in_Rk\Delta z} \end{bmatrix} = e^{i\psi} egin{bmatrix} e^{i\phi/2} & 0 \ 0 & e^{-i\phi/2} \end{bmatrix} pprox egin{bmatrix} e^{i\phi/2} & 0 \ 0 & e^{-i\phi/2} \end{bmatrix}$$

Jones Matrix in a linear polarization basis Transformation matrix

• use
$$\mathbf{P}_{\text{Cir}\rightarrow\text{Lin}} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ i & -i \end{bmatrix}$$
 transformation matrix

•
$$\mathbf{P}_{\mathrm{Cir}\to\mathrm{Lin}}M(\mathbf{P}_{\mathrm{Cir}\to\mathrm{Lin}})^{-1} = e^{i\Psi} \begin{bmatrix} \cos(\phi/2) & \sin(\phi/2) \\ -\sin(\phi/2) & \cos(\phi/2) \end{bmatrix}$$



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Homework

Show that an incoming linear polarisation is simply rotated by an angle proportional to the propagation distance

Jones Matrices Polarizers Linear and Circular Anisotropy Jones Matrices Composition

The physics of polarization optics

- Polarization states
- Jones Calculus
- Stokes parameters and the Poincare Sphere

2 Polarized light propagation

- Jones Matrices
- Polarizers
- Linear and Circular Anisotropy
- Jones Matrices Composition
- 3 Partially polarized light
 - Formalisms used
 - Propagation through optical devices



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Jones Matrices Polarizers Linear and Circular Anisotropy Jones Matrices Composition

Jones Matrices Composition

The Jones matrices of cascaded optical elements can be composed through Matrix multiplication

Matrix composition

- If a $\overrightarrow{J_0}$ incident light passes through M_1 and M_2 in that order
- First transmission: $M_1 \overrightarrow{J_0}$
- Second transmission: $M_2M_1\overline{J_0}$
- Composed Jones Matrix : M₂M₁

Beware of non commutativity

- Matrix product does not commute in general
- Think of the case of a linear anisotropy followed by optical activity
 - in that order
 - in the reverse order



Reversed order

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Formalisms used Propagation through optical devices

Stokes parameters for partially polarized light Generalize the coherent definition using the statistical average intensity



Stokes parameters for partially polarized light Generalize the coherent definition using the statistical average intensity



Stokes parameters for partially polarized light Generalize the coherent definition using the statistical average intensity



Formalisms used Propagation through optical devices

The Jones Coherence Matrix

Jones Vectors are out

- They describe phase differences
- Meaningless when not monochromatic

Jones Coherence Matrix

• If
$$\overrightarrow{J} = \begin{bmatrix} A_x(t) e^{i\varphi_x(t)} \\ A_y(t) e^{i\varphi_y(t)} \end{bmatrix}$$

• $\Gamma_{ij} = \langle \overrightarrow{J}_i(t) \overrightarrow{J}_j(t) \rangle$

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$$\bar{\Gamma} = \begin{bmatrix} \Gamma_{xx} & \Gamma_{xy} \\ \Gamma_{yx} & \Gamma_{yy} \end{bmatrix}$$



Formalisms used Propagation through optical devices

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Formalisms used Propagation through optical devices

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Formalisms used Propagation through optical devices

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Coherence Matrix: explicit formulation

$${f \Gamma} = egin{bmatrix} \langle |A_x\left(t
ight)|^2
angle & \langle A_x\left(t
ight)\overline{A_y\left(t
ight)}e^{i\left(arphi_x-arphi_y
ight)} \ \langle |A_y\left(t
ight)|^2
angle & \langle |A_y\left(t
ight)|^2
angle \end{pmatrix} \end{cases}$$



Formalisms used Propagation through optical devices

Jones Coherence Matrix: properties



Formalisms used Propagation through optical devices

Jones Coherence Matrix: properties


Formalisms used Propagation through optical devices

Jones Coherence Matrix: properties



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Formalisms used Propagation through optical devices

Jones Coherence Matrix: properties



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Formalisms used Propagation through optical devices

Coherence Matrix: further properties

Polarization degree

$$\rho = \sqrt{\frac{P_1^2 + P_2^2 + P_3^2}{P_0^2}} = \sqrt{1 - \frac{4(\Gamma_{xx}\Gamma_{yy} - \Gamma_{xy}\Gamma_{yx})}{(\Gamma_{xx} + \Gamma_{yy})^2}} = \sqrt{1 - \frac{4\mathrm{Det}(\Gamma)}{\mathrm{Tr}(\Gamma)^2}}$$

Decomposition in polarized and depolarized components

•
$$\Gamma = \Gamma_P + \Gamma_{NP}$$

 Find Γ_P and Γ_{NP} using the relationship with the Stokes parameters



Formalisms used Propagation through optical devices

Coherence Matrix: further properties

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3 Partially polarized light

- Formalisms used
- Propagation through optical devices



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Formalisms used Propagation through optical devices

Propagation of the Coherence Matrix

Jones Calculus

- If incoming polarization is $\overline{J(t)}$
- Output one is $\overrightarrow{J'(t)} = \mathbf{M}\overrightarrow{J(t)}$

Coherence Matrix if **M** is unitary

- M unitary means : linear and/or circular anisotropy only
- $\Gamma' = \langle \overrightarrow{J'(t)} \overrightarrow{J'(t)}^{\mathrm{t}} \rangle$
- $M^{(i)}_{i} = M(\overline{J(i)U(i)}) = M = M = M = M = M$

Basis change

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Formalisms used Propagation through optical devices

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Formalisms used Propagation through optical devices

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$$\Gamma' = \mathbf{M} \langle \overline{J(t)} \overline{J(t)}^{\mathrm{t}} \rangle \mathbf{M}^{-1}$$

Basis change

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Polarization degree

- Unaltered for unitary operators
 Tr and Det are unaltered
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Formalisms used Propagation through optical devices

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Formalisms used Propagation through optical devices

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Formalisms used Propagation through optical devices

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Propagating the Jones coherence matrix is difficult if the operator is not unitary

Jones Calculus raises some difficulties

- Coherence matrix OK for partially polarized light
- Propagation through unitary optical devices

(linear or circular anisotropy only)

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• Hard Times if Polarizers are present

- Describing intensity, they can be readily measurered
- We will show they can be propagated using 4 × 4 real matrices
- They are the Mueller matrices



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Formalisms used Propagation through optical devices

The projection on a polarization state Matrix of the polariazer with axis parallel to \vec{V}



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Formalisms used Propagation through optical devices

The projection on a polarization state Matrix of the polariazer with axis parallel to \vec{V}



Formalisms used Propagation through optical devices

The Pauli Matrices

A base for the 4D 2 \times 2 matrix vector space

$$\sigma_1 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \sigma_2 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}, \sigma_3 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \sigma_1 = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}$$

 ${\bf P}_{{\bf V}}$ decomposition

$$\mathbf{P}_{\mathbf{V}} = \frac{1}{2} \left(p_0 \sigma_0 + p_1 \sigma_1 + p_2 \sigma_2 + p_3 \sigma_3 \right)$$



Formalisms used Propagation through optical devices

Pv composition and Trace property Trace is the eigen values sum

Projection property

$$\overline{\overrightarrow{V}^{t}} \cdot \sigma_{j} \overrightarrow{V} = \left(\overline{\overrightarrow{V}^{t}} \overrightarrow{V}\right) \overline{\overrightarrow{V}^{t}} \cdot \sigma_{j} \overrightarrow{V}$$

Projection Trace in its eigen basis

- **P**_V eigenvalues : 0 & 1
- $\sim 10^{\circ} eigenvalues \sim 0.2 co$ $\sim 10^{\circ} c$
- Pyg eigenvectors are the same as Pyc.
 - V associated to eigenvalue or
- Project the projection
 - $\nabla^{k} \cdot \mathsf{Pyp} \nabla = \alpha$

Formalisms used Propagation through optical devices

Pv composition and Trace property Trace is the eigen values sum

Projection property

$$\overline{\overrightarrow{V}^{t}} \cdot \sigma_{\mathbf{j}} \overline{\overrightarrow{V}} = \left(\overline{\overrightarrow{V}^{t}} \overline{\overrightarrow{V}}\right) \overline{\overrightarrow{V}^{t}} \cdot \sigma_{\mathbf{j}} \overline{\overrightarrow{V}} = \overline{\overrightarrow{V}^{t}} \left(\overline{\overrightarrow{V}} \overline{\overrightarrow{V}^{t}}\right) \sigma_{j} \overline{\overrightarrow{V}}$$

Projection Trace in its eigen basis

- **P**_V eigenvalues : 0 & 1
- $P_V \sigma_1$ eigenvalues : $0 \ \& \ \alpha = --- \ \alpha \leq 1$
- Pygreigenvectors are the same as Pyc.
 - \circ V associated to eigenvalue α
- Project the projection
- \overline{V}^{1} . Pyg $\overline{V} = \alpha$

Formalisms used Propagation through optical devices

Pv composition and Trace property Trace is the eigen values sum

Projection property

$$\overline{\overrightarrow{V}^{t}} \cdot \sigma_{j} \overrightarrow{V} = \left(\overline{\overrightarrow{V}^{t}} \overrightarrow{V}\right) \overline{\overrightarrow{V}^{t}} \cdot \sigma_{j} \overrightarrow{V} = \overline{\overrightarrow{V}^{t}} \cdot \mathbf{P}_{V} \sigma_{j} \overrightarrow{V}$$

Projection Trace in its eigen basis

- P_V eigenvalues : 0 & 1
 - $\mathsf{P}_{\mathsf{V}}\sigma_{\mathsf{i}}$ eigenvalues : 0 & α $\alpha \leq 1$ $\operatorname{Tr}(P_{\mathsf{V}}\sigma_{\mathsf{i}}) = c$
- P_V or eigenvectors are the same as P_V:
 - \circ \overline{V} associated to eigenvalue lpha
- Project the projection
 - $\nabla^{1} \cdot \operatorname{Pvg} \nabla = \alpha$

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Formalisms used Propagation through optical devices

 $\operatorname{Tr}(P_V) = 1$

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Pv composition and Trace property Trace is the eigen values sum

Projection property

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Formalisms used Propagation through optical devices

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 - $ec{V}$ associated to eigenvalue lpha
- Project the projection

$$\overrightarrow{V}^{\mathrm{t}} \cdot \mathbf{P}_{\mathbf{V}} \sigma_{\mathbf{j}} \overrightarrow{V} = \alpha$$

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Formalisms used Propagation through optical devices

 $\operatorname{Tr}(P_V) = 1$ $\operatorname{Tr}(P_V\sigma_i) = \alpha$

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Formalisms used Propagation through optical devices

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Formalisms used Propagation through optical devices

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$$\overrightarrow{V}^{t} \cdot \mathbf{P}_{\mathbf{V}} \sigma_{\mathbf{j}} \overrightarrow{V} = \alpha = \operatorname{Tr} \left(\mathbf{P}_{\mathbf{V}} \sigma_{\mathbf{j}} \right)$$

Formalisms used Propagation through optical devices

 $\operatorname{Tr}(P_V) = 1$ $\operatorname{Tr}(P_V\sigma_i) = \alpha$

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Pv composition and Trace property Trace is the eigen values sum

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 - \overrightarrow{V} associated to eigenvalue α
- Project the projection

$$\overline{\overrightarrow{V}^{t}} \cdot \mathbf{P_{V}} \sigma_{\mathbf{j}} \overline{\overrightarrow{V}} = \alpha = \operatorname{Tr}\left(\mathbf{P_{V}} \sigma_{\mathbf{j}}\right) = \overline{\overrightarrow{V}^{t}} \cdot \sigma_{\mathbf{j}} \overline{\overrightarrow{V}}$$

Formalisms used Propagation through optical devices



Formalisms used Propagation through optical devices



$$\overrightarrow{V}^{t} \cdot \sigma_{j} \overrightarrow{V} = \operatorname{Tr} (P_{V} \sigma_{j}) \qquad \operatorname{Tr} (\sigma_{i} \sigma_{j}) = 2\delta_{ij}$$

$$\overrightarrow{V}^{t} \cdot \sigma_{j} \overrightarrow{V} = \operatorname{Tr} (P_{V} \sigma_{j}) = \frac{1}{2} \sum_{i} \operatorname{Tr} (\sigma_{i} \sigma_{j}) p_{i} = \frac{1}{2} \sum_{i} 2\delta_{ij} p_{i} = p_{j}$$
Project the base vectors on \overrightarrow{V}

$$\bullet \text{ Using } \overrightarrow{V} = A_{x} e^{-i\frac{\varphi}{2}} \overrightarrow{X} + A_{y} e^{i\frac{\varphi}{2}} \overrightarrow{Y}$$

$$\bullet P_{V} \overrightarrow{X} = A_{x}^{2} \overrightarrow{X} + A_{x} A_{y} e^{i\varphi} \overrightarrow{Y}$$

$$\bullet P_{V} \overrightarrow{Y} = A_{y}^{2} \overrightarrow{Y} + A_{x} A_{y} e^{-i\varphi} \overrightarrow{X}$$

$$\bullet \text{ Using the P_{V} decomposition on the Pauli Basis}$$

$$\bullet P_{V} \overrightarrow{X} = \frac{(\rho_{v} + \rho_{v})}{(\rho_{v} - \rho_{v})} \overrightarrow{Y} + \frac{(\rho_{v} - i\rho_{v})}{(\rho_{v} - i\rho_{v})} \overrightarrow{X}$$

$$\overrightarrow{\overrightarrow{V}}^{t} \cdot \sigma_{j} \overrightarrow{V} = \operatorname{Tr} (P_{V} \sigma_{j}) \qquad \operatorname{Tr} (\sigma_{i} \sigma_{j}) = 2\delta_{ij}$$

$$\overrightarrow{\overrightarrow{V}}^{t} \cdot \sigma_{j} \overrightarrow{V} = \operatorname{Tr} (P_{V} \sigma_{j}) = \frac{1}{2} \sum_{i} \operatorname{Tr} (\sigma_{i} \sigma_{j}) p_{i} = \frac{1}{2} \sum_{i} 2\delta_{ij} p_{i} = p_{j}$$
Project the base vectors on \overrightarrow{V}

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$$\bullet \text{ Using the } P_{V} \text{ decomposition on the Pauli Basis}$$

$$\bullet P_{V} \overrightarrow{X} = \frac{1}{2} (p_{0} + p_{1}) \overrightarrow{X} + \frac{1}{2} (p_{2} + ip_{3}) \overrightarrow{Y}$$

$$\bullet P_{V} \overrightarrow{Y} = \frac{1}{2} (p_{0} - p_{1}) \overrightarrow{Y} + \frac{1}{2} (p_{2} - ip_{3}) \overrightarrow{X}$$

$$\bullet \text{ Identify}$$

$$\overrightarrow{V}^{t} \cdot \sigma_{j} \overrightarrow{V} = \operatorname{Tr} (P_{V} \sigma_{j}) \qquad \operatorname{Tr} (\sigma_{i} \sigma_{j}) = 2\delta_{ij}$$

$$\overrightarrow{V}^{t} \cdot \sigma_{j} \overrightarrow{V} = \operatorname{Tr} (P_{V} \sigma_{j}) = \frac{1}{2} \sum_{i} \operatorname{Tr} (\sigma_{i} \sigma_{j}) p_{i} = \frac{1}{2} \sum_{i} 2\delta_{ij} p_{i} = p_{j}$$
Project the base vectors on \overrightarrow{V}

• Using $\overrightarrow{V} = A_{x} e^{-i\frac{\varphi}{2}} \overrightarrow{X} + A_{y} e^{i\frac{\varphi}{2}} \overrightarrow{Y}$

• $P_{V} \overrightarrow{X} = A_{x}^{2} \overrightarrow{X} + A_{x} A_{y} e^{i\varphi} \overrightarrow{Y}$

• $P_{V} \overrightarrow{Y} = A_{y}^{2} \overrightarrow{Y} + A_{x} A_{y} e^{-i\varphi} \overrightarrow{X}$

• Using the P_{V} decomposition on the Pauli Basis

• $P_{V} \overrightarrow{X} = \frac{1}{2} (p_{0} + p_{1}) \overrightarrow{X} + \frac{1}{2} (p_{2} + ip_{3}) \overrightarrow{Y}$

• $P_{V} \overrightarrow{Y} = \frac{1}{2} (p_{0} - p_{1}) \overrightarrow{Y} + \frac{1}{2} (p_{2} - ip_{3}) \overrightarrow{X}$

• Identify

$$\overrightarrow{V}^{t} \cdot \sigma_{j} \overrightarrow{V} = \operatorname{Tr} (P_{V} \sigma_{j}) \qquad \operatorname{Tr} (\sigma_{i} \sigma_{j}) = 2\delta_{ij}$$

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Project the base vectors on \overrightarrow{V}

• Using $\overrightarrow{V} = A_{x} e^{-i\frac{\varphi}{2}} \overrightarrow{X} + A_{y} e^{i\frac{\varphi}{2}} \overrightarrow{Y}$

• $P_{V} \overrightarrow{X} = A_{x}^{2} \overrightarrow{X} + A_{x} A_{y} e^{i\varphi} \overrightarrow{Y}$

• $P_{V} \overrightarrow{Y} = A_{y}^{2} \overrightarrow{Y} + A_{x} A_{y} e^{-i\varphi} \overrightarrow{X}$

• Using the P_{V} decomposition on the Pauli Basis

• $P_{V} \overrightarrow{X} = \frac{1}{2} (p_{0} + p_{1}) \overrightarrow{X} + \frac{1}{2} (p_{2} + ip_{3}) \overrightarrow{Y}$

• Identify

Formalisms used Propagation through optical devices

$\mathbf{P}_{\mathbf{V}}$ Pauli composition and Stokes parameters

Stokes parameters as $\mathbf{P}_{\mathbf{V}}$ decomposition on the Pauli base

•
$$p_0 = P_0 = A_x^2 - A_y^2 = I_x - I_y$$

•
$$p_1 = P_1 = A_x^2 - A_y^2 = I_x - I_y$$

•
$$p_2 = P_2 = 2A_x A_y \cos(\varphi) = I_{\pi/4} - I_{-\pi/4}$$

•
$$p_3 = P_3 = 2A_xA_y\sin(\varphi) = I_{\rm L} - I_{\rm R}$$



Formalisms used Propagation through optical devices

Propagating through devices: Mueller matrices $\vec{V'} = M_J \vec{V}$



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Formalisms used Propagation through optical devices

Propagating through devices: Mueller matrices $\vec{v'} = M_J \vec{v}$



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Formalisms used Propagation through optical devices

Propagating through devices: Mueller matrices $\vec{v'} = \mathbf{M}_{J}\vec{v}$



Formalisms used Propagation through optical devices

Propagating through devices: Mueller matrices $\vec{v'} = \mathbf{M}_{J}\vec{v}$



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Formalisms used Propagation through optical devices

Propagating through devices: Mueller matrices $\overrightarrow{v'} = M_J \overrightarrow{v}$

Projection on
$$\overrightarrow{V'}$$

 $\mathbf{P}_{\mathbf{V}'} = \overrightarrow{V'}\overrightarrow{V'}^{\mathrm{t}} = \mathbf{M}_{\mathbf{J}}\overrightarrow{V}\overrightarrow{V}^{\mathrm{t}}\mathbf{M}_{\mathbf{J}}^{\mathrm{t}} = \mathbf{M}_{\mathbf{J}}\mathbf{P}_{\mathbf{V}}\mathbf{M}_{\mathbf{J}}^{\mathrm{t}}$
Trace relationship
 $P'_{i} = \mathrm{Tr}\left(\mathbf{P}_{\mathbf{V}'}\sigma_{i}\right) = \mathrm{Tr}\left(\mathbf{M}_{\mathbf{J}}\mathbf{P}_{\mathbf{V}}\mathbf{M}_{\mathbf{J}}^{\mathrm{t}}\sigma_{i}\right)$
Mueller matrix
 $S' = \mathbf{M}_{\mathbf{M}}S$
 $\left(M_{\mathcal{M}}\right)_{ij} = \frac{1}{2}\mathrm{Tr}\left(\mathbf{M}_{\mathbf{J}}\sigma_{j}\mathbf{M}_{\mathbf{J}}^{\mathrm{t}}\sigma_{i}\right)$

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Formalisms used Propagation through optical devices

Propagating through devices: Mueller matrices $\vec{V'} = M_J \vec{V}$

Projection on $\overrightarrow{V'}$

$$\mathbf{P}_{\mathbf{V}'} = \overrightarrow{V'} \overrightarrow{V'}^{\mathrm{t}} = \mathbf{M}_{\mathbf{J}} \overrightarrow{V} \overrightarrow{V'}^{\mathrm{t}} \overline{\mathbf{M}_{\mathbf{J}}}^{\mathrm{t}} = \mathbf{M}_{\mathbf{J}} \mathbf{P}_{\mathbf{V}} \overline{\mathbf{M}_{\mathbf{J}}}^{\mathrm{t}}$$

Trace relationship

$$\begin{aligned} \mathcal{P}'_{i} &= \operatorname{Tr}\left(\mathbf{P}_{\mathbf{V}'}\sigma_{i}\right) = \operatorname{Tr}\left(\mathbf{M}_{\mathbf{J}}\mathbf{P}_{\mathbf{V}}\overline{\mathbf{M}_{\mathbf{J}}}^{\mathrm{t}}\sigma_{\mathbf{i}}\right) = \\ & \frac{1}{2}\sum_{j=0}^{3}\operatorname{Tr}\left(\mathbf{M}_{\mathbf{J}}\sigma_{\mathbf{j}}\overline{\mathbf{M}_{\mathbf{J}}}^{\mathrm{t}}\sigma_{\mathbf{i}}\right)\mathcal{P}_{j} \end{aligned}$$

Mueller matrix

 $S' = M_M S$

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$$\left(M_{M}\right)_{ij} = \frac{1}{2} \operatorname{Tr}\left(\mathbf{M}_{\mathbf{J}} \sigma_{\mathbf{j}} \overline{\mathbf{M}_{\mathbf{J}}}^{\mathrm{t}} \sigma_{\mathbf{i}}\right)$$

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Formalisms used Propagation through optical devices

Propagating through devices: Mueller matrices $\vec{V'} = M_J \vec{V}$

Projection on $\overrightarrow{V'}$

$$\mathbf{P}_{\mathbf{V}'} = \overrightarrow{V'} \overrightarrow{\overrightarrow{V'}^{\mathrm{t}}} = \mathbf{M}_{\mathbf{J}} \overrightarrow{V} \overrightarrow{\overrightarrow{V}^{\mathrm{t}}} \overline{\mathbf{M}_{\mathbf{J}}^{\mathrm{t}}} = \mathbf{M}_{\mathbf{J}} \mathbf{P}_{\mathbf{V}} \overline{\mathbf{M}_{\mathbf{J}}^{\mathrm{t}}}$$

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Mueller matrix

$$(M_M)_{ij} = \frac{1}{2} \operatorname{Tr} \left(\mathsf{M}_{\mathsf{J}} \sigma_{\mathsf{j}} \overline{\mathsf{M}_{\mathsf{J}}}^{\mathrm{t}} \sigma_{\mathsf{i}} \right)$$

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 $\overrightarrow{S'} = \mathbf{M}_{\mathbf{M}} \overrightarrow{S}$

Mueller matrices and partially polarized light Time average of the previous study

Mueller matrices are time independent

$$\langle \overrightarrow{S'}
angle = \mathbf{M}_{\mathbf{M}} \langle \overrightarrow{S}
angle$$

Mueller calculus can be extended to...

- Partially coherent light
- Cascaded optical devices

Final homework

Find the Mueller matrix of each :

- Polarizers along eigen axis or θ tilted
- half and quarter wave plates
- linearly and circularly birefringent crystal



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Mueller matrices and partially polarized light Time average of the previous study

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