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Polarization Optics

Nicolas Fressengeas

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Polarization Optics

N. Fressengeas

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Unité de Recherche commune à l'Université de Lorraine et à Supélec

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Further reading

[Hua94, GB94]



A. Gerrard and J.M. Burch.

Introduction to matrix methods in optics.

Dover, 1994.



S. Huard.

Polarisation de la lumière.

Masson, 1994.



Course Outline

- 1 The physics of polarization optics
 - Polarization states
 - Jones Calculus
 - Stokes parameters and the Poincare Sphere
- 2 Polarized light propagation
 - Jones Matrices Examples
 - Matrix, basis & eigen polarizations
 - Jones Matrices Composition
- 3 Partially polarized light
 - Formalisms used
 - Propagation through optical devices



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The vector nature of light

Optical wave can be polarized, sound waves cannot

The scalar monochromatic plane wave

The electric field reads:

$$A \cos(\omega t - kz - \varphi)$$

A vector monochromatic plane wave

- Electric field is orthogonal to wave and Poynting vectors
- Lies in the wave vector normal plane



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A **vector** monochromatic plane wave

- Electric field is orthogonal to wave and Poynting vectors
- Lies in the wave vector normal plane
- Needs 2 components
 - $E_x = A_x \cos(\omega t - kz - \varphi_x)$
 - $E_y = A_y \cos(\omega t - kz - \varphi_y)$



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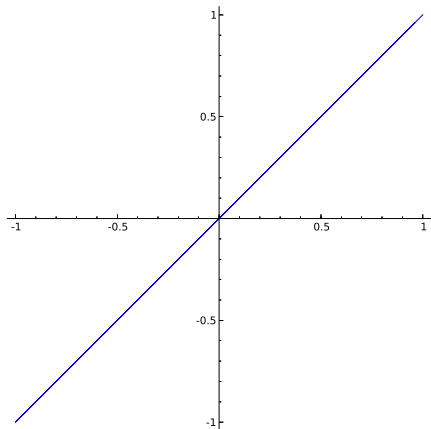
- $E_y = A_y \cos(\omega t - kz - \varphi_y)$



Linear and circular polarization states

In phase components

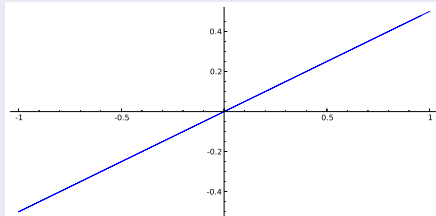
$$\varphi_y = \varphi_x$$



Linear and circular polarization states

In phase components

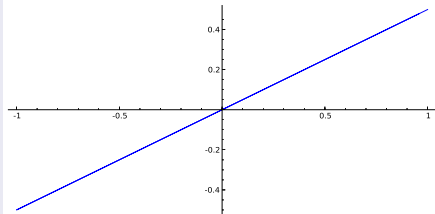
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Linear and circular polarization states

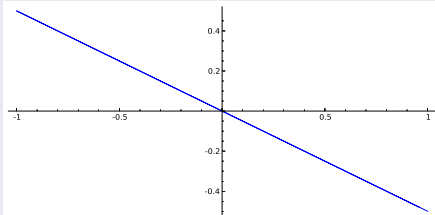
In phase components

$$\varphi_y = \varphi_x$$



π shift

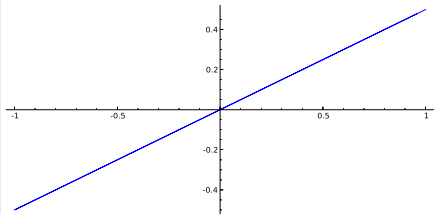
$$\varphi_y = \varphi_x + \pi$$



Linear and circular polarization states

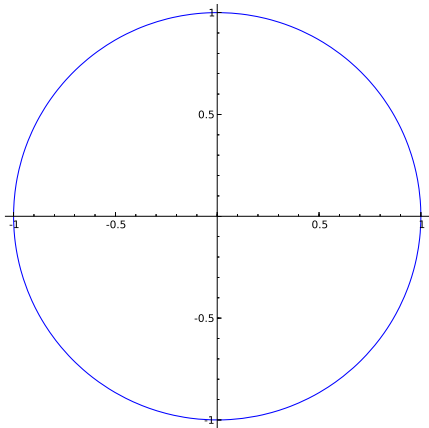
In phase components

$$\varphi_y = \varphi_x$$



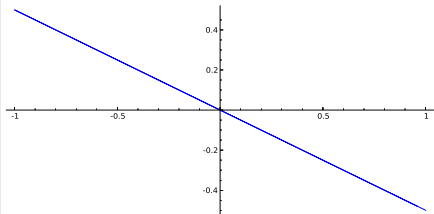
$\pi/2$ shift

$$\varphi_y = \varphi_x \pm \pi/2$$



π shift

$$\varphi_y = \varphi_x + \pi$$

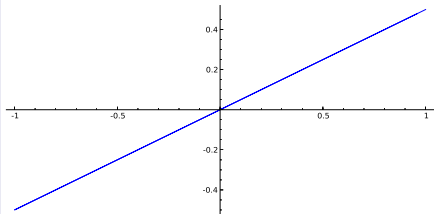


Left or Right

Linear and circular polarization states

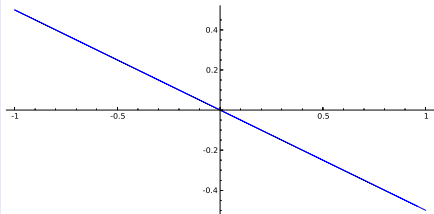
In phase components

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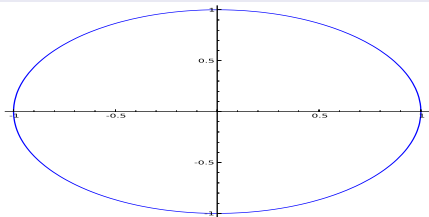
π shift

$$\varphi_y = \varphi_x + \pi$$



$\pi/2$ shift

$$\varphi_y = \varphi_x \pm \pi/2$$



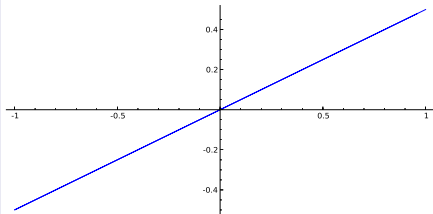
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Linear and circular polarization states

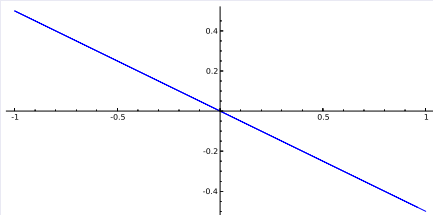
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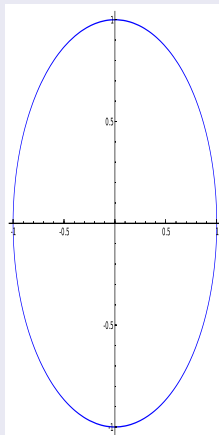
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$\pi/2$ shift

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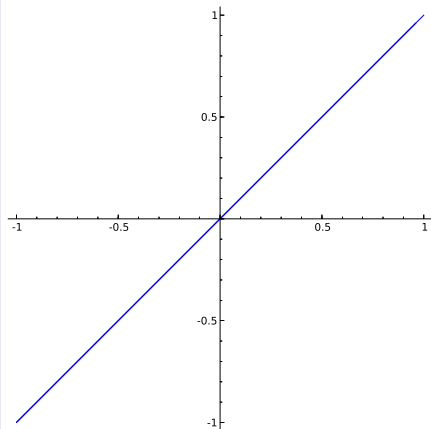


Left or Right

The elliptic polarization state

The polarization state of ANY monochromatic wave

$$\varphi_y - \varphi_x = \pm 0$$



Electric field

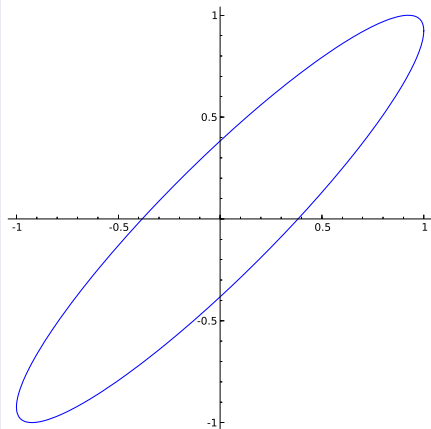
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The elliptic polarization state

The polarization state of ANY monochromatic wave

$$\varphi_y - \varphi_x = \pm\pi/8$$



Electric field

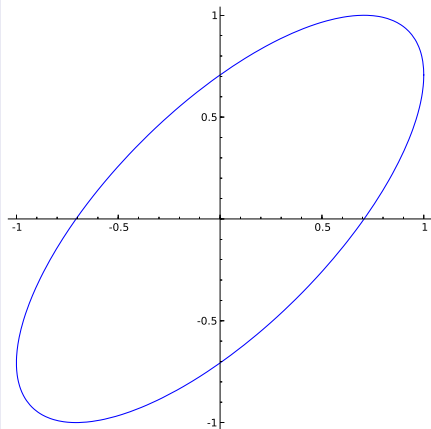
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The elliptic polarization state

The polarization state of ANY monochromatic wave

$$\varphi_y - \varphi_x = \pm\pi/4$$



Electric field

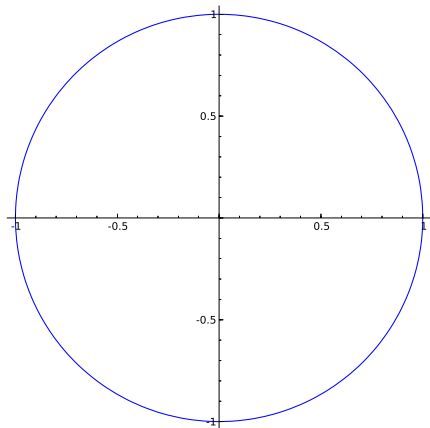
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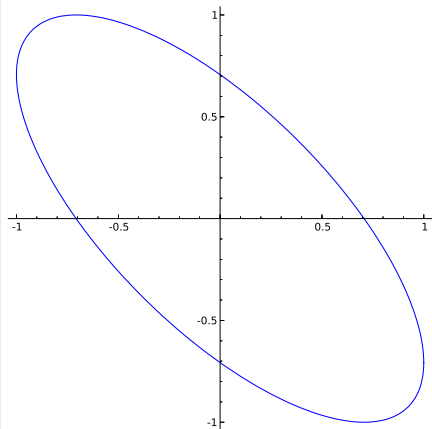
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The elliptic polarization state

The polarization state of ANY monochromatic wave

$$\varphi_y - \varphi_x = \pm 3\pi/4$$



Electric field

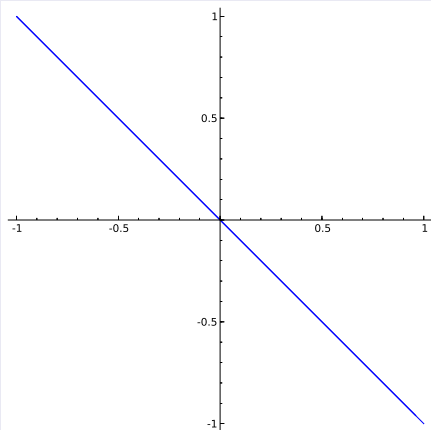
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The elliptic polarization state

The polarization state of ANY monochromatic wave

$$\varphi_y - \varphi_x = \pm\pi$$



Electric field

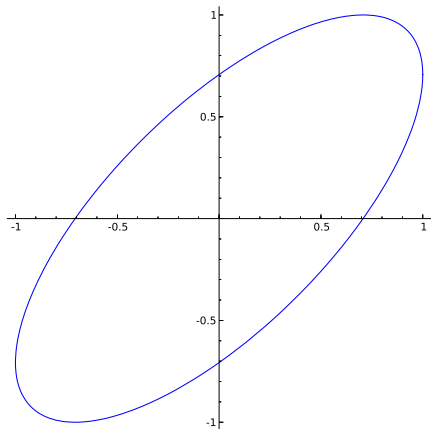
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The elliptic polarization state

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- $E_y = A_y \cos(\omega t - kz - \varphi_y)$

4 real numbers

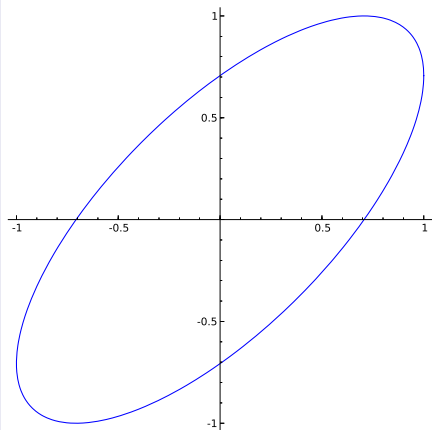
- A_x, φ_x
- A_y, φ_y



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4 real numbers

- A_x, φ_x
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2 complex numbers

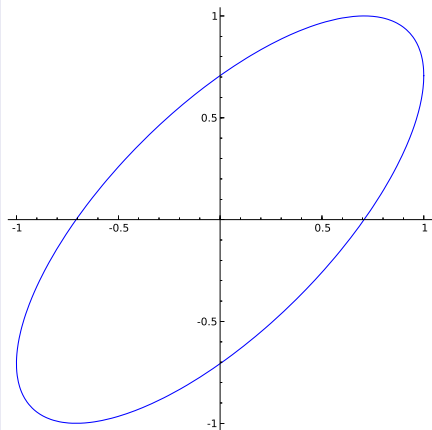
- $A_x \exp(-i\varphi_x)$
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Polarization states are vectors

Monochromatic polarizations belong to a 2D vector space based on the Complex Ring

ANY elliptic polarization state \iff Two complex numbers

A set of **two** ordered complex numbers is **one** 2D complex vector

Canonical Basis

$$\left(\begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix} \right)$$

Link with optics ?

- These two vectors represent two polarization states
- We must decide which ones !

Polarization Basis

Two independent polarizations :

- Crossed Linear
- Reversed circular
- ...
- **YOUR** choice



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Examples : Linear Polarizations

Canonical Basis Choice

- $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$: horizontal linear polarization
- $\begin{bmatrix} 0 \\ 1 \end{bmatrix}$: vertical linear polarization



Examples : Linear Polarizations

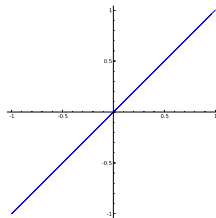
Canonical Basis Choice

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Tilt

$$\theta = \pi/4$$

$$\frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$



Examples : Linear Polarizations

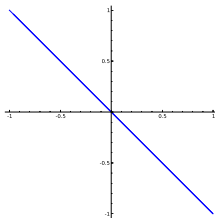
Canonical Basis Choice

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Tilt

$$\theta = 3\pi/4$$

$$\frac{1}{\sqrt{2}} \begin{bmatrix} -1 \\ 1 \end{bmatrix}$$



Examples : Linear Polarizations

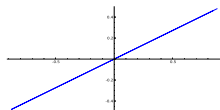
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$$\begin{bmatrix} \cos(\theta) \\ \sin(\theta) \end{bmatrix}$$



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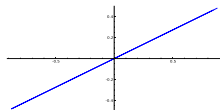
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Linear polarization Jones vector in a linear polarization basis

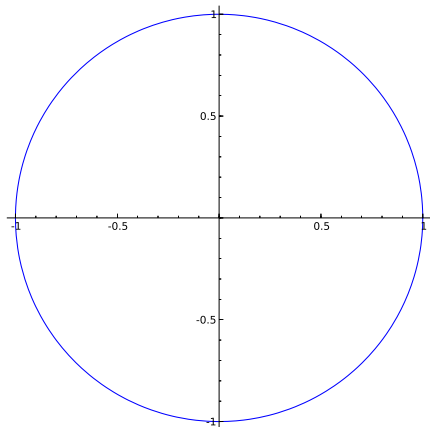
Linear Polarization : two in phase components



Examples : Circular Polarizations

In the same canonical basis choice : linear polarizations

$$\varphi_y - \varphi_x = \pm\pi/2$$



Electric field

- $E_x = A_x \cos(\omega t - kz - \varphi_x)$
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Jones vector

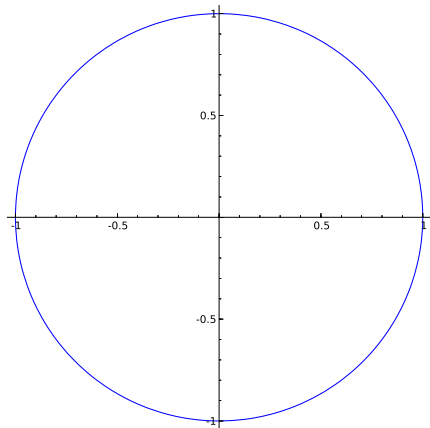
$$\frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ \pm i \end{bmatrix}$$



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About changing basis

A polarization state Jones vector is basis dependent

Some elementary algebra

- The polarization vector space dimension is 2
- Therefore : two non colinear vectors form a basis
- Any polarization state can be expressed as the sum of two non colinear other states
- Remark : two colinear polarization states are identical

Homework

Find the transformation matrix between between the two following bases :

- Horizontal and Vertical Linear Polarizations
- Right and Left Circular Polarizations



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Relationship between Jones and Poynting vectors

Jones vectors also provide information about intensity

Choose an orthonormal basis

(J_1, J_2)

- Hermitian product is null : $\overline{J_1} \cdot J_2 = 0$
- Each vector norm is unity : $\overline{J_1} \cdot J_1 = \overline{J_2} \cdot J_2 = 1$

Hermitian Norm is Intensity

Simple calculations show that :

- If each Jones component is one complex electric field component
- The Hermitian norm is proportional to beam intensity



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The Stokes parameters

A set of 4 dependent real parameters that can be measured

P_0 Overall Intensity

$$P_0 = I$$

P_1 Intensity Différence

$$P_1 = I_x - I_y$$

P_2 in a $\pi/4$ Tilted Basis

$$P_2 = I_{\pi/4} - I_{-\pi/4}$$

P_3 in a Circular Basis

$$P_3 = I_L - I_R$$



The Stokes parameters

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Relationship between Jones and Stockes

Sample Jones Vector

$$J = \begin{bmatrix} A_x \exp(+i\varphi/2) \\ A_y \exp(-i\varphi/2) \end{bmatrix}$$

P_0 Overall Intensity

$$P_0 = I = A_x^2 + A_y^2$$

P_2 in a $\pi/4$ Tilted Basis

$$J_{\pi/4} = \begin{bmatrix} \cos\left(-\frac{\pi}{4}\right) & -\sin\left(-\frac{\pi}{4}\right) \\ \sin\left(-\frac{\pi}{4}\right) & \cos\left(-\frac{\pi}{4}\right) \end{bmatrix} J$$

4 dependent parameters

$$P_0^2 = P_1^2 + P_2^2 + P_3^2$$

P_1 Intensity Difference

$$P_1 = I_x - I_y = A_x^2 - A_y^2$$

P_3 in a Circular Basis

$$J_{\text{Cir}} = \frac{1}{\sqrt{2}} \begin{bmatrix} A_x e^{+i\varphi/2} - iA_y e^{-i\varphi/2} \\ A_x e^{+i\varphi/2} + iA_y e^{-i\varphi/2} \end{bmatrix}$$

$$P_3 = \overline{J_{\text{Cir}}^x} \cdot J_{\text{Cir}}^x - \overline{J_{\text{Cir}}^y} \cdot J_{\text{Cir}}^y = 2A_x A_y \sin(\varphi)$$

Relationship between Jones and Stockes

Sample Jones Vector

$$J = \begin{bmatrix} A_x \exp(+i\varphi/2) \\ A_y \exp(-i\varphi/2) \end{bmatrix}$$

P_0 Overall Intensity

$$P_0 = I = A_x^2 + A_y^2$$

P_2 in a $\pi/4$ Tilted Basis

$$J_{\pi/4} = \begin{bmatrix} \cos\left(-\frac{\pi}{4}\right) & -\sin\left(-\frac{\pi}{4}\right) \\ \sin\left(-\frac{\pi}{4}\right) & \cos\left(-\frac{\pi}{4}\right) \end{bmatrix} J$$

4 dependent parameters

$$P_0^2 = P_1^2 + P_2^2 + P_3^2$$

P_1 Intensity Difference

$$P_1 = I_x - I_y = A_x^2 - A_y^2$$

P_3 in a Circular Basis

$$J_{\text{Cir}} = \frac{1}{\sqrt{2}} \begin{bmatrix} A_x e^{+i\varphi/2} - iA_y e^{-i\varphi/2} \\ A_x e^{+i\varphi/2} + iA_y e^{-i\varphi/2} \end{bmatrix}$$

$$P_3 = \overline{J_{\text{Cir}}^x} \cdot J_{\text{Cir}}^x - \overline{J_{\text{Cir}}^y} \cdot J_{\text{Cir}}^y = 2A_x A_y \sin(\varphi)$$

Relationship between Jones and Stockes

Sample Jones Vector

$$J = \begin{bmatrix} A_x \exp(+i\varphi/2) \\ A_y \exp(-i\varphi/2) \end{bmatrix}$$

P_0 Overall Intensity

$$P_0 = I = A_x^2 + A_y^2$$

P_2 in a $\pi/4$ Tilted Basis

$$J_{\pi/4} = \begin{bmatrix} \cos\left(-\frac{\pi}{4}\right) & -\sin\left(-\frac{\pi}{4}\right) \\ \sin\left(-\frac{\pi}{4}\right) & \cos\left(-\frac{\pi}{4}\right) \end{bmatrix} J$$

4 dependent parameters

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$$J_{\pi/4} = \frac{\sqrt{2}}{2} \begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix} J$$

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The Poincare Sphere

Polarization states can be described geometrically on a sphere

Recall the Stokes parameters

$$P_0^2 = P_1^2 + P_2^2 + P_3^2$$

Normalized Stokes parameters

$$S_i = P_i / P_0$$

Unit Radius Sphere

$$\sum_{i=1}^3 S_i^2 = 1$$



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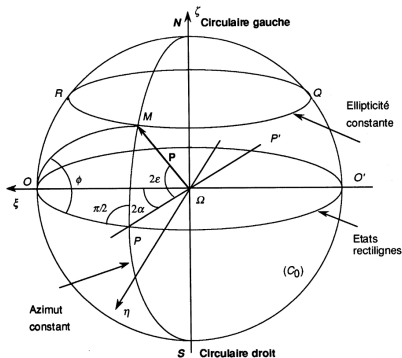
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(S_1, S_2, S_3) on a unit radius sphere



Figures from [Hua94]

The Poincare Sphere

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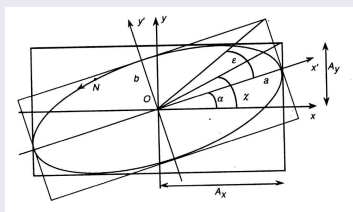
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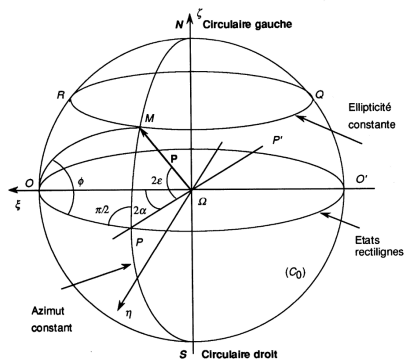
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General Polarisation



(S_1, S_2, S_3) on a unit radius sphere



Figures from [Hua94]

- 1 The physics of polarization optics
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A polarizer lets one component through

Polarizer aligned with x : its action on two orthogonal polarizations

- Lets through the linear polarization along x : $\begin{bmatrix} 1 \\ 0 \end{bmatrix} \rightarrow \begin{bmatrix} 1 \\ 0 \end{bmatrix}$
- Blocks the linear polarization along y : $\begin{bmatrix} 0 \\ 1 \end{bmatrix} \rightarrow \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

x polarizer Jones matrix

in *this* basis

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A quarter wave plate adds a $\pi/2$ phase shift

Birefringent material: n_1 along x and n_2 along y thickness e

- Linear polarization along x : phase shift is $ke = k_0 n_1 e$
- Linear polarization along y : phase shift is $ke = k_0 n_2 e$

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$$\begin{bmatrix} e^{ik_0 n_1 e} & 0 \\ 0 & e^{ik_0 n_2 e} \end{bmatrix} = e^{ik_0 n_1 e} \begin{bmatrix} 1 & 0 \\ 0 & e^{ik_0 (n_2 - n_1) e} \end{bmatrix}$$



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Homework

Think about Half Wave Plates



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Eigen Polarizations

Eigen polarization are polarizations that do not change upon propagation

Eigen Vectors

$\lambda \in \mathbb{C}$

$M \cdot v = \lambda v \Leftrightarrow$ v is an eigen vector
 λ is **its** eigen value

Polarization unchanged

- J and λJ describe the same polarization
- Intensity changes

Handy basis

A matrix is diagonal in its eigen basis

→ M is diagonal in the basis of its eigen polarizations
→ M is diagonal in the basis of its axes

Homework

Find the eigen polarizations for an *optically active* material that rotates any linear polarisation by an angle ϕ



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• Polarizer eigen basis is along its axes

• Diagonalization of any matrix is along its axes

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A polarizer in a rotated basis

In its eigen basis

Eigen basis Jones matrix : $\mathbf{P}_x = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$

When transmitted polarization is θ tilted

Change base through $-\theta$ rotation Transformation Matrix

$$\mathbf{R}(\theta) = \begin{bmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{bmatrix}$$



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Homework

- Find the same result with physics arguments only
- Find the power transmitted through such a polarizer from a x polarized plane wave



Changing basis in the general case

Using the Transformation Matrix

- If basis B_1 is deduced from basis B_0 by transformation P :
$$B_1 = P B_0$$
- Jones Matrix is transformed using $J_1 = P^{-1} J_0 P$

From linear to circular

example

- Optically Active media in a linear basis :

$$J = \begin{bmatrix} \cos(\phi) & \sin(\phi) \\ -\sin(\phi) & \cos(\phi) \end{bmatrix}$$

- Transformation Matrix to a circular basis $P = \begin{bmatrix} 1 & 1 \\ i & -i \end{bmatrix}$

- $P^{-1}MP = \begin{bmatrix} e^{i\phi} & 0 \\ 0 & e^{-i\phi} \end{bmatrix}$



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Anisotropy can be linear and circular

Linear Anisotropy

- Orthogonal eigen **linear** polarizations
- Different index n_1 & n_2

Eigen Jones Matrix

$$\begin{bmatrix} 1 & 0 \\ 0 & e^{i\theta} \end{bmatrix}$$

Orthogonal **linear** polarisations basis

Circular Anisotropy

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Eigen Jones Matrix

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Orthogonal **Circular** basis

Back to linear basis

$$\begin{bmatrix} \cos\left(\frac{\theta}{2}\right) & \sin\left(\frac{\theta}{2}\right) \\ -\sin\left(\frac{\theta}{2}\right) & \cos\left(\frac{\theta}{2}\right) \end{bmatrix}$$

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Orthogonal **Circular** basis

Back to linear basis

$$\begin{bmatrix} \cos\left(\frac{\theta}{2}\right) & \sin\left(\frac{\theta}{2}\right) \\ -\sin\left(\frac{\theta}{2}\right) & \cos\left(\frac{\theta}{2}\right) \end{bmatrix}$$

Optically Active media

- 1 The physics of polarization optics
 - Polarization states
 - Jones Calculus
 - Stokes parameters and the Poincare Sphere
- 2 Polarized light propagation**
 - Jones Matrices Examples
 - Matrix, basis & eigen polarizations
 - Jones Matrices Composition**
- 3 Partially polarized light
 - Formalisms used
 - Propagation through optical devices



Jones Matrices Composition

The Jones matrices of cascaded optical elements can be composed through Matrix multiplication

Matrix composition

- If a \vec{J}_0 incident light passes through \mathbf{M}_1 and \mathbf{M}_2 in that order
- First transmission: $\mathbf{M}_1 \vec{J}_0$
- Second transmission: $\mathbf{M}_2 \mathbf{M}_1 \vec{J}_0$
- Composed Jones Matrix : $\mathbf{M}_2 \mathbf{M}_1$ Reversed order

Beware of non commutativity

- Matrix product does not commute in general
- Think of the case of a linear anisotropy followed by optical activity
 - in that order
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Stokes parameters for partially polarized light

Generalize the coherent definition using the statistical average intensity

Stokes Vector

$$\vec{S} = \begin{bmatrix} P_0 \\ P_1 \\ P_2 \\ P_3 \end{bmatrix} = \begin{bmatrix} \langle I_x + I_y \rangle \\ \langle I_x - I_y \rangle \\ \langle I_{\pi/4} - I_{-\pi/4} \rangle \\ \langle I_L - I_R \rangle \end{bmatrix}$$

Polarization degree $0 \leq p \leq 1$

$$p = \frac{\sqrt{P_1^2 + P_2^2 + P_3^2}}{P_0}$$

Stokes decomposition

Polarized and depolarized sum

$$\vec{S} = \begin{bmatrix} P_0 \\ P_1 \\ P_2 \\ P_3 \end{bmatrix} = \begin{bmatrix} pP_0 \\ P_1 \\ P_2 \\ P_3 \end{bmatrix} + \begin{bmatrix} (1-p)P_0 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \vec{S}_P + \vec{S}_{NP}$$



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The Jones Coherence Matrix

Jones Vectors are out

- They describe phase differences
- Meaningless when not monochromatic

Jones Coherence Matrix

- If $\vec{J} = \begin{bmatrix} A_x(t) e^{i\varphi_x(t)} \\ A_y(t) e^{i\varphi_y(t)} \end{bmatrix}$
- $\Gamma_{ij} = \langle \vec{J}_i(t) \overline{\vec{J}_j(t)} \rangle$
- $\Gamma = \langle \vec{J}(t) \overline{\vec{J}(t)}^t \rangle$

Coherence Matrix: explicit formulation

$$\Gamma = \begin{bmatrix} \Gamma_{xx} & \Gamma_{xy} \\ \Gamma_{yx} & \Gamma_{yy} \end{bmatrix}$$



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Jones Coherence Matrix: properties

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Trace is Intensity

$$\text{Tr}(\Gamma) = I$$

Base change

Transformation P

$$\mathbf{P}^{-1} \Gamma \mathbf{P}$$

Relationship with Stokes parameters

from definition

$$\begin{bmatrix} P_0 \\ P_1 \\ P_2 \\ P_3 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & -i & i \end{bmatrix} \begin{bmatrix} \Gamma_{xx} \\ \Gamma_{yy} \\ \Gamma_{xy} \\ \Gamma_{yx} \end{bmatrix}$$



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Inverse relationship

$$\begin{bmatrix} \Gamma_{xx} \\ \Gamma_{yy} \\ \Gamma_{xy} \\ \Gamma_{yx} \end{bmatrix} = \frac{1}{2} \begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & i \\ 0 & 0 & 1 & -i \end{bmatrix} \begin{bmatrix} P_0 \\ P_1 \\ P_2 \\ P_3 \end{bmatrix}$$

Coherence Matrix: further properties

Polarization degree

$$p = \sqrt{\frac{P_1^2 + P_2^2 + P_3^2}{P_0^2}} = \sqrt{1 - \frac{4(\Gamma_{xx}\Gamma_{yy} - \Gamma_{xy}\Gamma_{yx})}{(\Gamma_{xx} + \Gamma_{yy})^2}} = \sqrt{1 - \frac{4\text{Det}(\Gamma)}{\text{Tr}(\Gamma)^2}}$$

Γ Decomposition in polarized and depolarized components

- $\Gamma = \Gamma_P + \Gamma_{NP}$
- Find Γ_P and Γ_{NP} using the relationship with the Stokes parameters



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Propagation of the Coherence Matrix

Jones Calculus

- If incoming polarization is $\vec{J}(t)$
- Output one is $\vec{J}'(t) = \mathbf{M}\vec{J}(t)$

Coherence Matrix if M is unitary

- M unitary means a linear and/or circular anisotropy only

$$\vec{J}' = \mathbf{M}\vec{J}$$

$$\mathbf{C}' = \mathbf{M}\mathbf{C}\mathbf{M}^\dagger$$



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- Unaltered for unitary operators (Tr and Det are unaltered)

- Unaltered for Hermitian operators (Tr and Det are unaltered)



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Mueller Calculus

Propagating the Jones coherence matrix is difficult if the operator is not unitary

Jones Calculus raises some difficulties

- Coherence matrix OK for partially polarized light
- Propagation through unitary optical devices
(linear or circular anisotropy only)
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The Stokes parameters may be an alternative

- Describing intensity, they can be readily measured



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The projection on a polarization state

Matrix of the polarizer with axis parallel to \vec{V}

Projection on \vec{V} in Jones Basis

P_V

- Orthogonal Linear Polarizations Basis: \vec{X} and \vec{Y}
- Normed Projection Base Vector :
 - $\vec{V} = A_x e^{-i\frac{\varphi}{2}} \vec{X} + A_y e^{i\frac{\varphi}{2}} \vec{Y}$
 - $\vec{V}^t \vec{V} = 1$
- $P_V = \vec{V} \vec{V}^t$ ^a

^aEasy to check in the projection eigen basis





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The Pauli Matrices

A base for the 4D 2×2 matrix vector space

$$\sigma_0 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \sigma_1 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}, \sigma_2 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \sigma_3 = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}$$

\mathbf{P}_V decomposition

$$\mathbf{P}_V = \frac{1}{2} (p_0 \sigma_0 + p_1 \sigma_1 + p_2 \sigma_2 + p_3 \sigma_3)$$



P_V composition and Trace property

Trace is the eigen values sum

Projection property

$$\vec{V}^t \cdot \sigma_j \vec{V} = \left(\vec{V}^t \vec{V} \right) \vec{V}^t \cdot \sigma_j \vec{V}$$

Projection Trace in its eigen basis

- P_V eigenvalues : 0 & 1

$$\text{Tr}(P_V) = 1$$



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P_V Pauli components and physical meaning

Express p_i as a function of \vec{V} and the Pauli matrices, then find their signification

$$\vec{V}^{\dagger} \cdot \sigma_j \vec{V} = \text{Tr}(P_V \sigma_j) \qquad \text{Tr}(\sigma_i \sigma_j) = 2\delta_{ij}$$

$$\text{Tr}(P_V \sigma_j) = \frac{1}{2} \sum_i \text{Tr}(\sigma_i \sigma_j) p_i$$

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P_V Pauli components and physical meaning

Express p_i as a function of \vec{V} and the Pauli matrices, then find their signification

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\mathbf{P}_V Pauli composition and Stokes parameters

Stokes parameters as \mathbf{P}_V decomposition on the Pauli base

- $p_0 = P_0 = A_x^2 - A_y^2 = I_x - I_y$
- $p_1 = P_1 = A_x^2 - A_y^2 = I_x - I_y$
- $p_2 = P_2 = 2A_x A_y \cos(\varphi) = I_{\pi/4} - I_{-\pi/4}$
- $p_3 = P_3 = 2A_x A_y \sin(\varphi) = I_L - I_R$



Propagating through devices: Mueller matrices

$$\vec{V}' = \mathbf{M}_J \vec{V}$$

Projection on \vec{V}'

$$\mathbf{P}_{V'} = \vec{V}' \vec{V}'^t$$

Trace relationship

$$P'_i = \text{Tr}(\mathbf{P}_{V'} \sigma_i)$$

Mueller matrix

$$\vec{S}' = \mathbf{M}_M \vec{S}$$

$$(M_M)_{ij} = \frac{1}{2} \text{Tr}(\mathbf{M}_J \sigma_j \overline{\mathbf{M}_J^t} \sigma_i)$$



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Mueller matrices and partially polarized light

Time average of the previous study

Mueller matrices are time independent

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Mueller calculus can be extended to...

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- Cascaded optical devices

Final homework

Find the Mueller matrix of each :

- Polarizers along eigen axis or θ tilted
- half and quarter wave plates
- linearly and circularly birefringent crystal



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