Low energy $\bar{P}$P physics

E. Predazzi

LOW ENERGY $\bar{P}P$ PHYSICS

E. PREDAZZI

Istituto di Fisica Teorica, Università di Torino, Torino (Italie)

et

Istituto Nazionale di Fisica Nucleare, Sezione di Torino (Italie)

INTRODUCTION AND MOTIVATIONS FOR $\bar{p}p$ STUDIES.

The interest in $N\bar{N}$ physics (particularly $p\bar{p}$) has always been deemed by the difficulty of having a suitable tool of investigation, namely, a reasonably intense beam of antiprotons. This interest has so far been confined to the verification of the asymptotic theorems in the high energy domain and to the spectroscopic analysis of the baryonium both in its theoretical and experimental aspects.

The recent development of cooling techniques opens new perspectives ranging from the very low to the very high energy domain. Among the main issues from low energy data, many consequences of general theorems, such as CPT are going to be verified experimentally, the $B\bar{B}$ resonance spectrum (baryonium) and very low energy cross-sections are going to be measured. In the high energy domain, aside from the already mentioned verification of the asymptotic theorems for pp and $p\bar{p}$, an exciting new field has been the search for $W^{\pm}$ and Z^{o} on the one hand (at the level of our understanding of the fundamental properties of the basic interactions) and the verification of the hints provided by cosmic ray data, on the other hand, which has greatly renewed the interest for hadronic physics.

All these developments have not been accessible so far with the use of the conventional (low intensity) beam of antiprotons that have been traditionally available and whose contamination from pions has always been very large. Furthermore, the range of available energies has so far been limited and strongly correlated to that of the primary (proton) beams. This can be seen, as an example, from the momentum spectrum of antiprotons produced in the forward direction by a 23 GeV/c beam of protons on a lead target (Fig. 1). In Fig. 1 the distribution peaks at 3.5 GeV/c corresponding to production at rest in the C.M.

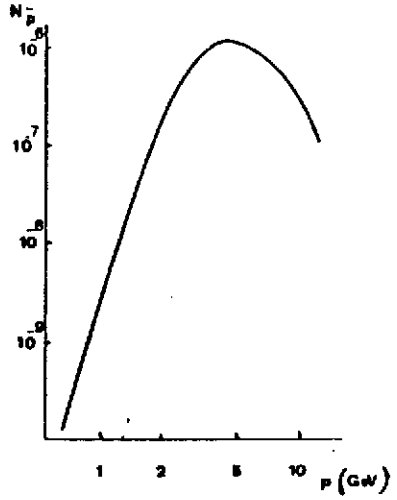


FIG.1

The situation is soon going to be drastically modified by the new $\bar{p}$ beams obtained with cooling techniques which are going to provide very intense sources of antiprotons. The latter, produced at the peak energy of Fig. 1 will be stocked in an accumulator ring and the momentum spread of each incoming bunch ($\Delta p/p \simeq 1\%$) is going to be reduced or "cooled down" to $\Delta p/p \simeq 10^{-4}$. The present expectation is

of stocking $\lesssim 10^{12}$ antiprotons in a day time at CERN by means of a specific antiprotons accumulator ring (AA) to be then utilized in the new devices called LEAR and ICE; they will be discussed together with the planned physics program in the lectures by U. Gastaldi (these proceedings).

The options of using relatively low energy beams (up to a maximum of few GeV/c per beam) are both being pursued at CERN while the high energy option only is presently being considered at FNAL.

In the high energy option at CERN, the antiprotons stocked in the AA ring are first accelerated at the PS up to 26 GeV/c and have then either been injected in the SPS used as collider (i.e. accelerating at the same time beams of protons and antiprotons circulating in opposite directions) or in the ISR. The latter alternative is now out due to the shut down of the ISR. In the low energy option, the antiprotons are first decelerated down to ~0.6 GeV/c and then transferred to a small storage ring (LEAR) which will provide beams whose energy will range from 0.1 GeV/c to 2 GeV/c with $\Delta p/p \simeq 10^{-4}$.

The maximum energy of 270x2 = 540 GeV/c has been reached in the collider (i.e. nine times the maximum energy attained at the ISR) leading to the discovery of the vector bosons $W^{\pm}$ and Z^{o} mediating the electroweak interaction. The experimental finding of $m_W \simeq 82$ GeV/c^2 and $m_Z = 93$ GeV/c^2 is the best confirmation of the so called standard (or Weinberg-Salam) model unifying the weak and the electromagnetic forces. Thus, once again, the first actual discovery of $W^{\pm}$ and Z^{o} has been the product of hadronic machines. Quite probably, however, the $\bar{p}p$ collider will not be able to carry out the analysis of the spectroscopic properties of the vector bosons; for this, the new generation of e^+e^- machines (such as LEP) will be necessary. The Collider has also been useful for measuring $\sigma_t(\bar{p}p)$ and for extracting other data.

Much information has also been gathered from the second high energy option at CERN, i.e. injecting the antiprotons in the ISR. This has allowed one to study the $\bar{p}p$ elastic and total cross-sections up to the present highest energies at which they have been measured for pp ($\sqrt{s}$ = 22 to 63 GeV) and to verify both their expected analogies and the validity of the Pomeranchuk theorem as well as their appearent differences[1].

While the high energy proton beam developed at CERN will presumably loose its competitivity once the FNAL collider will be in operation, a longer lifetime should be expected for the CERN low energy facility of antiprotons (LEAR) which is going to be a very flexible tool by providing: first of all, a very clean external $\bar{p}$ beam with an average intensity of $\lesssim 10^6$ $\bar{p}$/sec and high duty cycle; secondly, an internal beam to be used on a gas jet target; thirdly, the simultaneous stocking of H^- and $\bar{p}$ beams circulating in the same direction so that very low energy collisions should be possible; lastly, the work of a standard collider accumulating p and $\bar{p}$ circulating in opposite directions to investigate low and medium $\bar{p}p$ physics.

More specifically, at low energies we expect the following issues to be important in connection with intense $\bar{p}$ beams:

A/ Annihilation processes

B/ The baryonium (or q q $\bar{q}$ $\bar{q}$) states

C/ Quasinuclear states

D/ The protonium states

E/ The study of charmonium.

As already mentioned, this physics is going to be studied at LEAR with a high intensity ($\lesssim 10^6$ $\bar{p}$/sec) high duty cycle, high momentum resolution $\Delta p/p \simeq 10^{-4}$, low energy (0.1 ÷ 2 GeV/c) extracted beam or within a storage ring operating with 10^9 to $\lesssim 10^{12}$ $\bar{p}$. In this mode of operation dense targets are required to provide large stopping rates above $\sim$0.1 GeV/c. Alternatively, the $p\bar{p}$ spectroscopy can be studied with an internal beam on a gas jet target and an extremely high momentum resolution ($\Delta p/p \simeq 10^{-5}$).

A/ Annihilation processes.

All modes of annihilation can be studied by stopping virtually all of the 10^6 $\bar{p}$/sec in a small volume (30 cm long hydrogen target). Also annihilation into e^+e^- pair should give the large yield of $\sim$300 events/hour (to be compared with the present statistics of $\sim$26 events detected at the PS in the ELPAR experiment).

Annihilation is going to be investigated using degradation in matter and tagging with a spectrometer or time of flight measurements. Below 0.2 GeV/c this field is totally unexplored.

B/ The baryonium states.

An extremely important issue that can be studied with either an external beam or an internal gas jet target is that of baryonium states; namely, of those mesonic states that are weakly coupled to mesons and should strongly effect $p\bar{p}$ channels at low energies. These states, expected from duality considerations extended from meson-meson and meson-baryon to baryon-antibaryon, should be q q $\bar{q}$ $\bar{q}$ states whose experimental (and theoretical) evidence has been a rather controversial issue in connection with the colour degree of freedom and they could be manifestations of diquark-antidiquark systems belonging to 3 $\bar{3}$ (d $\bar{d}$) and 6 $\bar{6}$ (d $\bar{d}$) representations (recall that if q belongs to 3 and $\bar{q}$ to $\bar{3}$, than a qq state can give 3x3=$\bar{3}$+6 with a coupling which is $-\frac{4}{3}$ and $\frac{2}{3}$ respectively). These states are expected to be weakly coupled to meson-meson channels and should lead to narrow states below or near the 3 $\bar{3}$ threshold and narrow states above the 6 $\bar{6}$ threshold which should however be rather difficult to excite.

Generally, baryonium states should appear in formation experiments ($p\bar{p} \longrightarrow X$) or missing mass experiments ($p\bar{p} \longrightarrow X + \pi\zeta$).

Experimentally, the situation is presently rather obscure. Many states have been reported in the past by various groups but, so far, none has been firmly established. Therefore, this point arouses great expectations. This point will be taken up briefly in Secs. III.5 and IV.2.

C/ Quasinuclear states.

From the study of N N interactions one expects short range attraction in the N $\bar{N}$ system giving rise to resonances (quasinuclear states) around threshold. These states are, in princi-

ple, expected to be $(q\ q\ q)$ $(\bar{q}\ \bar{q}\ \bar{q})$ systems as compared to the previous $(q\ q\ \bar{q}\ \bar{q})$ baryonium states. In practice, however, it may be very difficult to discriminate among these two kinds of states.

D/ Protonium states.

These would correspond to the formation of hydrogen-like systems bound together by a Coulomb force. Given the theoretical predictions, the issue here is to compare the latter with high precision measurements of level shifts and widths associated with strong interactions in the low angular momentum states. They can be studied both with stopped $\bar{p}$ in a gas targed (LEAR) or by $\bar{p}$ H^- interactions in flight; one expects intense beams of protonium to be obtained from the straight sections of LEAR (typically, 10^4 atoms from 10^9 $\bar{p}$ and H^- stored in the LEAR ring).

E/ Charmonium.

The whole charmonium family can be studied using LEAR as a collider with a high momentum resolution $\frac{\Delta p}{p} \simeq 10^{-4}$. The main point is, of course, to finally settle the various questions which are still open[2].

The plan of these lectures is the following: In Part I we first recall some general properties of the basic symmetries C,P,T and we then discuss the quantum numbers and the selection rules of the $\bar{N}N$ system; Part II is devoted to some kinematics of NN and $\bar{N}N$ in the simplest reaction modes; Part III deals with a few aspects of the experimental side of low energy "conventional" $\bar{N}N$ phenomenology; Part IV, finally, takes up very briefly the discussion of some theoretical results in low energy $B\bar{B}$ phenomenology.

Part of the material used in these notes follows the presentation by L. Bertocchi[3] and, for the last part, we have borrowed from R. Vinh Man[4].

PART I

GENERAL PROPERTIES

SPACE INVERSION, CHARGE CONJUGATION, TIME REVERSAL, PCT THEOREM, G CONJUGATIONS, QUANTUM NUMBERS

I.1 SPACE INVERSION

I.1.1 Introduction:

In ordinary Quantum Mechanics, the invariance of the Hamiltonian under space inversion transformation $(x,y,z) \longrightarrow (-x,-y,-z)$ leads to the possibility of classifying states of the system according to a parity quantum number ± 1. More precisely, there exists a unitary operator P such that coordinates, momentum and spin transform according to

$$\begin{aligned} \vec{x}' &= P^{-1}\,\vec{x}\,P = -\vec{x} \\ \vec{p}' &= P^{-1}\,\vec{p}\,P = -\vec{p} \\ \vec{J} &= P^{-1}\,\vec{J}\,P = \vec{J} \end{aligned} \qquad \text{(I.1.1)}$$

while state vectors with definite parity satisfy

$$P\,|\,\rangle = \pm\,|\,\rangle \qquad \text{(I.1.2)}$$

Eq. (I.1.1) preserves the commutation rules and the equations of motions.

In field theory there is a generalization corresponding to eq. (I.1.1) for the field operators which we will briefly recall here.

I.1.2 Electromagnetic field:

The charge of a particle is assumed to be a true scalar quantity. Thus the following space-inversion laws obtain

$$\begin{aligned} \rho'(\vec{x}) &= \rho(-\vec{x}) \\ \vec{j}'(\vec{x}) &= -\vec{j}(-\vec{x}) \end{aligned} \qquad \text{(I.1.3)}$$

From Maxwell equations invariance under space inversion, the electric field is known to be a polar vector and the magnetic field to be an axial vector

$$\begin{aligned} \vec{E}'(\vec{x}) &= -\vec{E}(-\vec{x}) \\ \vec{B}'(\vec{x}) &= \vec{B}(-\vec{x}) \end{aligned} \qquad \text{(I.1.4)}$$

Thus, the four-vector potential transforms according to

$$\begin{aligned} \vec{A}'(\vec{x}) &= -\vec{A}(-\vec{x}) \\ \phi'(\vec{x}) &= \phi(-\vec{x}) \end{aligned} \qquad \text{(I.1.5)}$$

The quantum mechanical corresponding forms are all of the type

$$E_k'(\vec{x}) = P^{-1}\,E_k(x)\,P = -E_k(-\vec{x}) \qquad (k=1,2,3) \qquad \text{(I.1.6)}$$

and the two Lorentz invariant combinations of the electromagnetic fields $(\vec{E}^2-\vec{B}^2)$, $\vec{E}\cdot\vec{B}$ have therefore opposite parities

$$P^{-1}(\vec{E}^2-\vec{B}^2)P = \vec{E}^2-\vec{B}^2$$
$$P^{-1}(\vec{E}\cdot\vec{B})P = -\vec{E}\cdot\vec{B} \qquad \text{(I.1.7)}$$

I.1.3 Scalar and pseudoscalar fields:

Spinless bosons are described by field operators $\phi(\vec{x},t)$ which are rotationally invariant. Their space inversion transformations, however, depend on whether they correspond to scalar (S) or pseudoscalar (P) operators. In the first case:

$$\phi_S'(\vec{x},t) \equiv P^{-1}\phi_S(\vec{x},t)P = \phi_S(-\vec{x},t) \qquad \text{(I.1.8)}$$

while

$$\phi_P'(\vec{x},t) \equiv P^{-1}\phi_P(\vec{x},t)P = -\phi_P(-\vec{x},t) \qquad \text{(I.1.9)}$$

When the field operator is expanded in an orthonormal set of plane waves, P acts only on the creation and destruction operators. Thus, given

$$\phi(\vec{x},t) = \frac{1}{V^{1/2}}\sum_k \frac{1}{\sqrt{2\omega_k}}\left[a_k e^{i\vec{k}\cdot\vec{x}-i\omega_k t} + b_k^+ e^{-i\vec{k}\cdot\vec{x}+i\omega_k t}\right] \qquad \text{(I.1.10)}$$

the transformed field $\phi'(\vec{x},t)$ is given by

$$\phi'(\vec{x},t) = \frac{1}{V^{1/2}}\sum_k \frac{1}{\sqrt{2\omega_k}}\left[P^{-1}a_k P\, e^{i\vec{k}\cdot x-i\omega_k t} + P^{-1}b_k^+ P\, e^{-i\vec{k}\cdot\vec{x}+i\omega_k t}\right] \qquad \text{(I.1.11)}$$

Comparing with eqs. (I.1.8,9) we have for the transformation properties of a_k, b_k^+

$$P^{-1}a_k P = \eta_P a_{-k}$$
$$P^{-1}b_k^+ P = \eta_P b_{-k}^+ \qquad \text{(I.1.12)}$$

where

$$\eta_P = +1 \quad \text{for scalar particles}$$
$$\eta_P = -1 \quad \text{for pseudoscalar particles} \qquad \text{(I.1.13)}$$

I.1.4 Dirac fields:

The single particle Dirac wave functions transform according to

$$\psi'(\vec{x}) = \beta\,\psi(-\vec{x}) \qquad \text{(I.1.14)}$$

where

$$\beta = \gamma_4 \qquad \text{(I.1.15)}$$

being

$$\left(\gamma_\mu \frac{\partial}{\partial x^\mu} + m\right)\psi = 0 \qquad \text{(I.1.16)}$$

the basic Dirac equation where we have chosen

$$\vec{\alpha} = \begin{pmatrix} 0 & \vec{\sigma} \\ \vec{\sigma} & 0 \end{pmatrix} \quad , \quad \beta = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \qquad \text{(I.1.17)}$$

(being $\vec{\sigma}$ the usual Pauli matrices) and where

$$\begin{aligned} &\vec{\gamma} = -i\beta\vec{\alpha} \qquad \gamma_4 = \beta \\ &\gamma_5 = \gamma_1\gamma_2\gamma_3\gamma_4 = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix} \\ &\{\gamma_\alpha, \gamma_\beta\} = 2\delta_{\alpha\beta} \, . \end{aligned} \qquad \text{(I.1.18)}$$

The adjoint field

$$\bar{\psi} = \psi^+ \gamma_4 \qquad \text{(I.1.19)}$$

satisfies the Dirac equation

$$-\frac{\partial \bar{\psi}}{\partial x^\mu}\gamma_\mu + m\bar{\psi} = 0 \qquad \text{(I.1.20)}$$

The plane wave solutions (spinors) of the free particle equation (I.1.16) can be written:

i) Positive energy:

$$u_{1,2}(\vec{p}) = \sqrt{\frac{E+m}{2m}} \begin{pmatrix} \chi_{1,2} \\ \frac{\vec{\sigma}\cdot\vec{p}}{E+m}\chi_{1,2} \end{pmatrix} \qquad \text{(I.1.21)}$$

ii) Negative energy:

$$v_{1,2}(\vec{p}) = \pm\sqrt{\frac{E+m}{2m}} \begin{pmatrix} \frac{\vec{\sigma}\cdot\vec{p}}{E+m}\chi_{1,2} \\ \chi_{1,2} \end{pmatrix} \qquad \text{(I.1.22)}$$

where E is positive and the Pauli spinors $\chi_{1,2}$ are defined as

$$\chi_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad , \quad \chi_2 = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \qquad \text{(I.1.23)}$$

The spinor $u_j(p)$ is a positive energy spinor representing a particle with momentum $\vec{p}$ and spin j. Similarly, $v_j(p)$ represents a negative energy particle with momentum $-\vec{p}$ and spin j. The charge conjugate spinor $v^c_j(p)$ represents a positive energy antiparticle of momentum $\vec{p}$ and opposite spin.

The plane wave spinors $u(p)$ and $v(p)$ obey the free particles equations of motions

$$\begin{cases} (i\gamma\cdot p + m)\, u(p) = 0 \\ (i\gamma\cdot p - m)\, v(p) = 0 \end{cases} \qquad \text{(I.1.24)}$$

while their adjoints obey

$$\bar{u}(p)\,(i\gamma\cdot p + m) = 0$$
$$\bar{v}(p)\,(i\gamma p - m) = 0 \qquad \text{(I.1.25)}$$

Finally, the normalization of these spinors is

$$\bar{u}_i(p)\,u_j(p) = \delta_{ij}$$
$$\bar{u}_i(p)\,v_j(p) = 0 \qquad \text{(I.1.26)}$$
$$\bar{v}_i(p)\,v_j(p) = -\delta_{ij}$$

and the projection operators for positive and negative energy states are

$$\Lambda_+(p) \equiv \sum_{j=1}^{2} u_j(p)\,\bar{u}_j(p) = \frac{m - i\gamma p}{2m}$$
$$\Lambda_-(p) \equiv \sum_{j=1}^{2} v_j(p)\,\bar{v}_j(p) = -\frac{m + i\gamma p}{2m} \qquad \text{(I.1.27)}$$

Going back to the transformation properties (I.1.14) (which follows directly from the equation of motion (I.1.16) using also (I.1.15) and (I.1.18) in terms of spinors (I.1.21,22) we have

$$u'_j(\vec{p}) = \beta\, u_j(\vec{p}) = u_j(-\vec{p})$$
$$v'_j(\vec{p}) = \beta\, v_j(\vec{p}) = -v_j(-\vec{p}) \qquad \text{(I.1.28)}$$

Rewriting eq. (I.1.14) for a Dirac spinor field

$$\psi'(\vec{x},t) \equiv P^{-1}\,\psi(\vec{x},t)\,P = \beta\,\psi(-\vec{x},t) \qquad \text{(I.1.29)}$$

and making the usual plane wave decomposition in terms of creation and annihilation operators

$$\psi(\vec{x},t) = \frac{1}{V^{1/2}} \sum_{p,j} \left(\frac{m}{E}\right)^{1/2} \left[u_j(p)\, e^{i\vec{p}\cdot\vec{x} - iEt}\, a_{p,j} + v_j(p)\, e^{-i\vec{p}\cdot\vec{x} + iEt}\, b^{+}_{p,j} \right] \qquad \text{(I.1.30)}$$

we finally have, using both (I.1.28) and (I.1.29)

$$P^{-1} a_{p,j}\, P = a_{-p,j}$$
$$P^{-1} b^{+}_{p,j}\, P = -b^{+}_{-p,j} \qquad \text{(I.1.31)}$$

with similar relations for their adjoints. The relative minus sign between the particle and antiparticle forms leads to the conclusion that a particle-antiparticle pair has an intrinsic odd parity.

This is the only relevant statement that can be made in the sense that the concept of "absolute parity" is meaningless for a fermion. Thus, if we choose, by convention, the parity of a spin 1/2 particle to be positive, its antiparticle will have negative parity.

To see how this checks with the experiment, let us consider the annihilation at threshold

$$e^+ e^- \longrightarrow 2\gamma$$

At threshold, the reaction proceeds via S-wave (L=0 of the e^+e^- system). Furthermore, it is found that the γ's angular distribution is isotropic which implies a total angular momentum J=0 for the initial state. As a consequence of J=L+S, and of L=0, J=0, we get also S = 0 (i.e. the initial state is a singlet). Thus, the reaction takes place in a state

$$^{2S+1}L_J = {}^1S_0$$

If the intrinsic e^+e^- state is negative, this, in turn, implies that the total parity of the initial state is

$$P = (-1)^{L+1} = -1 \qquad (\text{since } L = 0)$$

So, the initial state is invariant under rotation (J=0) but has negative parity: it must be, therefore, a pseudoscalar.

Let us now see what this implies for the final state which must again be a pseudoscalar owing to the fact that parity is conserved. To construct a pseudoscalar we have the following elements to take into account (we work in the C.M.):

i) the product of the intrinsic parities of the γ's is +1 since they are identical bosons;

ii) the two photons are real and therefore transverse (i.e.: $\vec{e}_1.\vec{k} = 0$, $\vec{e}_2.\vec{k} = 0$ where $\vec{k}$ is the photon momentum, $\vec{e}_1$ and $\vec{e}_2$ are the polarization vectors of the two photons).

Thus, the only pseudoscalar that we can form is $\vec{e}_1 \times \vec{e}_2 . \vec{k}$ predicting orthogonal polarization for the two γ rays. Notice that had the relative e^+e^- parity been positive, the state would have been scalar which could be represented by a form $\vec{e}_1 . \vec{e}_2$ predicting parallel polarization.

The data lead to orthogonal polarizations of the two final γ's thus confirming the relative negative e^+e^- parity.

The analogous test for the $N\bar{N}$ system would be that of a two-pion annihilation

$$N\bar{N} \rightarrow 2\pi$$

The steps of the proof would be:

i) to prove that the reaction proceeds from an S-wave (for instance through its energy dependence at threshold):

ii) to prove that it proceeds from an isotropic state J=0 so that the initial state should again be a $^1S_0$ state.

In this case, however, the check that the $N\bar{N}$ system is in a negative relative state would require the absence of the reaction $N\bar{N} \rightarrow 2\pi$ for the case where the two pions have isotropic angular distribution and have an energy dependence consistent with an S-wave. In the present case, in fact, the pions are spinless and we cannot form a pseudoscalar with the variables of the final state.

We shall briefly discuss in part III, the evidence for S-wave annihilation in $N\bar{N} \rightarrow 2\pi$, $K\bar{K}$ versus the P-wave annihilation (§ III.4).

I.1.5 Bilinear forms:

Recall now that the most general 4x4 matrix needs 16 elements to characterize it and its most general form can be made up with combinations of Dirac matrice. They can always be organized in 5 groups according to their Lorentz transformation: scalar, vector, tensor, axial vector and pseudoscalar which will denoted by O_i (i=1,2,3,4,5). Explicitly, one introduces the Hermitian operators ($O_i^+ = O_i$)

i	Lorentz property	O_i
1	S (scalar)	1
2	V (vector)	γ_α ; $\alpha = 1,2,3,4$
3	T (tensor)	$\sigma_{\alpha\beta} = \frac{1}{2i}(\gamma_\alpha\gamma_\beta - \gamma_\beta\gamma_\alpha)$; $\alpha,\beta = 1,2,3,4$; $\alpha \neq \beta$
4	A (axial vector	$i\gamma_\alpha\gamma_5$; $\alpha = 1,2,3,4$
5	P (pseudo-scalar)	γ_5

The adopted terminology is just consequence of the space transformation which we are now going to discuss.

From the transformation properties just discussed for a Dirac field (I.1.29), it follows that

$$P^{-1}\left(\bar{\psi}_a(\vec{x})\, O_{i\alpha}\, \psi_b(\vec{x})\right)P = \eta^P_{i\alpha}\left(\bar{\psi}_a(-\vec{x})\, O_{i\alpha}\, \psi_b(-\vec{x})\right) \qquad \text{(I.1.32)}$$

where the subscript α denotes the appropriate Lorentz index or indices. The phase factors $\eta^P_{i\alpha}$ are given by

Scalar		$+1$	
Vector	-1 for $\alpha = 1,2,3$	$+1$ for	$\alpha = 4$
Tensor	$+1$ for $\alpha,\beta = 1,2,3$	-1 for	α or $\beta = 4$
Axial vector	$+1$ for $\alpha = 1,2,3$	-1 for	$\alpha = 4$
Pseudoscalar		-1	

(I.1.33)

Notice that it is the space components of the above forms which transform according to their names.

I.2 CHARGE CONJUGATION

I.2.1 Introduction:

The definition of charge conjugation arose from the symmetry of Dirac equations for electrons and positrons interacting with the electromagnetic field. Its meaning is now broader since it applies also to electrically neutral systems (like K°, $\bar{K}^\circ$) but the name has survived.

I.2.2 Charged scalar or pseudoscalar field:

A complex scalar or pseudoscalar field $\phi = \phi_1 + i\phi_2$ (ϕ_i i=1,2 being Hermitian operators) describes spinless charged particles with an electromagnetic charge current 4-vector

$$j_\alpha = ie\left[(\partial_\alpha \phi^+)\,\phi - \phi^+(\partial_\alpha \phi)\right] \tag{I.2.1}$$

where $\partial_\alpha \equiv \partial/\partial x^\alpha$ and $\phi^+ = \phi_1 - i\phi_2$. The field ϕ (ϕ^+) destroys (creates) particles and creates (destroys) antiparticles. We define the unitary operator C which interchanges the roles of particles and antiparticles through the transformation properties

$$\begin{cases} \phi^C \equiv C^{-1}\phi C = \phi^+ \\ \phi^{+C} \equiv C^{-1}\phi^+ C = \phi \end{cases} \tag{I.2.2}$$

$$j'_\alpha \equiv C^{-1} j_\alpha C = -j_\alpha \tag{I.2.3}$$

Examples of complex boson fields are of course those used to describe the pions being π^+ the antiparticle of π^-. Another example is the pair of neutral K mesons (K° and $\bar{K}^\circ$) which differ in their strangeness.

From (I.1.10) one finds for the creation and destruction operators

$$\begin{aligned} C^{-1} a_k C &= b_k \\ C^{-1} b_k^+ C &= a_k^+ \end{aligned} \tag{I.2.4}$$

and analogous for their adjoints.

I.2.3 Self conjugate scalar and pseudoscalar field:

Neutral particles with no distinguishing quantum numbers (like baryon number of strangeness), such as π° are described by an Hermitian field ϕ_o which transforms into itself

$$\phi_o^C \equiv C^{-1}\phi_o C = \pm\phi_o \tag{I.2.5}$$

In the case of pions, the plus sign is chosen to make invariant under charge conjugation the charge symmetric coupling of pions to nucleons $\bar{\psi}\vec{\tau}\cdot\vec{\phi}\psi$.

I.2.4 The electromagnetic field:

From the behavior of the e.m. 4 current under charge conjugation it follows that the e.m. fields and the 4 vector potential transform as

$$\begin{aligned} \vec{E}^C &\equiv C^{-1}\vec{E}C = -\vec{E} \\ \vec{B}^C &\equiv C^{-1}\vec{B}C = -\vec{B} \\ A_\alpha^C &\equiv C^{-1}A_\alpha C = -A_\alpha \end{aligned} \tag{I.2.6}$$

which implies that the Lorentz invariant forms E^2-B^2 and $\vec{E}\cdot\vec{B}$ are even under C. This is one further reason to choose the sign + in the transformation of ϕ_o (eq.(I.2.5) for the π° field since the effective interaction $(\vec{E}\cdot\vec{B})\phi_o$ responsible for the decay $\pi^\circ \to 2\gamma$ is then even under C.

I.2.5 Dirac field:

For the single particle Dirac equation with electromagnetic coupling, the transformation which changes the sign of the e.m. coupling is

$$\psi \rightarrow \psi' = \gamma_2 \psi^* \tag{I.2.7}$$

where the operator γ_2 is peculiar to the representation of γ matrices chosen in § I.1.4.

In terms of the plane wave spinors $u_j(p)$ and $v_j(p)$, we have from (I.1.17,21,22)

$$\begin{aligned} u_1'(p) &\equiv \gamma_2 u_1^* = v_2(p) \quad ; \quad v_1'(p) \equiv \gamma_2 v_1^* = u_2 \\ u_2'(p) &\equiv \gamma_2 u_2^* = v_1(p) \quad ; \quad v_2'(p) \equiv \gamma_2 v_2^* = u_1 \end{aligned} \tag{I.2.8}$$

For the quantized Dirac field there exists a unitary operator C such that the charge conjugated field is given by

$$\psi^c \equiv C^{-1} \psi C = \gamma_2 \tilde{\psi}^+ \tag{I.2.9}$$

where the sign of transposed applies only to the spinors not to the creation and destruction operators; in other terms, the symbol $\tilde{\psi}^+$ implies adjoints for the creation and destruction operators and complex conjugates for the plane waves. Using the expansion (I.1.30) we have

$$\begin{aligned} C^{-1} a_{p,1} C &= b_{p,2} \quad ; \quad C^{-1} a_{p,2} C = b_{p,1} \\ C^{-1} b^+_{p,1} C &= a^+_{p,2} \quad ; \quad C^{-1} b^+_{p,2} C = a^+_{p,1} \end{aligned} \tag{I.2.10}$$

together with the adjoint relations.

It will be convenient to introduce the short-hand notation

$$\psi \equiv \psi_a \quad ; \quad \psi^c \equiv \psi_{\bar{a}} \tag{I.2.11}$$

to imply that if ψ is the spinor field of a given particle a, its charge conjugated ψ^c is the field of the antiparticle $\bar{a}$.

I.2.6 Bilinear forms:

Applying the transformations (I.2.9) or, equivalently, (I.2.10), the charge conjugation of the bilinear covariant forms $\bar{\psi}_a O_i \psi_b$ can be proved to be

$$(\bar{\psi}_{\bar{a}} O_i \psi_{\bar{b}}) \equiv C^{-1} (\bar{\psi}_a O_i \psi_b) C = \eta_i^c (\bar{\psi}_b O_i \psi_a) \tag{I.2.12}$$

where η_i^c = -1 for V,T and +1 for S,A,P.

Anticommutation of the fields is assumed in order to derive these results.

I.3 TIME REVERSAL

I.3.1 Introduction:

Time reversal is, notoriously, represented by an antiunitary operator T defined by the properties

$$T|\alpha\rangle = \langle\alpha_T|$$
$$T(c_1|\alpha_1\rangle + c_2|\alpha_2\rangle) = c_1\langle\alpha_{1T}| + c_2\langle\alpha_{2T}| \qquad (I.3.1)$$
$$\langle\beta_T|\alpha_T\rangle = \langle\alpha|T^{-1}T|\beta\rangle = \langle\beta|\alpha\rangle^*$$

The definition of the time reversed state $|\alpha_T\rangle$ is that it has <u>all momenta and angular momenta</u> reversed as compared to the state $|\alpha\rangle$.

Reversing bras with kets under T, amounts to complex conjugation of the wave functions. This is understandable if we consider the process A $\longrightarrow$ B whose S-matrix element will be $\langle {}^{B}_{out}|{}^{A}_{in}\rangle$. The time reversed situation will correspond to the process B' $\longrightarrow$ A' (where the momenta and spins of A', B' will be reversed as compared to the states A, B) with S-matrix amplitude $\langle {}^{A'}_{out}|{}^{B'}_{in}\rangle$.

Consider a general operator A and its time reversed counterpart $A' = T^{-1}AT$. The last relation in (I.3.1) can then be used to relate matrix elements of A in time reversed states, to matrix elements of A' in the original states. We have

$$\langle\beta|A'|\alpha\rangle^* = \langle\alpha|A'|\beta\rangle = \langle\alpha|T^{-1}AT|\beta\rangle = \langle\beta_T|A|\alpha_T\rangle \qquad (I.3.2)$$

In simple cases when A' and A are simply related as well as $|\alpha\rangle$ and $|\alpha_T\rangle$, $|\beta\rangle$ and $|\beta_T\rangle$ definite phase relations obtain implying, normally, that certain form factors are purely real (or imaginary).

In ordinary quantum mechanics of spinless particles, the time reversed wave function is given by

$$\psi'(\vec{x},t) = \psi^*(\vec{x},-t) \qquad (I.3.3)$$

while for Pauli spinors

$$\psi'(\vec{x},t) = i\sigma_2\,\psi^*(\vec{x},-t) \qquad (I.3.4)$$

whose relativistic generalization for Dirac particles

$$\psi'(\vec{x},t) = \gamma_3\gamma_1\,\psi^*(\vec{x},-t) \qquad (I.3.5)$$

is, once again peculiar to the choice of γ matrices used in § I.1.4. The plane wave spinors (I.1.21,22) transform as

$$\begin{aligned} u'_1(\vec{p}) &\equiv \gamma_3\gamma_1 u_1^*(\vec{p}) = -u_2(-\vec{p}) \\ u'_2(\vec{p}) &= u_1(-\vec{p}) \\ v'_1(\vec{p}) &= v_2(-\vec{p}) \\ v'_2(\vec{p}) &= -v_1(-\vec{p}). \end{aligned} \qquad (I.3.6)$$

In ordinary quantum mechanics the operators transform as

$$\vec{x}' \equiv T^{-1}\,\vec{x}\,T = \vec{x}$$
$$\vec{p}' \equiv T^{-1}\,\vec{p}\,T = -\vec{p} \qquad \text{(I.3.7)}$$
$$\vec{J}' \equiv T^{-1}\,\vec{J}\,T = -\vec{J}$$

For the classical e.m. field, the transformation of charge and current densities under time reversal ($\rho \to \rho$, $\vec{J} \to -\vec{J}$) implies

$$\vec{E}'(\vec{x},t) = \vec{E}(\vec{x},-t)$$
$$\vec{B}'(\vec{x},t) = -\vec{B}(\vec{x},-t) \qquad \text{(I.3.8)}$$
$$\vec{A}'(\vec{x},t) = -\vec{A}(\vec{x},-t)$$
$$\phi'(\vec{x},t) = \phi(\vec{x},-t).$$

Let us now revert to the transformation of field operators.

I.3.2 Scalar or pseudoscalar fields:

The generalization of (I.3.3) is

$$\phi^T(\vec{x},t) \equiv T^{-1}\phi(\vec{x},t)\,T = \eta_T\,\phi^+(\vec{x},-t) \qquad \text{(I.3.9)}$$

where η_T is a phase to be chosen ($\eta_T = -1$ for pions).

Inserting the expansion (1.1.10) one finds

$$T^{-1}\,a_k\,T = \eta_T\,a^+_{-k}$$
$$T^{-1}\,b^+_k\,T = \eta_T\,b_{-k} \qquad \text{(I.3.10)}$$

Notice that momenta are reversed and creation (destruction) operators turn into destruction (creation) operators. This is the counterpart of kets (bras) turning into bras (kets).

I.3.3 The electromagnetic field:

The operator analog of Eqs. (I.3.8) are

$$\vec{E}^T \equiv T^{-1}\,\vec{E}(\vec{x},t)\,T = \vec{E}(\vec{x},-t) \qquad \text{(I.3.11)}$$

and similarly for $\vec{B}$, $\vec{A}$, ϕ. Thus

$$T^{-1}(\vec{E}^2 - \vec{B}^2)\,T = \vec{E}^2 - \vec{B}^2$$
$$T^{-1}(\vec{E}\cdot\vec{B})\,T = -\vec{E}\cdot\vec{B}$$

I.3.4 Dirac fields:

Eq. (I.3.5) becomes, for a spinor field,

$$\psi^T \equiv T^{-1}\,\psi(\vec{x},t)\,T = \gamma_3\gamma_1\,\tilde{\psi}^+(\vec{x},-t) \qquad \text{(I.3.12)}$$

where, again, the transpose acts only on the C-number spinors.

Using the plane wave expansion (I.1.30) together with (I.3.36), we find, in term of

creation and annihilation operators

$$T^{-1} a_{p,1} T = a^{+}_{-p,2} \quad ; \quad T^{-1} a_{p,2} T = - a^{+}_{-p,1}$$
$$T^{-1} b^{+}_{p,1} T = - b_{-p,2} \quad ; \quad T^{-1} b^{+}_{p,2} T = b_{-p,1} \qquad \text{(I.3.13)}$$

with their analog for the adjoint operators. Again, the antiunitary character of T turns creation $\rightleftarrows$ destruction operators while reversing spins and momenta.

I.3.5 Bilinear forms:

The bilinear forms of field operators can be shown to transform under T according to

$$(\bar{\psi}_a^T O_{i\alpha} \psi_b^T) = \eta^T_{i\alpha} (\bar{\psi}_b O_{i\alpha} \psi_a) \qquad \text{(I.3.14)}$$

where the notation is similar to (I.1.32) and the phase factors $\eta^T_{i\alpha}$ have the values

Scalar			+ 1			
Vector	-1 for	α = 1,2,3		+1 for	α = 4	
Tensor	-1 for	α,β = 1,2,3		+1 for	α or β	= 4
Axial vector	-1 for	α = 1,2,3		+1 for	α = 4	
Pseudoscalar			- 1			

(I.3.15)

Here the spinor fields are treated as commuting since the interchange of ψ_a and ψ_b in (I.3.14) is connected with the antiunitarity of (I.3.1).

A somewhat simpler expression obtains taking the adjoint of the left hand side of the bilinear form in (I.3.14)

$$(\bar{\psi}_a^T O_i \psi_b^T)^{+} = \eta^T_i (\bar{\psi}_a O_i \psi_b) \qquad \text{(I.3.16)}$$

where now η^T_i = +1 for S, V, P and -1 for T, A. Notice that (I.3.16) is all that is needed in connection with the T properties of matrix elements of the type (I.3.2).

Equation (I.3.16) translates directly into spinor matrix elements

$$(\bar{u}^T_a O_i u^T_b) = \eta^T_i (\bar{u}_a O_i u_b)^{*} \qquad \text{(I.3.17)}$$

where the time reversed spinor u^T coincides with what was denoted u' in (I.3.6).

I.4 PCT THEOREM

The ensemble of symmetry operations P, C, T can be combined in one single operation P C T which: i) reverses the sign of both space and ii) time coordinates while iii) converting particles into antiparticles. This operation commutes with all proper homogeneous Lorentz transformations, so that a Lorentz invariant theory will be invariant under PCT. A necessary proviso is the connection between spin and statistics, i.e. boson (spinor) fields commute (anticommute).

For creation and destruction operators of a boson field, the combination of (I.1.12), (I.2.4) and (I.3.10) gives

$$(PCT)^{-1}\, a_k\, (PCT) = \eta_P \eta_T\, b_k^+$$
$$(PCT)^{-1}\, b_k\, (PCT) = \eta_P \eta_T\, a_k^+ \qquad \text{(I.4.1)}$$

where $\eta_P = \pm 1$ for scalar/pseudoscalar particles and η_T was defined in (I.3.9).

For Dirac fields, Eqs. (I.1.31), (I.2.10) and (I.3.13) give

$$(PCT)^{-1}\, a_{p,j}\, (PCT) = \pm\, b_{p,j}^+$$
$$(PCT)^{-1}\, b_{p,j}\, (PCT) = \pm\, a_{p,j}^+ \qquad \text{(I.4.2)}$$

where the sign + (-) goes with j = 1 (j = 2).

From Eqs. (I.4.1,2) we see that the PCT operation converts a state with particles of definite spins and momentum into a dual state with antiparticles of the same spins and momenta.

For the bilinear forms, from Eqs. (I.1.33), (I.2.12) and (I.3.15) we find

$$(PCT)(\bar\psi_a O_i \psi_b)\, PCT = \eta_i (\bar\psi_{\bar a} O_i \psi_{\bar b}) \qquad \text{(I.4.3)}$$

where

$$\eta_i = +1 \qquad (S, T, P)$$
$$\eta_i = -1 \qquad (V, A) \qquad \text{(I.4.4)}$$

The above result shows an example of a completely general theorem, i.e. even (odd) rank tensors are even (odd) under PCT.

We list in the following a few of the most remarkable consequences of PCT invariance:

a) the mass of a (stable) particle is exactly equal to the mass of its antiparticle;

b) the lifetimes of unstable particles and antiparticles are equal;

c) the magnetic moments of particle and antiparticle are equal in magnitude but of opposite sign;

e) a Lagrangian which is not invariant under one of the operations P, C, T (say P for weak interactions) is necessarily not invariant under at least another one (PC is known not to be conserved while not enough precise measurements so far exist to say whether or not also T is violated but is should if PCT is to be valid).

I.5 G CONJUGATION

Strong interactions possess isotopic spin invariance (I, I_3) implying invariance under rotations in isospin space (charge independence). This symmetry of strong interactions leads to introduce a new symmetry operation, G parity, which combines charge conjugation with isospin rotations. We shall discuss the case of pions as an example of isospin triplet and of nucleons as an isospin doublet. The properties of other isospin multiplets can be inferred from these two; thus (K^+, K^0) will transform like (p, n) and (K^0, K^-) like (n, p).

The charge conjugation properties of pions and nucleons are

$$C^{-1} p C = \bar p \qquad C^{-1} \pi^\pm C = \pi^\mp$$
$$C^{-1} n C = \bar n \qquad C^{-1} \pi^0 C = \pi^0 \qquad \text{(I.5.1)}$$

where the symbols stay for the corresponding fields. The pions are expressed by

$$\pi^{\pm} = \frac{1}{\sqrt{2}} (\phi_1 \pm i \phi_2)$$
$$\pi^{0} = \phi_3 \qquad \text{(I.5.2)}$$

and from (I.5.1) we get

$$C^{-1} \phi_1 C = \phi_1$$
$$C^{-1} \phi_2 C = -\phi_2 \qquad \text{(I.5.3)}$$
$$C^{-1} \phi_3 C = \phi_3$$

We define "G conjugation" or "G-parity", the unitary operator

$$G = e^{i\pi I_2} C \qquad \text{(I.5.4)}$$

i.e. charge conjugation followed by a rotation of 180° around the y axis of isospace.

Before considering the effect of G, let us examine the rotation in isospin space

$$R = e^{i\pi I_2}$$

For a system with $I=\frac{1}{2}$, $I = \frac{1}{2}\vec{\tau}$ ($\vec{\tau}$ is the Pauli isospin operator). Then $R = i\tau_2$

This means that for the nucleon isodoublet field we have the transformation

$$R^{-1} p R = -n \qquad R^{-1} \bar{p} R = -\bar{n}$$
$$R^{-1} n R = p \qquad R^{-1} \bar{n} R = \bar{p} \qquad \text{(I.5.5)}$$

with $R^2 = -1$. Notice that the nucleon and antinucleon doublets with $I_z = \pm 1/2$ are

$$(|p\rangle, |n\rangle), (|\bar{n}\rangle, |-\bar{p}\rangle).$$

For a system with I=1, the simplest way is to study the effects of R on a cartesian vector with components ϕ_1, ϕ_2, ϕ_3 in isospin space. We find

$$R^{-1} \phi_1 R = -\phi_1$$
$$R^{-1} \phi_2 R = \phi_2 \qquad \text{(I.5.6)}$$
$$R^{-1} \phi_3 R = -\phi_3$$

If we combine the effects of R (I.5.5,6) with that of C (I.5.1) we find the transformation properties of nucleon and pion fields under G

$$G^{-1} p G = -\bar{n} \qquad G^{-1} \bar{p} G = -n$$
$$G^{-1} n G = \bar{p} \qquad G^{-1} \bar{n} G = p \qquad \text{(I.5.7)}$$
$$G^{-1} \pi G = -\pi$$

In the case of π fields, the concept of G conjugation is particularly useful in ruling out loops or vertices with a purely odd number of pion lines (this is the analog of Furry's theorem in electrodynamics).

G conjugation is also useful for classifying states with zero baryon and zero strangeness quantum numbers ($K\bar{K}$, $N\bar{N}$,...) as we shall see in detail for the $N\bar{N}$ case.

The behavior under G transformation of bilinear forms made up with nucleon fields can be

derived from (I.2.12) and (I.5.5). We find

$$\begin{aligned} G^{-1}(\bar{\psi}_p O_i \psi_n)G &= \eta_i^G (\bar{\psi}_p O_i \psi_n) \\ G^{-1}(\bar{\psi}_n O_i \psi_p)G &= \eta_i^G (\bar{\psi}_n O_i \psi_p) \\ G^{-1}(\bar{\psi}_p O_i \psi_p)G &= -\eta_i^G (\bar{\psi}_n O_i \psi_n) \end{aligned} \qquad \text{(I.5.8)}$$

with $\eta_i^G = -\eta_i^C = +1$ for V, T and $\eta_i^G = -1$ for S,A,P.

Similar results hold for hyperons in bilinear forms

$$\begin{aligned} G^{-1}(\bar{\psi}_{\Sigma^+} O_i \psi_\Lambda)G &= \eta_i^G (\bar{\psi}_\Lambda O_i \psi_{\Sigma^-}) \\ G^{-1}(\bar{\psi}_\Lambda O_i \psi_{\Sigma^-})G &= \eta_i^G (\bar{\psi}_{\Sigma^+} O_i \psi_\Lambda) \\ G^{-1}(\bar{\psi}_{\Sigma^+} O_i \psi_{\Sigma^0})G &= -\eta_i^G (\bar{\psi}_{\Sigma^0} O_i \psi_{\Sigma^-}) \end{aligned} \qquad \text{(I.5.9)}$$

I.6 MISCELLANEOUS PROPERTIES OF NUCLEONS AND ANTINUCLEONS

Mass

As we have seen, by the CPT theorem, the mass of a particle and its antiparticle are the same. Experimentally (in MeV)

$$\begin{aligned} m_p &= 938.2796 \pm .0027 \\ m_{\bar{p}} &= 938.229 \pm .049 \\ m_n &= 939.5371 \pm .0027 \\ m_{\bar{n}} &= \text{essentially unknown} \end{aligned} \qquad \begin{aligned} m_{\pi^+} \simeq m_{\pi^-} &\simeq 139.580 \\ m_{\pi^0} &\simeq 134.974 \end{aligned} \qquad \text{(I.6.1)}$$

Lifetime

CTP again tells us that particles and antiparticles have the same lifetime. Here, however, we are in an even worse condition from the view-point of the experimental verification since assuming protons and antiprotons to be stable, the limits on their lifetimes are

$$\begin{aligned} \tau_p &\gtrsim 10^{32} \text{ years} \\ \tau_{\bar{p}} &> 1.2 * 10^{-4} \text{ sec.} \end{aligned} \qquad \text{(I.6.2)}$$

(see, however, "baryon number" below).

Neutrons (and antineutrons) can decay via weak interaction (β-decay)

$$\begin{aligned} n &\to p + e^- + \bar{\nu} \\ \bar{n} &\to \bar{p} + e^+ + \nu \end{aligned}$$

and experimentally,

$$\tau_n = 918 \pm 14 \text{ sec} \qquad \text{(I.6.3)}$$

while, again $\tau_{\bar{n}}$ is experimentally unknown.

Spin

It is always CPT which tells us that

$$S_N = S_{\bar{N}} = \hbar/2. \qquad \text{(I.6.4)}$$

While no doubt exists that $S_N = \hbar/2$, the proof that $S_{\bar{N}}$ is only indirect. If we recall the comment following eq. (I.1.31) that particle-antiparticle pairs have intrinsic odd parity (i.e. if

we assume parity to be a good quantum number), the argument that $\bar{p}$ as spin $\hbar/2$ follows from the fact that at low energy the decays $p\bar{p} \to 2\pi$ and $p\bar{p} \to K\bar{K}$ are largely in S-wave (see § III.4).

Baryon number

If matter is stable (remember that doubts are raised in this respect by Grand Unification Schemes) the number of protons and neutrons must remain constant. This requires

$$B_p = B_n = +1 \qquad \text{(I.6.5)}$$

Similarly, one should assume that the total number of isolated antinucleons remain stable and since we know that the reaction $N + \bar{N} \to n\pi$ takes place, we must assign negative baryon number to antinucleons

$$B_{\bar{p}} = B_{\bar{n}} = -1 \qquad \text{(I.6.6)}$$

The much smaller lifetime of $\bar{p}$ as compared to p comes from the practical impossibility of isolating antiprotons from protons so as to prevent their annihilation into pions.

Electric charge

Experimentally, the opposite deflection of nucleons and antinucleons in a magnetic field proves that particles and antiparticles have opposite electric charge. In particular

$$Q_p = -Q_{\bar{p}} = +1 \quad , \quad Q_n = Q_{\bar{n}} = 0 \qquad \text{(I.6.7)}$$

(in units of e) as required by charge conjugation.

Magnetic moment

Charge conjugation, i.e. the operation of particle - antiparticle conjugation changes the sign of electric charge and therefore the direction of the magnetic moment of a particle, but not its spin. Thus, the relative orientation of magnetic moment and spin will be opposite for particles and antiparticles. In units of $e\hbar/2cm_p$ we have the experimental values

$$\begin{aligned} \mu_p &= 2.7928456 \pm 0.0000011 \\ \mu_{\bar{p}} &= -2.791 \pm 0.021 \\ \mu_n &= -1.913148 \pm 0.000066 \\ \mu_{\bar{n}} &= \text{unknown} \end{aligned} \qquad \text{(I.6.8)}$$

Isospin

From the Gell Mann - Nishijima formula (without charm)

$$Q = I_3 + (B+S)/2 = I_3 + Y/2$$

using $S = 0$, $Q_p = B_p = 1$, $Q_n = 0$, $B_n = 1$ we find

$I_3 = +1/2$ for the proton

$I_3 = -1/2$ for the neutron

Thus, p, n form an isodoublet.

Assuming the same rule to be valid for antinucleons, we have

$$Q_{\bar{p}} = -1 \quad , B_{\bar{p}} = -1 \Rightarrow I_{3\bar{p}} = -1/2$$
$$Q_{\bar{n}} = 0 \quad , B_{\bar{n}} = -1 \Rightarrow I_{3\bar{n}} = +1/2 \qquad (I.6.9)$$

so that also $\bar{n}$, $\bar{p}$ form an isodoublet.

The above result is in agreement with charge conjugation inverting the sign of I_3

$$I_3(p) = \tfrac{1}{2} \qquad I_3(\bar{p}) = -\tfrac{1}{2}$$
$$I_3(n) = -\tfrac{1}{2} \qquad I_3(\bar{n}) = \tfrac{1}{2} \qquad (I.6.10)$$

I.7 ISOSPIN OF THE NN̄ SYSTEM.

If we consider the isospin of a N $\bar{N}$ system we see that it results from the vector combination of two isospins $\frac{\vec{1}}{2}$ and can therefore go into either an isotriplet (I=1) or an isosinglet (I = 0). A system p $\bar{n}$ has $I_3 = I$ and a system $\bar{p}$ n has $I_3 = -1$, so that they will both belong to a pure isotriplet (I=1) state; on the contrary, systems $\bar{p}$ p and $\bar{n}$ n have $I_3 = 0$ and will therefore belong to a combination of an isotriplet (I = 1) and an isosinglet (I = 0).

To find the isospin decomposition of the N $\bar{N}$ elastic amplitudes we can apply the usual technique of projection operators. The projection operators for the N $\bar{N}$ state can be formally constructed in the same way as for the NN states since in both cases we have two particles with isospin 1/2.

Let Q_i be the eigenvalues of the (total) isospin operator $\hat{Q}=I^2$ to which the i-th eigenstate belongs. We have

$$Q_0 = 0 \quad , Q_1 = I(I+1)\big|_{I=1} = 2 \qquad (I.7.1)$$

together with

$$\hat{Q} = \frac{1}{4}(\vec{\tau}_1 + \vec{\tau}_2)^2 = \frac{\vec{\tau}_1^{\,2}}{4} + \frac{\vec{\tau}_2^{\,2}}{4} + \frac{\vec{\tau}_1 \cdot \vec{\tau}_2}{2} = \frac{3}{2} + \frac{1}{2}\vec{\tau}_1 \cdot \vec{\tau}_2 = \frac{1}{2}\left(3 + \vec{\tau}_1 \cdot \vec{\tau}_2\right) \qquad (I.7.2)$$

where $\vec{\tau}_i$ are the usual (isospin) Pauli matrices. From a general rule, the projection operators $\hat{P}_i$ are given by

$$\hat{P}_i = \prod_{j \neq i} (\hat{Q} - Q_j) \Big/ \prod_{i \neq j} (Q_i - Q_j) \qquad (I.7.3)$$

Thus, we have

$$\hat{P}_0 = \left[\frac{1}{2}\left(3 + \vec{\tau}_1 \cdot \vec{\tau}_2\right) - 2\right]/(-2) = \frac{1}{4}\left(1 - \vec{\tau}_1 \cdot \vec{\tau}_2\right)$$
$$\hat{P}_1 = \left[\frac{1}{2}\left(3 + \vec{\tau}_1 \cdot \vec{\tau}_2\right)\right]/2 = \frac{1}{4}\left(3 + \vec{\tau}_1 \cdot \vec{\tau}_2\right) \qquad (I.7.4)$$

Now, recalling that

$$(\bar{p}\,n)\begin{pmatrix}\bar{p}\\ n\end{pmatrix} = (\bar{p}\,p)\begin{pmatrix}\bar{p}\\ p\end{pmatrix} = (\bar{n}\,n)\begin{pmatrix}\bar{n}\\ n\end{pmatrix} = 1$$

$$(\bar{p}\,n)(\vec{\tau}_1\cdot\vec{\tau}_2)\begin{pmatrix}\bar{p}\\ n\end{pmatrix} = (\bar{p}\,n)(\tau_{1z}\tau_{2z})\begin{pmatrix}\bar{p}\\ n\end{pmatrix} = (\bar{p}\,n)[(-1)\cdot(-1)]\begin{pmatrix}\bar{p}\\ n\end{pmatrix} = 1$$

$$(\bar{p}\,p)(\vec{\tau}_1\cdot\vec{\tau}_2)\begin{pmatrix}\bar{p}\\ p\end{pmatrix} = (\bar{p}\,p)(\tau_{1z}\tau_{2z})\begin{pmatrix}\bar{p}\\ p\end{pmatrix} = (\bar{p}\,p)[(-1)\cdot(1)]\begin{pmatrix}\bar{p}\\ p\end{pmatrix} = -1$$

$$(\bar{p}\,p)(\vec{\tau}_1\cdot\vec{\tau}_2)\begin{pmatrix}\bar{n}\\ n\end{pmatrix} = (\bar{p}\,p)(\tau_{1x}\tau_{2x}+\tau_{1y}\tau_{2y})\begin{pmatrix}\bar{n}\\ n\end{pmatrix} = (\bar{p}\,p)[-1-1]\begin{pmatrix}\bar{p}\\ p\end{pmatrix} = -2$$

and expressing the $N\bar{N}$ amplitudes in terms of those with a definite total isospin in the s-channel T_0 and T_1, we have

$$\begin{aligned} T(\bar{p}n \to \bar{p}n) &= T(\bar{n}p \to \bar{n}p) = T_1 \\ T(\bar{p}p \to \bar{p}p) &= T(\bar{n}n \to \bar{n}n) = \tfrac{1}{2}(T_0 + T_1) \\ T(\bar{p}p \to \bar{n}n) &= T(\bar{n}n \to \bar{p}p) = \tfrac{1}{2}(T_0 - T_1) \end{aligned} \qquad \text{(I.7.5)}$$

I.8 ISOSPIN, CHARGE CONJUGATION AND G-PARITY FOR THE N $\bar{N}$ SYSTEM

We now consider in more general terms the quantum numbers problem for the N $\bar{N}$ system.

We shall formally deal with the N $\bar{N}$ system but all considerations apply to all other isodoublets (such as K mesons). Notice, however, that states with one meson and one nucleon cannot be eigenstates of G since isospin changes, say $p \rightleftarrows n$, $K^+ \rightleftarrows K^\circ$ while C turns $p \rightleftarrows \bar{p}$, $K^+ \rightleftarrows K^-$.

Let us recall that for an isospin $\frac{1}{2}$ doublet the isospin operators I_i can be described by the Pauli matrices

$$I_i = \frac{\tau_i}{2} \qquad \text{(I.8.1)}$$

where

$$\tau_1 = \begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}, \quad \tau_2 = \begin{pmatrix}0 & -i\\ i & 0\end{pmatrix}, \quad \tau_3 = \begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix} \qquad \text{(I.8.2)}$$

and the proton and neutron states are described by $\binom{1}{0}$, $\binom{0}{1}$ respectively. To deal with an N $\bar{N}$ system we enlarge our notation by replacing (I.8.1,2) by

$$I_1 = \frac{1}{2}\begin{pmatrix}0&1&0&0\\1&0&0&0\\0&0&0&1\\0&0&1&0\end{pmatrix}, \quad I_2 = \frac{1}{2}\begin{pmatrix}0&-i&0&0\\i&0&0&0\\0&0&0&-i\\0&0&i&0\end{pmatrix}, \quad I_3 = \frac{1}{2}\begin{pmatrix}1&0&0&0\\0&-1&0&0\\0&0&1&0\\0&0&0&-1\end{pmatrix} \qquad \text{(I.8.3)}$$

Owing to (I.6.10), the various states can be chosen either as

$$p=\begin{pmatrix}1\\0\\0\\0\end{pmatrix}\quad,\quad n=\begin{pmatrix}0\\1\\0\\0\end{pmatrix}\quad,\quad \bar{n}=\begin{pmatrix}0\\0\\1\\0\end{pmatrix}\quad,\quad \bar{p}=\begin{pmatrix}0\\0\\0\\1\end{pmatrix} \tag{I.8.4}$$

or, given that the states are defined only up to a phase, as

$$p=\begin{pmatrix}1\\0\\0\\0\end{pmatrix}\quad,\quad n=\begin{pmatrix}0\\1\\0\\0\end{pmatrix}\quad,\quad \bar{n}=\begin{pmatrix}0\\0\\1\\0\end{pmatrix}\quad,\quad \bar{p}=\begin{pmatrix}0\\0\\0\\-1\end{pmatrix} \tag{I.8.5}$$

For both choices (I.6.10) are satisfied.

To choose among (I.8.4) and (I.8.5), we first impose (I.5.1) which we rewrite as

$$C|p\rangle=|\bar{p}\rangle\ ,\ C|n\rangle=|\bar{n}\rangle\ ,\ C|\bar{n}\rangle=|n\rangle\ ,\ C|\bar{p}\rangle=|p\rangle \tag{I.8.6}$$

It is immediate to see that this requires C to be of the form

$$C=\begin{pmatrix}0&0&0&c_1\\0&0&c_2&0\\0&c_3&0&0\\c_4&0&0&0\end{pmatrix} \tag{I.8.7}$$

Furthermore, from (I.8.6) we see also that

$$C^2=1 \tag{I.8.8}$$

Imposing (I.8.6) we see that $C_1C_4=1$, $C_2C_3=1$ which leaves as possible solutions

$$\begin{aligned} &a)\ c_i=1 \quad (i=1,\dots 4) && c)\ c_i=-1 \quad (i=1,\dots 4)\\ &b)\ c_1=c_4=-1\ ;\ c_2=c_3=1 && d)\ c_1=c_4=1\ ;\ c_2=c_3=-1 \end{aligned} \tag{I.8.9}$$

Solutions a,b are equivalent to c,d under the change $C \rightleftharpoons -C$ so we can limit to consider the two possibilities (a) and (b).

It is straightforward to see that with the choice

$$C=\begin{pmatrix}0&0&0&1\\0&0&1&0\\0&1&0&0\\1&0&0&0\end{pmatrix} \tag{I.8.10}$$

one has the "commutation rules"

$$I_1C-CI_1=0\ ;\ I_2C+CI_2=0\ ;\ I_3C+CI_3=0 \tag{I.8.11}$$

and corresponds to using (I.8.4) for the states, whereas the choice

$$C=\begin{pmatrix}0&0&0&-1\\0&0&1&0\\0&1&0&0\\-1&0&0&0\end{pmatrix} \tag{I.8.12}$$

leads to

$$I_1C+CI_1=0\ ;\ I_2C-CI_2=0\ ;\ I_3C+CI_3=0 \tag{I.8.13}$$

and corresponds to using (I.8.5) for the states.

If we recall the definition of G (I.5.4)

$$G = C\, e^{i\pi I_2} \tag{I.8.14}$$

and we want C to commute with I_2 we have to choose

$$C = \begin{pmatrix} 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{pmatrix} \tag{I.8.12}$$

and

$$p = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} \; ; \; n = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix} \quad ; \quad \bar{n} = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix} \; ; \; \bar{p} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ -1 \end{pmatrix} \tag{I.8.5}$$

while, from (recall $\tau_2^2 = 1$)

$$e^{i\pi I_2} = e^{i\pi \tau_2/2} = \cos\frac{\pi}{2} + i\tau_2 \sin\frac{\pi}{2} = i\tau_2 = \begin{pmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \end{pmatrix}$$

we have

$$G = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{pmatrix} \tag{I.8.15}$$

From (I.8.15) and (I.8.3)

$$[I_i, G] = 0 \qquad i = 1,2,3 \tag{I.8.16}$$

so that G commutes with all the components of isospin (whereas C does not (I.8.13)).

The baryon number operator is diagonal and, to comply with (I.6.5,6) is given by

$$B = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \tag{I.8.17}$$

Notice that also B commutes with all component of isospin

$$[B, I_i] = 0 \qquad i = 1,2,3 \tag{I.8.18}$$

but anticommutes with both C

$$\{B, C\} = 0 \tag{I.8.19}$$

and G

$$\{B, G\} = 0 \tag{I.8.20}$$

Thus, B and G cannot be measured together (unless B=0).

We know, however, that in a system containing nucleons and antinucleons, B is always con-

served (it satisfies a superselection rule). Thus, for a state of baryons, the concept of G parity is useful only in the case B=0. This is just the case of an N $\bar{N}$ system for which B=0 and we can therefore measure I, I_3 and G. The eigenvalues of G for such a system can be ± 1 since $G^+G=1$ and all the members of an isomultiplet have the same value of G.

To determine the eigenvalues of G for the various N $\bar{N}$ systems, we recall that from (I.5.7) we have (see also (I.8.5) and (I.8.15))

$$G|p\rangle = -|\bar{n}\rangle \; ; \; G|n\rangle = |\bar{p}\rangle \; ; \; G|\bar{n}\rangle = |p\rangle \; ; \; G|\bar{p}\rangle = -|n\rangle \qquad \text{(I.8.21)}$$

so that there follows

$$\begin{cases} G|\bar{p}p\rangle = |n\bar{n}\rangle \\ G|\bar{n}n\rangle = |p\bar{p}\rangle \\ G|\bar{p}n\rangle = -|n\bar{p}\rangle \\ G|p\bar{n}\rangle = -|\bar{n}p\rangle \end{cases} \Rightarrow \begin{cases} G|\bar{p}p + \bar{n}n\rangle = |\bar{n}n + \bar{p}p\rangle \\ G|\bar{p}p - \bar{n}n\rangle = -|\bar{p}p - \bar{n}n\rangle \end{cases} \qquad \text{(I.8.22)}$$

Thus, to find the eigenvalues of G, one has simply to compute the effect of the exchange N $\rightleftarrows$ $\bar{N}$ with respect to the other quantum numbers.

The previous situation can be easily applied to states of K mesons where the role of B is now replaced by strangeness S. S, like B, is an additive quantum number which commutes with both B and I_i

$$[B, S] = 0 \quad , \quad [S, I_i] = 0 \qquad (i=1,2,3) \qquad \text{(I.8.23)}$$

but anticommutes with G

$$\{S, G\} = 0 \qquad \text{(I.8.24)}$$

Thus, here again, G is a good quantum number only if S=0 (i.e. for K $\bar{K}$ systems).

I.9 C and G EIGENVALUES FOR THE N $\bar{N}$ SYSTEM

Considering an N $\bar{N}$ system, we have to implement the effects of C and G operations (I.8.6) and (I.8.21)

$$C|p\rangle = |\bar{p}\rangle \; , \; C|n\rangle = |\bar{n}\rangle \; , \; C|\bar{n}\rangle = |n\rangle \; , \; C|\bar{p}\rangle = |p\rangle$$
$$G|p\rangle = -|\bar{n}\rangle \; , \; G|n\rangle = |\bar{p}\rangle \; , \; G|\bar{n}\rangle = |p\rangle \; , \; G|\bar{p}\rangle = -|n\rangle \qquad \text{(I.9.1)}$$

with their effect on an N $\bar{N}$ state with respect to the other degrees of freedom such as spin and space coordinates. First of all, we recall that while G is always a good quantum number for a state with B=0 (such as any N $\bar{N}$ system) (I.8.16,20), C is a good quantum number only if I_3=0. Thus, as far as C is concerned, we shall consider neutral N$\bar{N}$ systems (such as p $\bar{p}$ or n $\bar{n}$) for which I_3=0.

If we denote by $\phi_{N\bar{N}}(\vec{r}_1, \vec{r}_2) \simeq \phi_N(\vec{r}_1)\,\phi_{\bar{N}}(\vec{r}_2)$ the space dependent N and $\bar{N}$ wave

funcrtion and by $\chi_S^{N\bar{N}}$ the spin wave function where S denotes the total spin of the system (S = 1 for triplet or S=0 for singlet), the general form of the (neutral) N $\bar{N}$ system will be

$$\psi_S^{N\bar{N}} = \chi_S^{N\bar{N}} \phi_{N\bar{N}}(\vec{z}_1, \vec{z}_2) \tag{I.9.2}$$

Operating with C

$$C\,\psi_S^{N\bar{N}} = \psi_S^{\bar{N}N} = \chi_S^{\bar{N}N} \phi_{\bar{N}N}(\vec{z}_1, \vec{z}_2) \tag{I.9.3}$$

Recall now that the triplet (singlet) wave function is symmetric (antisymmetric) under N $\rightleftarrows$ $\bar{N}$:

$$\chi_S^{N\bar{N}} = (-1)^{S+1} \chi_S^{\bar{N}N} \tag{I.9.4}$$

Next, remember that, owing to the negative intrinsic parity of an N $\bar{N}$ system

$$\phi_{\bar{N}N}(\vec{z}_1, \vec{z}_2) = (-1)^{L+1} \phi_{N\bar{N}}(\vec{z}_1, \vec{z}_2) \tag{I.9.5}$$

Thus,

$$C\,\psi_S^{N\bar{N}} = (-1)^{L+S} \psi_S^{N\bar{N}} \tag{I.9.6}$$

or, the C eigenvalues for a neutral N $\bar{N}$ system are

$$C = (-1)^{L+S} \tag{I.9.7}$$

Let us now consider the G eigenvalues for an N $\bar{N}$ state (neutral or not). Recall that

$$e^{i\pi I_2}|p\rangle = -|n\rangle \;;\; e^{i\pi I_2}|n\rangle = |p\rangle$$
$$e^{i\pi I_2}|\bar{n}\rangle = |\bar{p}\rangle \;;\; e^{i\pi I_2}|\bar{p}\rangle = -|\bar{n}\rangle$$

so that for a state with I = 1 ($I_3 = \pm 1, 0$)

$$e^{i\pi I_2}|p\bar{n}\rangle = -|n\bar{p}\rangle \;;\; e^{i\pi I_2}|n\bar{p}\rangle = -|p\bar{n}\rangle$$
$$e^{i\pi I_2}|p\bar{p}+n\bar{n}\rangle = -|p\bar{p}+n\bar{n}\rangle \tag{I.9.8}$$

whereas for a state with I = 0:

$$e^{i\pi I_2}|p\bar{p}-n\bar{n}\rangle = |p\bar{p}-n\bar{n}\rangle \tag{I.9.9}$$

Combining (I.9.6) with (I.9.8,9) (the argument holds also for non-neutral states as can be seen by repeating for the space and spin parts of the wave function the argument leading to (I.9.6)):

$$G\,\psi_S^{N\bar{N}} = (-1)^{L+S+I} \psi_S^{N\bar{N}} \tag{I.9.10}$$

which proves that the G eigenvalues are

$$G = (-1)^{L+S+I} \tag{I.9.11}$$

I.10 QUANTUM NUMBERS FOR THE $N\bar{N}$ SYSTEM

P, C, G, J, L, I and S are the quantum numbers to be used to classify the possible eigenvalues of the $N\bar{N}$ system.

J^{PC} states:

i) Singlet (S=0) states. In this case J=L and

$$P = (-1)^{J+1} \quad ; \quad C = (-1)^{J} \tag{I.10.1}$$

so that

$$C = -P \tag{I.10.2}$$

Thus, with increasing J=0,1, ... we have

$$J=0 \quad \begin{cases} P=- \\ C=+ \end{cases}$$

$$J=1 \quad \begin{cases} P=+ \\ C=- \end{cases}$$

Thus, in the singlet case, we have

$$J^{PC} = 0^{-+}, 1^{+-}, 2^{-+}, 3^{+-}, \ldots \tag{I.10.3}$$

ii) Triplet (S=1) states.

We turn now to the triplet S=1 when J = L + S. In this case we can have two opposite parities with the same J:

$$J = L \qquad P = (-1)^{J+1} \quad ; \quad C = (-1)^{J+1}$$

$$\underline{P = C}$$

$$J = L \pm 1 \qquad P = (-1)^{J} \quad ; \quad C = (-1)^{J}$$

$$\underline{P = C}$$

In both cases

$$P = C \tag{I.10.4}$$

What distinguishes singlet from triplet states is therefore that

$$P = -C \qquad \text{(singlet)}$$

$$P = C \qquad \text{(triplet)}$$

Next, notice that for the triplet J = L, L = 0 (=J) is excluded since L = 0, S = 1 can only give

$J = L + S = 1$. Thus, we have the series

$\underline{J = L}$ $(S = 1)$ $\quad J^{PC} = 1^{++}, 2^{--}, 3^{++}, \ldots$

On the other hand, for the triplet S=1, $J = L \pm 1$ we have the series

$\underline{J = L \pm 1}$ $(S = 1)$ $J^{PC} = 0^{++}, 1^{--}, 2^{++}, \ldots$

Remark that among all these states, only the 1^{--} triplet state can be explored directly when studying the reaction

$$e^+e^- \rightleftarrows N\bar{N}$$

to the extent that this reaction proceeds via virtual γ production

$$e^+e^- \rightarrow \gamma \rightarrow N\bar{N}$$

or via vector meson (i.e. J/ψ, Υ) 1^{--} production.

Singlet (pseudoscalar 0^{-+}) can be hunted for (in e^+e^-) by first emitting one γ. Similarly one can also reach other states than 1^{--} but this is not "natural" in an e^+e^- reaction whereas they are all present when studying $N\bar{N}$ annihilation.

In the spectroscopic notation $^{2S+1}L_J$, we have the following possibilities

$\underline{\text{Singlet}}$

$J = L$ $\quad ^{2S+1}L_J = {}^1S_0, {}^1P_1, {}^1D_2, {}^1F_3, \ldots$

$\underline{\text{Triplet}}$

$J = L$ $\quad ^{2S+1}L_J = {}^3P_1, {}^3D_2, {}^3F_3, \ldots$

$J = L+1$ $\quad ^{2S+1}L_J = {}^3S_1, {}^3P_2, {}^3D_3, {}^3F_4, \ldots$

$J = L-1$ $\quad ^{2S+1}L_J = {}^3P_0, {}^3D_1, {}^3F_2, \ldots$

In this notation, the states which are directly accessible from e^+e^- are again those with $J^{PC} = 1^{--}$, i.e. ${}^3S_1$, ${}^3D_1$.

For every series, we have also to specify the two possibilities I=0, I=1 and we have therefore to add I^G to specify the state completely. Recalling that

$$G = (-1)^{L+S+I}$$

we have then the following relations between C, P and G:

$\underline{\text{Singlet}}$

$(S=0)$ $\quad J = L$; $C = -P$, $G = P(-1)^{I+1} = (-1)^{J+I}$

$\underline{\text{Triplet}}$

$(S=1)$ $\begin{cases} J = L \; ; \; C = P \; , \; G = P(-1)^I = (-1)^{J+I+1} \\ J = L \pm 1 ; \; C = P \; , \; G = P(-1)^I = (-1)^{J+I} \end{cases}$

According to whether one chooses S=0 and I=0 or I=1, the following mesonic (intermediate) states

(ω, ρ, π, η) can occur:

L S-WAVE	Spin	J	I	G	P	C	MESON
0	1	1	0	−	−	−	ω
0	1	1	1	+	−	−	ρ
0	0	0	0	+	−	+	η
0	0	0	1	−	−	+	π

This is a useful rule when working with the quark model.

I.11 SELECTION RULES FOR THE N $\bar{N}$ SYSTEM → $\pi\pi$

The selection rules are particularly useful in the low energy domain when the N $\bar{N}$ system annihilates into a small number of spinless bosons. We begin with the case $N\bar{N} \longrightarrow 2\pi$

I.11a) Selection rules for the process $N\bar{N} \longrightarrow 2\pi$:

Parity: Let us denote by P_j $(j = 1,2)$ the intrinsic parities of the two particles in the final state (in our case $P_1 = P_2$). If we denote by L_f and L_i the orbital angular momenta of the final and initial states, the initial parity is

$$P_i = (-1)^{L_i+1} \tag{I.11.1}$$

whereas the final state parity is

$$P_f = (-1)^{L_f} P_1 P_2 = (-1)^J P_1 P_2 \tag{I.11.2}$$

since the final particles are spinless bosons.

Conservation of parity in the initial and final states ($P_i = P_f$) requires then (using $P_1 = P_2$)

$$(-1)^J = (-1)^{L_i+1} \tag{I.11.3}$$

Thus, only triplet $J = L \pm 1$ states are allowed and both the singlet and triplet $J = L$ states are forbidden.

The only allowed states are, therefore,

$$J^{PC} = 0^{++}, 1^{--}, 2^{++}, 3^{--}, 4^{++}, \ldots \tag{I.11.4}$$

or

$$^{2S+1}L_J = {}^3S_1, {}^3P_0, {}^3P_2, {}^3D_1, {}^3D_3, {}^3F_2, {}^3F_4, \ldots \tag{I.11.5}$$

If, in particular, the two spinless final bosons are identical particles (such as $\pi^0\pi^0$), J must be even and only

$$J^{PC} = 0^{++}, 2^{++}, 4^{++}, \ldots$$
$$^{2S+1}L_J = {}^3P_0, {}^3P_2, {}^3F_2, {}^3F_4, \ldots \tag{I.11.6}$$

are allowed.

Notice that $N\bar{N} \rightarrow 2\pi$ is altogether forbidden in the singlet $^1S_0$ case and also that the 1^{--} state (i.e. the one with the photon quantum numbers) cannot produce $2\pi^0$'s.

Charge conjugation: A new selection rule arises in the case when $P_1 = P_2$ but $C_1 = -C_2$ (such as in the case $N\bar{N} \rightarrow K_o^1 K_o^2$, not $\pi^+\pi^-$ since a pion is not an eigenstate of C). In this case, $C_{fin} = -1$ and one has to select among (I.11.4,5) only those states with $C_{in} = -1$.
Thus, the possible states in this case can be only

$$J^{PC} = 1^{--}, 3^{--}, 5^{--}, \ldots$$

$$^{2S+1}L_J = {}^3S_1, {}^3D_1, {}^3D_3, {}^3G_3, {}^3G_5, \ldots$$

G-parity selection rules: A useful selection rule obtains when all the final particles are eigenstates of G as in the case under discussion ($N\bar{N} \rightarrow 2\pi$).

As seen previously (I.11.3), we have to consider only the triplet $J = L \pm 1$ states for which

$$G = (-1)^{J+I} = C\,(-1)^I \qquad \text{(I.11.7)}$$

Therefore, we can only have the series

$$\underline{N\bar{N} \rightarrow 2\pi\,,\ I=0\,,\ C=G}$$

$$J^{PC} = 0^{++}, 2^{++}, 4^{++} \qquad \text{(I.11.8)}$$

$$^{2S+1}L_J = {}^3P_0, {}^3P_2, {}^3F_2, \ldots$$

(implying, in particular, that also $N\bar{N} \rightarrow 2\pi$ in the 1^{--} state with I=0 is forbidden)and

$$\underline{N\bar{N} \rightarrow 2\pi\,,\ I=1\,,\ C=-G}$$

$$J^{PC} = 1^{--}, 3^{--}, 5^{--} \qquad \text{(I.11.9)}$$

$$^{2S+1}L_J = {}^3S_1, {}^3D_1, {}^3D_3, {}^3G_3, {}^3G_5, \ldots$$

In conclusion, only odd waves for the $N\bar{N}$ system are allowed to decay into 2π's in the I = 0 case and only even waves in the I = 1 case.

I.11b) Selection rules for $N\bar{N} \rightarrow m\pi$:

Much less detailed conclusions can be reached here.

G-parity: Recalling (I.9.11)

$$G = (-1)^{L+S+I}$$

we can say that

$$L+S+I \quad \text{even} \rightarrow m \text{ even}$$

$$L+S+I \quad \text{odd} \rightarrow m \text{ odd} \qquad \text{(I.11.10)}$$

C conjugation: Recalling that (2.8.7) $C = (-1)^{L+S}$

L + S odd requires C odd. If only π^0's are produced, $C_f = +1$ so $N\bar{N} \to m\pi^0$ is allowed only if m is even.

A general method to study the kinematical dependence of an annihilation matrix element can be illustrated in the case $N\bar{N} \to 3\pi$ and can be used to prove that this transition is forbidden if we start from the state $^3P_0$. To show this we use parity conservation.

In the initial state, $P = (-1)^{L+1} = 1$ for L = 1 and J = 0.

In the final state each pion has odd intrinsic parity and given that the initial total angular momentum J = 0, to match the parities of the final and initial states, with the three momenta of the final pions $\vec{k}_1$, $\vec{k}_2$, $\vec{k}_3$ we should be able to form a pseudoscalar. The only such pseudoscalar is $\vec{k}_1 \times \vec{k}_2 \cdot \vec{k}_3$ which is zero since in the C.M. system $\vec{k}_1$, $\vec{k}_2$, $\vec{k}_3$ are in a plane ($\vec{k}_1 + \vec{k}_2 + \vec{k}_3 = 0$) and the triple product vanishes.

In the next Tables we summarize the transitions $N\bar{N} \to m\pi$ which are forbidden (X totally forbidden, † forbidden by G-parity) for the two cases p $\bar{p}$ (or n $\bar{n}$) and $\bar{p}n \to m\pi$ ($m \leq 6$).

As a last comment on the consequences of G-invariance, we notice that in the case when an $N\bar{N}$ state is a pure eigenstate of I (like $\bar{p}$ n or p $\bar{n}$) and decays into an eigenstate of G (such a non strange mesonic state), G invariance tells us that the angular distribution must be target-beam symmetric or, in other words, that it must be forward-backward symmetric in the C.M. This is trivially true in the case of decay into 2π's when, as we have seen, only odd waves contribute in the case I = 0 and only even waves in the case I = 1. Since in both case there is no interference between odd and even waves, the angular distribution is indeed forward-backward symmetric in the C.M..

This theorem, which can be proven quite generally, gives testable consequences only in the case of $\bar{p}$ n annihilation which is the only accessible state of definite I = 1 ($I_z = -1$) and does not apply to neither $\bar{p}$ p nor $\bar{n}$ n which are not pure eigenstates of I and G.

I.12 ELECTROMAGNETIC DECAYS OF THE $N\bar{N}$ SYSTEM

So far we have considered only strong interaction decays of $N\bar{N}$ when G, P, C, I may be conserved. We now turn briefly to the case when also photons can be emitted and the smallness of the fine structure constant α guarantees that the most probable transitions will involve just one photon emission which will accordingly be depressed compared to processes involving just hadrons.

In the case of e.m. transitions, G and I are not conserved any longer but C and P are. Useful selection rules obtain for transitions from neutral $N\bar{N}$ systems (such as p $\bar{p}$) which are eigenstates of C belonging to the eigenvalues (I.9.7)

$$C = (-1)^{L+S} \tag{I.12.1}$$

If we consider the transition

$$p\bar{p} \to \pi^0 \gamma \tag{I.12.2}$$

$\bar{p}n$ state	spin parity	I	G	$\pi^-\pi^0$	$2\pi^-\pi^+$	$\pi^-2\pi^0$	$2\pi^-\pi^+\pi^0$	$\pi^-3\pi^0$	$3\pi^-2\pi^+$	$2\pi^-\pi^+2\pi^0$	$\pi^-4\pi^0$
$^1S_0$	0^-	1	−	×			†	†			
$^3S_1$	1^-	1	+		†	†			†	†	†
$^1P_1$	1^+	1	+	×	†	†			†	†	†
$^3P_0$	0^+	1	−	†	×	×	†	†			
$^3P_1$	1^+	1	−	×			†	†			
$^3P_2$	2^+	1	−	†			†	†			

$p\bar{p}/n\bar{n}$ state	spin parity	C	I	G	$2\pi^0$	$\eta\pi^-$	$3\pi^0$	$\pi^+\pi^-\pi^0$	$4\pi^0$	$\pi^+\pi^-2\pi^0$	$2\pi^+2\pi^-$	$5\pi^0$	$\pi^+\pi^-3\pi^0$	$2\pi^+2\pi^-\pi^0$
$^1S_0$	0^-	+	0	+	×	×	†	†				†	†	†
			1	−	×	×			†	†	†			
$^3S_1$	1^-	−	0	−	×	†	×		×	†	†	×		
			1	+	×		×	†	×			×	†	†
$^1P_1$	1^+	−	0	−	×	×	×		×	†	†	×		
			1	+	×	×	×	†	×			×	†	†
$^3P_0$	0^+	+	0	+			×	×				†	†	†
			1	−	†	†	×	×	†	†	†			
$^3P_1$	1^+	+	0	+	×	×	†	†				†	†	†
			1	−	×	×			†	†	†			
$^3P_2$	2^+	+	0	+			†	†				†	†	†
			1	−	†	†			†	†	†			

where $C_{fin} = -1$ $(C_{\pi^0} = +1,\ C_\gamma = -1)$ we see that only <u>L + S = odd</u> transitions are allowed.

On the contrary, in the case

$$p\bar{p} \to \varrho^0\gamma\ ,\ \omega^0\gamma \tag{I.12.3}$$

where $C_{fin} = +1$ $(C_{\varrho^0} = C_{\omega^0} = -1,\ C_\gamma = -1)$ the only allowed transitions require <u>L+S=even.</u>

If we now consider

$$p\bar{p} \to \pi\pi\gamma \tag{I.12.4}$$

we have to distinguish the two possibilities $C_{p\bar{p}} = +1$ and $C_{p\bar{p}} = -1$ (or L+S even, L+S=odd respectively).

Remembering that $C_{\pi^0\pi^0} = 1$ we see that the case $C_{p\bar{p}} = -1$ contributes to $\pi^0\pi^0\gamma$ and $\pi^+\pi^-\gamma$ decays ($C_{\pi^+\pi^-}$ can be either +1 or -1 and the case $C_{\pi^+\pi^-} = 1$ has to be selected since $C_\gamma = -1$). On the other hand, the choice $C_{p\bar{p}} = +1$ can contribute only to $\pi^+\pi^-\gamma$ (where we select $C_{\pi^+\pi^-} = -1$) and the $\pi^0\pi^0\gamma$ transition is strictly forbidden.

Similarly, $p\bar{p} \longrightarrow K_1^0 K_2^0 \gamma$ is allowed only for L+S even and $p\bar{p} \longrightarrow K_o^1 K_o^1 \gamma$, $K_o^2 K_o^2 \gamma$ is allowed only for L+S odd.

PART II

BASIC KINEMATICS OF $N\bar{N}$ DECAYS

In this part we shall discuss the kinematics of the basic reaction modes for the N $\bar{N}$ system such as elastic N $\bar{N}$ reactions, two and three pion annihilations. The main point will be of providing the theoretical tools to study renonances in formation experiments (i.e. states formed in N $\bar{N}$ decay) and to find the allowed quantum numbers. Though somewhat tedious, this exercise is particularly useful in studying low energy N $\bar{N}$ physics.

II.1 ELEMENTS OF KINEMATICS OF N $\bar{N}$ ELASTIC SCATTERING

Most of the kinematics of N $\bar{N}$ elastic scattering can be borrowed directly from that for N N scattering. A number of important differences must, however, be emphasized:

i) In N N scattering, below pion production threshold the only open channel is the elastic one and the phase shifts are correspondingly real. In N $\bar{N}$, on the contrary, we have a large number of annihilation channels open such as

$$p\bar{p} \to n\pi \quad , \; n\pi + m(K\bar{K})$$

In particular, $2m/\mu \approx 13$ is the number of π's that can be produced by N $\bar{N}$ at rest. As a consequence, the phase shifts in N $\bar{N}$ are always complex (absorption is always present).

ii) In N N the symmetry for nucleon-nucleon interchange (Pauli symmetry) reduces the number of partial waves to one half (once the total spin and isospin are given, only either even ℓ or odd ℓ waves are present). Again, this is not true in N $\bar{N}$ when both even and odd waves are present. Being these complex (as compared to their being real in the case N N), this makes four times larger the number of real parameters needed to describe N $\bar{N}$ elastic scattering at low energy as compared to the N N case.

iii) Last but not least, the simplest N N elastic reaction (p p) is a pure I = 1 state whereas the simplest N $\bar{N}$ reaction (p $\bar{p}$) is a mixture of both I = 0 and I = 1, so that a further doubling of parameters is involved.

In what concerns the kinematics of N $\bar{N}$ $\longrightarrow$ N $\bar{N}$ reaction

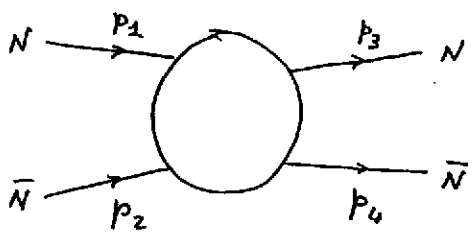


Fig. II.1

its structure differs very little from the kinematics of NN $\longrightarrow$ NN (other than in the points previously stressed). In particular, we still have energy momentum conservation which reads

$$p_1 + p_2 = p_3 + p_4 \qquad \text{(II.1.1)}$$

We introduce the so called Mandelstam variables

$$\begin{aligned} s &= (p_1+p_2)^2 \\ t &= (p_1-p_3)^2 \\ u &= (p_1-p_4)^2 \end{aligned} \qquad s+t+u = 4m^2 \tag{II.1.2}$$

In the Lab system $[p_1 \equiv (m;0,0,0)]$, $p_2 \equiv (E_2;0,0,p_2)]$ we have

$$s = 2m^2 + 2mE_2 = 4m^2 + 2mT \tag{II.1.3}$$

being T the $\bar{N}$ kinetic energy $(E_2 = m+T = (m^2+\vec{p}_2^{\,2})^{1/2})$ while in the CM system $(\vec{p}_1+\vec{p}_2=0\,,\ \vec{p}_1 = \vec{k} = -\vec{p}_2)$

$$\begin{cases} s = 4(m^2+\vec{k}^2) \\ t = -2k^2(1-\cos\theta) \\ u = -2k^2(1+\cos\theta) \end{cases} \tag{II.1.4}$$

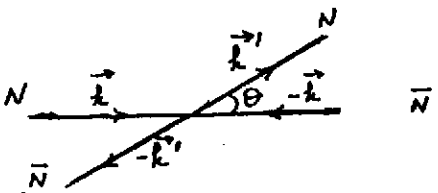


Fig. II.2

We see that we go from the N $\bar{N}$ case (Fig. II.1) to the N N reaction

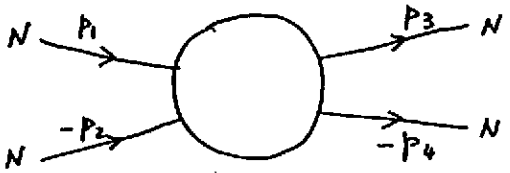


Fig. II.3

by just reverting the corresponding arrow lines, i.e. to making the inversions $p_2 \longrightarrow -p_2$, $p_4 \longrightarrow -p_4$. In this case, the variable s (II.1.2) which played the role of the total squared C.M. energy in the N $\bar{N}$ reaction becomes a momentum transfer while the momentum transfer u plays now the role of the total squared C.M. energy for the N N reaction. All in all s $\rightleftarrows$ u, t $\longleftrightarrow$ t, and we have, in the respective C.M. frames ($\vec{p}_1 + \vec{p}_2 = 0$ for N $\bar{N}$ and $\vec{p}_1 + \vec{p}_4 = 0$ for N N)

$$\overset{NN}{\begin{cases} s = -2k_N^2(1+\cos\theta_N) \\ t = -2k_N^2(1-\cos\theta_N) \\ u = 4(k_N^2+m^2) \end{cases}} \qquad \overset{N\bar{N}}{\begin{cases} s = 4(k_A^2+m^2) \\ t = -2k_A^2(1-\cos\theta_A) \\ u = -2k_A^2(1+\cos\theta_A) \end{cases}} \tag{II.1.4'}$$

It is useful to introduce instead of $p_1 \ldots p_4$ four vectors which have simpler properties under crossing $p_2 \rightleftarrows -p_4$ such as

$$P = \frac{1}{2}(p_1+p_3)\,,\quad N = \frac{1}{2}(p_2+p_4)\,,\quad \Delta = p_1-p_3 = p_4-p_2 \tag{II.1.5}$$

which under $p_2 \longrightarrow -p_4$, $p_4 \longrightarrow -p_2$ give

$$P, \Delta \longrightarrow P, \Delta \qquad N \longrightarrow -N$$

It is trivial to show that P, N are orthogonal to Δ as four vectors

$$P\cdot\Delta = N\cdot\Delta = 0 \qquad \text{(II.1.6)}$$

$$N\Delta = \tfrac{1}{2}(p_2+p_4)(p_4-p_2) = \tfrac{1}{2}(p_4^2-p_2^2) = 0 \qquad (p_2^2=p_4^2=m^2).$$

Furthermore, comparing with (II.1.2)

$$\Delta^2 = t \;,\; P^2 = N^2 = \tfrac{1}{4}(s+u) \qquad \text{(II.1.7)}$$

as can be seen from

$$P^2 = \tfrac{1}{4}(p_1+p_3)^2 = \tfrac{1}{4}(p_1^2+p_3^2+2p_1p_3) = \tfrac{1}{4}\left(2p_1^2+2p_3^2-(p_1-p_3)^2\right) =$$
$$= \tfrac{1}{4}(4m^2-t) = \tfrac{1}{4}(s+u)$$

having used

$$s+t+u = 4m^2$$

Spin properties.

As in the elastic N N case, we end up with five invariant amplitudes. This can be seen as follows for N N. Let us, for convenience, imagine first that the nucleon of momentum p_1 "flows" into the nucleon of momentum p_3 and that the nucleon of momentum p_2 "flows" into p_4 identifying two sets of Lorentz covariants for "space 1" and "space 2" corresponding to the matrix elements of all possible γ matrices

$$1,\ \gamma_5,\ \gamma_\mu,\ \gamma_5\gamma_\mu,\ \sigma_{\mu\nu} = \frac{i}{2}(\gamma_\mu\gamma_\nu - \gamma_\nu\gamma_\mu)$$

between the corresponding spinors $\bar{u}(p_3) \ldots u(p_1)$ (for space 1) and $\bar{u}(-p_4) \ldots u(-p_2)$ (for space 2). Every index γ_μ can only be saturated with P_μ, N_μ (II.1.5).

The matrix elements involving $\sigma_{\mu\nu}$ however, are absent because $\sigma_{\mu\nu}$ is antisymmetric in $\mu\nu$ and the contraction with the symmetric tensor $N_\mu N_\nu$ or $P_\mu P_\nu$ gives zero. Thus, a priori, in "space 1" we have the following structure

$$a_1 S^{(1)} + b_1 V^{(1)} + c_1 PS^{(1)} + d_1 PV^{(1)} \qquad \text{(II.1.8)}$$

(and a similar structure holds for space 2) where a_1, b_1, c_1, d_1 are invariant scalar functions and

$$S^{(1)} \equiv \bar{u}(p_3)u(p_1) \qquad V^{(1)} \equiv \bar{u}(p_3)\,i\gamma_\mu^{(1)}N_\mu\,u(p_1)$$
$$PS^{(1)} \equiv \bar{u}(p_3)\gamma_5^{(1)}u(p_1) \qquad PS^{(1)} \equiv \bar{u}(p_3)\,i\gamma_5^{(1)}\gamma_\mu^{(1)}N_\mu\,u(p_1) \qquad \text{(II.1.9)}$$

where the first line gives true Lorentz scalars and the second line Lorentz pseudo-scalars.

Notice that the Dirac equations (I.1.24, 25) for $u, \bar{u}$

$$(i\gamma p + m)u(p) = 0$$
$$\bar{u}(p)(i\gamma p + m) = 0$$

guarantees that only saturation with the four vector N_μ gives non zero (when particles 1,3 are in play) and only saturation with P_μ gives non zero (when particles 2,4 are in play).

Multiplying the two structures (II.1.8) for spaces 1 and 2 we are a priori left with eight

possible terms of the form

$$S^{(1)}S^{(2)}\ ,\ S^{(1)}V^{(2)}\ ,\ S^{(2)}V^{(1)}\ ,\ V^{(1)}V^{(2)}$$
$$PS^{(1)}\ PS^{(2)}\ ,\ PS^{(1)}\ PV^{(2)}\ ,\ PS^{(2)}\ PV^{(1)}\ ,\ PV^{(1)}\ PV^{(2)} \qquad \text{(II.1.10)}$$

since, of course, contracting Lorentz scalars with Lorentz pseudo-scalars gives no contribution because of parity conservation.

Having imposed already parity, we still have to enforce time reversal conservation. Rather than working out the time reversal transformation of each term, we use the following simple physical arguments (see, however, eq. (I.3.16)).

Scalars, pseudo-scalars and vectors are unaffected by changing the direction of time and, therefore, S, PS and V are invariant under time reversal. This is not the case for pseudo-vectors which are the analog of the magnetic moment generated by the current circulating in a ring. Changing $t \longrightarrow -t$ inverts the sign of the current, and therefore, of the magnetic moment. In other words, pseudo-vectors are odd under time reversal (remark that, having already imposed P, by the PCT theorem, T invariance is the same as C conjugation. The latter, changing the sign of the electric charge of the particle circulating in the ring would have had the same effects). Thus, time reversal kills the two terms $PS^{(1)}\ PV^{(2)}$ and $PS^{(2)}\ PV^{(1)}$ in (II.1.10).

Finally, the identity of protons for p p scattering (or charge symmetry for p n) demands that if we exchange particles 1 (3) and 2 (4) both in spin space $(\gamma^{(1)} \longleftrightarrow \gamma^{(2)})$ and momentum space (P $\longleftrightarrow$ N) the amplitude must remain the same. This is already the case for $S^{(1)}\ S^{(2)}$, $V^{(1)}\ V^{(2)}$, $PS^{(1)}\ PS^{(2)}$, $PV^{(1)}\ PV^{(2)}$ but requires the coefficients of $V^{(1)}\ S^{(2)}$ and of $V^{(2)}\ S^{(1)}$ to be the same so that we are finally left with a total of five independent scalar amplitudes to describe N N scattering which we can write in the general form

$$\bar{u}(p_3)\,\bar{u}(-p_4)\Big[G_1(s,t,u) + i(\gamma^{(1)}\!\cdot N + \gamma^{(2)}\!\cdot P)G_2(s,t,u) +$$
$$+ i(\gamma^{(1)}\!\cdot N)\,i(\gamma^{(2)}\!\cdot P)G_3 + (i\gamma_5^{(1)}\gamma^{(1)}\!\cdot N)(i\gamma_5^{(2)}\gamma^{(2)}\!\cdot P)G_4 + \gamma_5^{(1)}\gamma_5^{(2)}G_5(s,t,u)\Big]u(-p_2)\,u(p_1). \qquad \text{(II.1.11)}$$

Each of the above amplitudes comes, of course, in two isospin states I = 0, I = 1.

Crossing in spin space.

We denote the N N and N $\bar{\text{N}}$ matrix elements of the scattering operator by

$$T_{NN\to NN} = \bar{u}_i(p_3)\,\bar{u}_k(-p_4)\,M_{ij}^{kl}\,u_l(-p_2)\,u_j(p_1) \qquad \text{(II.1.12)}$$

$$T_{N\bar{N}\to N\bar{N}} = \bar{u}_i(p_3)\,\bar{v}_k(p_4)\,M_{ij}^{kl}\,v_l(p_2)\,u_j(p_1) \qquad \text{(II.1.13)}$$

where an ingoing antinucleon corresponds to an outgoing nucleon, v, $\bar{v}$ are the antiparticle spinors and i, j, k, l are spin indices. The matrix M has the γ structure given in (II.1.11).

The substitution law tells that the same matrix M describes the two processes provided the appropriate analytical continuation of the various scalar functions G_i (i = 1,...5) in the respective physical regions of reactions N N and N $\bar{\text{N}}$ (II.1.4') is made.

Crossing requires $p_2 \longleftrightarrow -p_4$ (i.e. P $\longrightarrow$ P, N $\longrightarrow$ -N). To see how this affects the scalar functions G_i, one has to recall how v(p) is related to u (-p). Recalling (I.1.24,25)

$$(i\gamma p + m)\, u(p) = 0 \qquad \bar{u}(p)\,(i\gamma p + m) = 0$$

$$(i\gamma p - m)\, v(p) = 0 \qquad \bar{v}(p)\,(i\gamma p - m) = 0$$

together with (I.1.19)

$$\bar{u} = u^{+}\gamma_4 \qquad \gamma_4 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

and

$$C\,\gamma_\mu^{T}\,C^{-1} = -\gamma_\mu$$

one finds under charge conjugation the following transformation properties

$$\begin{aligned}
&\bar{v}(p_4)\,v(p_2) = -\,\bar{u}(-p_4)\,u(-p_2) \\
&\bar{v}(p_4)\,\gamma_\mu^{(2)}\,v(p_2) = \bar{u}(-p_4)\,\gamma_\mu^{(2)}\,u(-p_2) \\
&\bar{v}(p_4)\,\gamma_5^{(2)}\,v(p_2) = -\,\bar{u}(-p_4)\,\gamma_5^{(2)}\,u(-p_2) \\
&\bar{v}(p_4)\,\gamma_5^{(1)}\gamma_\mu^{(1)}\,v(p_2) = -\,\bar{u}(-p_4)\,\gamma_5^{(2)}\gamma_\mu^{(2)}\,u(-p_2)
\end{aligned} \qquad \text{(II.1.14)}$$

Thus, using (II.1.14) and keeping in mind that under crossing P $\longrightarrow$ P, N $\longrightarrow$ -N, we conclude that

i) The coefficients of G_1, G_3, G_5 change sign.

ii) The coefficients of G_2, G_4 do not change sign.

Recalling that crossing on u, s, t amounts to

$$\begin{aligned}
&s \rightleftarrows u \\
&t \leftrightarrow t
\end{aligned} \qquad \text{(II.1.15)}$$

we finally have the following transformation properties for the scalar functions G_i (s, t, u) under crossing

$$G_i(s,t,u) = (-1)^i\, G_i(u,t,s) \qquad \text{(II.1.16)}$$

The non-relativistic limit.

We will recall that in the non-relativistic limit, the matrix elements between four dimensional spinors can be rewritten in terms of matrix elements between two dimensional Pauli spinors of expressions involving Pauli matrices (see Section I.1.4). The amplitude (for the two isospin values i = 0, 1) can be written as

$$\begin{aligned}
T^{(i)} = {} & \alpha^{(i)} + \beta^{(i)}\left(\vec{\sigma}^{(1)}\cdot\hat{\vec{n}}\right)\left(\vec{\sigma}^{(2)}\cdot\hat{\vec{n}}\right) + i\gamma^{(i)}\left(\vec{\sigma}^{(1)}+\vec{\sigma}^{(2)}\right)\cdot\hat{\vec{n}} \\
& + \delta^{(i)}\left(\vec{\sigma}^{(1)}\cdot\hat{\vec{m}}\right)\left(\vec{\sigma}^{(2)}\cdot\hat{\vec{m}}\right) + \varepsilon^{(i)}\left(\vec{\sigma}^{(1)}\cdot\hat{\vec{l}}\right)\left(\vec{\sigma}^{(2)}\cdot\hat{\vec{l}}\right)
\end{aligned} \qquad \text{(II.1.17)}$$

where $\hat{\vec{n}}$, $\hat{\vec{l}}$, $\hat{\vec{m}}$ are the unit vectors in the directions $\vec{l} = \vec{k} + \vec{k}'$, $\vec{m} = \vec{k} - \vec{k}'$, $\vec{n} = \vec{m} \times \vec{l}$ ($\vec{k}$ and $\vec{k}'$ where defined in Fig. (II.2) and are the initial and final nucleon three momenta in the N $\bar{N}$ c.m. system).

The five functions $\alpha, \beta, \gamma, \delta, \varepsilon$ can be expressed in terms of the G_i's (i=1,...5). We shall simply recall without proof[5] that introducing the auxiliary variables (in the C.M.)

$$E = (\vec{k}^2 + m^2)^{1/2} \quad ; \quad \lambda = \frac{1}{2}(2E^2 - m^2) \quad ; \quad \nu = \frac{1}{m} P \cdot N = \lambda + t/4m^2$$

$$A = 1 + \frac{t}{4m\lambda} \frac{2E+m}{E+m} \quad ; \quad B = 1 + \frac{t}{4m(E+m)}$$

and the 3 x 3 matrix

$$a = \begin{pmatrix} B^2 & -2\lambda AB & \lambda^2 A^2 \\ \frac{k^2 + t/4}{m^2(E+m)^2} & \frac{t}{4}\left[1 \;,\; 2(2E+m) \;,\; (2E+m)^2\right] & \\ \frac{\sqrt{-t(k^2+t/4)}}{2m(E+m)}\left[B \;,\; (m+2E)B - \lambda A \;,\; -(2E+m)\lambda A\right] & & \end{pmatrix} \qquad \text{(II.1.18)}$$

the relation between the two sets of amplitudes is given by

$$\begin{pmatrix} \alpha \\ \beta \\ \gamma \end{pmatrix} = a \begin{pmatrix} G_1 \\ G_2 \\ G_3 \end{pmatrix} \qquad \text{(II.1.19)}$$

$$\delta = \frac{t}{4m^2} G_4$$

$$\varepsilon = -\frac{4E^2}{m^2}\left(k^2 + \frac{t}{4}\right) G_5$$

Amplitudes with definite total spin.

We end the list of various amplitudes used in the literature, by introducing the amplitudes with definite total spin singlet (S=0) or triplet (S=1). There is just one singlet amplitude ($T^s(E, \theta)$) and, a priori five triplet amplitudes $T^t_{m'm}$ where m, m' are the projections of the total spin. The m m' dependence is relevant only for spin correlation parameters and not for the unpolarized cross-section. If we retain in $T^t_{mm'}$ only the θ dependence, their φ dependence obtains by multiplying them by $e^{i(m-m')\varphi}$. The $T^t_{-m,-m'}$ amplitudes are related to the $T^t_{m,m'}$ (via the properties of the d functions) so that, apparently, we are left with five triplet amplitudes

$$T^t_{00} \;,\; T^t_{10} \;,\; T^t_{01} \;,\; T^t_{11} \;,\; T^t_{1-1}$$

among which, however, there exists one relationship

$$T^t_{11} - T^t_{1-1} - T^t_{00} = \sqrt{2}\, \operatorname{ctg} \theta \quad (T^t_{10} + T^t_{01}) \qquad \text{(II.1.20)}$$

so that only 5 amplitudes altogether (singlet plus triplet) are actually linearly independent as it should be.

The following correspondences exist

$$\begin{cases} \alpha = \frac{E}{4m}\left(2\,T_{11}^{t} + T_{00}^{t} + T^{s}\right) \\ \beta = \frac{E}{4m}\left(-2T_{11}^{t} + T_{00}^{t} - T^{s}\right) \\ \gamma = \sqrt{2}\ \frac{E}{4m}\ \left(T_{10}^{t} - T_{01}^{t}\right) \\ \delta = \frac{E}{4m}\left\{\left(T_{11}^{t} + T_{1-1}^{t} - T^{s}\right) - \sec\theta\left(T_{11}^{t} - T_{1-1}^{t} - T_{00}^{t}\right)\right\} \\ \varepsilon = \frac{E}{4m}\left\{\left(T_{11}^{t} + T_{1-1}^{t} - T^{s}\right) + \sec\theta\left(T_{11}^{t} - T_{1-1}^{t} - T_{00}^{t}\right)\right\} \end{cases} \qquad \text{(II.1.21)}$$

Recall, one again, that two of these amplitudes exist for isospin I = 0, 1, respectively.

Partial wave expansions.

We end this section by just giving the expressions for the partial wave decompositions which are particularly useful in the low energy domain when few partial wave are relevant and, in particular, when resonances of given $^{2S+1}L_J$ quantum numbers can be produced. In this case, the proper amplitudes to use are the T^s and T^t amplitudes just introduced and the only formal difference between the N N and N $\bar{\text{N}}$ case, as already noticed, is that the latter case both even and odd waves contribute at the same time (besides, they are all complex).

The singlet case is very simple

$$T^{s}(k,\theta) = \frac{1}{k}\sum_{L=0}^{\infty}(2L+1)\,S_L(k)P_L(\cos\theta) \qquad \text{(II.1.22)}$$

where

$$S_L(k) = e^{i\delta_L^{(s)}(k)}\sin\delta_L^{(s)}(k) \qquad \text{(II.1.23)}$$

is the (singlet) S matrix element and $\delta_L^{(s)}(k)$ are the complex phase shifts for the singlet state.

The triplet case is more complex and it is convenient to consider separately the case L = J for which we introduce the partial wave amplitudes

$$\alpha_J^J = \sin\delta_J^J\ e^{i\delta_J^J} \qquad \text{(II.1.24)}$$

from the cases L = J $\pm$ 1. For the latter we introduce the partial wave amplitudes

$$\alpha_J = e^{i\delta_\alpha^J}\sin\delta_\alpha^J \quad , \quad \beta_J = e^{i\delta_\beta^J}\sin\delta_\beta^J \qquad \text{(II.1.25)}$$

and the so-called admixture parameter ε_J which allows the transition from L-1 $\longleftrightarrow$ L+1 at fixed J. With these definitions, the amplitudes a_{J-1}^J , b_{J-1}^J with L = J - 1 and a_{J+1}^J, b_{J+1}^J with L = J + 1 are given by

L = J - 1

$$a^J_{J-1} = \alpha_J \cos^2\varepsilon_J + \beta_J \sin^2\varepsilon_J - \sqrt{\frac{J}{J+1}}\ \frac{1}{2}(\alpha_J - \beta_J)\sin^2\varepsilon_J$$

$$b^J_{J-1} = \alpha_J \cos^2\varepsilon_J + \beta_J \sin^2\varepsilon_J + \sqrt{\frac{J+1}{J}}\ \frac{1}{2}(\alpha_J - \beta_J)\sin^2\varepsilon_J \qquad \text{(II.1.26)}$$

L = J + 1

$$a^J_{J+1} = \alpha_J \sin^2\varepsilon_J + \beta_J \cos^2\varepsilon_J - \sqrt{\frac{J+1}{J}}\ \frac{1}{2}(\alpha_J - \beta_J)\sin^2\varepsilon_J$$

$$b^J_{J+1} = \alpha_J \sin^2\varepsilon_J + \beta_J \cos^2\varepsilon_J + \sqrt{\frac{J}{J+1}}\ \frac{1}{2}(\alpha_J - \beta_J)\sin^2\varepsilon_J \qquad \text{(II.1.27)}$$

It is in terms of the amplitudes a^J_L , b^J_L (II.1.24, 26, 27) that the triplet amplitudes $T^t_{m'm}$ have the following (relatively) simple expansions in terms of Legendre polynomials $P_L(x)$ and of associated Legendre functions $P^{(m)}_L(x)$

$$P_\ell^m(x) = (1-x^2)^{m/2}\ \frac{d^m}{dx^m} P_\ell(x) \qquad \text{(II.1.28)}$$

$$T^t_{00} = \frac{1}{k}\sum_{L=0}^{\infty}\left\{(L+1)\,b_L^{L+1} + L\,b_L^{L-1}\right\}P_\ell(\cos\vartheta)$$

$$T^t_{11} = \frac{1}{2k}\sum_{L=0}^{\infty}\left\{(L+2)\,a_L^{L+1} + (2L+1)\,a_L^{L} + (L-1)\,a_L^{L-1}\right\}P_L(\cos\vartheta)$$

$$T^t_{10} = \frac{1}{\sqrt{2}\,k}\sum_{L=1}^{\infty}\left\{b_L^{L+1} - b_L^{L-1}\right\}P_L^{(1)}(\cos\vartheta)$$

$$T^t_{01} = \frac{1}{\sqrt{2}\,k}\sum_{L=1}^{\infty}\left\{-\frac{L+2}{L+1}\,a_L^{L+1} + \frac{2L+1}{L(L+1)}\,a_L^{L} + \frac{L-1}{L}\,a_L^{L-1}\right\}P_L^{(1)}(\cos\vartheta) \qquad \text{(II.1.29)}$$

$$T^t_{1-1} = \frac{1}{2k}\sum_{L=2}^{\infty}\left\{\frac{1}{L+2}\,a_L^{L+1} - \frac{2L+1}{L(L+1)}\,a_L^{L} + \frac{1}{L}\,a_L^{L-1}\right\}P_L^{(2)}(\cos\vartheta)$$

II.2 KINEMATICS OF TWO PION ANNIHILATIONS

Let us now consider the simplest case of $N\bar{N} \longrightarrow m\pi$ reactions, i.e. $N\bar{N} \to 2\pi$ (or, more generally, $N\bar{N} \longrightarrow M\,M$ where M is a pseudoscalar spinless boson, i.e. either π or K). This process is schematized by

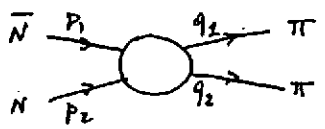


Fig. II.4

The Mandelstam variables are now

$$\begin{cases} s = (p_1+p_2)^2 = (q_1+q_2)^2 \\ t = (p_1-q_1)^2 = (p_2-q_2)^2 \\ u = (p_1-q_2)^2 = (p_2-q_1)^2 \end{cases} \qquad s+t+u = 2(m^2+\mu^2) \qquad \text{(II.2.1)}$$

Spin structure

The spin structure of $N\bar{N} \to 2\pi$ can be most easily obtained by noticing that it is obtained from $N\pi \to N\pi$ by crossing

$$q_2 \to -q_2 \qquad p_1 \to -p_1$$
$$s \leftrightarrow t \qquad u \text{ fixed} \tag{II.2.2}$$

In terms of the usual invariant amplitude decomposition for $\pi N \to \pi N$

$$T_{N\pi \to N\pi} = -A + iB\gamma_\mu \tfrac{1}{2}(q_1+q_2)_\mu \tag{II.2.3}$$

we get for $N\bar{N} \to \pi\pi$

$$T_{N\bar{N} \to \pi\pi} = -A + iB\gamma_\mu \tfrac{1}{2}(q_1-q_2)_\mu \tag{II.2.4}$$

where use has been made of (II.2.2) so that the invariant amplitudes A, B in (II.2.4) obtain from (II.2.3) by interchange of $s \rightleftarrows t$.

The invariant matrix element T_{fi} for the process of Fig. II.4

$$T_{fi} = \bar{u}(p_2)\left[-A + i\gamma\cdot\frac{(q_1-q_2)}{2}B\right]v(p_2) \tag{II.2.5}$$

can be rewritten here again using two dimensional Pauli matrices and spinors in the C.M. and one gets

$$T_{fi} = \chi_N^+ \left[h_1\,\vec{\sigma}\cdot\vec{k} + h_2\,\vec{\sigma}\cdot\vec{q}\right]\chi_{\bar{N}} \tag{II.2.6}$$

where

$$\begin{cases} h_1 = -\frac{1}{m}\left[A + B\,\frac{\vec{k}\cdot\vec{q}}{E+m}\right] \\ h_2 = \frac{E}{m}B \end{cases} \tag{II.2.7}$$

This form is the analog of the decomposition $\chi_N^+\left[f + ig\,\vec{\sigma}\cdot\hat{n}\right]\chi_N$ for πN scattering in terms of the spin - non flip and spin flip amplitudes f and g.

A further useful decomposition is in terms of helicity amplitudes $F_{\lambda\lambda'}$ where the indices refer to the quantization of the N, $\bar{N}$ spins along the direction of motion. Once again there are two independent spin amplitudes $F_{++} = F_{--}$, $F_{+-} = -F_{-+}$ whose partial wave expansion is given by

$$F_{++} = \frac{1}{2q}\sum_J (2J+1)\,T_+^J(s)\,P_J(\cos\theta)$$
$$F_{+-} = \frac{1}{2q}\sum_J \frac{2J+1}{[J(J+1)]^{1/2}}\,T_-^J(s)\,\sin\theta\,P_J'(\cos\theta) \tag{II.2.8}$$

and the $T_\pm^J$ partial wave amplitudes are given by the unitary S-matrix partial waves through

$$T_\pm^J = -i\left(\frac{q}{k}\right)^{1/2} S_\pm^J \tag{II.2.9}$$

The connection with the partial wave projections of the invariant amplitudes A, B used previously

$$(A_J, B_J) = \int_{-1}^{1} dz\, P_J(z)\, (A, B) \tag{II.2.10}$$

is given by

$$\begin{cases} T_+^J = \frac{2}{\sqrt{s}} \frac{q}{k} \frac{1}{8\pi} \left\{ -k^2 A_J + \frac{mqk}{2J+1} \left[(J+1) B_{J+1} + J B_{J-1} \right] \right\} \\ T_-^J = \frac{q}{k} \frac{1}{8\pi} \frac{\vec{k}\cdot\vec{q}}{} \frac{\left[J(J+1) \right]^{1/2}}{2J+1} (B_{J-1} - B_{J+1}) \end{cases} \tag{II.2.11}$$

A last final form on which we shall come back later one concerns the decomposition of spin amplitudes in terms of amplitudes of definite J = L+1 or J = L-1 (remember that J=L is forbidden as we have discussed previously (I.11.3). This decomposition will turn out to be particularly useful because, as we have discussed, Bose symmetry allows only even (odd) J values according to whether I is = 0 (I = 1).

Before doing this, we shall discuss the relation between the differential cross-section and the transition matrix elements.

Differential cross-section ($N\bar{N} \longrightarrow 2\pi$)

The T-matrix element between the initial (i) and final (f) state is the matrix element between a positive energy spinor $\bar{u}(p_2)$ for N and a negative energy spinor $v(p_1)$ for $\bar{N}$

$$T_{fi} = \bar{u}(p_2)\, T\, v(p_1) \tag{II.2.12}$$

where the usual relation holds between T_{fi} and the corresponding S-matrix element S_{fi}

$$S_{fi} = -i(2\pi)^4 \delta^{(4)}(p_1 + p_2 - q_1 - q_2) \frac{m}{(2\pi)^6} \frac{1}{\sqrt{E_1 E_2 \omega_1 \omega_2}} T_{fi} \tag{II.2.13}$$

where m is the nucleon mass, E_i (ω_i) the nucleons (pions) energy and the elastic channel is absent (no δ_{fi} term).

The above form corresponds to having chosen a plane wave for a scalar particle normalized to one particle in the volume $(2\pi)^3$, i.e.

$$f_q(\vec{x}) = \frac{1}{\sqrt{2\omega}} \frac{e^{i\vec{q}\cdot\vec{x}}}{(2\pi)^{3/2}} \tag{II.2.14}$$

giving

$$\langle q | q' \rangle = \delta^{(3)}(\vec{q} - \vec{q}') \tag{II.2.15}$$

together with

$$\begin{aligned} u^s_{\vec{p}}(\vec{x}) &= \left(\frac{m}{E}\right)^{1/2} \frac{u^s(p)}{(2\pi)^{3/2}} e^{i\vec{p}\cdot\vec{x}} \\ v^s_{\vec{p}}(\vec{x}) &= \left(\frac{m}{E}\right)^{1/2} \frac{v^s(p)}{(2\pi)^{3/2}} e^{-i\vec{p}\cdot\vec{x}} \end{aligned} \tag{II.2.16}$$

where the spinors normalization was given previously (I.1.26)

$$\begin{cases} \bar{u}^i(p)\, u^j(p) = -\bar{v}^i(p)\, v^j(p) = \delta_{ij} \\ \bar{u}^i(p)\, v^j(p) = 0 \end{cases} \tag{II.2.17}$$

corresponding again to

$$\langle p,z \mid p',s\rangle = \delta^{(3)}(\vec{p}-\vec{p}')\,\delta_{zs} \tag{II.2.18}$$

(i.e. we have one particle in the volume $(2\pi)^3$).

Normalizing in a box, we replace

$$\delta^{(3)}(\vec{p}_1-\vec{p}_2) = \frac{V}{(2\pi)^3}\,\delta_{\vec{p}_1,\vec{p}_2}$$
$$\delta^{(4)}(p_1-p_2) = \frac{V}{(2\pi)^4}\,\frac{\sin\frac{\Delta E}{2}T}{\Delta E/2}\,\delta_{\vec{p}_1,\vec{p}_2} \tag{II.2.19}$$

where V is the three dimensional volume of the box and T is the time interval.

If $\phi = |\vec{v}_1-\vec{v}_2|/(2\pi)^3$ is, with our normalization ($\vec{v}_i$ are the incident particle velocities), the incident flux, the cross-section is given by

$$\sigma_f = P_{if}/\phi \tag{II.2.20}$$

where P_{if} is the transition probability for unit time

$$P_{if} = \lim_{T\to\infty}\frac{|S_{if}|^2}{T} = \frac{(2\pi)^4}{(2\pi)^{12}}(2\pi)^3\frac{m}{E_1}\frac{m}{E_2}\frac{1}{2\omega_1}\frac{1}{2\omega_2}|T_{if}|^2\delta^{(4)}(p_1+p_2-q_1-q_2) \tag{II.2.21}$$

so that

$$\sigma_f = \frac{1}{4(2\pi)^2}\,\frac{|T_{if}|^2}{|\vec{v}_1-\vec{v}_2|}\,\frac{m^2}{E_1E_2\omega_1\omega_2}\,\delta^{(4)}(p_1+p_2-q_1-q_2) \tag{II.2.22}$$

Upon using

$$|\vec{v}_1-\vec{v}_2| = \frac{k}{E_1}+\frac{k}{E_2} = \frac{2k}{E} = 2\frac{\sqrt{E^2-m^2}}{E} = 2\frac{\sqrt{s-4m^2}}{\sqrt{s}} \tag{II.2.23}$$

we get for the cross-section $d\sigma$ for $p_1+p_2 \longrightarrow q_1+q_2$ in the C.M.

$$d\sigma = \frac{m^2}{(2\pi)^2 8Ek\omega_1\omega_2}\,|T_{if}|^2\,\delta^{(3)}(\vec{q}_1+\vec{q}_2)\,\delta(\omega_1+\omega_2-\sqrt{s})\,d^3q_1\,d^3q_2 \tag{II.2.24}$$

The differential cross-section for one particle to be diffused in the solid angle $d\Omega$ will therefore be

$$\frac{d\sigma}{d\Omega} = \int_0^\infty q_1^2\,dq_1\int d^3\vec{q}_2\;\delta^{(3)}(\vec{q}_1+\vec{q}_2)\,\delta(\omega_1+\omega_2-\sqrt{s})\,\frac{|T_{fi}|^2\,m^2}{8(2\pi)^2Ek\omega_1\omega_2} \tag{II.2.25}$$

Integrating over $d^3\vec{q}_2$ ($\Rightarrow \vec{q}_2 = -\vec{q}_1 \Rightarrow |\vec{q}_2| = |\vec{q}_1| \Rightarrow \omega_1=\omega_2=\sqrt{\vec{q}_1^2+\mu^2} \Rightarrow q_1\,dq_1 = \omega\,d\omega$) we get

$$\frac{d\sigma}{d\Omega} = \frac{q}{k}\,\frac{m^2}{16\pi^2 s}\,|T_{if}|^2 \tag{II.2.26}$$

where $q = \sqrt{\omega^2-\mu^2} = \sqrt{s/4-\mu^2}$.

Eq. (II.2.26) shows the usual $1/v$ law of exothermic processes since, at threshold ($s\to 4m^2$), $k = \sqrt{\frac{s}{4}-m^2} \longrightarrow 0$ while $q \to \sqrt{m^2-\mu^2}$.

In terms of invariant scalar variables, using

$$t = (p_1 - q_1)^2 = \left(\sqrt{\vec{k}^2 + m^2} - \sqrt{\vec{q}^2 + \mu^2}\right)^2 - (\vec{k} - \vec{q})^2$$

$$dt = 2kq\, d(\cos\theta) = \frac{kq}{\pi}\, d\Omega$$

we have for the angular distribution

$$\frac{d\sigma}{dt} = \frac{m^2}{4\pi s (s - 4m^2)} \; |T_{if}|^2 \qquad \text{(II.2.27)}$$

We can now return to the problem of expressing the angular distribution using the partial wave expansion.

This is most conveniently done using the T_J amplitudes with definite J since a resonance occurs at given values $^{2S+1}L_J$ which means a Breit Wigner pole in T_J. As we have already discussed, the only allowed quantum numbers for the reaction under study are $^3L_{J=L\pm1}$ so that only (triplet) amplitudes of $J = L \pm 1$ occur. Furthermore, remember that only even (odd) waves are present according to whether I = 0 (I = 1).

We have

$$\frac{d\sigma}{d\Omega} = \frac{\pi}{4k^2}\left\{ \left| \sum_{J=0}^{\infty} \left[\sqrt{J}\; T_{J=L+1} + \sqrt{J+1}\; T_{J=L-1}\right] Y_J^0(\theta,\varphi) \right|^2 + \left| \sum_{J=1}^{\infty} \left[\sqrt{J+1}\; T_{J=L+1} + \sqrt{J}\; T_{J=L-1}\right] Y_J^1(\theta,\varphi) \right|^2 \right\} \qquad \text{(II.2.28)}$$

where $T_{J=L\pm1} = \sin\delta_{L\pm1}\, \exp[i\delta_{L\pm1}]$ and the sums involve only J even or odd according to whether I = 0 or 1.

Isospin decomposition

Just like in πN scattering (i.e. the crossed channel of $N\bar{N} \to \pi\pi$) we introduce the decomposition of the scattering amplitude with respect to the isospin indices of the pion (α, β) into an even part $T^{(+)}$ (which can only be proportional to the symmetric tensor which one can construct with two isospin Pauli matrices, i.e. $\{\tau_\alpha, \tau_\beta\}$ or $\delta_{\alpha\beta}$) and into an odd part $T^{(-)}$ (which can only be proportional to the antisymmetric tensor $[\tau_\alpha, \tau_\beta]$)

$$T_{\alpha\beta} = \delta_{\alpha\beta}\, T^{(+)} + \frac{1}{2}\left[\tau_\alpha, \tau_\beta\right] T^{(-)} \qquad \text{(II.2.29)}$$

Working out the Clebsch-Gordan coefficients we have the proportionality relation

$$T^{(+)} = T_0 / \sqrt{6}$$
$$T^{(-)} = T_1 / 2 \qquad \text{(II.2.30)}$$

between the crossing even (odd) and the total isospin I = 0 (I = 1) in the $N\bar{N} \to \pi\pi$ channel.

For the two most useful processes ($p\bar{p} \to \pi\pi$ and $\bar{p}n \to \pi\pi$) we get

$$T_{p\bar{p}\to 2\pi} = T_{n\bar{n}\to 2\pi} = T^{(+)} + T^{(-)} = \frac{1}{\sqrt{6}} T_0 + \frac{1}{2} T_1$$
$$T_{p\bar{n}\to 2\pi} = T_{\bar{p}n\to 2\pi} = 2\, T^{(-)} = T_1 \qquad \text{(II.2.31)}$$

N $\bar{N}$ $\longrightarrow$ $\pi\pi$ polarization

The determination of the amplitudes in a given N $\bar{N}$ reaction will require also measurements of polarization. For this it will be useful the decomposition (II.2.6).

Define $\vec{k}$ and $\vec{q}$ as the CM momenta of the $\bar{p}$ and of the π^- in the reaction $p\bar{p} \to \pi^+\pi^-$ and let us introduce the usual vector

$$\vec{n} = \vec{k} \times \vec{q} \qquad \text{(II.2.32)}$$

normal to the scattering plane.

We call $\sigma_\uparrow$ and $\sigma_\downarrow$ the cross-sections for the case when the target proton is polarized parallel and, respectively, antiparallel to $\vec{n}$.

The (unpolarized) differential cross-section and the asymmetry parameter A (ϑ) will then be

$$\frac{d\sigma}{d\Omega} = \frac{1}{2}\left(\sigma_\uparrow + \sigma_\downarrow\right) \qquad \text{(II.2.33)}$$

$$A(\vartheta) = \frac{\sigma_\uparrow - \sigma_\downarrow}{\sigma_\uparrow + \sigma_\downarrow} \qquad \text{(II.2.34)}$$

Choose now the reference system in such a way that $\vec{k}$ is along the x axis and the y axis lies in the $(\vec{k}, \vec{q})$ plane, so that $\vec{n}$ is along z. In this case

$$\vec{\sigma}\cdot\vec{k} = \sigma_x k$$
$$\vec{\sigma}\cdot\vec{q} = q\left(\sigma_x \cos\vartheta + \sigma_y \sin\vartheta\right) \qquad \text{(II.2.35)}$$

which we use in (II.2.6) making use of the two-dimensional spinors

$$|+\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad , \quad |-\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \qquad \text{(II.2.36)}$$

The polarized cross-sections are

$$\begin{cases} \sigma_\uparrow = |\langle + | T | + \rangle|^2 + |\langle + | T | - \rangle|^2 \\ \sigma_\downarrow = |\langle - | T | + \rangle|^2 + |\langle - | T | - \rangle|^2 \end{cases} \qquad \text{(II.2.37)}$$

Using the explicit form of the Pauli matrices together with (II.2.36) we have

$$\begin{cases} \langle + | \sigma_x | + \rangle = \langle + | \sigma_y | + \rangle = \langle - | \sigma_x | - \rangle = \langle - | \sigma_y | - \rangle = 0 \\ \langle + | \sigma_x | - \rangle = \langle - | \sigma_x | + \rangle = 1 \\ -\langle + | \sigma_y | - \rangle = \langle - | \sigma_y | + \rangle = i \end{cases}$$

which imply

$$\langle +|T|+\rangle = \langle -|T|-\rangle = 0$$

$$\langle +|T|-\rangle = h_1 k + h_2 q (\cos\theta - i \sin\theta)$$

$$\langle -|T|+\rangle = h_1 k + h_2 q (\cos\theta + i \sin\theta)$$

so that

$$\sigma_{\uparrow,\downarrow} = |h_1|^2 k^2 + |h_2|^2 q^2 + 2 \mathrm{Re}\,(h_1 h_2^*)\, \vec{k}\cdot\vec{q} \pm 2q \sin\theta\ \mathrm{Im}\left\{ h_2 (k h_1 + q h_2 \cos\theta)^* \right\} \qquad \text{(II.2.38)}$$

Inserting (II.2.38) into (II.2.34) we finally get

$$\frac{d\sigma}{d\Omega} = \left| h_1 \vec{k} + h_2 \vec{q} \right|^2 \qquad \text{(II.2.39)}$$

$$A(\theta) = \frac{2q \sin\theta\ \mathrm{Im}\left\{ h_2 (k h_1 + q h_2 \cos\theta)^* \right\}}{d\sigma/d\Omega} \qquad \text{(II.2.40)}$$

II.3 THREE BODY ANNIHILATION ($N\bar{N} \to 3\pi$)

II.3.1 Kinematics of a + b $\longrightarrow$ 1+2+3:

We now consider in some detail the kinematics of a two - to three body reaction

$$p_a + p_b \longrightarrow p_1 + p_2 + p_3 \qquad \text{(II.3.1)}$$

where a, b are the initial particles (clearly, what we have in mind is to specialize to the three pions annihilation $N\bar{N} \longrightarrow 3\pi$).

The first question is the choice of variables. Remember that given a process involving N particles altogether, one is left with a total of 3N-10 scalar variables (4N components of four--vectors minus N mass shell constraints, minus 4 energy-momentum conservation constraints, minus 6 Euler angles in four dimensional space). It will be useful to choose the variables to use according to what one is looking for. If one is searching for quantum number effects, it will be convenient to use the variables suggested in Fig. II.5

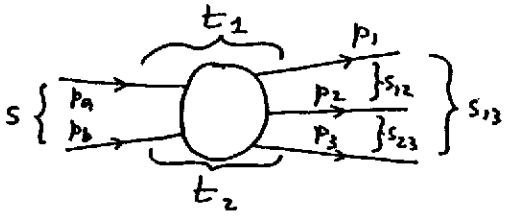


Fig. II.5

i.e.

$$s = (p_a + p_b)^2 = (p_1 + p_2 + p_3)^2 \qquad \text{(squared total energy in the C.M.)}$$

$$s_{12} = (p_1 + p_2)^2$$
$$s_{13} = (p_1 + p_3)^2$$
$$s_{23} = (p_2 + p_3)^2$$

} squared invariant masses of the different pairing of final particles

$$t_1 = (p_a - p_1)^2$$
$$t_2 = (p_b - p_3)^2$$

} squares of momentum transfer variables

As we must have only five independent variables, one of the previously defined ones must be expressible in terms of the others. In fact, from (II.3.1), squaring

$$s = s_{12} + s_{23} + s_{13} - m_1^2 - m_2^2 - m_3^2 \qquad \text{(II.3.2)}$$

where we have used $p_i^2 = m_i^2$

If we now write the invariant three body phase space $(\omega_i = \sqrt{\vec{p}_i + m_i^2})$

$$dS_3 = \frac{d^3p_1}{\omega_1}\,\frac{d^3p_2}{\omega_2}\,\frac{d^3p_3}{\omega_3}\,\delta^{(4)}(p_a + p_b - p_1 - p_2 - p_3) \qquad \text{(II.3.3)}$$

we shall prove that this can be rewritten as

$$dS_3 = \frac{\pi}{2s}\, ds_{13}\, ds_{23}\, d\cos\vartheta\, d\varphi \qquad \text{(II.3.4)}$$

where ϑ is the angle between $\vec{p}_1$ and $\vec{p}_b$ (as we shall see $-1 \le \cos\vartheta \le 1$) and φ is the angle between the planes $\vec{p}_2\,\vec{p}_3$ and $\vec{p}_1\,\vec{p}_b$ (it will turn out that $0 \le \varphi \le 2\pi$). We shall also introduce the angle β between $\vec{p}_1$ and $\vec{p}_2$ and the overall azimuth around the direction of flight of the incident particles ($\vec{p}_a$ or $\vec{p}_b$).

Upon integrating over this last angle (which gives a factor of 2π) and over $\vec{p}_3$ one gets $\left(\omega_3 = \left[(\vec{p}_1 + \vec{p}_2)^2 + m_3^2\right]^{1/2}\right)$

$$dS_3 = \frac{2\pi}{\omega_3}\,\frac{p_1^2\,dp_1}{\omega_1}\,\frac{p_2^2\,dp_2}{\omega_2}\,d\cos\vartheta\,d\varphi\,\delta(\sqrt{s} - \omega_1 - \omega_2 - \omega_3)\,d\cos\beta. \qquad \text{(II.3.5)}$$

Recalling the definition of β, $d\cos\beta = \frac{\omega_3}{p_1 p_2}\, d\omega_3$ one can perform the β integration to obtain

$$dS_3 = 2\pi\, d\omega_1\, d\omega_2\, d\cos\vartheta\, d\varphi \qquad \text{(II.3.6)}$$

where we have also used $p_1\,dp_1 = \omega_1\,d\omega_1$, $p_2\,dp_2 = \omega_2\,d\omega_2$. Since all conservation constraints have been used, $0 \le \vartheta \le \pi$, $0 \le \varphi \le 2\pi$.

To get (II.3.4) we notice that, in the C.M.

$$s_{23} = (p_2 + p_3)^2 = (p_a + p_b - p_1)^2 = \left[(p_a + p_b - p_1)_a\right]^2 = s + m_1^2 - 2\omega_1\sqrt{s} \qquad \text{(II.3.7)}$$

so that

$$d\omega_i = -\frac{1}{2\sqrt{s}}\, ds_{jk} \qquad (i, j, k = \text{cyclic p. of } 1, 2, 3)$$

II.3.2 Dalitz plot:

So far we have only imposed energy momentum conservation and integrated over an irrelevant angular variable so that the result could be simply multiplied by the proper matrix element to get the differential cross section. Suppose now that the matrix element is independent of Θ and β. Integrating over the latter variables, one sees that the phase space is uniform in s_{13}, s_{23} therefore, a Dalitz plot where one plots the events as function of the invariant masses would be uniformely populated in what concerns the contribution coming from the phase space and any departure from uniform density is due to the matrix element and must reflect some dynamical property such as a resonance formation in one (or more) subchannel.

The boundaries of the physical region in s_{13}, s_{23} can be found using $-1 \leq \cos\beta \leq 1$ Squaring

$$\omega_3 = \sqrt{s} - \omega_1 - \omega_2 = \left[\vec{p}_1^{\,2} + \vec{p}_2^{\,2} + m_3^2 + 2|\vec{p}_1||\vec{p}_2|\cos\beta\right]^{1/2}$$

one gets

$$0 \leq \left[(\sqrt{s} - \omega_1 - \omega_2)^2 - \vec{p}_1^{\,2} - \vec{p}_2^{\,2} - m_3^2\right]^2 \leq 4\,\vec{p}_1^{\,2}\,\vec{p}_2^{\,2} \qquad \text{(II.3.8)}$$

Using (II.3.7) with $\omega_i^2 = \vec{p}_i^{\,2} + m_i^2$ we find after some algebra the curve

$$\left[(s - s_{23} + m_1^2)^2 - 4s\,m_1^2\right]\left[(s - s_{13} + m_2^2)^2 - 4s\,m_2^2\right]$$

whose shape is approximately given by the one in Fig. (II.6) and whose boundaries (dotted lines) obtain noticing that s_{ij} is maximum when $\vec{p}_k$ is at rest in the C.M., i.e. for $\omega_k = m_k$ and is minimum when $\vec{p}_i = \vec{p}_j = 0$. Thus

$$s_{ij}^{max} = s + m_k^2 - 2m_k\sqrt{s} = (\sqrt{s} - m_k)^2 \qquad \text{(II.3.9)}$$

$$s_{ij}^{min} = (m_i + m_j)^2$$

The wavy lines indicate where a more dense population of events in the Dalitz plot would be expected in case of resonances in the various subchannels (1) $s_{23} = (p_2 + p_3)^2$, (2) $s_{13} = (p_1 + p_3)^2$ (3) $s_{12} = (p_1 + p_2)^2$

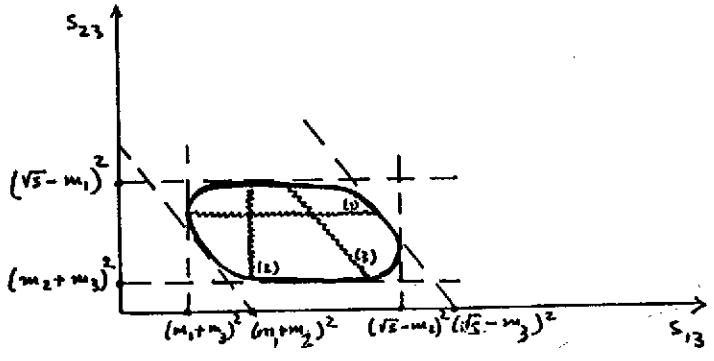


Fig. II.6

Often it is more convenient to introduce the kinetic energies $T_i = \omega_i - m_i$ related to s_{jk}

$$s_{jk} = s - m_i^2 - 2\omega_i\sqrt{s} = (\sqrt{s} - m_i)^2 - 2\sqrt{s}\,T_i \qquad \text{(II.3.10)}$$

Owing to the linear relationship between the s_{ij} and T_k variables, a uniform distribution in the s_{ij} variables entails also a uniform distribution in the T_k's. In the case of interest to us ($N\bar{N} \rightarrow 3\pi$) where $m_1=m_2=m_3=\mu$, the constraint (II.3.2) becomes

$$\sqrt{s} = 3\mu + \sum_{i=1}^{3} T_i$$

implying that the total energy is the sum of the kinetic energies and of the masses.

In terms of the Q value

$$Q = \sqrt{s} - 3\mu \tag{II.3.11}$$

the boundary value of the Dalitz plot becomes

$$\left[Q^2 + 2Q\mu - 2Q(T_1+T_2) + 2T_1T_2 - 4\mu(T_1+T_2)\right]^2 = \\ = T_1T_2(T_1+2\mu)(T_2+2\mu) \tag{II.3.12}$$

whose non-relativistic limit $T_i \ll \mu$ gives a circle of radius Q/2 centered in the plane (T_1,T_2) (Fig. II.7):

$$\left[Q - 2(T_1+T_2)\right]^2 = T_1T_2 \tag{II.3.13}$$

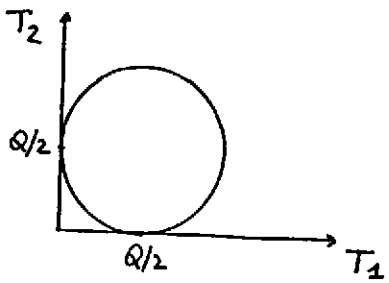


Fig. II.7

In the opposite limit $Q \gg \mu$ it is more useful to introduce the variables T_3 and $x = \frac{1}{\sqrt{3}}(T_1 - T_2)$ so that substituting in (II.3.12):

$$T_1 + T_2 = Q - T_3$$
$$T_1 - T_2 = \sqrt{3}\,x$$

one finds

$$\left\{2Q(T_3 - Q/2) + \frac{1}{2}\left[(Q-T_3)^2 - 3x^2\right]\right\}^2 \simeq \frac{1}{16}\left[(Q-T_3)^2 - 3x^2\right]^2 \tag{II.3.14}$$

leading to the rounded triangle of Fig. II.8 where again, the wavy lines simulate dynamical effects such as given, for instance, by resonances in the various channels.

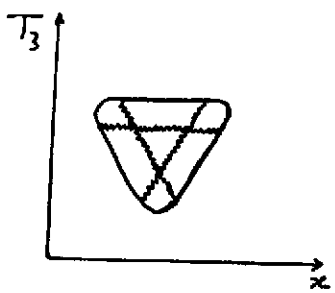


Fig. II.8

II.4 QUANTUM NUMBERS AND THE DALITZ PLOT

In the present context where we are investigating N $\bar{N}$ annihilation, the emphasis is, of course, on S-channel resonances and it is therefore of interest to study how (formation of resonances in) particular sub channels with given quantum numbers can give kinematical restrictions which can be extracted from the transition matrix element. What we are interested in is to see how selection rules can affect the Dalitz plot which, as we just saw, would be uniformely populated in the three final subchannels were it not for either kinematical constraints or dynamical effects (which we are not going to consider in the following). We shall not discuss in detail the general case but rather give a few examples related to the case of three body decay and, in particular, $N\bar{N} \to 3\pi$.

II.4.1 Quantum numbers effects in the case $N\bar{N} \to 3\pi$.

If we restrict ourselves to consider the angular momenta L = 0 and L = 1 only, we recall that the result of § I.11b showed that three pions annihilation can occur only if the N $\bar{N}$ systems is in one of the following states

$$I = 0 \quad \begin{cases} {}^3S_1 \\ {}^1P_1 \end{cases} \qquad I = 1 \quad \begin{cases} {}^1S_0 \\ {}^3P_1 \\ {}^3P_2 \end{cases}$$

and since we have three identical particles in the final state, they must be in a totally symmetric configuration of both isospin and space coordinates.

II.4.1a I = 0

Let $\vec{I}_i$ (i=1,2,3) be the isospin vectors of the final pions. The only rotationally invariant form (in isospin space) which corresponds to $\vec{I} = 0$ is $(\vec{I}_1 \times \vec{I}_2) \cdot \vec{I}_3$. The latter, however, is totally antisymmetric and this requires the amplitude to be correspondingly antisymmetric under the exchange of $\vec{p}_i$ (i=1,2,3).

Let us consider first the case of a state ${}^3S_1$ when J = 1, L = 0 and P = -1 in the initial state. Therefore, given that the three final pions have already negative intrinsic parity, the matrix element must have the rotational properties of a vector (since $\vec{J} = 1$) and the parity of an axial vector (+1). If $\vec{J}$ is the initial spin, the only allowed form for the matrix element will thus be

$$M \propto \vec{J} \cdot \left[\vec{p}_1 \times \vec{p}_2 + \vec{p}_2 \times \vec{p}_3 + \vec{p}_3 \times \vec{p}_1 \right] \qquad \text{(II.4.1)}$$

or, given that in the C.M. $\vec{p}_1 + \vec{p}_2 + \vec{p}_3 = 0$, we can choose $\vec{p}_1$ and $\vec{p}_2$ as independent vectors and we have

$$M \propto \vec{J} \cdot (\vec{p}_1 \times \vec{p}_2) \qquad \text{(II.4.2)}$$

The effect on the Dalitz plot distribution will therefore be a kinematical form of the kind

$$|M_{(^3S_1)}|^2 \propto |\vec{p}_1 \times \vec{p}_2|^2 = p_1^2 p_2^2 \sin^2\beta \,. \qquad \text{(II.4.3)}$$

Let us now consider the case of an initial state $^1P_1$ corresponding to J=1, L=1, P= +1, so that the final state must have the rotational properties and the parity of a vector. Furthermore, it must be completely antisymmetric in $\vec{p}_1$, $\vec{p}_2$, $\vec{p}_3$ as we have already remarked. Thus, the matrix element must be proportional to

$$M_{(^1P_1)} \propto \vec{J} \cdot \left[\vec{p}_1 (\vec{p}_2^{\,2} - \vec{p}_3^{\,2}) + \vec{p}_2 (\vec{p}_3^{\,2} - \vec{p}_1^{\,2}) + \vec{p}_3 (\vec{p}_1^{\,2} - \vec{p}_2^{\,2}) \right]$$

or, using the two independent vectors

$$\vec{P} = (\vec{p}_1 + \vec{p}_2)/2 \qquad \vec{q} = (\vec{p}_1 - \vec{p}_2)/2 \tag{II.4.4}$$

and momentum conservation

$$M_{(^1P_1)} \propto \vec{J} \cdot \left[\vec{P} (\vec{P} \cdot \vec{q}) + \vec{q} \left(\tfrac{3}{2} \vec{P}^{\,2} - \tfrac{1}{4} \vec{q}^{\,2} \right) \right] \tag{II.4.5}$$

whose modulus squared

$$\sim (\vec{P} \cdot \vec{q})^2 \left(4\vec{P}^{\,2} - \tfrac{1}{2} \vec{q}^{\,2} \right) + \vec{q}^{\,2} \left(\tfrac{3}{2} \vec{P}^{\,2} - \tfrac{1}{4} \vec{q}^{\,2} \right)^2 \tag{II.4.6}$$

tells us how we should expect the Dalitz plot to be correspondingly affected.

II.4.1b $\underline{I = 1}$

We have now to combine the three pion isospin vectors $\vec{I}_i$ (i = 1,2,3) to form a vector in isospin space. This can be done in three different ways, i.e. combining $\vec{I}_1$ and $\vec{I}_2$ to give isospin zero to the subsystem $\vec{I}_{12} = \vec{I}_1 + \vec{I}_2$ and leaving to $\vec{I}_3$ the vector character, or permuting cyclically 1,2,3. Thus we have the three possibilities:

$$\vec{I}_3 (\vec{I}_1 \cdot \vec{I}_2) \quad , \quad \vec{I}_1 (\vec{I}_2 \cdot \vec{I}_3) \quad , \quad \vec{I}_2 (\vec{I}_3 \cdot \vec{I}_1) \tag{II.4.7}$$

Notice that one could similarly form combinations which are antisymmetric in the various pairs, that is

$$(\vec{I}_1 \times \vec{I}_2) \times \vec{I}_3 \quad , \quad (\vec{I}_2 \times \vec{I}_3) \times \vec{I}_1 \quad , (\vec{I}_3 \times \vec{I}_1) \times \vec{I}_2$$

They would correspond to have, say $\vec{I}_{12}$ in an isospin I = 1 and to combining it with $\vec{I}_3$ to get again I = 1. The above forms c a n, however, be expressed in terms of the previous ones (II.4.7) since, for instance,

$$\vec{I}_1 \times (\vec{I}_2 \times \vec{I}_3) = \vec{I}_2 (\vec{I}_1 \cdot \vec{I}_3) - \vec{I}_3 (\vec{I}_1 \cdot \vec{I}_2)$$

To get a matrix element totally symmetric in the combination of isospin and momenta, we can either take

$$M \propto \alpha_{23} \vec{I}_1 (\vec{I}_2 \cdot \vec{I}_3) + \alpha_{31} \vec{I}_2 (\vec{I}_3 \cdot \vec{I}_1) + \alpha_{12} \vec{I}_3 (\vec{I}_1 \cdot \vec{I}_2) \tag{II.4.8}$$

with $\alpha_{ij} = \alpha_{ji}$ symmetric for the exchanges $\vec{p}_i \rightleftarrows \vec{p}_j$ or

$$M \propto \beta_{23} \vec{I}_1 \times (\vec{I}_2 \times \vec{I}_3) + \beta_{31} \vec{I}_2 \times (\vec{I}_3 \times \vec{I}_1) + \beta_{12} \vec{I}_3 \times (\vec{I}_1 \times \vec{I}_2) \tag{II.4.9}$$

with $\beta_{ij} = -\beta_{ji}$ antisymmetric under the exchanges $\vec{p}_i \rightleftarrows \vec{p}_j$ or one can make more general combinations of mixed space and isospin symmetry.

Let us now investigate the momentum dependence of the matrix element for the lowest permissible state $^1S_0$ when J = L = 0, P = -1. Since the 3 pions final state has already intrinsic negative parity, and J = 0, the simplest choices to have a true scalar in the symmetric and antisymmetric combinations, respectively, are

$$\alpha_{ij} = c_1 + c_2\, \vec{p}_i \cdot \vec{p}_j + \cdots$$
$$\beta_{ij} = \vec{p}_i^{\,2} - \vec{p}_j^{\,2} \qquad \text{(II.4.10)}$$

In the state $^3P_1$ when J = L = 1, P = +1 the matrix element will have to be proportional to $\vec{J} \cdot \vec{V}$ where $\vec{V}$ is a true vector. Thus, the simplest choices are

$$\alpha_{ij} \propto (\vec{p}_i + \vec{p}_j)$$
$$\beta_{ij} \propto (\vec{p}_i - \vec{p}_j) \qquad \text{(II.4.11)}$$

Finally, in the state $^3P_2$ where L = 1, J = 2, P = +1, the matrix element must have in character of a second order tensor in its rotational properties, being pseudo tensor in its space inversion properties. This tensor must be constructed using the vectors $\vec{p}_i$ and the pseudo vectors $\vec{p}_i \times \vec{p}_j$ and must be saturated by the analogous tensor describing the angular momentum of a state $^3P_2$. If μ, ν are the Lorentz indices and i, j the usual pion indices, the following quantities

$$\alpha_{ij}^{\mu\nu} \propto \left[(\vec{p}_i - \vec{p}_j)^\mu (\vec{p}_i \times \vec{p}_j)^\nu + (\vec{p}_i - \vec{p}_j)^\nu (\vec{p}_i \times \vec{p}_j)^\mu\right]$$
$$\beta_{ij}^{\mu\nu} \propto \left[(\vec{p}_i + \vec{p}_j)^\mu (\vec{p}_i \times \vec{p}_j)^\nu + (\vec{p}_i + \vec{p}_j)^\nu (\vec{p}_i \times \vec{p}_j)^\mu\right] \qquad \text{(II.4.12)}$$

are both traceless $(\alpha^{\mu\nu}\delta_{\mu\nu} = \beta^{\mu\nu}\delta_{\mu\nu} = 0)$ symmetric tensors in the μ,ν indices and their properties differ only for the exchange of the pion indices i j. Both are pseudotensors, i.e. $\alpha^{\mu\nu}(\vec{p}_i, \vec{p}_j) = -\alpha^{\mu\nu}(-\vec{p}_i, -\vec{p}_j)$ symmetric and antisymmetric respectively under $\vec{p}_i \rightleftarrows \vec{p}_j$.

II.4.2 General method ($N\bar{N} \rightarrow 3\pi$):

The cases discussed earlier are special examples of the general case of three pion annihilations with unrestricted angular momenta values which can be treated by a general method of spherical tensors due to Zemach[6].

Let the initial state have a definite J^P and be described by a traceless irreducible tensor of order J (scalar when J=0, vector when J=1, tensor when J=2, ...) if the parity is $(-1)^J$ or by a traceless irreducible pseudotensor or order J if the parity is $(-1)^{J+1}$ constructed with the spins degrees of freedom of the initial state.

In writing down the most general form of a matrix element, owing to the intrinsic negative parity of the three pions in the final state, this tensor (or pseudotensor) will have to be

contracted and saturated with a similar pseudotensor (or tensor) constructed with the final momenta. This means, the final state will contribute with a pseudotensor if L is odd and with a tensor if L is even. Furthermore, the combined space-isospin symmetry will have to be even (the isospin analysis is the same as in § II.4.1).

Thus, if we denote by $S^{(J)}_{ijl\ldots}(\vec{S})$ the traceless, irreducible tensor of order J describing the angular momentum of the initial state, by I $(\vec{I})$ the isospin factor (discussed previously, i.e. $(\vec{I}_1 \times \vec{I}_2) \cdot \vec{I}_3$ for I = 0), and by $P^{(J)}_{ijl\ldots}(\vec{p}_i)$ the traceless tensor (or pseudotensor as the case, i.e. the parity may be) made with the final momenta $\vec{p}_i$, the matrix element will be a combination of the form

$$M = S^{(J)}_{ijl\ldots}(\vec{S})\; I(\vec{I})\; P^{(J)}_{ijl\ldots}(\vec{p}_i)\; F(\vec{p}_i^{\,2}) \qquad \text{(II.4.13)}$$

Here $F(\vec{p}_i^{\,2})$ is a form factor depending only upon the pion energy variables which contains all information which comes neither from kinematics nor from general properties, i.e. contains all the dynamical information. In general, there may be several of the above expressions for a given decay depending on the symmetry of I $(\vec{I})$ and $P^{(J)}(\vec{p})$ and so several form factors will also have to be used.

The question is then how to construct the most general tensor $P^{(J)}$ of a given parity having at one's disposal the momenta $\vec{p}_i$'s (i = 1,2,3) of which only two are independent (say $\vec{p}_1$, $\vec{p}_2$) in the three body final state owing to momentum conservation and the pseudovectors $\vec{p}_i \times \vec{p}_j$ of which only $\vec{q} = \vec{p}_1 \times \vec{p}_2$ is independent given our previous choice. The latter cannot appear at powers higher then one since all even povers can, obviously, be reexpressed in terms of powers of the momenta themselves.

Thus, if the state is $J^P = 0^-, 1^+, 2^-, \ldots$ (which are usually termed "unnatural parity" states) the tensor $P^{(J)}_{ijl\ldots}$ can be constructed using only the vectors $\vec{p}_i$ (i = 1,2) whereas the pseudovector $\vec{q}$ will have to appear at power one in the case of "natural parity" states $J = 1^-, 2^+, \ldots$ (recall 0^+ cannot go into 3π).

II.4.3 Connection with general properties in the Dalitz plot:

The previous analysis enables one to study in detail what kind of population one should expect in the Dalitz plot in the various cases.

As an example, we consider the case I = 0, when the matrix element was found to be completely antisymmetric in $\vec{p}_1$, $\vec{p}_2$, $\vec{p}_3$ implying that the rate (i.e. the modulus squared) will be completely symmetric in $\vec{p}_1$, $\vec{p}_2$, $\vec{p}_3$. This means that if we divide the Dalitz plot with the lines $\vec{p}_i = 0$ (i = 1,2,3), each sextant thus obtained will be uniformely populated.

Next, recall that the periphery of the Dalitz plot corresponds to $\vec{p}_1 = \pm \vec{p}_2$ which leads to $\vec{q} = \vec{p}_1 \times \vec{p}_2 = 0$. Therefore, the natural parity series $J^P = 1^-, 2^+, \ldots$ which was seen to depend linearly on $\vec{q}$ (see § II.4.2) is depopulated at the border of the Dalitz plot whereas the unnatural parity series $0^-, 1^+, 2^-, \ldots$ is not.

II.4.4 Covariant formulation:

So far we have used three dimensional vectors p_i to construct the tensor $P^{(J)}_{ijk}$ ... to use in Zemach method and this is good so long as one sticks to the CM system. In most cases of interest, however, it is convenient to visualize the production mechanism as following from a sequential decay of successive resonances

$$a + b \longrightarrow A + B$$
$$\hookrightarrow C + D + E$$
$$\hookrightarrow F + G + \cdots$$
$$\hookrightarrow \cdots$$

whereby after having studied in the CM the first reaction, one would want to study the subsequent resonance decays in the respective rest systems of the various decaying resonances.

It is therefore convenient to generalize Zemach's method to use four vectors rather than three dimensional vectors. The trick is to introduce four vectors p_i^μ orthogonal to the total momentum P^μ (just as $\vec{p}_i$ (i=1,2) in the C.M. having only space components were orthogonal to P^μ which in the c.m. had only non zero time component $P_0 = \sqrt{s}$, $\vec{P} = 0$.

It is straightforward to see that the four vectors

$$P_i^\mu = p_i^\mu - P^\mu \frac{P \cdot p_i}{P^2} \qquad \text{(II.4.14)}$$

are orthogonal to P^μ

$$P^\mu P_i^\mu = 0$$

and reduce to $\vec{p}_i$ in the rest frame where P_i^0 vanish as can be checked directly from(II.4.14).

Similarly, the generalization of $\vec{q}$ to its covariant form is given by

$$Q^\mu = \varepsilon^{\mu\nu\rho\sigma} p_1^\nu p_2^\rho P^\sigma / \sqrt{s} \qquad \text{(II.4.15)}$$

where $\varepsilon^{\mu\nu\rho\sigma}$ is the usual Levi-Civita antisymmetric tensor. Again $Q^\mu P^\mu = 0$ and $Q^0 = 0$, $\vec{Q} = \vec{q}$ in the C.M..

Finally, the tensor $\delta^{\mu\nu}$ is replaced by

$$g^{\mu\nu} = \delta^{\mu\nu} - \frac{P^\mu P^\nu}{s} \qquad \text{(II.4.16)}$$

which satisfies

$$P_1^\mu P_2^\nu g^{\mu\nu} = p_1^\mu p_2^\nu \delta^{\mu\nu} = p_1^\mu p_2^\mu \qquad \text{(II.4.17)}$$

The covariant use of Zemach method amounts therefore to the use of (II.4.14,15,16) instead of $\vec{p}_i$, $\vec{q}$, $\delta^{\mu\nu}$.

II.5 CASCADE PRODUCTION WITH RESONANCE FORMATION

The method outlined above is very useful in connection with a decay analysis where resonances are formed sequentially and then decay

$$N\bar{N} \longrightarrow A + R \qquad R \longrightarrow B + R' \qquad R' \longrightarrow \ldots \tag{II.5.1}$$

Specific examples could be

$$N\bar{N} \longrightarrow \pi + \rho \qquad \rho \longrightarrow \pi+\pi \tag{II.5.2}$$

$$N\bar{N} \longrightarrow \pi + \omega \qquad \omega \longrightarrow \pi+\pi+\pi \tag{II.5.3}$$

$$N\bar{N} \longrightarrow \pi + A_1 \qquad A_1 \longrightarrow \pi+\pi+\pi \tag{II.5.4}$$

II.5.1 Two pions resonance:

As a specific example, we will investigate reaction (II.5.1) starting from a state of given quantum numbers decaying via a two pion resonance with definite quantum numbers $(I)_R$, $(J^P)_R$.

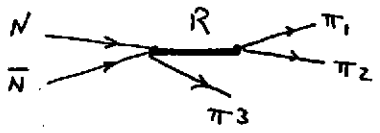
For simplicity, if the reaction is $N\bar{N} \longrightarrow \pi_1 \pi_2 \pi_3$ we will suppose that the process goes via

$$N\bar{N} \longrightarrow (\pi_1 \pi_2)_R \pi_3$$

so that the invariant mass of R is $s_{12} = (p_1 + p_2)^2$ and the matrix element close to the mass of the resonance will be proportional to

$$\frac{1}{\sqrt{s} - m_R + i\,\Gamma_R/2} \tag{II.5.5}$$

Diagrammatically, the reaction proceeds via



and we have to consider the isospin and momentum structure of the vertices (1,2) and (1,2)3.

II.5.1a Isospin structure

If $I_R = 1$, the isospin structure of (1,2) is a vector

$$\vec{I}_{1,2} = \vec{I}_1 \times \vec{I}_2$$

whereas if $I_R = 0$, the isospin structure of (1,2) is a scalar

$$I_{1,2} = \vec{I}_1 \cdot \vec{I}_2$$

We can now distinguish two cases for the isospin structure of the complete vertex (1,2)3 according two whether $I_{N\bar{N}} = 0$ or 1.

A/ $I_{N\bar{N}} = 0$. We have two possibilities:

i) $I_R = 1$

The vertex (1,2) 3 will have the isospin form

$$(\vec{I}_1 \times \vec{I}_2) \cdot \vec{I}_3 \qquad \text{(II.5.6)}$$

ii) $I_R = 0$

The reaction is forbidden because with $I_R = 0$ we can only form $I_{N\bar{N}} = 1$.

B/ $I_{N\bar{N}} = 1$. We have two possibilities:

i) $I_R = 1$

leads to the (1,2) 3 isospin structure

$$(\vec{I}_1 \times \vec{I}_2) \times \vec{I}_3 \qquad \text{(II.5.7)}$$

ii) $I_R = 0$

gives the (1,2)3 vertex

$$(\vec{I}_1 \cdot \vec{I}_2)\ \vec{I}_3 \qquad \text{(II.5.8)}$$

II.5.1b Momentum structure ($N\bar{N} \longrightarrow \pi\varrho$)

We recall that for a two pion state, J even (odd) implies I even (odd).

Reaction $N\bar{N} \longrightarrow (\pi_1\pi_2)\pi_3$ can proceed with the two pions π_1, π_2 in any state of the natural parity series 0^+, 1^-, 2^+,..., but we will for simplicity limit ourselves to the case $(J^P)_R = 1^-$. As this requires $I_R = 1$, this means that the resonance R has the quantum numbers of the ϱ meson.

First we write the vertex in the R rest frame and we then transform it in the covariant formulation to boost it in the $N\bar{N}$ CMS. It turns out that this has no effect in the present example, as we will see.

Notice that being R a true vector (1^-), the momentum structure will have the form $\vec{S}_R \cdot (\vec{P}_1 - \vec{P}_2)$ in the R rest frame. We now boost the vertex from the R frame to the N $\bar{N}$ CM. For this we rewrite $\vec{P}_1 - \vec{P}_2$ in a covariant way using P_1^μ, P_2^μ (II.4.14). We will then take the space component of $(P_1 - P_2)^\mu$ and use $P_\mu = (p_1 + p_2)_\mu$, $P_\mu^2 = m_R^2$. We find

$$P_1^\mu - P_2^\mu = (p_1 - p_2)^\mu - \frac{P^\mu}{m_R^2}\left[P^\rho (p_1 - p_2)^\rho\right] = (p_1 - p_2)^\mu - \frac{P^\mu}{m_R^2}(p_1^2 - p_2^2) = (p_1 - p_2)^\mu$$

which proves that the boost has no relevance in this case. Summing up the previous consideration, according to the $I(J^P)$ of the N $\bar{N}$ system we have for the various matrix elements the following form of amplitudes

$$1(0^-) \Rightarrow (\vec{I}_1 \times \vec{I}_2) \times \vec{I}_3 \quad (\vec{p}_1 - \vec{p}_2) \cdot \vec{p}_3$$

$$0(1^-) \Rightarrow (\vec{I}_1 \times \vec{I}_2) \cdot \vec{I}_3 \quad (\vec{p}_1 - \vec{p}_2) \times \vec{p}_3 =$$

$$= -(\vec{I}_1 \times \vec{I}_2) \cdot \vec{I}_3 \quad (\vec{p}_1 - \vec{p}_2) \times (\vec{p}_1 + \vec{p}_2) =$$

$$= 2(\vec{I}_1 \times \vec{I}_2) \cdot \vec{I}_3 \quad \vec{p}_2 \times \vec{p}_1$$

$$0(1^+) \Rightarrow (\vec{I}_1 \times \vec{I}_2) \cdot \vec{I}_3 \quad (\vec{p}_1 - \vec{p}_2)$$

$$1(1^+) \Rightarrow (\vec{I}_1 \times \vec{I}_2) \times \vec{I}_3 \quad (\vec{p}_1 - \vec{p}_2)$$

$$1(2^+) \Rightarrow (\vec{I}_1 \times \vec{I}_2) \times \vec{I}_3 \left[(\vec{p}_1 - \vec{p}_2)_\mu (\vec{p}_1 \times \vec{p}_2)_\nu + (\vec{p}_1 - \vec{p}_2)_\nu (\vec{p}_1 \times \vec{p}_2)_\mu\right]$$

Proper symmetrization between the final pions should also be performed if the resonance is not formed by a specific couple of pions.

PART III

THE EXPERIMENTAL SIDE OF LOW ENERGY N $\bar{N}$ PHENOMENOLOGY

III.1 INTRODUCTION

While it is always rather difficult to define "low energy" in an elastic reaction, this becomes almost impossible in a reaction such as N $\bar{N}$. Elastic reactions at low energy are in fact usually dominated by resonances implying rapid variations and oscillations of the cross-sections (this is not the case of exotic channel such as p p $\longrightarrow$ p p where the variations may be non negligible but there are essentially no oscillations). In N $\bar{N}$ reactions, resonances may be present (and if they are, they are an interesting phenomenon) but the dominant mechanism is that of annihilation as we shall now discuss.

We will assume that "low energy" does not exceed a few GeV where the cross-section has dropped of almost one order of magnitude from the lowest energy measured so far ($p_{Lab} \sim 300$ MeV; the coming in operation of the new CERN devices will no doubt lower very much this limit).

III.2 THE p $\bar{p}$ CROSS-SECTION

As we have mentioned at the beginning, the low (as well as the high) energy region of N $\bar{N}$ has been very little investigated before LEAR. In particular, the total cross-section $\sigma_t(p\bar{p})$ has been measured at $p_{lab} \sim 300$ MeV/c as the lowest value giving there $\sigma_t(p\bar{p}) \simeq 300$ mb. $\sigma_t(p\bar{p})$ decreases quickly with increasing p_{lab} dropping without major oscillations to about 80 mb at $p_{lab} \sim 2.5$ GeV/c and continuing to drop at a much slower pace ($\sigma_t \sim 42$ mb at $p_{lab} \sim 300$ GeV/c). After a broad minimum, $\sigma_t(p\bar{p})$ starts then growing and grows to about 62 mb at Collider energies ($\sqrt{s} = 546$ GeV) where the comparison with cosmic rays gives only a very rough indication for $\sigma_t(pp) \simeq 70 \pm 10$ mb. Even at the highest accelerator energies for which a comparison is meaningful (the ISR) $\sigma_t(p\bar{p})$ is higher than $\sigma_t(pp)$ (and it is one of the contentions of Pomeranchuk theorem that they should tend to the same asymptotic limit[1]) and this fact is interpreted with being p $\bar{p}$ more absorptive than p p because of the annihilation channels. At lower energies, however, this factor is enormous: $\sigma_t(p\bar{p})$ is about 80 mb at $p_{lab} \sim 2.5$ GeV/c, i.e. twice as large compared with $\sigma_t(pp)$. The gap reduces enormously at high energy; for instance σ_t (p $\bar{p}$) $\sim$ 42 mb and σ_t(p p) $\sim$ 40 mb at $p_{Lab} \sim$ 300 GeV/c.

From a qualitative point of view, we recall that exothermic reactions are expected to proceed via the famour 1/v law which holds if the matrix element is finite at v=0. This law seems indeed to represent the behavior of σ_t(p $\bar{p}$) in the low energy domain (i.e. down to $\sim$300 MeV/c) where is well parametrized by

$$\sigma_t(p\bar{p}) \simeq \left(66 + \frac{26}{k}\right) \text{mb} \qquad \text{(III.2.1)}$$

being k the C.M. momentum (in GeV/c).

If one now looks at σ_{el} (p $\bar{p}$) one sees that again it decreases rather smoothly with increasing energy, it is $\sim$ 80 mb at $p_{lab} \sim 300$ MeV/c and goes down to $\sim$ 30 mb at $p_{lab} \sim 2.5$ GeV/c.

The charge exchange reaction p $\bar{p} \longrightarrow$ n $\bar{n}$ is a very minor contribution (about 10 mb at 300 MeV/c and 1 mb at 2.5 GeV/c). Therefore, we conclude that the large difference between the total and the elastic $\bar{p}$ p cross sections comes from the annihilation channels p $\bar{p} \longrightarrow$ mesons which are present in p $\bar{p}$ and absent in p p.

Aside from the 1/v law of exothermic reactions, already mentioned, the following qualitative remarks can be made to shed some light on the situation:

a) if appears that p $\bar{p}$ cannot be represented in terms of scattering off a black sphere in spite of the inelastic (annihilation) contribution being large. In this case, in fact, one would have $\sigma_t / \sigma_{in} \simeq 1$ which is not the case in p $\bar{p}$ where, at 300 MeV/c, $\sigma_t/\sigma_{in} \simeq \frac{300}{200} \simeq 1.5$ (decreasing with increasing energy);

b) Analyzing the partial wave amplitudes of the annihilation cross-section one finds that the unitarity limit is essentially saturated

$$\sigma_\ell^{(ann.)} \simeq (2\ell+1)\,\pi/k^2 \,. \qquad \text{(III.2.2)}$$

c) The angular distribution of the elastic p $\bar{p}$ reaction shows the typical diffractive peak in the momentum transfer t from an object of radius R $\sim$ 1-1.4 fermi implying that the number of effective partial waves in the elastic channel is of the order of $\ell \simeq k R$.

The above seemingly contradictory remarks have been interpreted as suggesting that in p $\bar{p}$ there are two dimensions; one (R_1 = 1 − 1.4 f) responsible for elastic scattering determining the shape of $d\sigma(p\bar{p})/dt$ through unitarity and the other, much smaller ($R_2 \sim 0.2$ f) responsible for annihilation and determining through the optical theorem

$$Im\, f_{el}(k,0) = \frac{k}{4\pi}\,\sigma_t = \frac{k}{4\pi}\,(\sigma_{el} + \sigma_{ann.}) \qquad \text{(III.2.3)}$$

the optical point

$$\left.\frac{d\sigma_{el}}{dt}\right|_{t=0} \geq \frac{1}{16\pi}\,(\sigma_{el} + \sigma_{ann})^2 \qquad \text{(III.2.4)}$$

In conclusion, annihilation represents the major contribution to the total p $\bar{p}$ cross-section at low energy. Of this, about 95% consists of pions and resonances decaying into pions.

It is perhaps a fortuitous (but exciting) coincidence that the inner radius (R $\sim$ 0.2 f) has the same rough value of the estimated "effective" quark radius.

III.3 MULTIPLICITY

Although annihilation into pions appears as the dominant process of p $\bar{p}$, still be observed multiplicity is much lower than that allowed by energy conservation. The relevance of this statement can better be seen from Table III.1.

p_{lab} (GeV/c)	Q (GeV)	(allowed by energy conservation)	observed $\langle n_\pi \rangle$	R
0	1.88	13	5.0 ± 0.15	60%
7	3.85	27	6.7 ± 0.30	75%

Table III.1

where

$$R = \frac{Q - \langle n_\pi \rangle \mu}{Q} \qquad \text{(III.3.1)}$$

is the fraction of energy that goes into kinetic energy of the produced pions. Although the latter is very large, it is still relatively small compared with that of particles produced at high energy in p p collisions where not more than 10% is used to produce the mass of particles; differently stated, annihilation and production appeare quite different mechanisms. This consideration is corroborated by the fact that two-body or quasi two-body annihilation

$$p\bar{p} \rightarrow M_1 M_2$$

where M_1, M_2 are particles or resonances, does not seem to contribute more than 7% to the observed annihilation (at rest). By contrast, high energy particle production is known to be mostly due to quasi two-body processes (although the ambiguities in such an analysis are many and varied).

The multiplicity distribution is fairly narrow; for instance, at rest where $\langle n_\pi \rangle \simeq 5$, the various branching ratios are $\sim$ 45% for n = 5, $\sim$20% for n = 4,6, $\sim$8% for n = 3,7, $\sim$1% for n = 2,8 (confirming the smallness of two pion annihilation).

III.4 RELATIVE IMPORTANCE OF S vs. P WAVES AT REST

Even through the phase shifts are complex in N $\bar{N}$, the kinematic behavior $\delta_{\ell} \underset{k \to 0}{\sim} k^{2\ell+1}$ leads to conjecture that in the low energy region S waves should dominate. This can be checked to be so in the ratio

$$R_K = \frac{p\bar{p} \rightarrow K_0 \bar{K}_0 \quad (C=+1)}{p\bar{p} \rightarrow K_0 \bar{K}_0 \quad (C=-1)} \qquad \text{(III.4.1)}$$

since we recall that the $C = \pm 1$ channels receive their contribution only from $\begin{cases} \text{odd} \\ \text{even} \end{cases}$ partial waves respectively thus, at rest, by $\begin{cases} P \\ S \end{cases}$ waves. The ratio (III.4.1) is, experimentally, very small at rest $\lesssim$ 1.5% proving that S wave indeed dominate at low energy but is already $\sim$40% at 1.2 GeV/c.

If, however, we look at the ratio

$$R_\pi \simeq \frac{p\bar{p} \rightarrow 2\pi^0}{p\bar{p} \rightarrow \pi^+ \pi^-} \qquad \text{(III.4.2)}$$

where, we recall, the numerator gets contribution only from odd waves, we find the apparently surprising result that this ratio, at rest, is of order 40% (i.e. almost 40% of the $2\pi^-$ production goes into $2\pi^0$) implying a large contribution from P waves. This is, however, only partly surprising since in the annihilation $p\bar{p}$ at rest, a pair of π's has a much larger share of relative momentum than a pair of K's.

III.5 DETECTION OF RESONANCES

In the $N\bar{N}$ system, the process of multipion annihilation appears as the dominant mechanism. It is interesting to analyze the methods by which pionic resonances can be detected in the reaction

$$p\bar{p} \rightarrow n\pi \qquad (III.5.1)$$

and some experimental evidence for such resonances. For convenience we consider separately the cases of resonances with mass $M \lesssim 2m_N$ and $M > 2m_N$.

III.5.1 Resonances below threshold $M \lesssim 2m_N$:

III.5.1a γ DE EXCITATION OF $p\bar{p}$

In $p\bar{p}$ annihilation at rest, one measures the γ-ray spectrum in the reaction

$$p\bar{p} \rightarrow \gamma X \qquad (III.5.1)$$

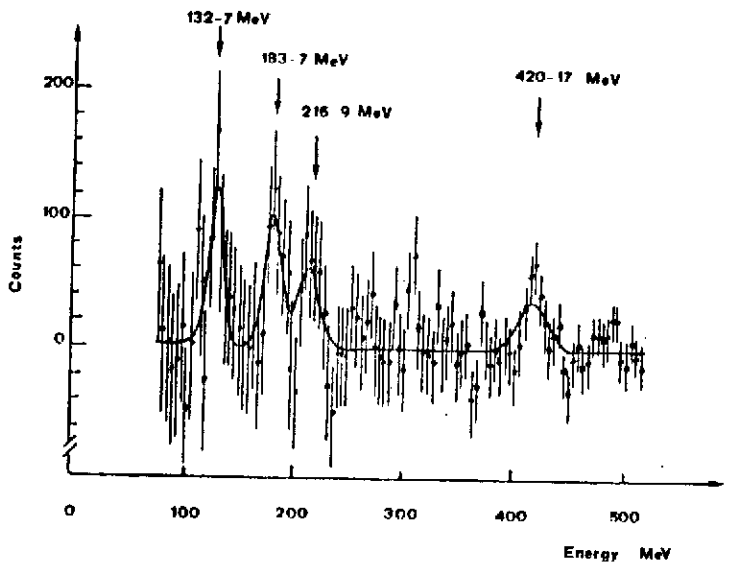


FIG. III.1

Superimposed to the continuum, at least three narrow states have been found with γ energies of 183, 216 and 420 MeV (Fig. III.1).

The peak at 132 MeV is the line of $\pi^- p$ radiative capture.

These lines correspond to masses of the X state $(M_X = 2m_N - E_\gamma)$ of 1456, 1660 and 1693 fitted with resonances (continuous curve) in Fig. III.1. These states are quite narrow and the width quoted in Fig. III.1 are due to the limitations of the apparatus.

III.5.1b Missing mass.

Consider the reaction

$$\bar{p}d \rightarrow p_{slow} + \text{pions} \qquad (III.5.2)$$

where the colliding $\bar{p}$ annihilates with the neutron in the deuteron and the emerging proton is "slow" (momentum $\lesssim$ 150 MeV/c). By varying the momentum of the slow proton one can study the yields for various values of the invariant mass of the final pions. Let p, d, q be the momenta of the colliding antiproton, of the deuteron and of the slow emerging proton respectively so that the system of final pions has fourmomentum X = p + d − q whose invariant mass X^2, in the laboratory system, neglecting the deuteron binding energy and assuming both p and q to be non-relativistic $E_{\bar{p}} \simeq m_N + \frac{\vec{p}^2}{2m_N}$, $E_q \simeq m_N + \frac{\vec{q}^2}{2m_N}$ (where $\vec{p}$ and $\vec{q}$ are the $\bar{p}$ and the p momenta in the laboratory system) becomes

$$M_X^2 = X^2 = (p + d - q)^2 = (E_{\bar{p}} + 2m_N - E_q)^2 - (\vec{p} - \vec{q})^2 \simeq \left(2m_N + \frac{\vec{p}^2 - \vec{q}^2}{2m_N}\right)^2 - (\vec{p} - \vec{q})^2 \qquad \text{(III.5.3)}$$

Retaining only terms up to order $\vec{q}^2$ and $\vec{p}^2$ we get from (III.5.3)

$$M_X^2 \simeq 4m_N^2 - 4\vec{q}^2 + (\vec{p} + \vec{q})^2$$

and

$$M_X \simeq 2m_N - \frac{\vec{q}^2}{m_N} + \frac{(\vec{p} + \vec{q})^2}{4m_N} \qquad \text{(III.5.4)}$$

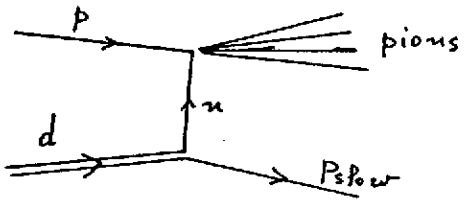
Therefore, one can vary the missing mass M_X by varying the momentum of the final slow proton and its direction relative to $\vec{p}$. Notice that if the kinetic energy of the final proton obeys the relationship

$$T = \frac{\vec{q}^2}{2m_N} > \frac{(\vec{p} + \vec{q})^2}{8m_N} \qquad \text{(III.5.5)}$$

we will be in the kinematical configuration when

$$M_X < 2m_N$$

Sticking to the case of a slow recoiling proton has the advantage that the process in this case is dominated by the virtual exchange of a neutron



with the deuteron's proton acting as a spectator and the final pion system coming from the interaction of the incident antiproton with the virtual neutron. This is the case, however, if we can assume that the above diagram dominates, which happens if $M_X < 2m_N$ i.e. if the dominant contribution comes indeed from the (unphysical) neutron's pole when $(p - d)^2 = -m_N^2$. It can be shown that this would require the kinetic energy of the final proton to be

$\simeq -\frac{B}{2} = -1.1$ MeV/c (B being the deuteron binding energy $\simeq$ 2.2 MeV/c). It is quite obvious that the final proton being a physical particle cannot have this unphysical (negative) value for its kinetic energy. However we shall be the closest to this unphysical value when the final proton is at rest or, in defect, very slow.

With the method just discussed, evidence of a further resonance below threshold (at M_X = 1794 MeV, $\Gamma \lesssim$ 8 MeV) has been given. Furthermore, evidence has also been given of another celebrated resonance above threshold at 1936 MeV ($\Gamma \simeq$ 5 MeV). All these "evidences" are still very much controversial and much clarification is expected from LEAR.

III.5.2 Resonances above threshold ($M > 2\,m_N$):

These resonances can be studied with a variety of methods either in "formation" experiments when the resonance is found in the S-channel as an intermediate state or in "production" experiments when the resonance is formed among a group of final particles.

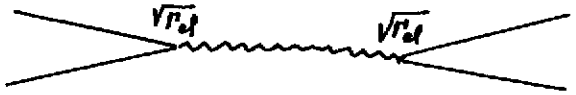
In the first cathegory we list the study of the energy variation of integrated quantities (such as total, elastic, charge exchange, total annihilation and particular annihilation channels cross-sections) and the study of the energy variation of angular distributions and polarizations (elastic and particular two-body annihilation channels). In the second, we recall off shell $N\bar{N}$ interactions in various channels through backward production and a variety of other production experiments.

Partial wave analyses are also occasionally useful to provide further information.

In should be stressed, however, that hunting for resonances is always a very ambiguous game where one can easily confuse kinematical effects (such as thresholds, the Deck effects etc.) for bona fide resonances and the $N\bar{N}$ system is no exception to this general rule. The point is that the only way to actually see a resonance would be to sit on the corresponding pole; this, however, in the traditional language of a Breit Wigner form lies at physical values of angular momentum and unphysical (complex) energy values or, in the Regge language, at physical (real) energies but unphysical (complex) angular momenta so that in any case we lack direct evidence. Further difficulties arise if either the resonance has too large a width (whatever this may mean) or too low an elasticity (i.e. coupling constant). In the way of ambiguities, the $N\bar{N}$ case is emblematic since a lot of resonances have been reported above threshold and all of them have successively been questioned so that it remains entirely to the new facilities to clear the field.

III.5.2a Energy variation of integrated quantities near a resonance

Let us recall how a resonance is usually studied and let us for simplicity consider the elastic scattering of two neutral spinless particles. A resonance of spin J, mass M, total width Γ, elastic width Γ_{el} is conventionally attributed to a diagram of the kind



and represented by a Breit Wigner amplitude of the form (in the C.M. system)

$$f_J(E,\theta) = \frac{2J+1}{2k} \frac{\Gamma_{el}}{E_0 - E - i\,\Gamma/2} P_J(\cos\theta) \qquad \text{(III.5.6)}$$

contributing to the J-th partial wave of the elastic amplitude (whose modulus squared is the differential cross section). In (III.5.6) the energy position of the resonance E_0 is related to the mass via

$$M^2 = 4\left(m^2 + k_0^2\right) \quad ; \quad k_0^2 = (E_0 + m)^2 - m^2 = E_0^2 + 2m\,E_0 \qquad \text{(III.5.7)}$$

Γ is the total width of the resonance(possibly contributing to several channels) and among the many ambiguities connected with the use of a Breit Wigner (a "relativistic" or "non-relativistic" form, such as (III.5.6)) we recall that the width is usually endowed with a threshold behavior accounting for the so-called "centrifugal barrier" which is usually parametrized as

$$\Gamma = \Gamma_0 \left(\frac{k}{k_0}\right)^{2J+1} \frac{E}{E_0} \qquad \text{(III.5.8)}$$

with Γ_0 taken to be a constant. The form (III.5.8), convenient for low values of k, becomes rapidly a practical problem if one wants to take into account the "tail" of a resonance (i.e. when k increases) where, however, the concept of a resonance looses meaning. The "proper" treatment of the tail of a resonance was recognized as one of the unsolved problems at the time of the duality program.

Γ_{el} (i.e. the width of the resonance in the elastic channel) acts as the coupling constant and should also be parametrized in a form (III.5.8) to take into account threshold effects. It is customary to introduce the elasticity parameter

$$x = \Gamma_{el}/\Gamma \qquad \text{(III.5.9)}$$

which essentially tells us the relative strength of the resonance in the elastic channels as compared with all other channels.

Taking the modulus squared of (III.5.6) and integrating over $\cos\theta$ one gets the resonant contribution to the elastic cross-section

$$\sigma_{el}^{res} = \frac{4\pi}{k^2}(2J+1)\frac{\Gamma_{el}^2}{4}\frac{1}{(E-E_0)^2 + \Gamma^2/4} = \frac{4\pi}{k^2}(2J+1)B(E)x^2 \qquad \text{(III.5.10)}$$

where we have used (III.5.9) and introduced the Breit Wigner factor

$$B(E) = \frac{\Gamma^2/4}{(E - E_o)^2 + \Gamma^2/4} \qquad \text{(III.5.11)}$$

In general interefence terms may appear if a background is superimposed to the resonant partial wave or when two (or more) resonances lie close-by.

From the optical theorem, the imaginary part of (III.5.6) gives for the contribution of the resonance to the total cross-section

$$\sigma_{tot}^{res} = \frac{4\pi}{k} \operatorname{Im} f_J(k,0) = \frac{\pi}{k^2}(2J+1)\frac{\Gamma_{el}\,\Gamma}{(E-E_0)^2 + \Gamma^2/4} = \frac{4\pi}{k^2}(2J+1)\,B(E)\,x \qquad \text{(III.5.12)}$$

Recalling the definition

$$\sigma_{tot} = \sigma_{el} + \sigma_{in} \qquad \text{(III.5.13)}$$

the resonant inelastic cross-section is now given by

$$\sigma_{in}^{res.} = \frac{4\pi}{k^2}(2J+1)\,B(E)\,x\,(1-x) \qquad \text{(III.5.14)}$$

Notice also that (III.5.10,12) give

$$x = \frac{\sigma_{el}^{res.}}{\sigma_{tot}^{res.}} \qquad \text{(III.5.15)}$$

In the case of a resonance in a p $\bar{p}$ channel, we must remember that the reaction has the isospin decomposition $(T_0+T_1)/2$. Thus, if the resonance has a definite isospin, a factor of 1/2 appears in (III.5.6,8,11) and a factor of 1/4 in (III.5.10). Furthermore, the cross-section must be divided by another factor of 4 coming from $(2S_1+1)(2S_2+1)$ being S_1, S_2 the spins of the incoming protons; furthermore, there is a charge exchange contribution. In summary, we get instead of (III.5.10,12,14) (now $x = 2\sigma_{el}^{res}/\sigma_{tot}^{res} = \frac{\sigma_{el}^{res} + \sigma_{c.ex}^{res}}{\sigma_{tot}^{res}}$)

$$\sigma_{el}^{res} = \frac{\pi}{4k^2}(2J+1)\,B(E)\,x^2$$

$$\sigma_{tot}^{res} = \frac{\pi}{2k^2}(2J+1)\,B(E)\,x \qquad \text{(III.5.16)}$$

$$\sigma_{ann}^{res} \equiv \sigma_{in}^{res} \equiv \sigma_{tot}^{res} - \sigma_{el}^{res} - \sigma_{ch.ex}^{res} = \frac{\pi}{2k^2}(2J+1)\,B(E)\,x(1-x).$$

Ideally, the study of a resonance would thus require; to study the energy position E_o of the resonance from the various cross-sections and to measure $\sigma_{tot}, \sigma_{el}, \sigma_{in}$ at $E = E_o$ to determine J and X.

In practice, if the resonance is not very narrow, the location of E_o is already difficult because of the interference between the resonance and the continuum or background. The latter is particularly relevant in the case of p $\bar{p}$ where a large background is present (the resonance effect is never larger than some 10 mb out of a σ_{tot} of $\gtrsim$ 100 mb.

Taking the data on cross-sections at face values, "evidence" was given in the past for the existence of several resonances.

Resonance $p\bar{p}$	"S"	"T"	"U"
Mass (MeV)	1936 ± 1	2190 ± 10	2350 ± 15
Width (MeV)	4 + 8	20 + 90	20 + 160
σ_t (mb)	10.6 ± 2.4	2 + 4	~ 2
Isospin	1 (0)	1	0 or 1
σ_{el} (mb)	7.0 ± 1.4	$\geqslant 2.12$	$\geqslant 2.18$
$\sigma_{ch.ex}$ (mb)	$< 0.3 \pm 0.3$	< 0.2	< 0.2

Table III.2

and, possibly, of more complicated structures both in the isospin I=0 and I=1 channels.

Assuming each contribution seen in σ_{tot} and σ_{el} to come from a single resonance, one would come to the conclusion for X and J:

	J	X
"S"	0	1
"T"	1	0.74
"U"	2	0.85

Table III.3

These conclusions have, however been challenged by several subsequent experiments. In particular, for the "S" resonance, the charge exchange cross-section is much too small as compared with σ_{el}^{res} while they should be the same if the resonance has a definite isospin value (remember $T_{el} = (T_0 + T_1)/2$, $T_{ch.ex.} = (T_0 - T_1)/2$).

This difficulty has been investigated and at least three different (or concomitant) explanations can be offered:

i) The bumps seen in $\sigma_{tot}(\bar{p}p)$ and $\sigma_{el}(\bar{p}p)$ do not contain a single resonance, but two resonances with the same J^{PC} but opposite I^G (0^+ and 1^-) are present almost degenerate in mass. In this case their effect would add up in $T_{el}(p\bar{p}) = \frac{1}{2}(T_0 + T_1)$ while they would cancel in $T_{ch.ex.} = \frac{1}{2}(T_0 - T_1)$. In this case, the fit gives $J^{PC} = 2^{-+}$.

This solution will be tested (when data will become available) by looking at $p\bar{p} \to m\pi$. If in fact two resonances are present one should see their effect in both $p\bar{p} \longrightarrow$ even π's and $p\bar{p} \longrightarrow$ odd π's whereas their effect would show up in only one class if the resonance is in a definite state of I^G.

ii) A second, more ad hoc explanation assumes the resonance to interfere in a different way with the large background present in elastic and charge exchange amplitudes. In this case, one gets also a larger J value and one could have either J = 1, X $\simeq$ 0.37 or J = 2, X $\simeq$ 0.22.

iii) A third, more elegant explanation, could come from the assumption that the "S" resonance is

a diquark-antidiquark state. In this case, the resonance instead of being in a pure isospin state would become a pure 2 quark - 2 antiquark system that could consist of either $(uu)(\bar{u}\bar{u})$ or $(dd)(\bar{d}\bar{d})$. In the former case, the S resonance could couple strongly only to the $p\bar{p} \equiv (uud)(\bar{u}\bar{u}\bar{d})$ channel but not to the $n\bar{n} \equiv (udd)(\bar{u}\bar{d}\bar{d})$ (see § III.6).

III.5.2b Angular distributions

A classical way of hunting for resonances starting from angular distributions is to expand them in Legendre polynomials

$$\frac{d\sigma}{dt} = \sum_n A_n(s) P_n(\cos\theta) \qquad \text{(III.5.17)}$$

and to look for rapid energy variations of $A_n(s)$. The addition theorem of Legendre polynomial tells us that $A_n(s)$ is a combination of partial waves up to n = 2J. Thus, a resonant wave of angular momentum J affects all coefficients $A_n(s)$ up to n = 2J. This method may be useful, in practice, only for low angular momentum resonances and can easily be applied to data coming from elastic scattering, polarization and annihilation into two mesons $p\bar{p} \to M_1 M_2$.

Another indirect way of looking for resonant effects is to analyze the energy variation of the diffraction slope $b(s)$ in the distribution $\exp[b(s)\,t]$. It is known that at intermediate energies ($p_{lab} \sim 1 + 2$ GeV/c), $b(s)$ oscillates around resonances and these oscillations are conspicuous for relatively large J ($J \sim 2, 3$) and fairly large elasticities.

A more largely used approach is to look at the backward direction where the diffractive background should largely cancel. The difficulty is how to properly extrapolate the resonance tail. In the particular case of $p\bar{p}$ elastic scattering, the overall difficulty of all partial wave analyses resides in the large complexity of independent amplitudes we have to deal with. We just recall that a total of 10 independent amplitudes are present in the combination of spin (5 amplitudes) - isospin space (2 components).

To add to these practical difficulties, in the charge exchange channel we are confronted with the further difficulty that most $p\bar{p}$ resonances seem to be weakly coupled to the $p\bar{p} \longrightarrow n\bar{n}$ process.

More promising seems to be the analysis of the yield in the $p\bar{p} \longrightarrow M_1 M_2$ annihilations channel. The reason for this it at least three-fold. First of all, like in any inelastic channel, there is no diffractive contribution present. Secondly, if one looks at $p\bar{p} \longrightarrow 2\pi$ being the two final pions spinless particles, only two (complex) spin amplitudes are present for each of the two isospin amplitudes. Thirdly, as we have seen previously, very often strict selection rules allow only definite states of S-channel quantum numbers to be present.

The shortcoming of this process is, of course, the smallness of the relative cross-section in this channel so that better data are needed and should be provided by the new low energy $p\bar{p}$ facilities.

Among the reactions that have been investigated we notice that

$$\begin{aligned} p\bar{p} &\to \pi^+\pi^- \quad (a) \\ &\to \pi^0\pi^0 \quad (b) \end{aligned} \qquad \text{(III.5.18)}$$

have the quantum numbers discussed previously (§ I.11a), proceed both in a state of positive G parity and can contribute to either isospin I = 0 or 1 with the decomposition (II.2.31):

$$T(p\bar{p} \to \pi^+\pi^-) = T^{(+)} + T^{(-)} = \frac{1}{\sqrt{6}} T_0 + \frac{1}{\sqrt{4}} T_1 \qquad \text{(II.5.19)}$$

$$T(p\bar{p} \to \pi^0\pi^0) = T^{(+)} = \frac{1}{\sqrt{6}} T_0 \qquad \text{(III.5.20)}$$

Furthermore, in the case $N\bar{N} \to \pi^+\pi^-$ the two isospin amplitudes do not interfere in the integrated cross-section since, we recall (I.11.8,9) only even (odd) waves contribute to I=0 (I=1).

We define

$$\begin{aligned} \sigma_{+-} &\equiv \int |T(p\bar{p} \to \pi^+\pi^-)|^2 d\Omega = \frac{1}{6}\sigma_0 + \frac{1}{4}\sigma_1 \\ \sigma_{00} &\equiv \int |T(p\bar{p} \to \pi^0\pi^0)|^2 d\Omega = \frac{1}{6}\sigma_0 \end{aligned} \qquad \text{(III.5.21)}$$

where

$$\begin{aligned} \sigma_0 &= \int |T_0|^2 d\Omega \\ \sigma_1 &= \int |T_1|^2 d\Omega \end{aligned} \qquad \text{(III.5.22)}$$

Thus, at least in principle, a clean separation of the two isospin channels in two pion annihilation is possible

$$\begin{aligned} \sigma_0 &= 6\,\sigma_{00} \\ \sigma_1 &= 4\,(\sigma_{+-} - \sigma_{00}) \end{aligned} \qquad \text{(III.5.23)}$$

and this has led to claiming evidence for the following situation:

J^{PC}	I^G	Mass (MeV)	Width (MeV)	Reaction
3^{--}	1^+	2150	200	$p\bar{p} \to \pi^+\pi^-$
4^{++}	0^+	2310	210	$p\bar{p} \to \begin{cases} \pi^+\pi^- \\ \pi^0\pi^0 \end{cases}$
5^{--}	1^+	2480	280	$p\bar{p} \to \pi^+\pi^-$

TABLE III.4

where, perhaps, the first resonance could be identified with the "T" resonance reported in

Table III.3 while the second two could be the two isospin states of the "U" resonances of Table III.3. Notice, however, that none of the resonances in Table III.4 is narrow contrary to the indications of Table III.3. Furthermore, the effects under discussion are just of the order of few nanobarns.

Another reaction were some data exist is

$$p\bar{p} \to \eta^0 \pi^0$$

whose I^G quantum numbers are 1^- (since $G_{\eta^0} = +$) and where some evidence has been given of a resonance

J^{PC}	I^G	Mass (MeV)
2^{++}	1^-	2100 – 2400

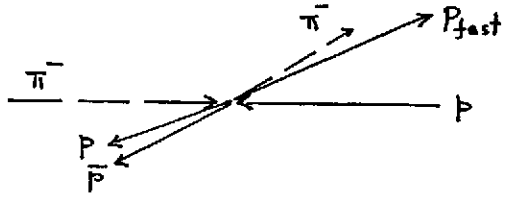
Other channels which can be studied are $p\bar{p} \to \eta^0\eta^0$ (whose $I^G = 0^+$) which should be complementary to the $\pi^0\pi^0$ channel and quasi two-body reactions such as $p\bar{p} \to \varrho^0\varrho^0$ ($I^{CG} = 0^{++}$),, $p\bar{p} \to \omega\varrho^0$ ($I^{CG} = 1^{+-}$), $\varrho^0 f^0$ ($I^{CG} = 1^{-+}$), ωf^0 ($I^{CG} = 0^{--}$) covering all interesting I^{CG} combinations (I^G of ϱ, ω, f being 1^+, 0^-, 0^+).

III.5.2c Off-shell backward production

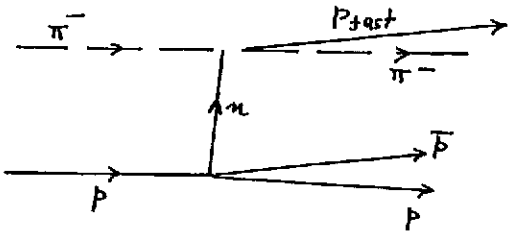
In a reaction like

$$\pi^- p \to p_{fast}\ \pi^-\ p\bar{p} \qquad \text{(III.5.25)}$$

the fast proton travels in the direction of the initial π^- (in the Lab. Sys.) and one looks at the mass spectrum of the p $\bar{p}$ state. In the C.M. the picture would look



and would be interpreted in terms of virtual n exchange



which should dominate the backward production.

The reaction (III.5.24) with a fast proton and π^- forward forming a Δ (1326) or N^*(1520) has been measured at 9 et 12 GeV/c finding three narrow peaks at 1930 MeV ($\Gamma \simeq 10$ MeV) (the S resonance?), at 2020 MeV ($\Gamma \simeq$ 12 + 24 MeV) and at 2204 MeV ($\Gamma \simeq 16^{+20}_{-12}$). The

presence of these narrow peaks has to be contrasted with the absence of narrow signals in the $\pi\pi$ channel and with their absence in the reaction $\pi^+ p$.
LEAR should clarify all this.

III.6 Conclusions

As we have seen, the resonance situation is the $\bar{B}$ B system is far from settled. The past years have witnessed a tremendously controversial series of statements on the so called baryonium states; a number of candidates have been reported and successively, their existence has invariably been denied. The first preliminary measurements from LEAR are, at the time of writing, still controversial. Undoubtedly, however, the situation will be soon clarified by the experimental groups working on the LEAR program.

PART IV

THEORETICAL TOOLS IN LOW ENERGY PHENOMENOLOGY

The introduction of subhadronic structures in the realm of theoretical hadronic interactions is in itself a very large subject[2] and is covered in another course at this school for what concerns the implications on nuclear physics. (see G. Ripka, these proceedings).

Thus, we shall not enter into this field beyond the very few ideas put forward in § IV.2. Also the more conventional approach in terms of meson and baryon degrees of freedom is covered in detail in yet another course at this school (see B. Desplanques, these proceedings). For this reason, we shall confine ourselves here to a very quick summary of the more conventional theoretical aspects of low energy N $\bar{N}$ phenomenology and we refer the interested reader to the specialized literature for a more comprehensive discussion on the subject[4].

IV.1 The potential approach to scattering data.

The traditional approach to low energy hadronic interactions views them as the result of the exchange of mesons and baryons so long, at least, as one is concerned with the long-range (LR) and medium-range (MR) parts of the intereaction responsible of those effects which involve large separations which are, presumably, the largest bulk of conventional low energy nuclear and particle physics. Those aspects which are expected to be dominated by the short-range (SR) part of the interaction where the quark and gluon contribution is fundamental will not be discussed here (other than for the very sketchy discussion of Sec. IV.2).

The conventional approach to the LR and MR parts of the interaction assumes dominance of one -, two - and three - pion exchanges by which one constructs an effective potential which is checked, usually, by comparing the calculated peripheral partial waves ($J > 2$) with the ones measured experimentally. The SR part of the interaction in this approach (i.e. distances smaller than $\sim$ 0.8 fm) is described phenomenologically bumping our ignorance in a few parameters to be adjusted from very low energy data. The above prescription has provided us with a quite satisfactory overall account of low energy N N scattering.

IV.1.1 The N $\bar{N}$ interaction: collision data.

As already mentioned (§ II.1), low energy N $\bar{N}$ differs from low energy N N mostly because of the presence of annihilation channels which make the phase shifts complex. We can then write, for the N $\bar{N}$ potential

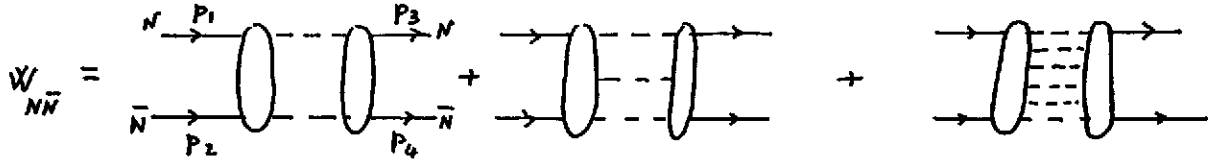
$$V_{N\bar{N}} = U_{N\bar{N}} - i W_{N\bar{N}} \qquad \text{(IV.1.1)}$$

where $U_{N\bar{N}}$ and $W_{N\bar{N}}$ are real and, below production threshold $W_{N\bar{N}}$ describes the annihilation.

As for the real part $U_{N\bar{N}}$, this can be taken, most simply, to be given by the contribution to the crossed N N channel along the lines discussed in § II.1 (G-parity rule) so that the LR and MR contributions can be derived from N N scattering. The SR contribution is still treated

phenomenologically. The main property which distinguishes the LR+MR parts is that this is strongly attractive in the N $\bar{N}$ as compared with the N N case. This is attributed[4] to the fact that the three-pion exchange simulated by ω-exchange (which was repulsive in the N N case) becomes attractive in the N $\bar{N}$ case whereas the two-pion contribution remains attractive. A more attractive potential produces, of course, a richer spectrum of resonances.

The widths of these resonances will be determined by the imaginary part $W_{N\bar{N}}$ which are given, from unitarity, by intermediate physical states among which the 4, 5 - pion states are expected to give the leading contribution (§ III.3). Symbolically, we have



We write $W_{N\bar{N}}$ in the form of a dispersion relation

$$W_{N\bar{N}}(s,t) = \sum_i \int_{4m_N^2}^{\infty} dt' \frac{\rho_i(s,t')}{t'-t} \qquad \text{(IV.1.2)}$$

where i denotes the intermediate states, s and t have been defined previously (II.1.2). As the spectral functions ρ_i are presumably strongly dependent on S, $W_{N\bar{N}}$ is expected to be correspondingly <u>non-local</u>. Moreover, as the threshold in (IV.1.2) is $(2m_N)^2$ for purely kinematic reasons, $W_{N\bar{N}}$ is expected to be quite short range.

We shall not enter in the game of describing the specific forms which have actually been used to fit the data and we refer to ref. 4 for details and literature on the subject. Suffices to say that adjusting the various parameters introduced, one fits a total of $\sim$ 1000 p $\bar{p}$ data points in the range $20 \lesssim T \lesssim 370$ MeV on i) total cross sections, ii) differential elastic cross section, iii) integrated charge exchange (CE) cross section, iv) differential charge exchange cross section, v) elastic polarization. The quality of the fits obtained is illustrated in fig. IV.1-5 (for the various data and theoretical models referred to in these figures, see ref. 4).

Again, the new data expected from LEAR in the coming few years will help greatly to improve our theoretical understanding of low energy N $\bar{N}$ phenomenology.

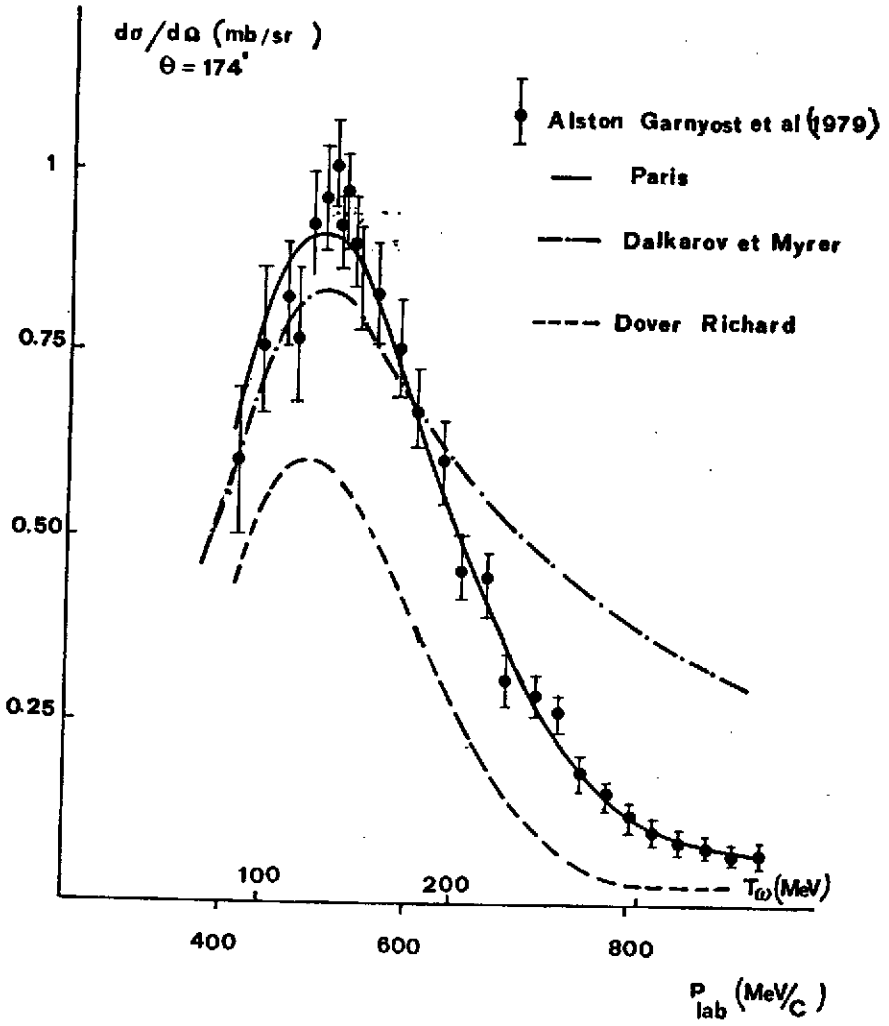


FIG. IV. 1

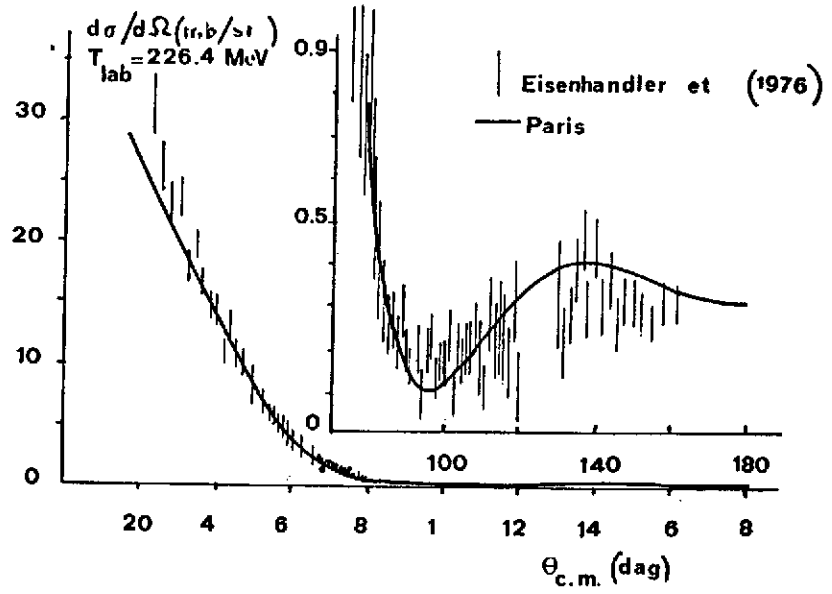


FIG. IV. 2

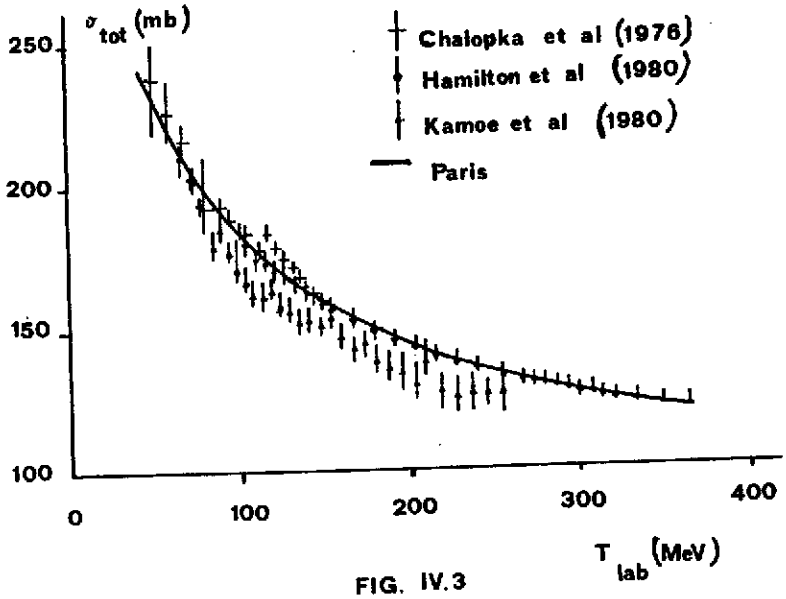


FIG. IV.3

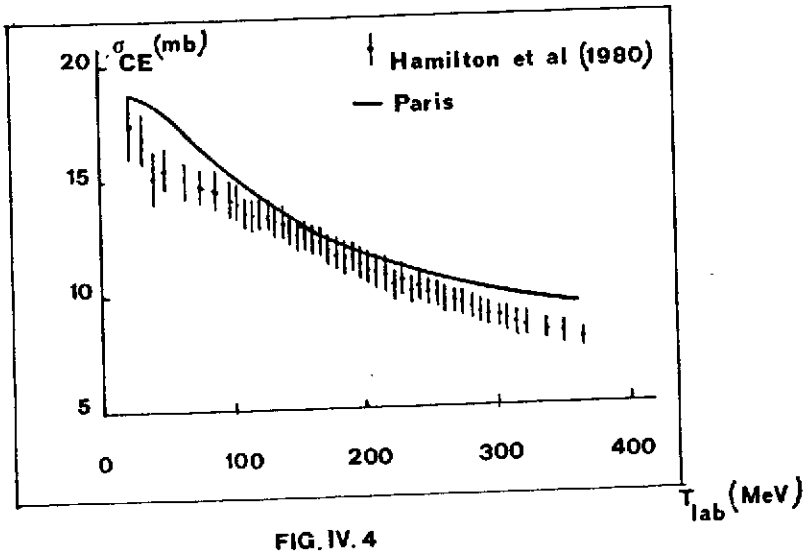


FIG. IV.4

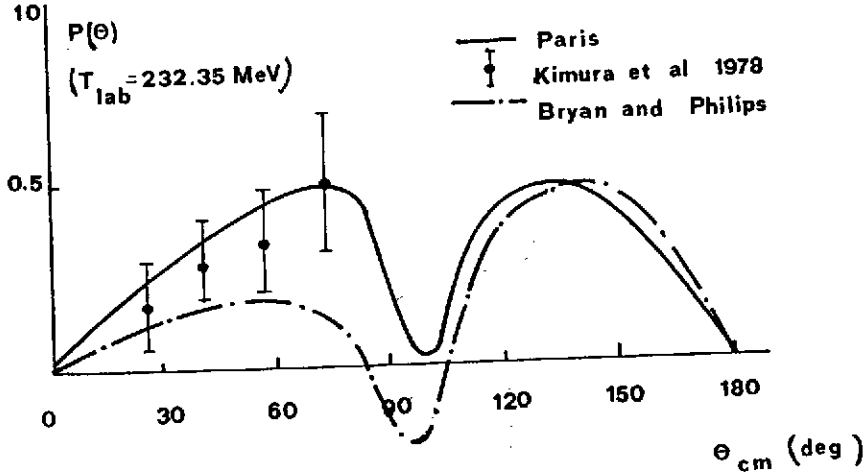


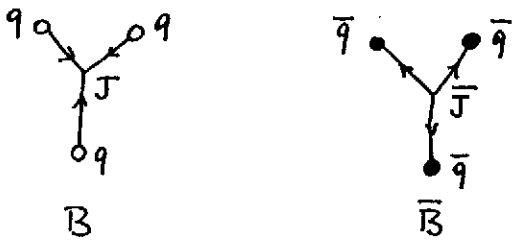
FIG. IV.5

IV.2 The case of baryonium.

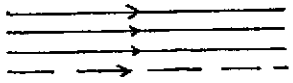
As we have seen (§ III.5), the resonance situation in the baryon-antibaryon system is highly controversial on the experimental side. It is also quite confused on the theoretical side where ideas range from potential model approaches to diquark-antidiquark pictures to the string and topological language[7] but a true predictive theory is (as so often is the case) actually missing.

We will not enter into any of the technicalities of the problem but let us recall very briefly what makes the baryonium such an interesting issue in low energy $N\bar{N}$ interaction.

To introduce the notion of baryonium, let us use the conventional diagrammatic string technique whereby a baryon (an antibaryon) is a color singlet "made" of three quarks (antiquarks) connected by a junction J (an antijunction $\bar{J}$) by which, loosely speaking, we mean the basycally unknown mechanism of what in a QCD language would be called "gluon binding"

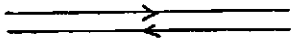


Conventionally, we will denote the effect of a junction by a dotted line so that a baryon's flow in the usual quark diagrams will be represented as



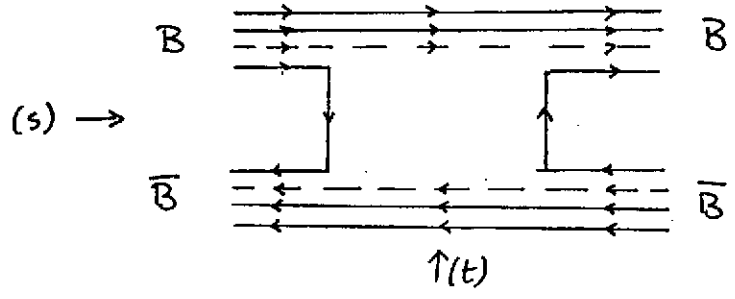
where the straight lines represent the quark's flow.

A meson i.e. a color singlet of a quark-antiquark system will be symbolized by a pair of quark-antiquark lines



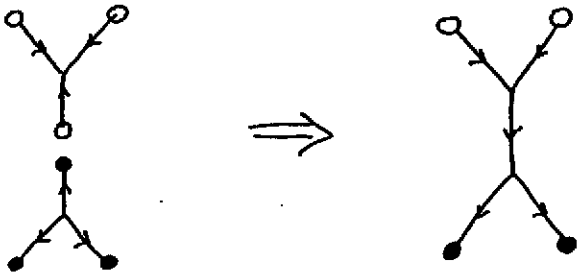
whith no junctions.

The process of $B\bar{B}$ scattering will then be represented as



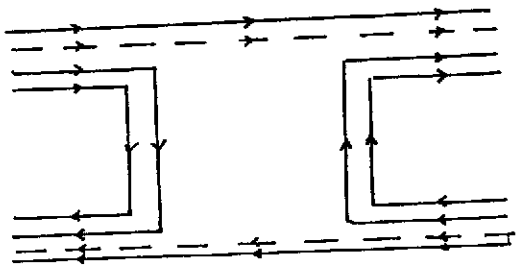
where the t-channel content is meson exchange while the s-channel "force" is a $q\bar{q}\bar{q}q$ system or a diquark-antidiquark $(qq)(\bar{q}\bar{q})$ state if one works within a quark-diquark approach.

From the string model point of view, one can visualize things as due to the merging of vertical lines

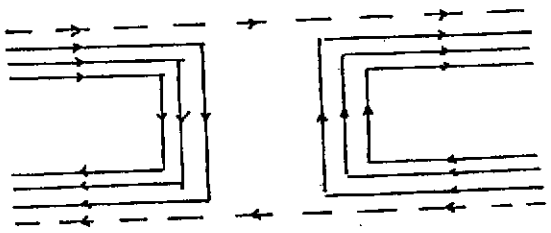


whereby the t-channel force is due to the exchange of an ordinary meson whereas the s-channel contribution is a $(qq\bar{q}\bar{q})$ "meson" (M_4, sometimes in the literature) which is not an ordinary meson $(q\bar{q})$ but can be seen as an "unusual meson" made of 2q, $2\bar{q}$ in a color singlet.

Another possible M_4-exchange contribution to $B\,\bar{B}$ scattering obtains if we now exchange $qq\ \bar{q}\bar{q}$ in the t-channel rather than in the s-channel as follows



More complicated (M_6) exchanges are also possible, i.e.



These new mesons are color singlets which do not decay into ordinary mesons because this would leave unsaturated the $J\,\bar{J}$ lines which are typical of baryons and not of mesons. They are usually referred as "baryonium" in the literature[7].

As so often is the case with strong interactions, the main problem with the theoretical side of baryonium is how to translate the above ideas into actual predictions based on some dynamical relativistic scheme.

As a possible indication we report the first few states predicted by one such model (unpublished)

$(u\,u)_{1S}\ (\bar{u}\,\bar{u})_{1S}$	= 1763 MeV
$(u\,u)_{1S}\ (\bar{u}\,\bar{u})_{2S}$	= 2088 MeV
$(u\,u)_{1S}\ (\bar{u}\,\bar{u})_{3S}$	= 2321 MeV
$u[u\,(\bar{u}\,\bar{u})_{1S}]_{1S}$	= 1769 MeV
$u[u\,(\bar{u}\,\bar{u})_{1S}]_{2S}$	= 2109 MeV
$u[u\,(\bar{u}\,\bar{u})_{1S}]_{3S}$	= 2348 MeV
$u[u\,(\bar{u}\,\bar{u})_{2S}]_{1S}$	= 2101 MeV
$u[u\,(\bar{u}\,\bar{u})_{2S}]_{2S}$	= 2438 MeV
$u[u\,(\bar{u}\,\bar{u})_{3S}]_{1S}$	= 2398 MeV

Table IV.1

where the notation is rather self-explanatory.

No parameters are used in deriving the numbers in Table IV.1 and the technique is an extension of the one used previously to give a very good account of the full hadron (both meson as well as baryon) spectroscopy[8].

Aside from the remarkable mass stability in the above predictions, note that, aside from the much controversial S-resonance (at 1936 MeV) quoted previously (§ III.5.1.b see also Table II), Table IV.1 predicts resonances not far from those for which "evidence" was given in the previous sections.

As already mentioned, however, the values quoted in Table IV.1 are only indicative and a much more careful analysis along the line of ref. 8 must be performed in one wants to really make a prediction concerning the spectrum of baryonium.

IV.3 Conclusions.

The presentation of the theoretical tools used in N $\bar{N}$ phenomenology has been confined to the bare minimum sufficient to give the interested reader enough motivation to pursue further this subject.

I strongly believe that low energy (as well as high energy) N $\bar{N}$ physics will be a main field of investigation of the present decade, both from the experimental and from the theoretical point of view. From both points of view this is a very promising and challenging field indeed on whose large potentiality we have just tried to draw the attention of the young physicists.

References

1) For a summary as well as for an extended literature, see:
M. Giffon and E. Predazzi: "High Energy Physics after the SPS Collider" Lyon preprint LYCEN 8402 (1984) to appear in the Rivista del Nuovo Cimento.

2) For a detailed discussion see, for instance:
E. Leader and E. Predazzi: An Introduction to Gauge Theories and the "New Physics"; Cambridge Univ. Press, 1982.

3) L. Bertocchi: "N $\bar{N}$ Physics" IC 78/61 (unpublished).

4) R. Vinh Mau: "Meson and Isobar Degrees of Freedom in Nuclear Forces"; Proceedings of the 1983 Indiana University Nuclear Physics Workshop on "Manifestations of Hadron Substructures in Nuclear Physics" (October 1983); see also the literature quoted in this paper.

5) M.L. Goldberger, Y. Nambu, R. Oehme: Ann. Phys. 2 226 (1957).

6) C. Zemach: Phys. Rev. 133 B 1201 (1964) and 140 B 97 (1965)

7) G. Rossi and G. Veneziano: Physics Reports 63 149 (1980).

8) D.B. Lichtenberg, W. Namgung, E. Predazzi and J.G. Wills: Phys. Rev. Lett. 48 1653 (1982); Zeit. f. Physik C19 19 (1983).