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Energy Harvesting from Ambient Vibrations

Frédéric Giraud

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Energy Harvesting from Ambient Vibrations

Frédéric Giraud

L2EP – University Lille1

November 27, 2012

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 - Introduction
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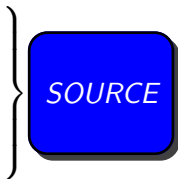
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We extract energy from an *ambient and free source*:

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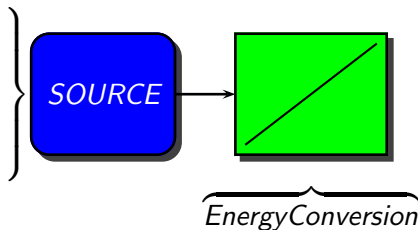
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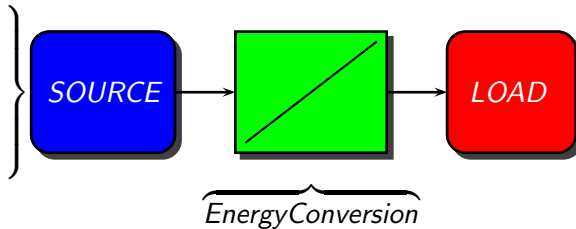
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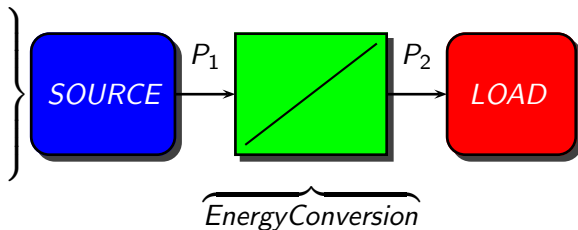
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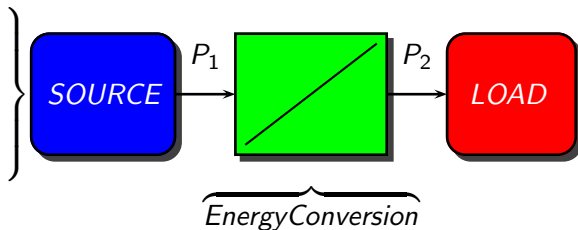
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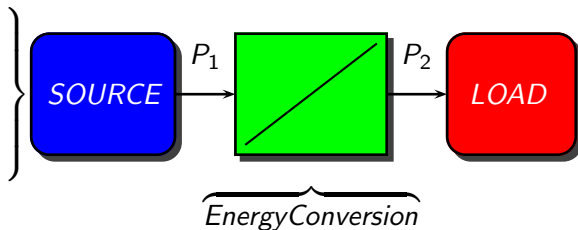


We talk about **Energy Harvesting** or also **energy scavenging** when the converted power is small, typically less than 1W.

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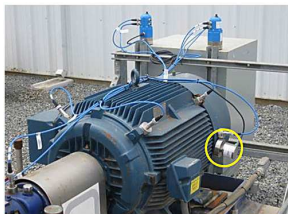
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$\eta = \frac{P_2}{P_1} = 1 - \frac{P_1 - P_2}{P_1} \rightarrow$ Losses in the energy converter should be as small as possible.

Objectives: Sensors Network

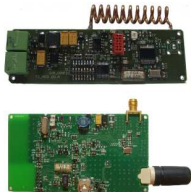


www.perpetuum.com

Objectives: Sensors Network



www.perpetuum.com



Gutiérrez, A Heterogeneous Wireless Identification Network for the Localization of Animals Based on Stochastic Movements

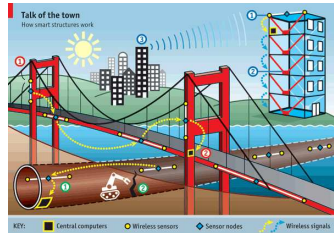
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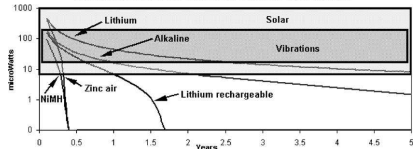


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<http://www.rfwirelessensors.com>, 2012

Continuous Power / cm³ vs. Life for Several Power Sources



Roundy et Al.: A study of low level vibrations as a power source for wireless sensor nodes.

Objectives: Power, just where you need it



<http://enocean.com>

- Wireless
- Reduce Cost, and is reconfigurable
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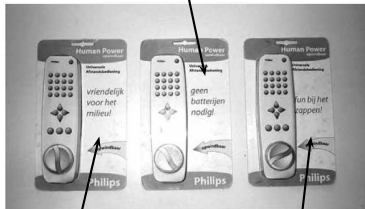


The economist – April 28th 2007

Objectives: Marketing Purpose

Experience: Same Remote controller energized by human power, but with different packaging.

"No need for Batteries"



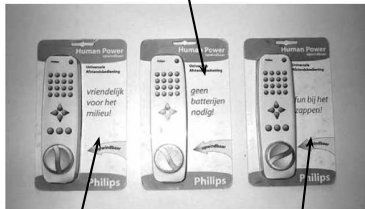
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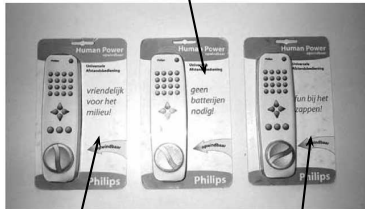
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54% of people Will choose the first one because it is eco-friendly.
(Jansen, Human power empirically explored)

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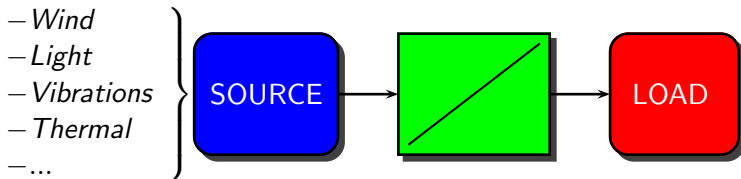
Several projects are born from this fact: **Metis** Produces energy from dancers' movements.



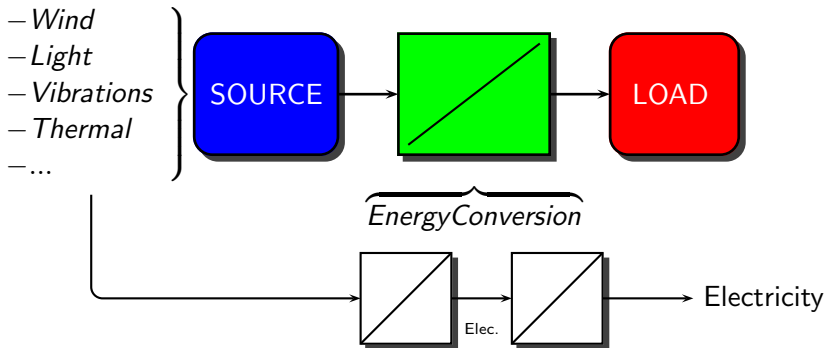
In Toulouse, the system **VIHA** proposes Smart Tiles to energies street lights.



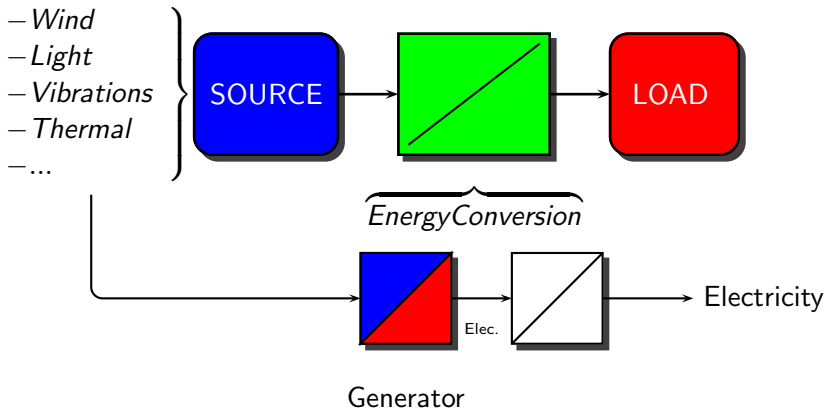
The energy converter



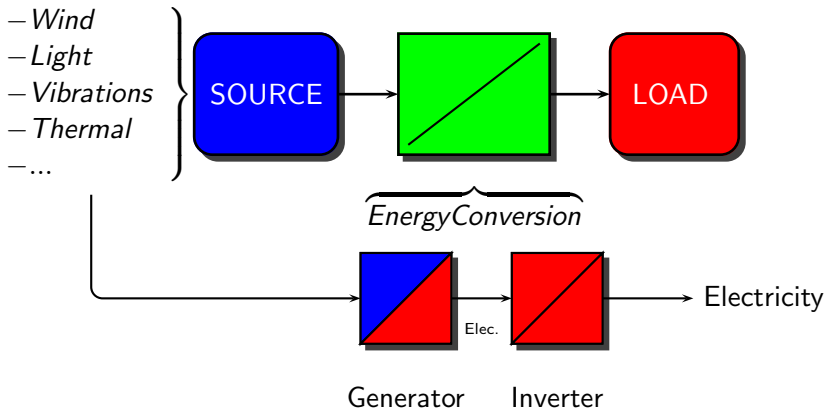
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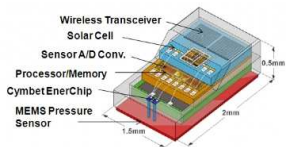


The energy converter



Solar Harvesters:

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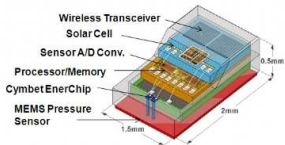


This sensor measures IntraOcular Pressure <http://cymbet.com>



Handbag to recharge electronic devices <http://www.neubers.de>

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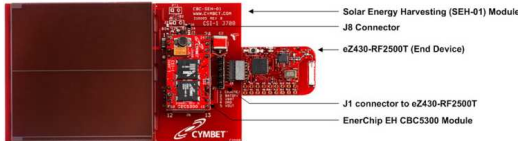
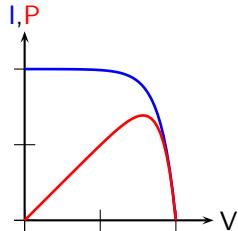


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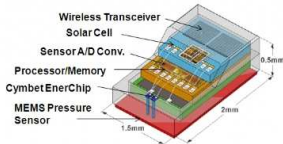
Handbag to recharge electronic devices <http://www.neubers.de>

Power and current as a function of voltage:



Solar Energy Harvester Evaluation Kit <http://ti.com>

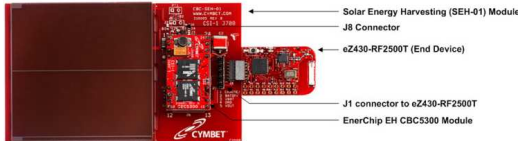
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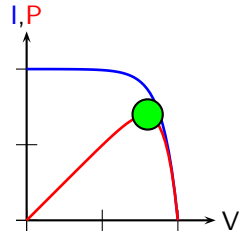


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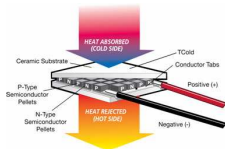
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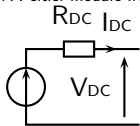


MPPT strategies, require Energy management of the system

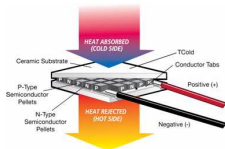
Thermoelectric



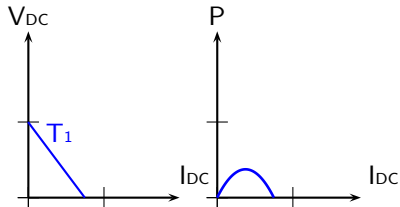
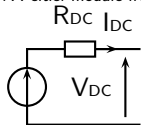
A Peltier module from <http://www.tellurex.com>



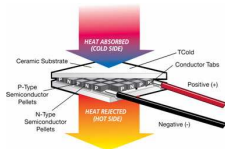
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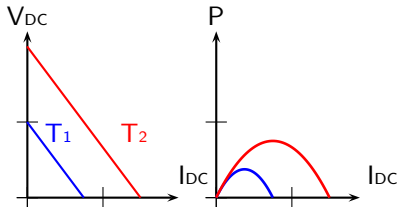
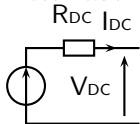
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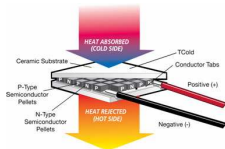
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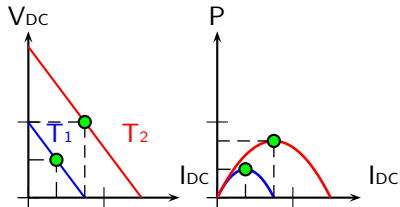
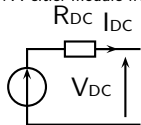
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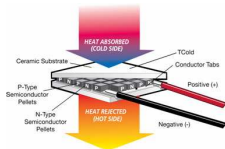
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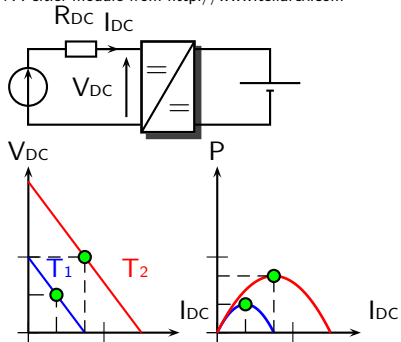
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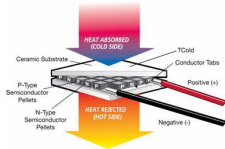
Thermoelectric



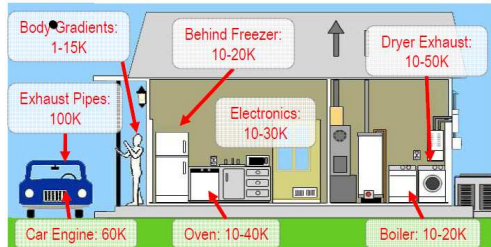
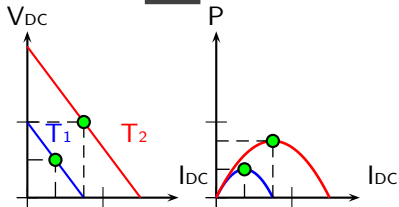
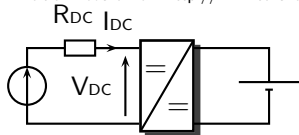
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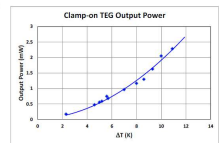


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Temperature Gradient (residential). Lindsay Miller,
<http://uc-ciee.org>

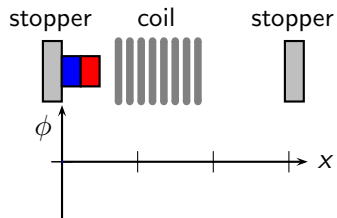
Clamp-on Drain Harvester



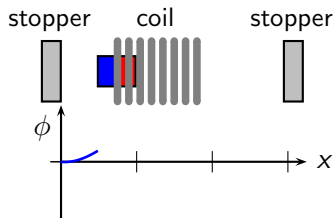
ΔT	V_{load} (mV)	P_{max} (mW)
5.0	38	0.56
8.0	55	1.16
10.0	73	2.05

plumbing application from <http://www.nextreme.com>

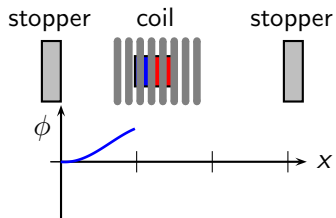
Magnetic



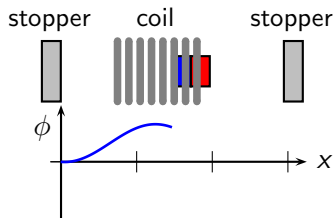
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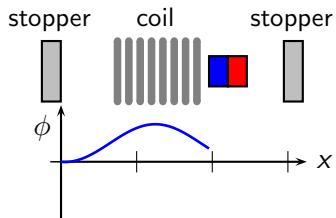
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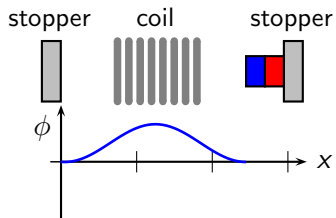
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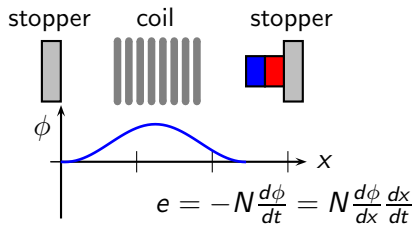
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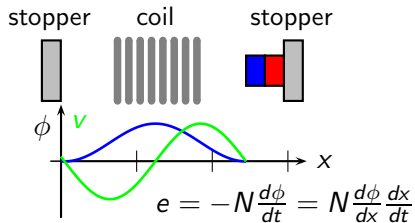
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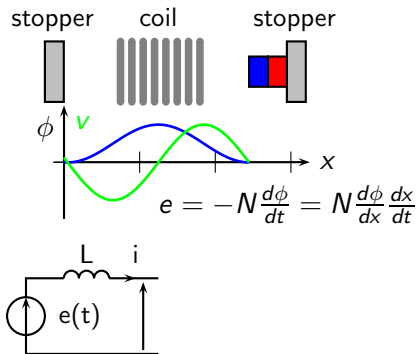
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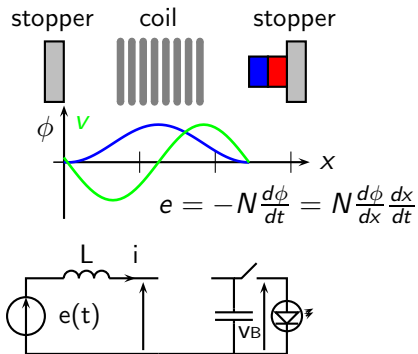
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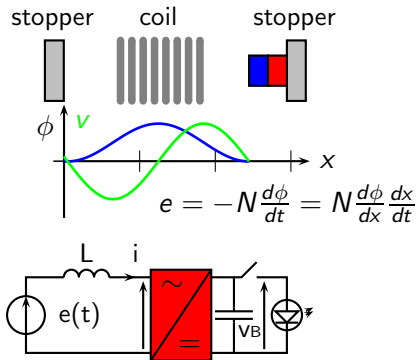
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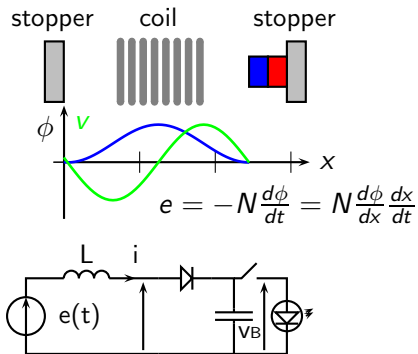
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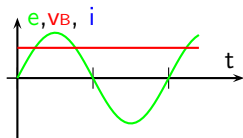
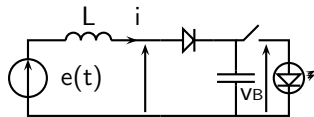
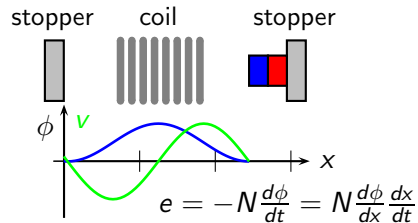
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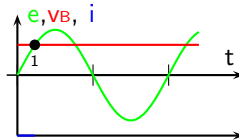
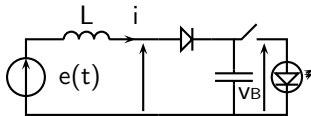
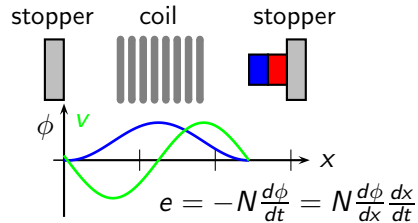
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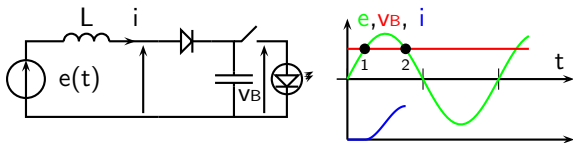
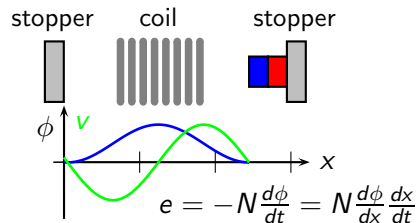


Magnetic



1) Diode turns ON

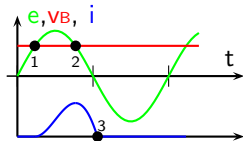
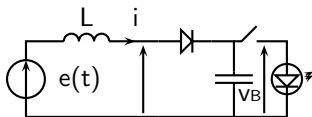
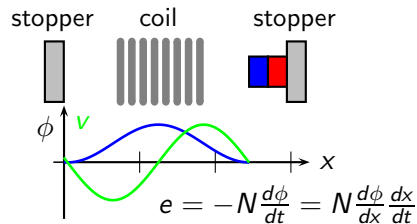
Magnetic



- 1) Diode turns ON
- 2) $\frac{di}{dt} = 0$ because $e - v_B = 0$



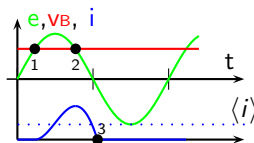
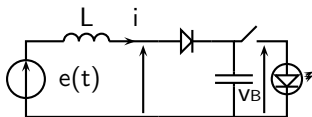
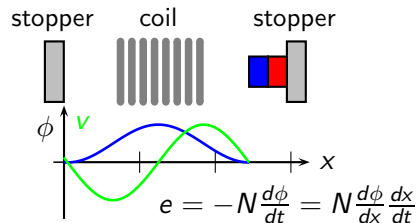
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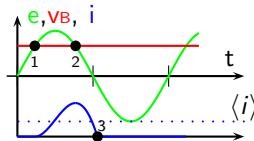
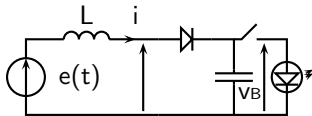
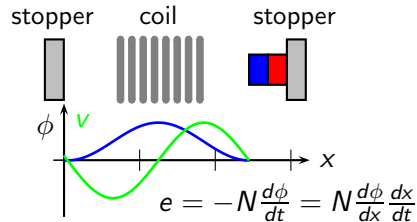


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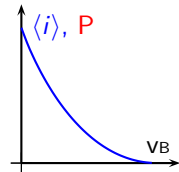


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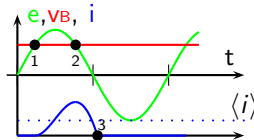
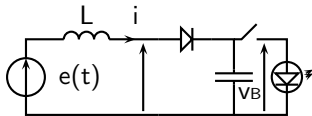
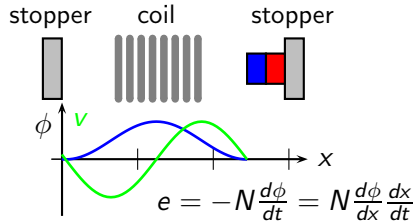
Magnetic



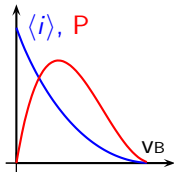
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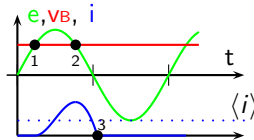
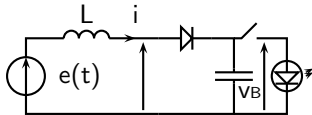
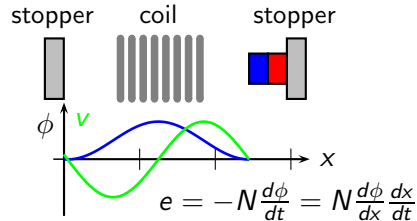


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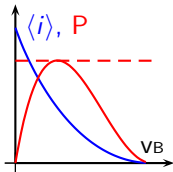


$$P = \langle v_B i \rangle = v_B \langle i \rangle$$

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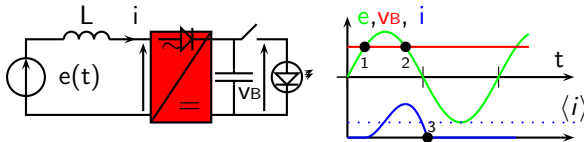
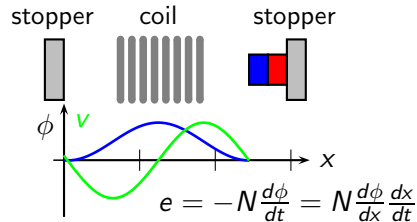


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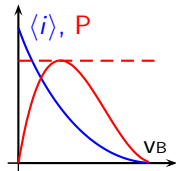


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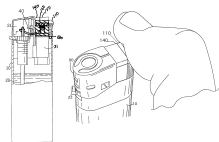


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Piezoelectric

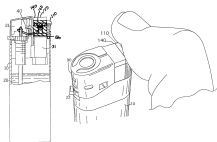


An electronic lighter,
<http://freepatentsonline.com>



Piezoelectric crystals

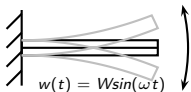
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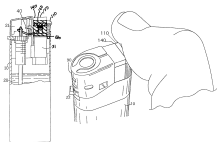
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Piezoelectric crystals
Cantilever beam



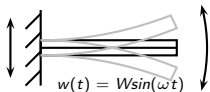
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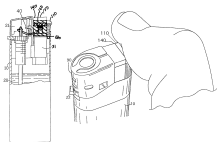
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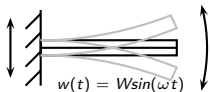


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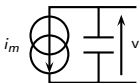


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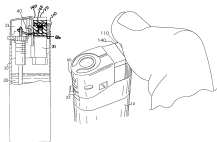


Equivalent electrical circuit



i_m is a current proportional
to the deformation speed

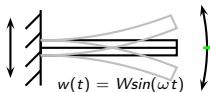
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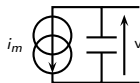
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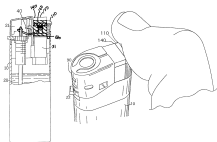


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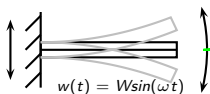
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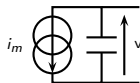
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Piezoelectric crystals
Cantilever beam



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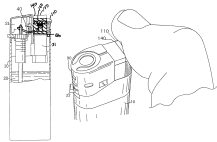


i_m is a current proportional
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Comparison Magn. Piezo

Magnetic	Piezo.
Voltage source	Current source
Inductive	capacitive
Large Stroke	Small Stroke
Remote action	

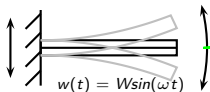
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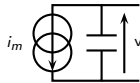
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Piezoelectric crystals
Cantilever beam

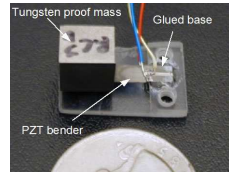


Equivalent electrical circuit



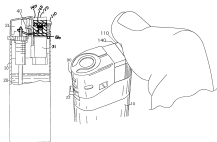
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Roundy, A piezoelectric vibration based generator for wireless electronics (2004)

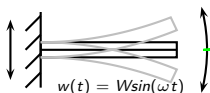
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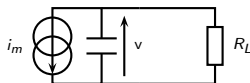
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Piezoelectric crystals
Cantilever beam

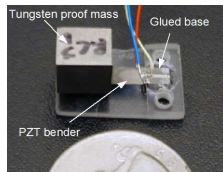


Equivalent electrical circuit



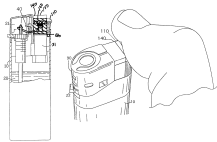
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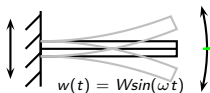
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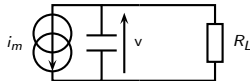
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Piezoelectric crystals
Cantilever beam

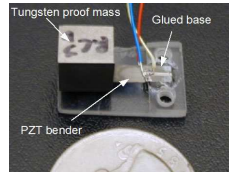


Equivalent electrical circuit

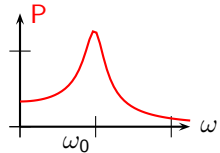


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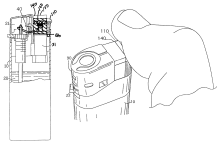
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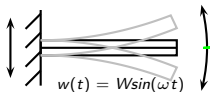
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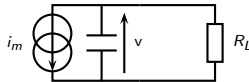
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Piezoelectric crystals
Cantilever beam

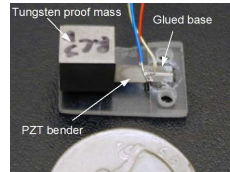


Equivalent electrical circuit

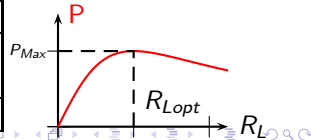
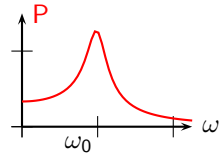


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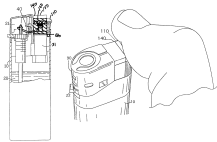
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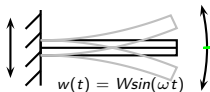
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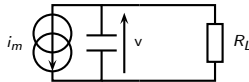


Piezoelectric crystals
Cantilever beam



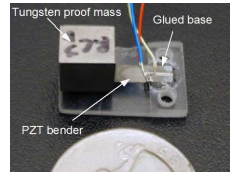
Energy Management

Equivalent electrical circuit

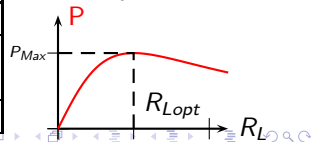
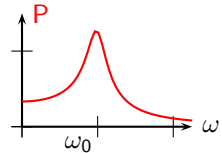


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Comparison Magn. Piezo

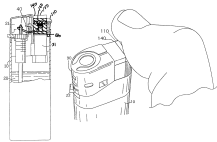
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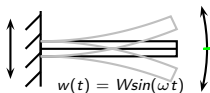
Piezoelectric



An electronic lighter,
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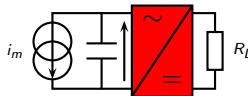


Piezoelectric crystals
Cantilever beam



Energy Management

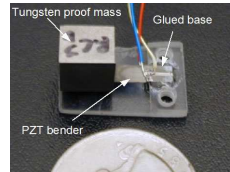
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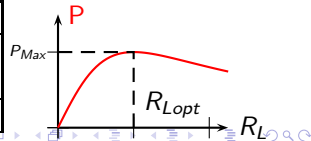
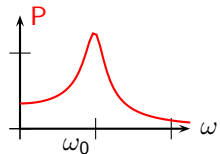
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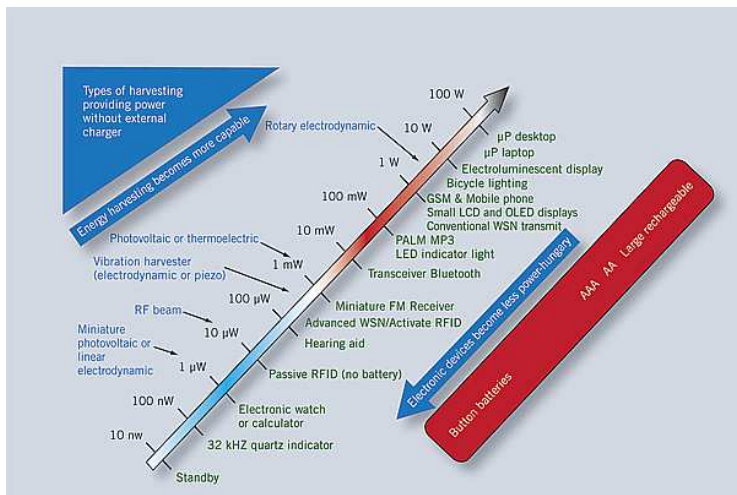
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Roundy, A piezoelectric vibration based generator for wireless electronics (2004)



There is no "one fit all" solution



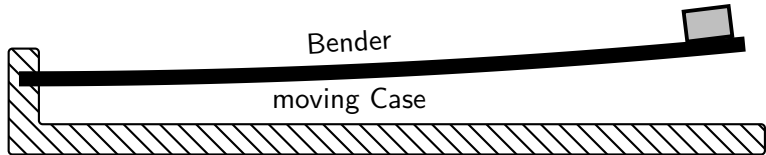
Each solution may be efficient in a certain range of Power.

Meanwhile, shrinking Chips consumption come at a time when energy harvesting becomes efficient and practical
(source: IDtechex.com)

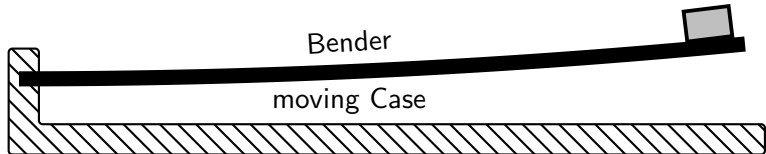
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 - EMR of the system
 - Power Extraction on a load resistor R_L from harmonic oscillation
- 3 An Example of inverter
 - Introduction
 - SSHI: Synchronized Switch Harvesting on Inductor

Coordinates and assumptions

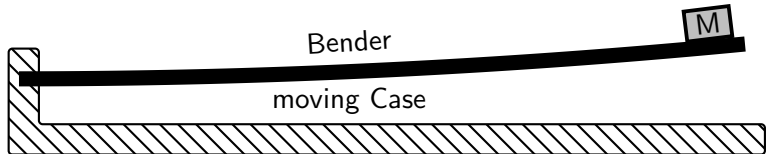


Coordinates and assumptions



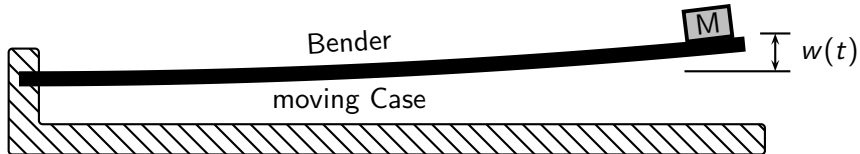
- The bender is attached to a vibrating and rigid case,

Coordinates and assumptions



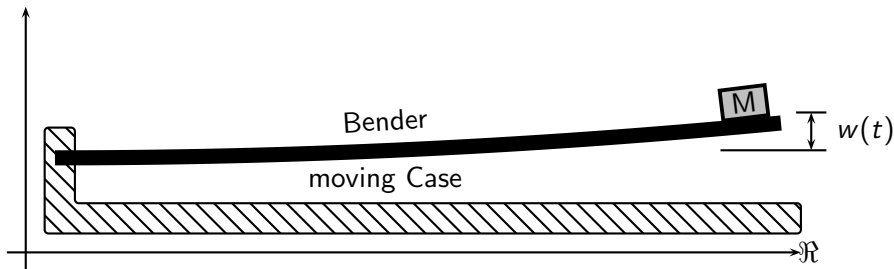
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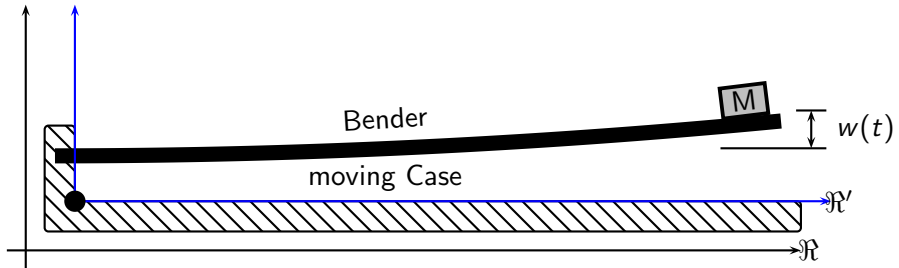
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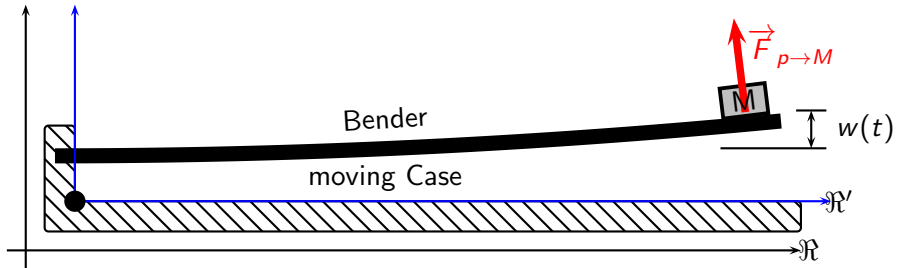
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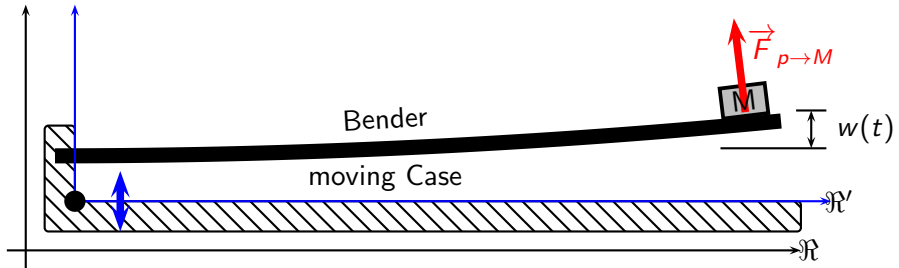
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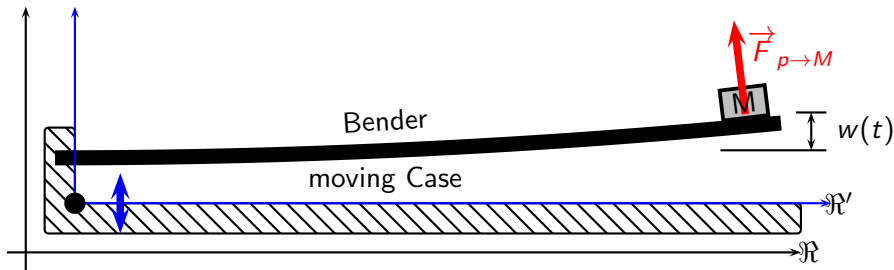
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Coordinates and assumptions



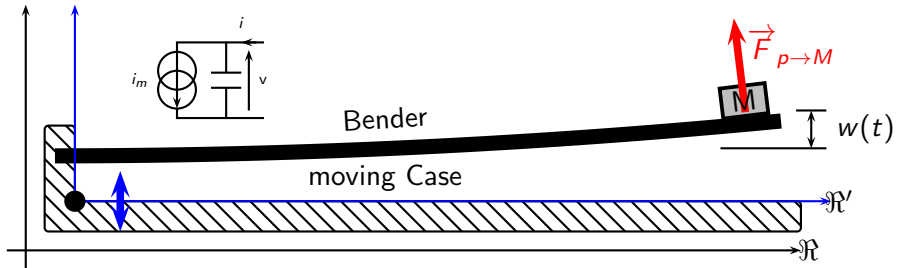
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- Actuator electrical convention.

Equations

- **Dynamic of the mass M :**

Glossary

- M the mass
- f , the force onto M
- A , vibration's amplitude
- ω , vibration's pulsation
- f_{acc} is the inertial force
- i current of the device (actuator convention)
- i_m motional current
- f_p inside piezo force
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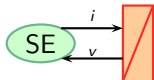
EMR of the system

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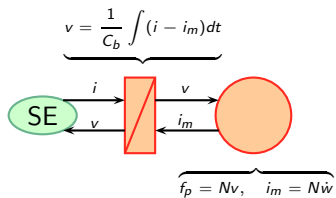


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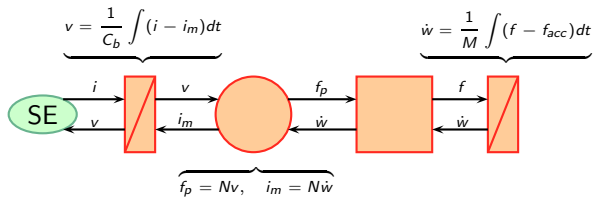
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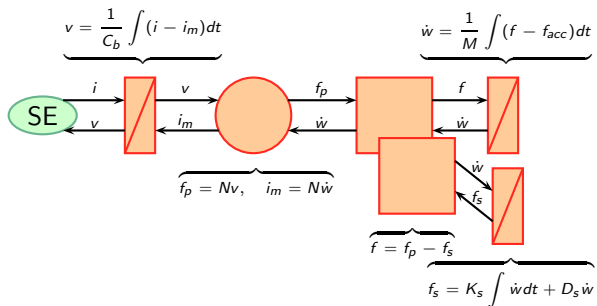
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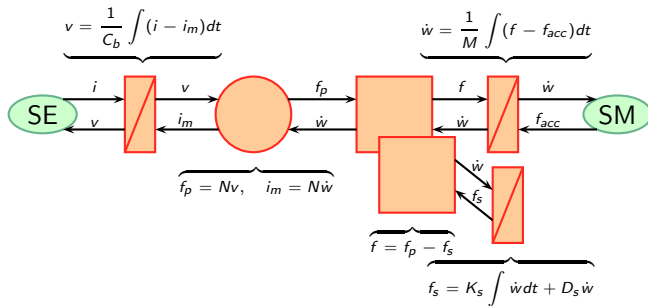
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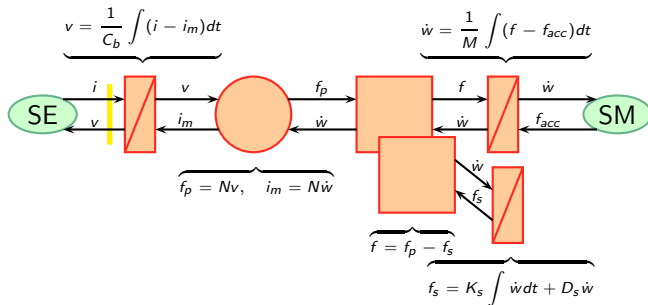
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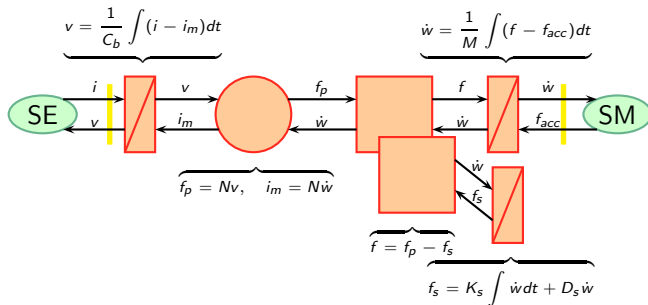


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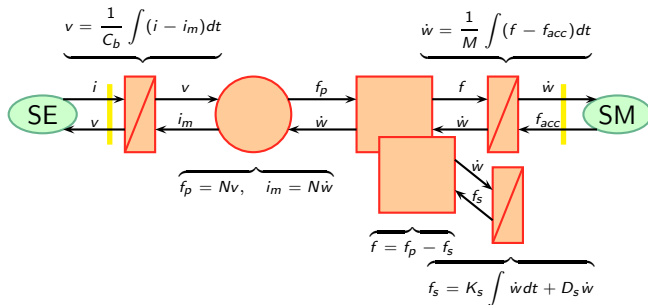
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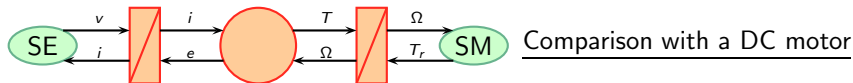


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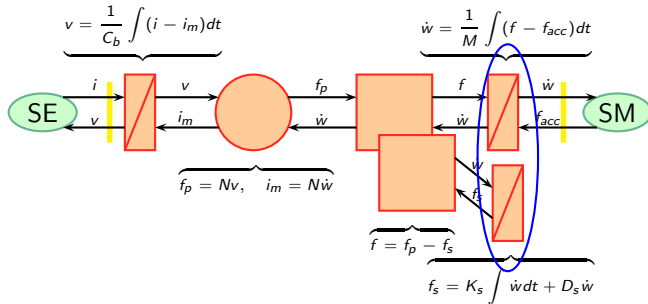


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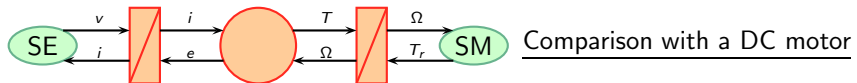


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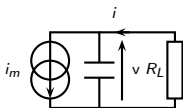
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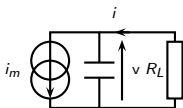
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And $R_L \ll \frac{1}{C_b\omega}$, yields
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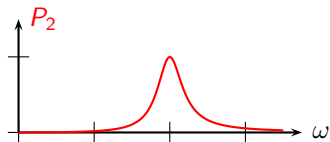
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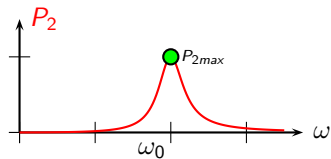
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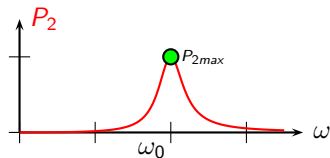
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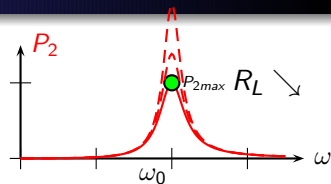


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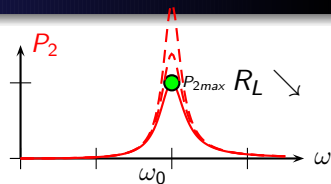


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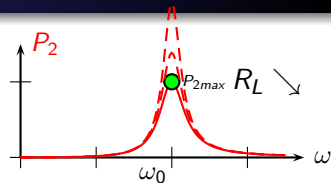


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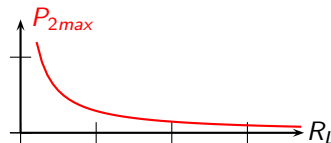
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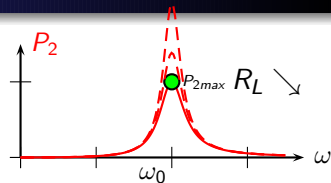
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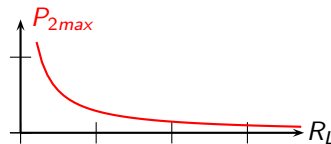
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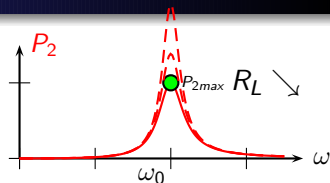
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We can harvest as much power as we want

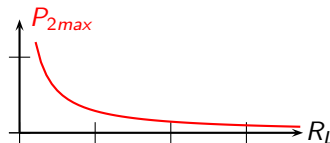
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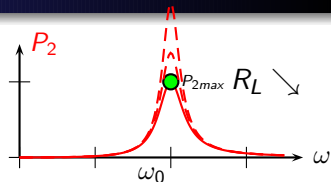
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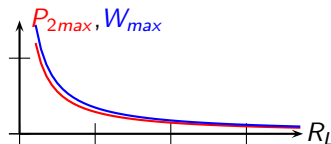
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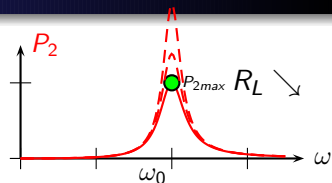
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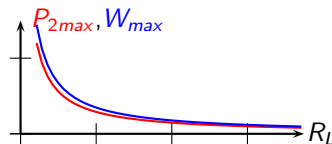
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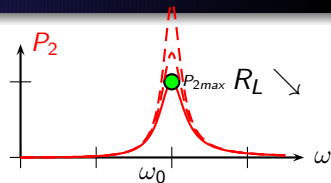
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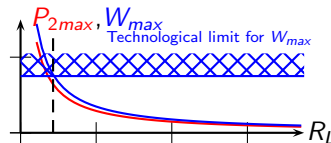
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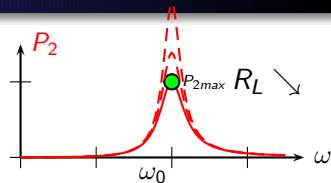
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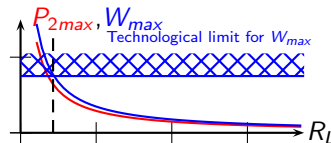
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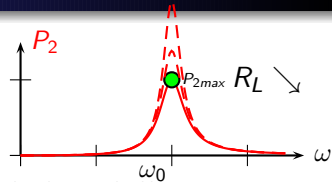
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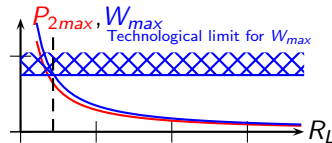
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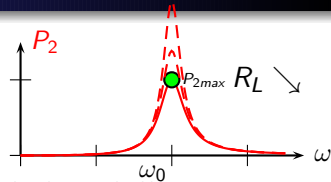
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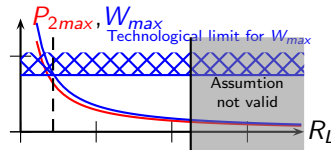
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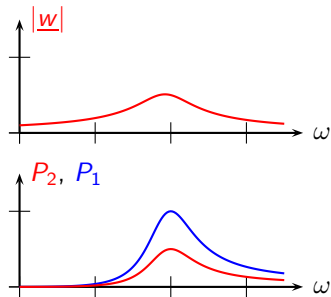
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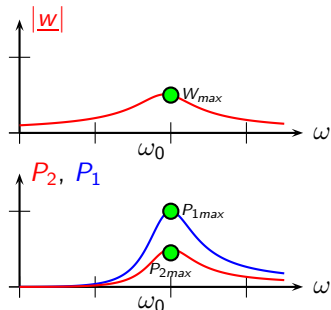
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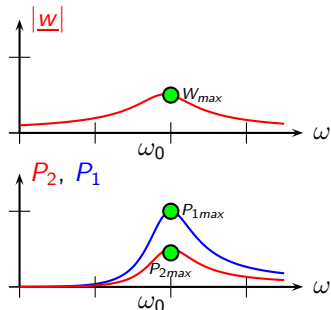
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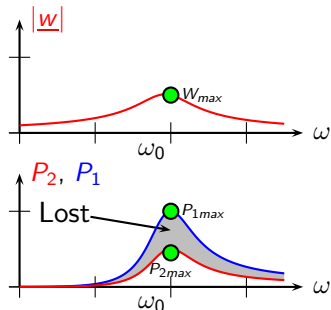
ω_0 should still be the vibration's pulsation.



For a real generator ($D_s \neq 0$)

- $\underline{f} = N\underline{v} - K_s\underline{w} - D_s\dot{\underline{w}}$
- $\underline{w} = \frac{\underline{f}_{acc}}{(K_s - M\omega^2) + j\omega(D_s + N^2R_L)}$
- $P_2 = -\frac{1}{2}R_L|i_m|^2$
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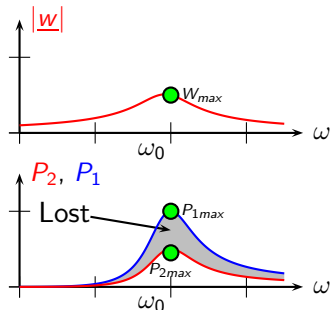
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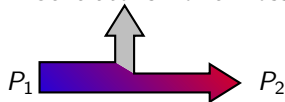
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Generator's Power Losses

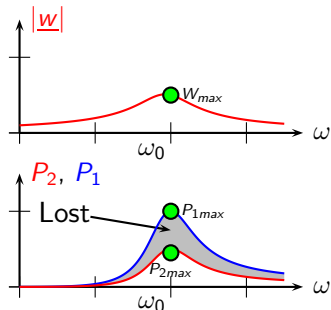


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Generator's Power Losses

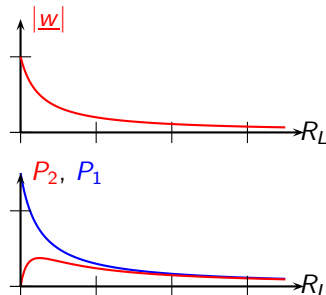


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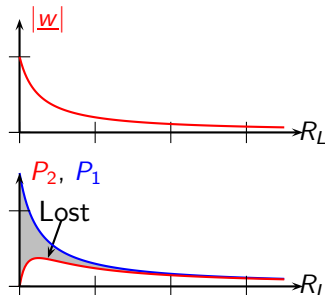
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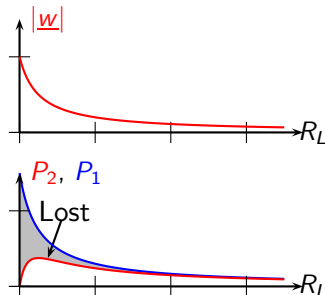


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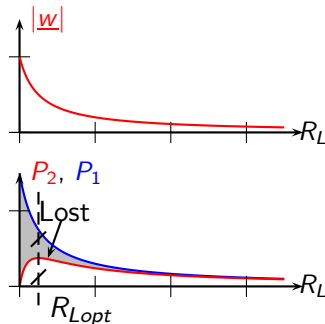


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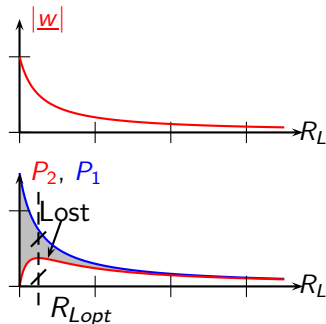


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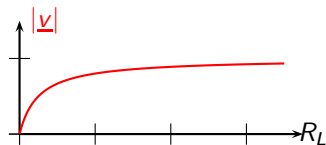
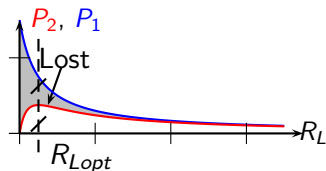
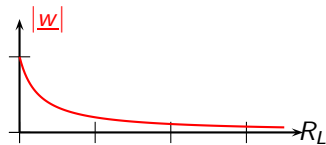


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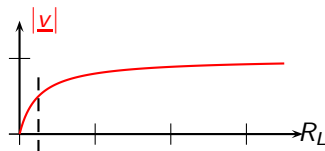
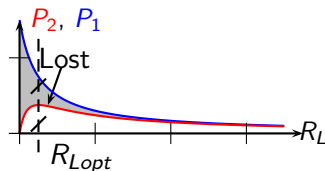
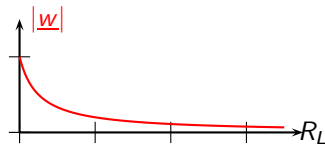


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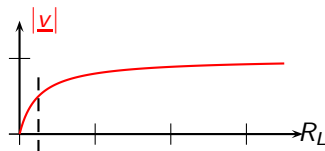
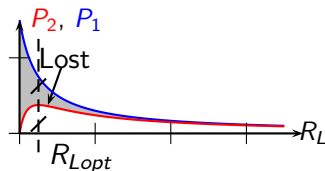
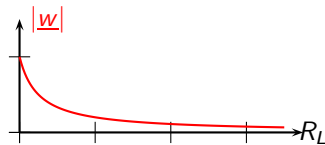


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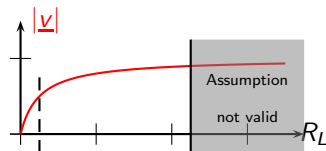
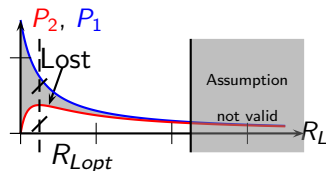
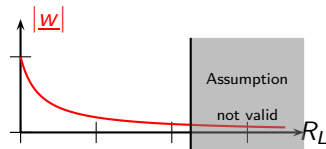


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Example

Device's properties

$$N = 0.012 \text{ N/V}, K_s = 6300 \text{ N/m}, C_b = 300 \text{ nF}, D_s = 0.17 \text{ Ns/m}, \\ M = 1 \text{ g}$$

Calculate for $A = 0.1 \text{ mm}$

- the best working frequency,
- the harvested P_2 power in the best case,
- the power of the source P_1 in such best case,
- the optimal resistor R_L ,
- the deflection amplitude of the bender,
- the voltage for this working point.

Validation

Is $v = -R_L i_m$ a valid assumption?

Answers

- The best working frequency is given by

$$f_0 = \frac{1}{2\pi} \sqrt{\frac{K_s}{M}} = \frac{1}{2\pi} \sqrt{\frac{6300}{1.10^{-3}}} = \underline{400Hz}$$

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$$P_2 = \frac{|f_{acc}|^2}{8D_s} = \frac{(1 \cdot 10^{-3} \cdot 1 \cdot 10^{-4} \cdot (2\pi \cdot 400)^2)^2}{8 \cdot 0,17} = \underline{464\text{mW}}$$

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$$W_{max} = \frac{|f_{acc}|}{(D_s + N^2 R_L)\omega_0} = \frac{(1.10^{-3} \cdot 1.10^{-4} \cdot (2\pi \cdot 400)^2)}{2 \cdot 0,17 \cdot 2\pi \cdot 400} = \underline{738\mu\text{m!}}$$

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- The voltage is then given by

$$v_{max} = N R_L \omega_0 |w| = 0,012 \cdot 1180 \cdot 2\pi \cdot 400 \cdot 738 \cdot 10^{-6} = \underline{26\text{V}}$$

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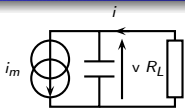
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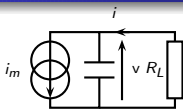
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- $Z_{Cb} = \frac{1}{C_b \omega} = \frac{1}{2\pi \cdot 400 \cdot 300 \cdot 10^{-9}} = \underline{1990\Omega} \approx R_L, \rightarrow$ **Calculations are NOT valid!**

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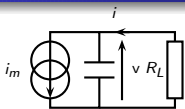


We can show:

$$\underline{v} = -\frac{R_L}{1+j\omega R_L C_b} \underline{i}_m = -j\omega N R_L \frac{1-j\omega R_L C_b}{1+(R_L C_b \omega)^2} \underline{w}_m$$

and we write: $\underline{v} = -j\omega N r_{Leq} \underline{w} - N k_{eq} \underline{w}$

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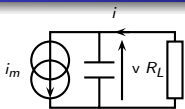
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2 Cases to consider, since ω should be ω_0 :

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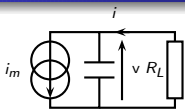
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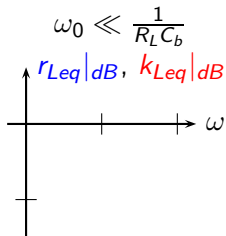


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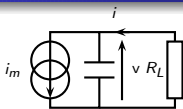
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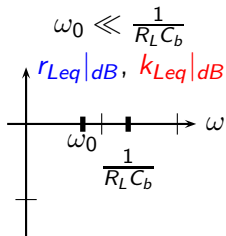


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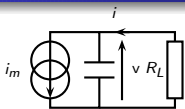
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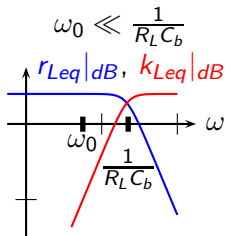


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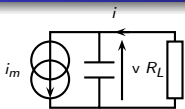
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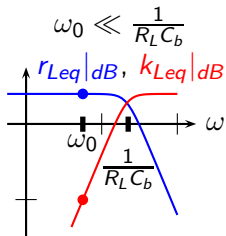


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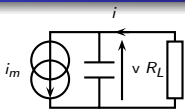
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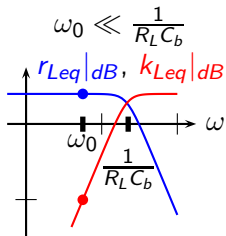


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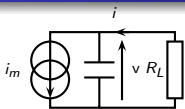
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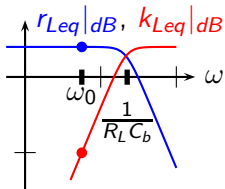
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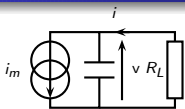
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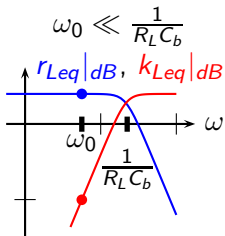


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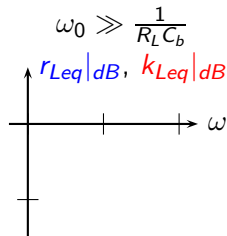
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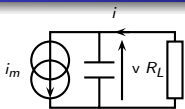
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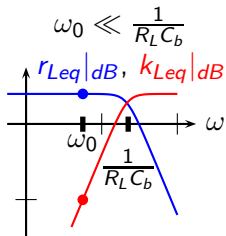


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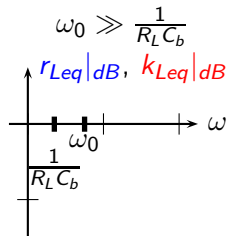
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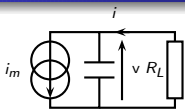
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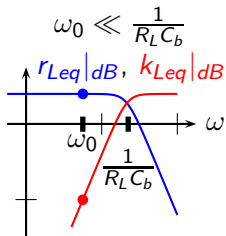


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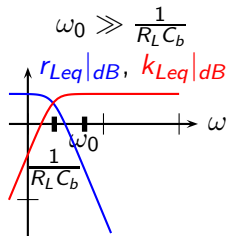
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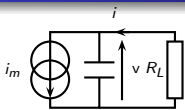
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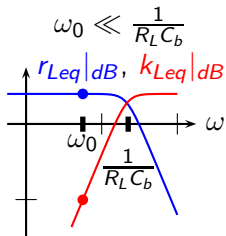


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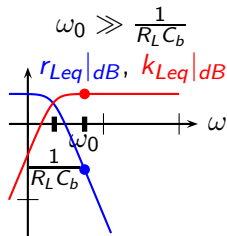
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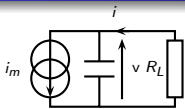
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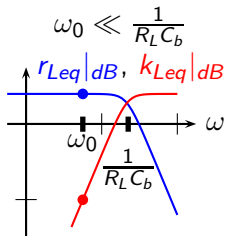


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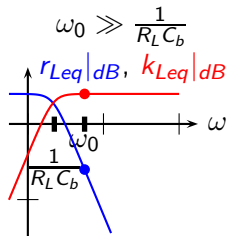
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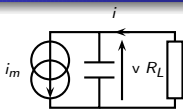


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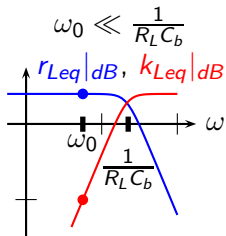


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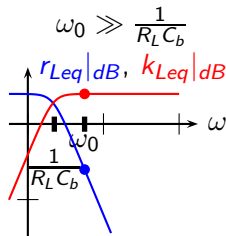
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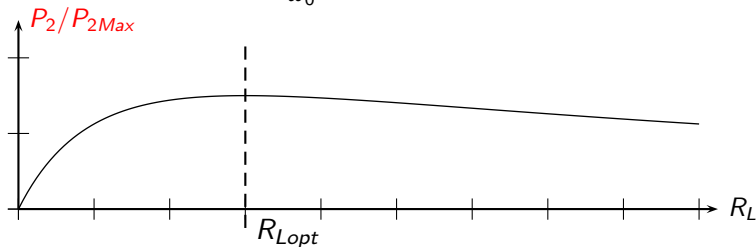
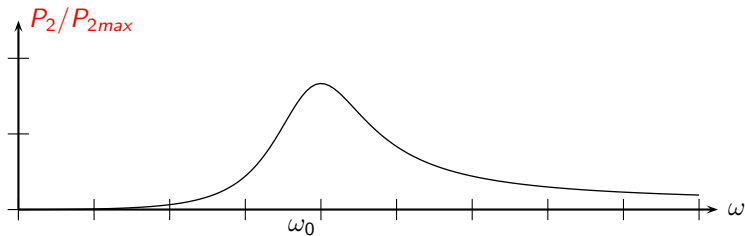


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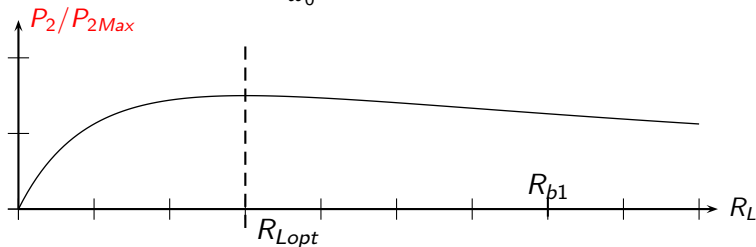
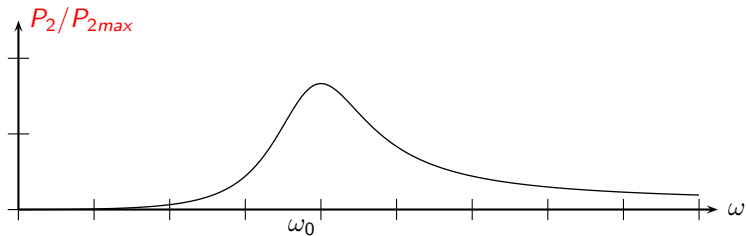
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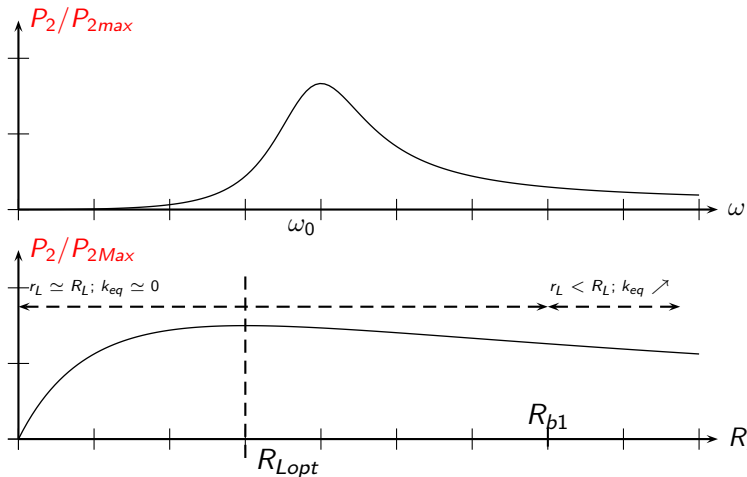
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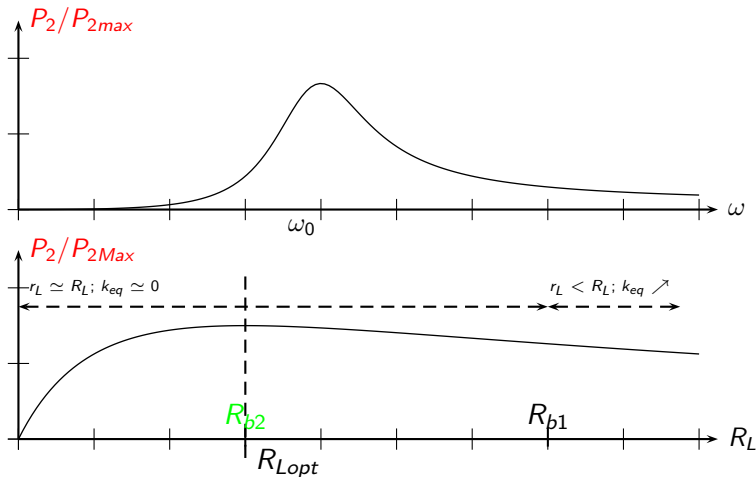
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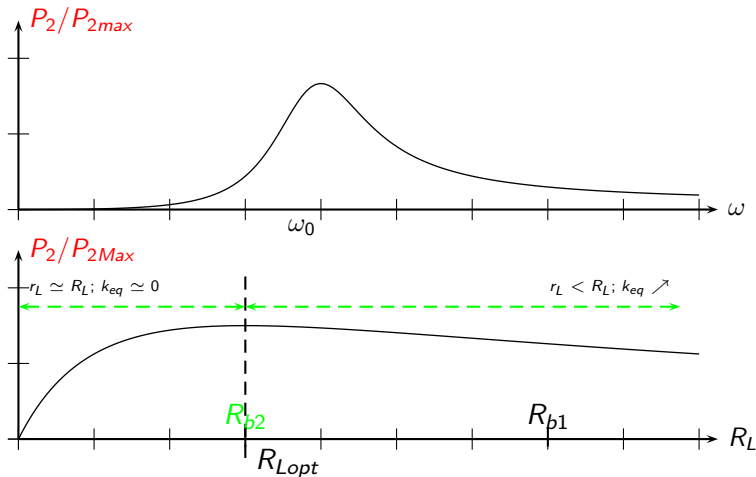
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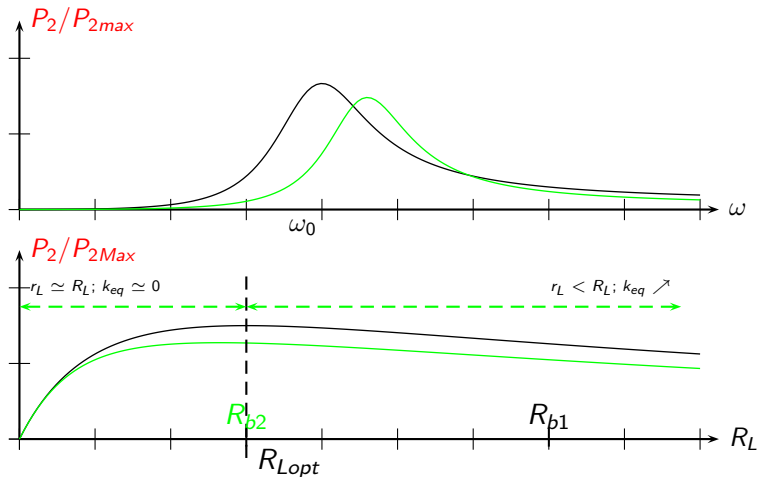
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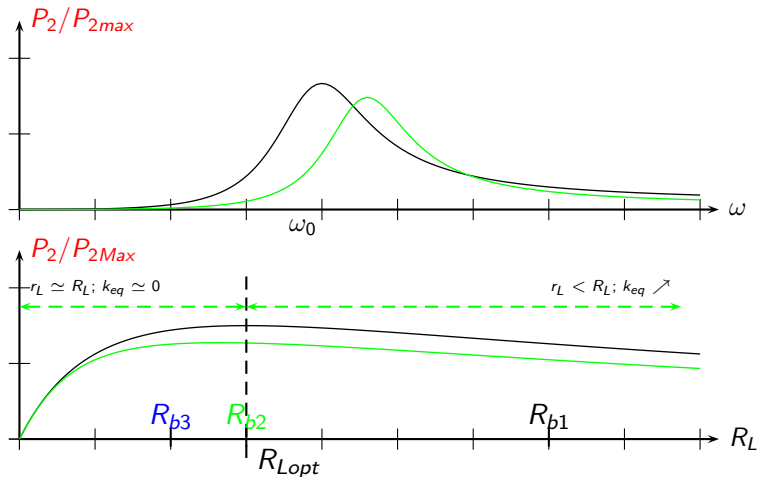
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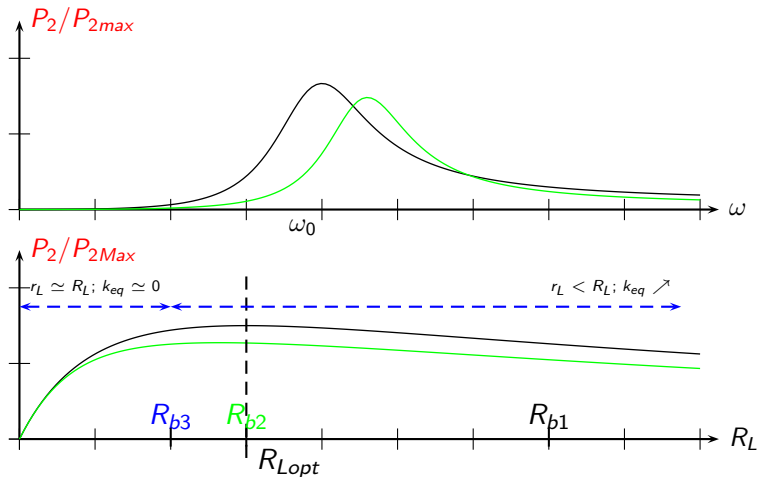
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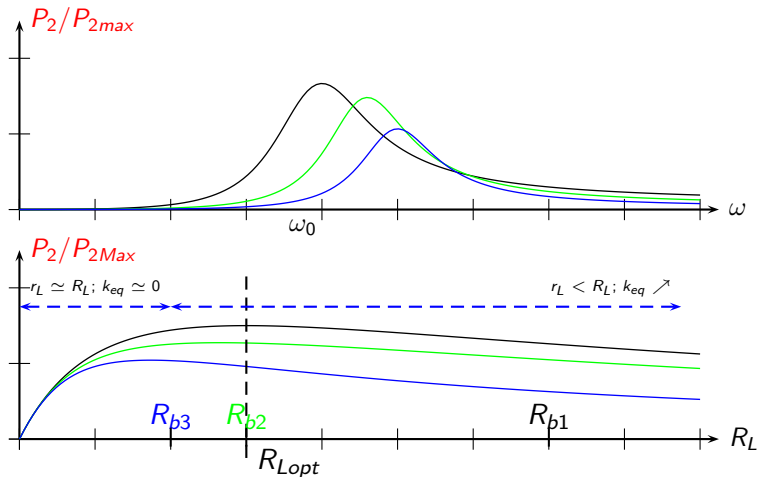
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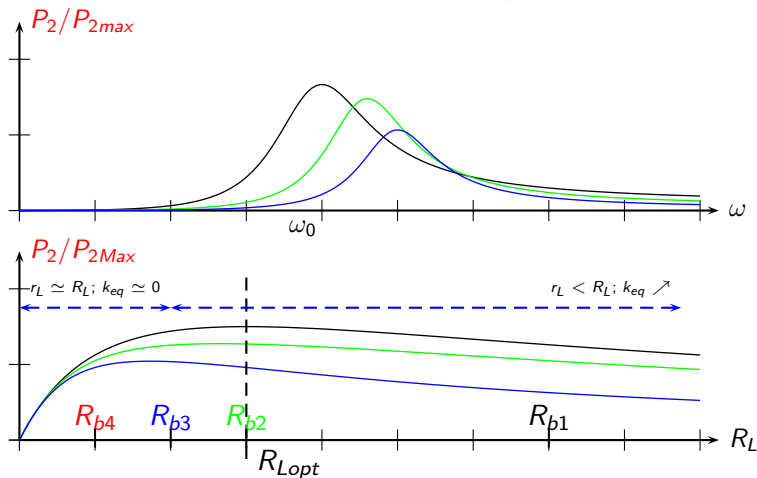
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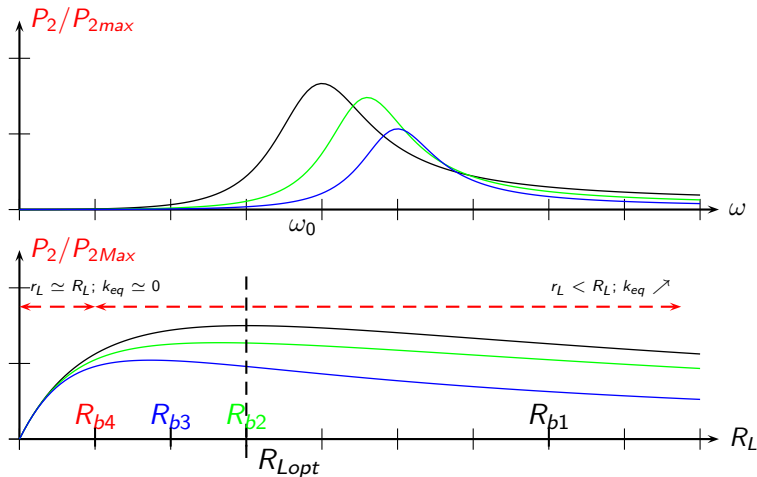
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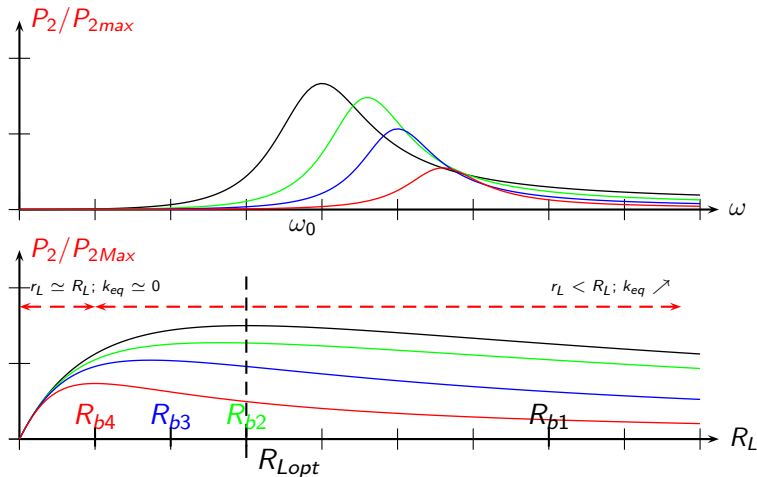
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If $R_{opt} > R_b$

The power is not well extracted because $P_2 < P_{2max}$. This happens for high damped mechanisms.

A simple resistor cannot recover the power optimally (and this is due to C_b).

What happens if $R_{opt} < R_b$ and $R_L \gg R_b$?

resonance is shifted

- $r_{Leq} = \frac{R_L}{1+(R_L C_b \omega)^2} \simeq \frac{R_L}{(R_L C_b \omega)^2}$

- $k_{eq} = R_L \frac{\omega^2 R_L C_b}{1+(R_L C_b \omega)^2} \simeq \frac{1}{C_b}$

$((K_s + N^2 k_{eq}) - M\omega^2) + j\omega(D_s + N^2 r_{Leq})\underline{w} = \underline{f}_{acc}$ leads to:

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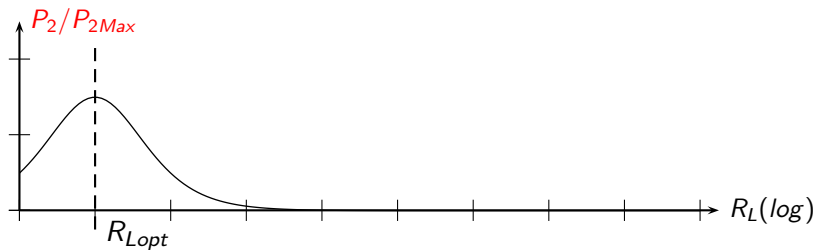
Another optimal resistor

The optimal power is harvested is $N^2 r_{Leq} = D_s$, leading to:

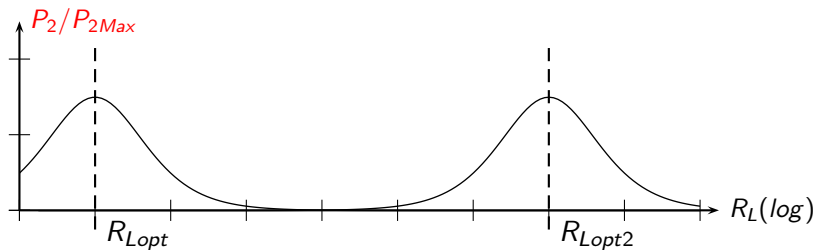
$$\frac{N^2 R_L}{(R_L C_b \omega'_0)^2} = \frac{N^2}{R_L C_b^2 \frac{K_s + \frac{N^2}{C_b}}{M}} = D_s$$

$$R_{Lopt2} = \frac{N^2}{D_s} \frac{1}{C_b^2 \frac{K_s}{M} (1 + \frac{N^2}{K_s C_b})} = \frac{R_b^2}{R_{Lopt}} \frac{1}{1 + \frac{N^2}{K_s C_b}}$$

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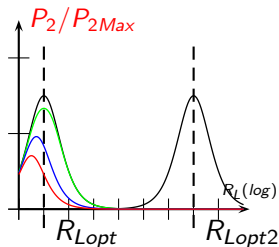


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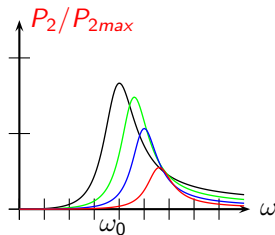
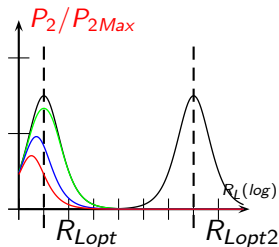
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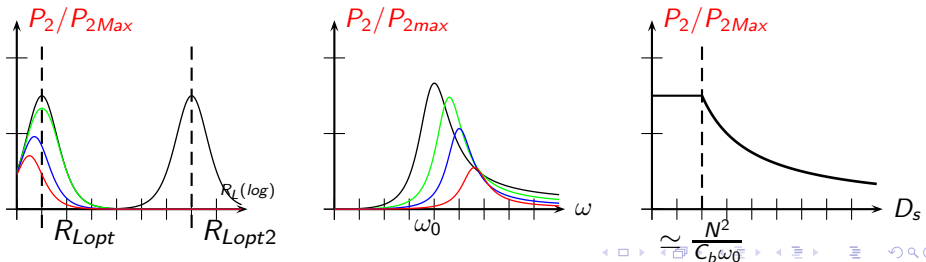
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- For the realistic case, there is one or two resistive loads which allow to extract the maximum of power,
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- we always want to harvest the maximum of power!

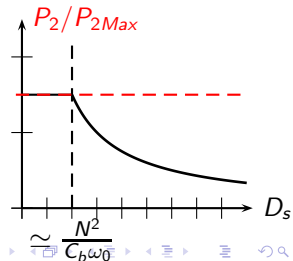
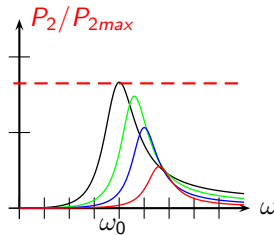
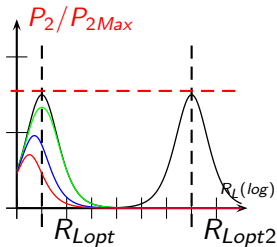
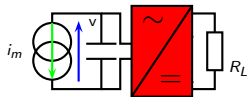


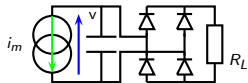
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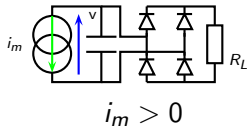
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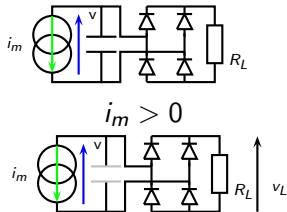
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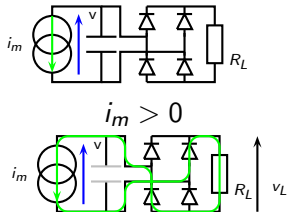
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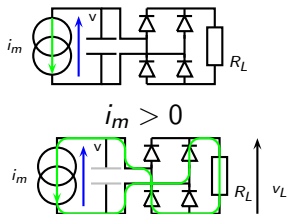
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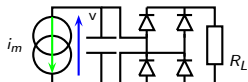
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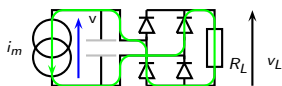
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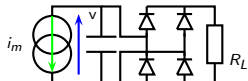


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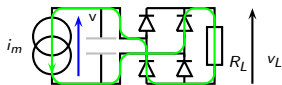
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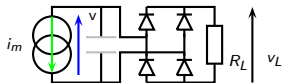
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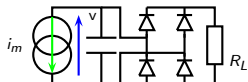
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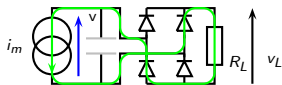
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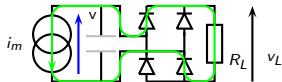
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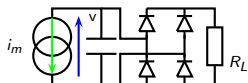
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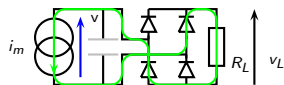
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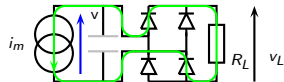
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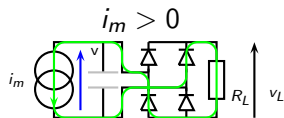
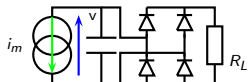


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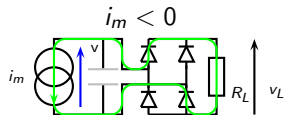
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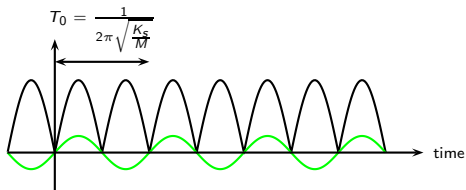
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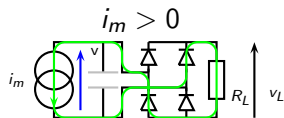
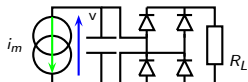
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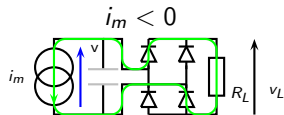


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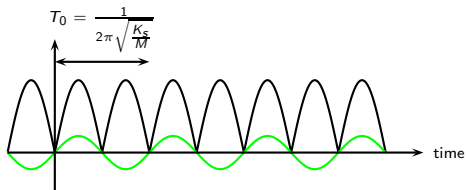
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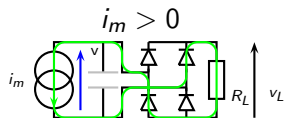
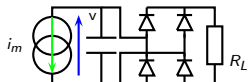
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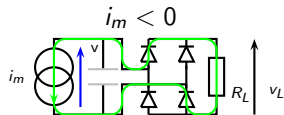
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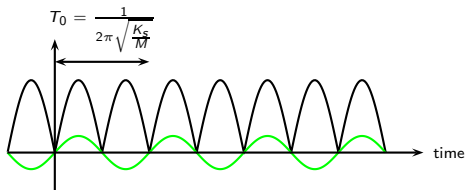
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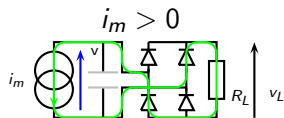
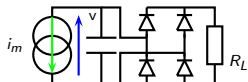
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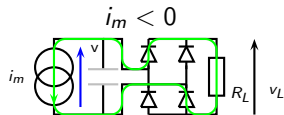
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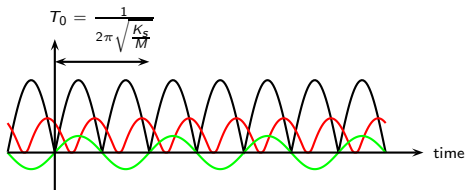
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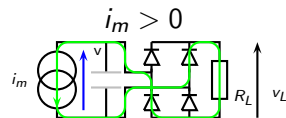
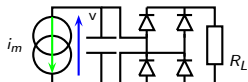
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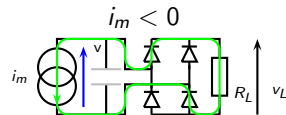
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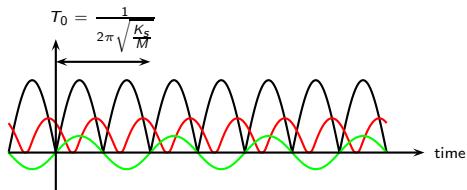
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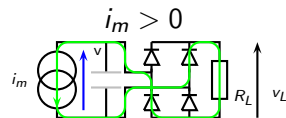
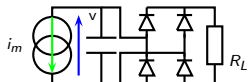
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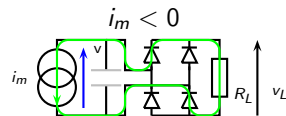


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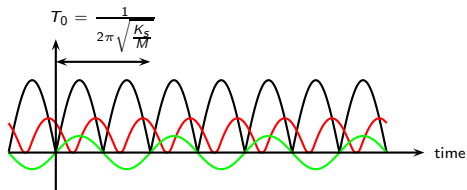
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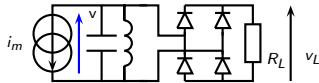
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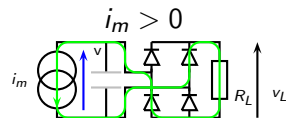
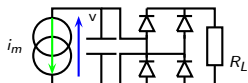
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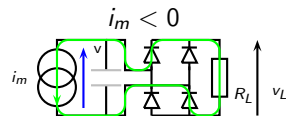
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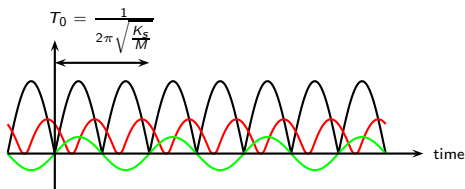
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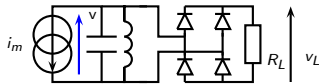
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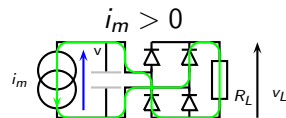
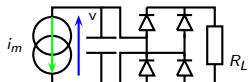


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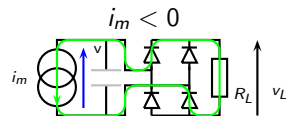


This is a bad solution because it works only for ω_0 (what if the frequency shifts?), and the Inductor is large (because ω_0 usually is small)

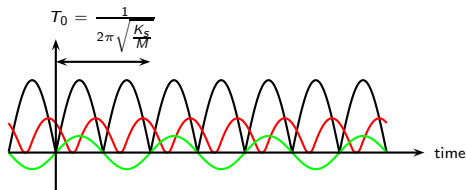
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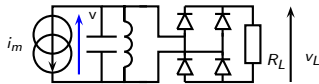
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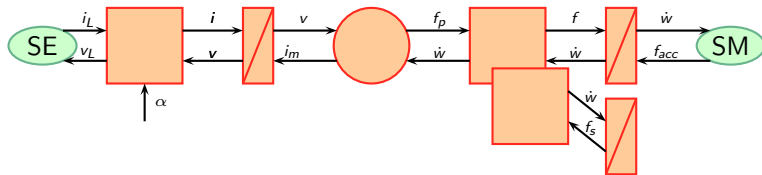
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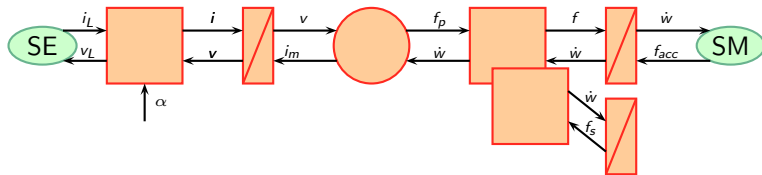
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→ Non linear Techniques

Why Synchronized

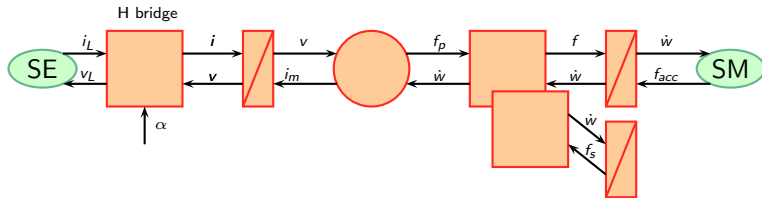
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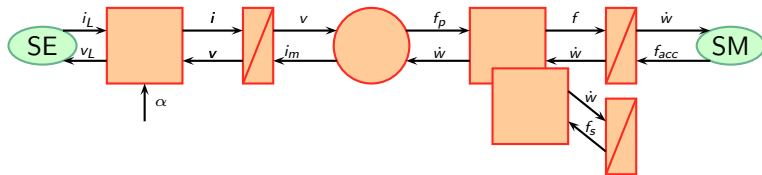
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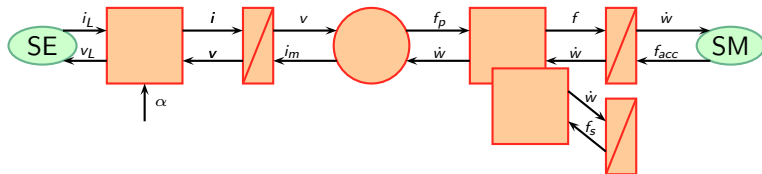


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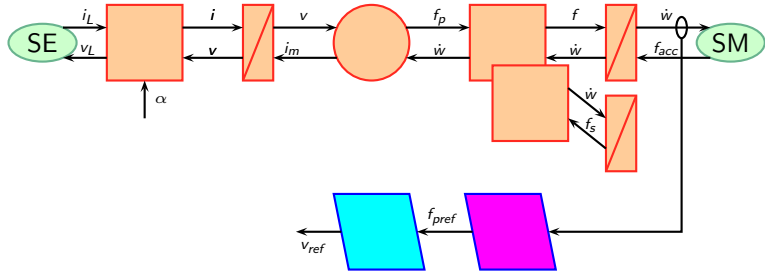
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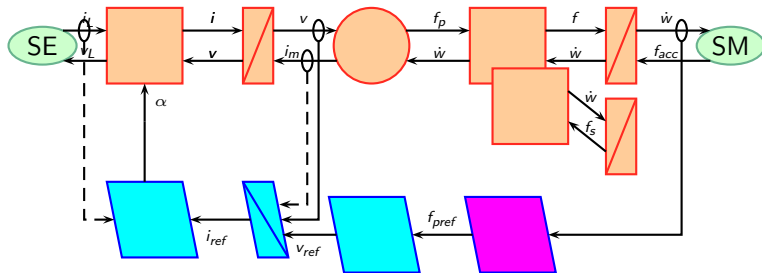
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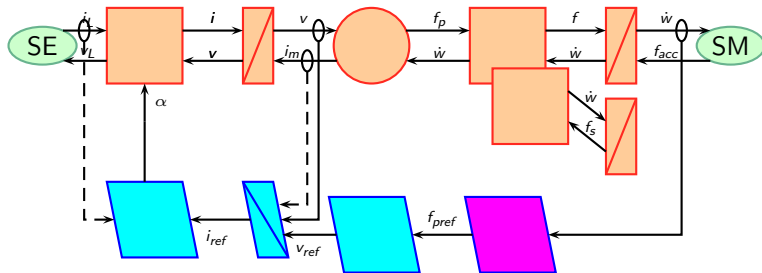
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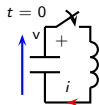
How it works

LC oscillations



How it works

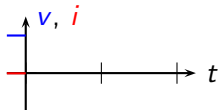
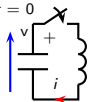
LC oscillations



How it works

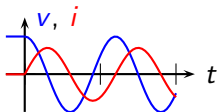
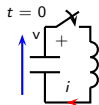
LC oscillations

$t = 0$



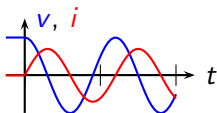
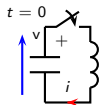
How it works

LC oscillations



How it works

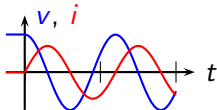
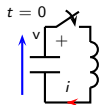
LC oscillations



Switched inductor

How it works

LC oscillations

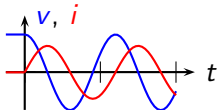
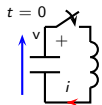


Switched inductor

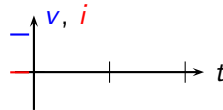
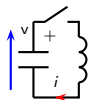


How it works

LC oscillations

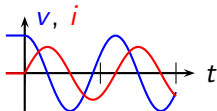
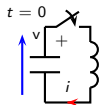


Switched inductor

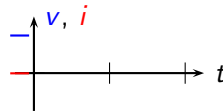


How it works

LC oscillations

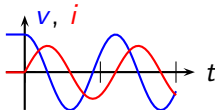
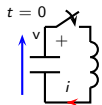


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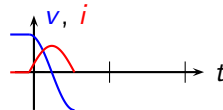


How it works

LC oscillations

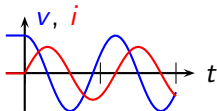
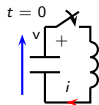


Switched inductor

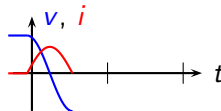
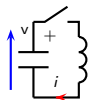


How it works

LC oscillations

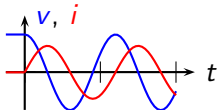
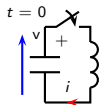


Switched inductor

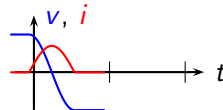
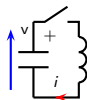


How it works

LC oscillations

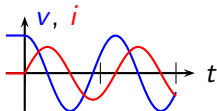
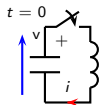


Switched inductor

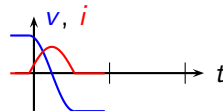


How it works

LC oscillations

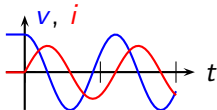
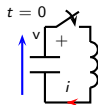


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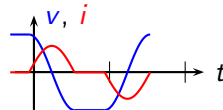


How it works

LC oscillations

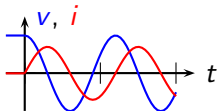
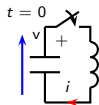


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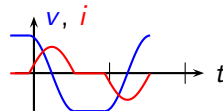
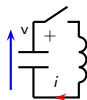


How it works

LC oscillations

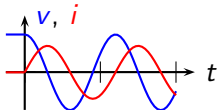
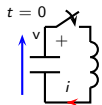


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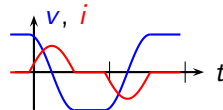
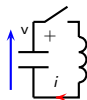


How it works

LC oscillations

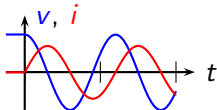
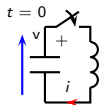


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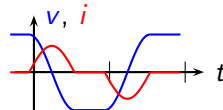
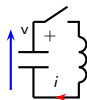


How it works

LC oscillations



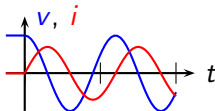
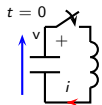
Switched inductor



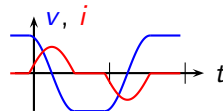
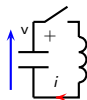
SSHI

How it works

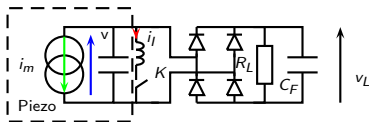
LC oscillations



Switched inductor

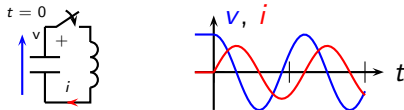


SSHI

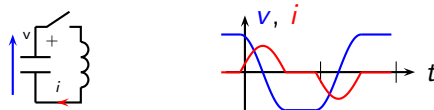


How it works

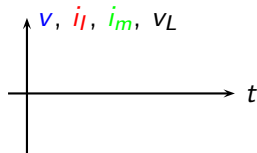
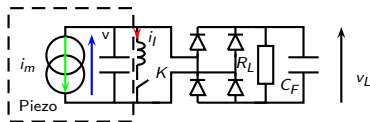
LC oscillations



Switched inductor

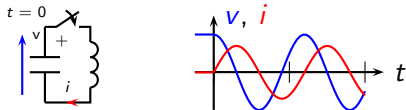


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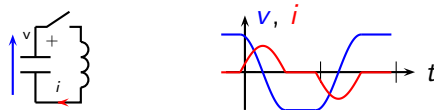


How it works

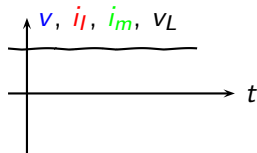
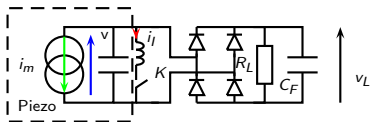
LC oscillations



Switched inductor

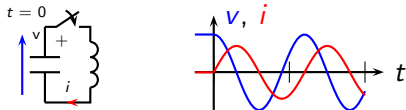


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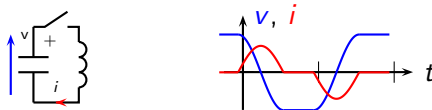


How it works

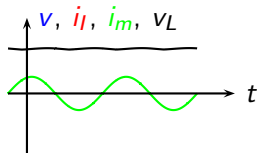
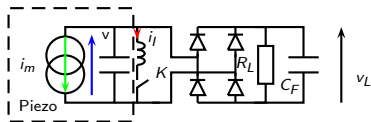
LC oscillations



Switched inductor

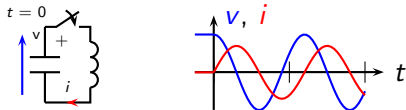


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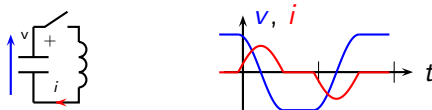


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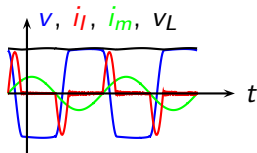
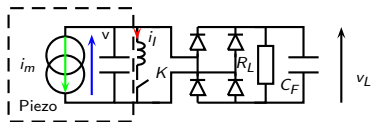
LC oscillations



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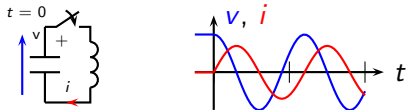


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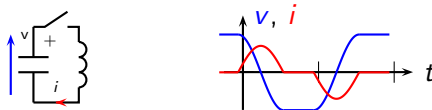


How it works

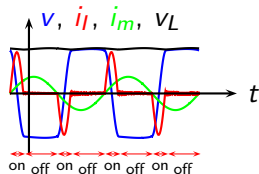
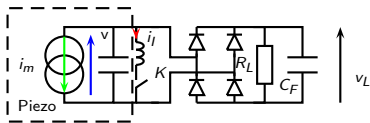
LC oscillations



Switched inductor



SSHI

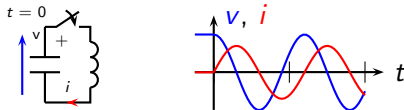


Switching is synchronized on \dot{w} , or

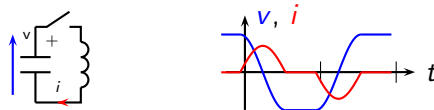
$$i_m = N\dot{w}.$$

How it works

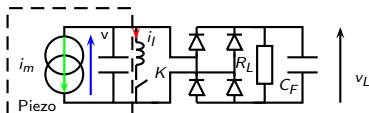
LC oscillations



Switched inductor

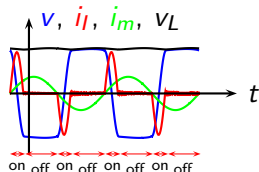


SSHI



Operating point: calculate v_L from i_m

1st Harmonic assumption: $P = \frac{1}{2} \frac{V_L^2}{R_L} \simeq \frac{1}{2} \frac{4V_L I_m}{\pi}$, $V_L = \frac{4}{\pi} R_L I_m$

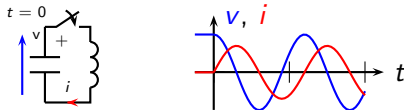


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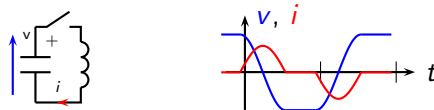
$$i_m = N\dot{w}$$

How it works

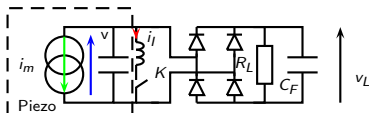
LC oscillations



Switched inductor



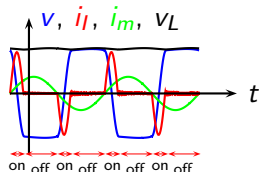
SSHI



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SSHI controls v and synchronises it, but doesn't impose $f_p = -D_s \dot{w}$.



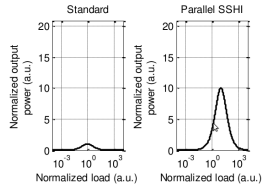
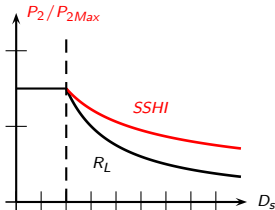
Switching is synchronized on \dot{w} , or

$$i_m = N \dot{w}$$

Conclusion

Performances

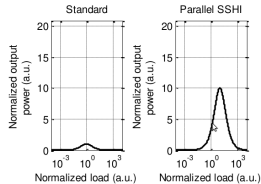
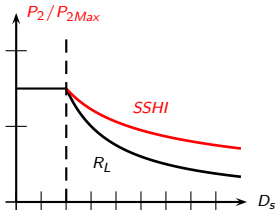
- SSHI can extract energy more efficiently than a resistor when damping is important,



Conclusion

Performances

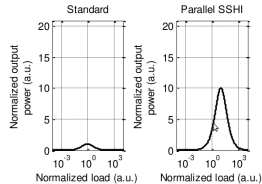
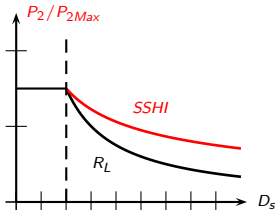
- SSHI can extract energy more efficiently than a resistor when damping is important,
- But Power extraction still depends on the load,



Conclusion

Performances

- SSHI can extract energy more efficiently than a resistor when damping is important,
- But Power extraction still depends on the load,
- Needs to measure bender's deflection $w(t)$.



General conclusion




In this presentation, applications of Energy Harvesting were shown. The modelling of a piezoelectric generator has shown that the power source needs an adaptation:

- in frequency,
- in load.

to maximize the harvested power.

The key energy management rules were presented through the analysis of the EMR of the system. A typical power electronic circuit was also presented, but the bibliography shows a lot of example.

References I

-  S Adhikari, M I Friswell, and D J Inman, *Piezoelectric energy harvesting from broadband random vibrations*, Smart Materials and Structures **18** (2009), no. 11, 115005.
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End of the presentation

Questions?