



**HAL**  
open science

# Energy Harvesting from Ambient Vibrations

Frédéric Giraud

► **To cite this version:**

| Frédéric Giraud. Energy Harvesting from Ambient Vibrations. Master. France. 2012. hal-02495300

**HAL Id: hal-02495300**

**<https://cel.hal.science/hal-02495300>**

Submitted on 1 Mar 2020

**HAL** is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

# Energy Harvesting from Ambient Vibrations

Frédéric Giraud

L2EP – University Lille1

November 27, 2012

# Table of contents

- 1 Introduction
  - What is Energy Harvesting ?
  - Generator Technologies
  - Summary
- 2 Modelling of a piezoelectric energy harvester
  - Presentation of the system
  - EMR of the system
  - Power Extraction on a load resistor  $R_L$  from harmonic oscillation
- 3 An Example of inverter
  - Introduction
  - SSHI: Synchronized Switch Harvesting on Inductor

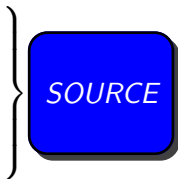
# What is Energy Harvesting ?

We extract energy from an *ambient and free source*:

# What is Energy Harvesting ?

We extract energy from an *ambient and free source*:

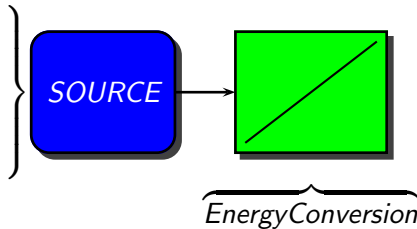
- *Wind*
- *Light*
- *Vibrations*
- *Thermal*
- ...



# What is Energy Harvesting ?

We extract energy from an *ambient and free source*:

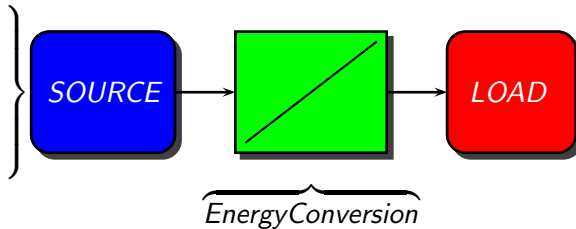
- Wind
- Light
- Vibrations
- Thermal
- ...



# What is Energy Harvesting ?

We extract energy from an *ambient and free source*:

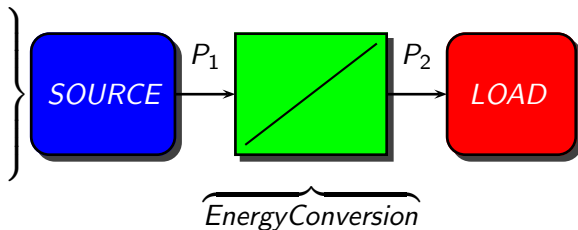
- Wind
- Light
- Vibrations
- Thermal
- ...



# What is Energy Harvesting ?

We extract energy from an *ambient and free source*:

- Wind
- Light
- Vibrations
- Thermal
- ...

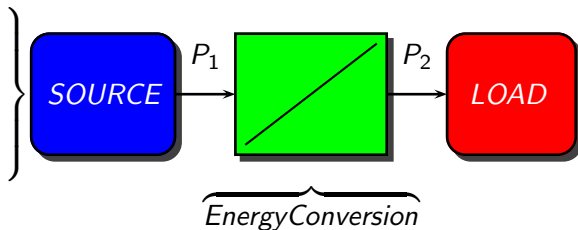




# What is Energy Harvesting ?

We extract energy from an *ambient and free source*:

- Wind
- Light
- Vibrations
- Thermal
- ...

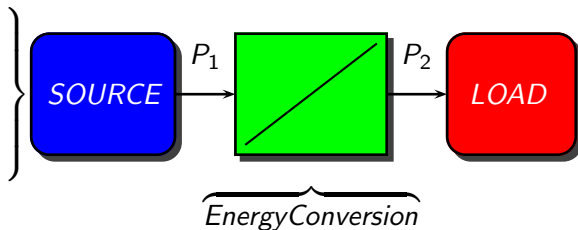


We talk about **Energy Harvesting** or also **energy scavenging** when the converted power is small, typically less than 1W.

# What is Energy Harvesting ?

We extract energy from an *ambient and free source*:

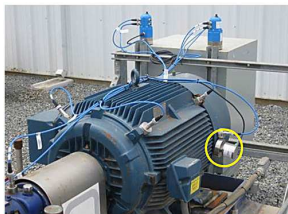
- Wind
- Light
- Vibrations
- Thermal
- ...



We talk about **Energy Harvesting** or also **energy scavenging** when the converted power is small, typically less than 1W.

$\eta = \frac{P_2}{P_1} = 1 - \frac{P_1 - P_2}{P_1} \rightarrow$  Losses in the energy converter should be as small as possible.

# Objectives: Sensors Network



[www.perpetuum.com](http://www.perpetuum.com)

# Objectives: Sensors Network



[www.perpetuum.com](http://www.perpetuum.com)



Gutiérrez, A Heterogeneous Wireless Identification Network for the Localization of Animals Based on Stochastic Movements

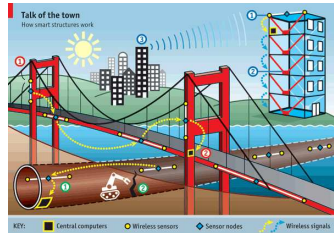
# Objectives: Sensors Network



www.perpetuum.com

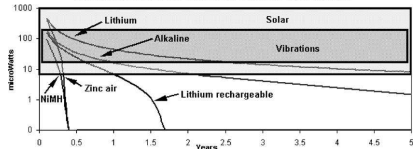


Gutiérrez, A Heterogeneous Wireless Identification Network for the Localization of Animals Based on Stochastic Movements



<http://www.rfwirelessensors.com>, 2012

Continuous Power / cm<sup>2</sup> vs. Life for Several Power Sources



Roundy et Al.: A study of low level vibrations as a power source for wireless sensor nodes.

# Objectives: Power, just where you need it



<http://enocean.com>

- Wireless
- Reduce Cost, and is reconfigurable
- Better Waste Cycle (Information from EnOcean)

# Objectives: Power, just where you need it

Innowattech's systems produces power with vehicles



<http://enocean.com>

- Wireless
- Reduce Cost, and is reconfigurable
- Better Waste Cycle (Information from Enocean)



<http://www.innowattech.com>

# Objectives: Power, just where you need it

Innowattech's systems produces power with vehicles



<http://enocean.com>

- Wireless
- Reduce Cost, and is reconfigurable
- Better Waste Cycle (Information from Enocean)



<http://www.innowattech.com>



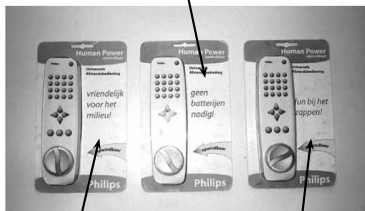
The economist – April 28th 2007



# Objectives: Marketing Purpose

Experience: Same Remote controller energized by human power, but with different packaging.

"No need for Batteries"



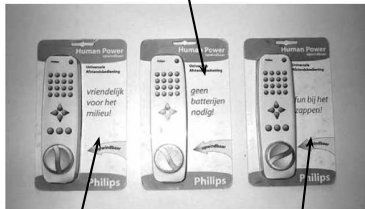
"Green"

"Fun"

# Objectives: Marketing Purpose

Experience: Same Remote controller energized by human power, but with different packaging.

"No need for Batteries"



"Green"

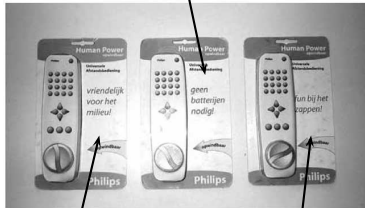
"Fun"

54% of people Will choose the first one because it is eco-friendly.  
(Jansen, Human power empirically explored)

# Objectives: Marketing Purpose

Experience: Same Remote controller energized by human power, but with different packaging.

"No need for Batteries"



"Green"

"Fun"

54% of people Will choose the first one because it is eco-friendly.  
(Jansen, Human power empirically explored)

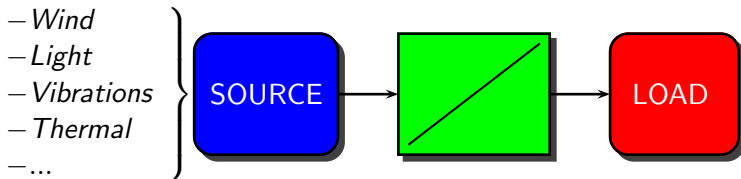
Several projects are born from this fact: **Metis** Produces energy from dancers' movements.



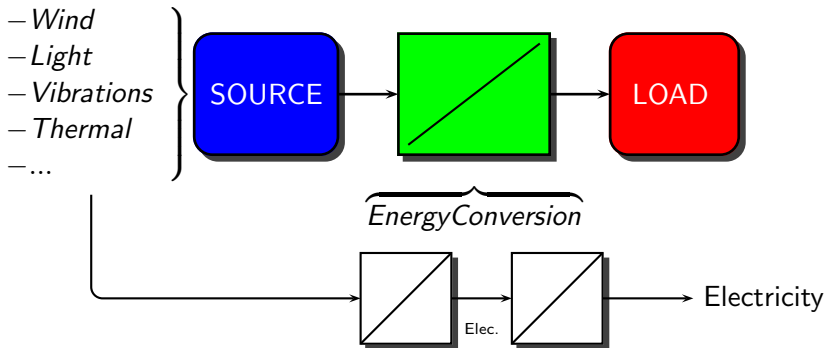
In Toulouse, the system **VIHA** proposes Smart Tiles to energize street lights.



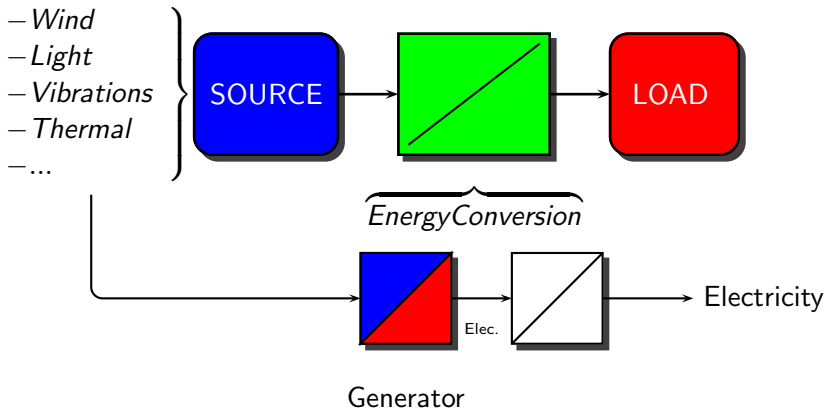
# The energy converter



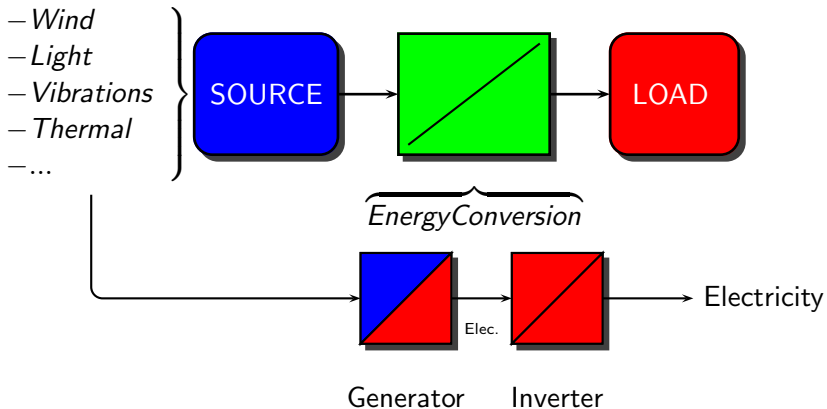
# The energy converter



# The energy converter



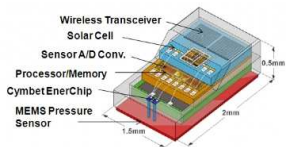
# The energy converter



# Solar Harvesters:



# Solar Harvesters:

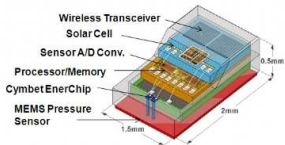


This sensor measures IntraOcular Pressure <http://cymbet.com>



Handbag to recharge electronic devices <http://www.neubers.de>

## Solar Harvesters:

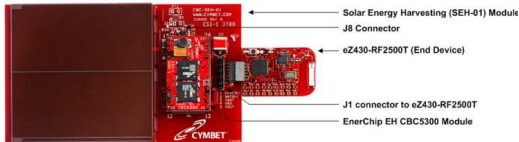
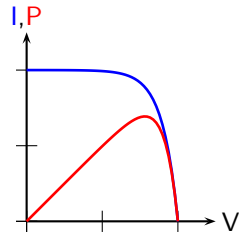


This sensor measures IntraOcular Pressure <http://cymbet.com>



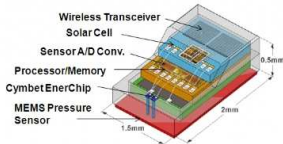
Handbag to recharge electronic devices <http://www.neubers.de>

Power and current as a function of voltage:



Solar Energy Harvester Evaluation Kit <http://ti.com>

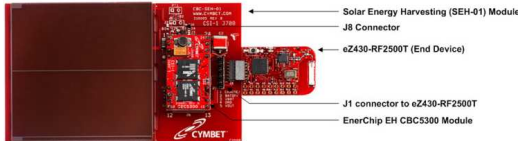
# Solar Harvesters:



This sensor measures IntraOcular Pressure <http://cymbet.com>

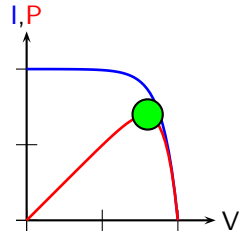


Handbag to recharge electronic devices <http://www.neubers.de>



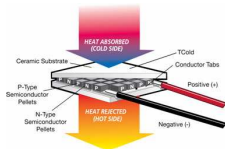
Solar Energy Harvester Evaluation Kit <http://ti.com>

Power and current as a function of voltage:

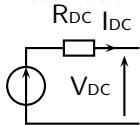


MPPT strategies, require Energy management of the system

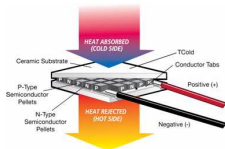
# Thermoelectric



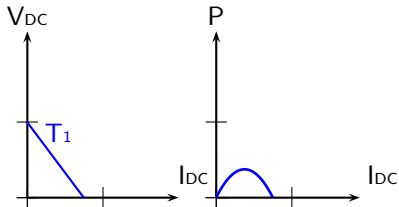
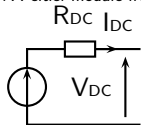
A Peltier module from <http://www.tellurex.com>



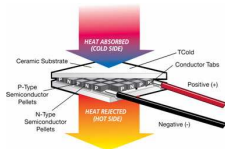
# Thermoelectric



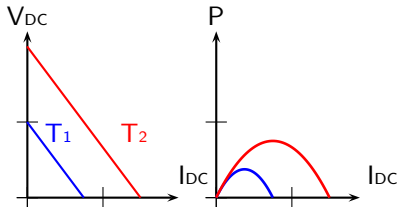
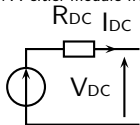
A Peltier module from <http://www.tellurex.com>



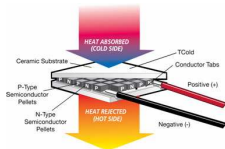
# Thermoelectric



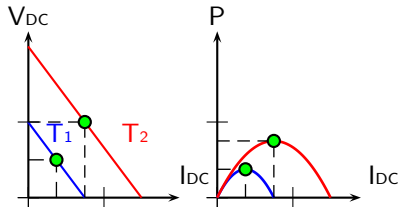
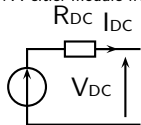
A Peltier module from <http://www.tellurex.com>



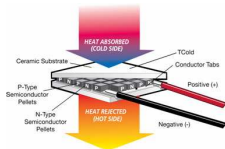
# Thermoelectric



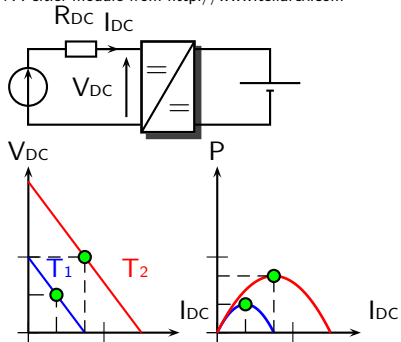
A Peltier module from <http://www.tellurex.com>



# Thermoelectric

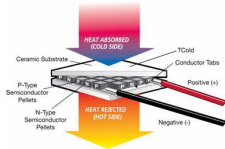


A Peltier module from <http://www.tellurex.com>

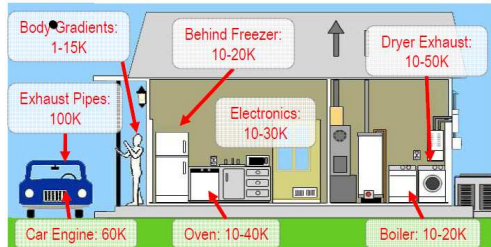
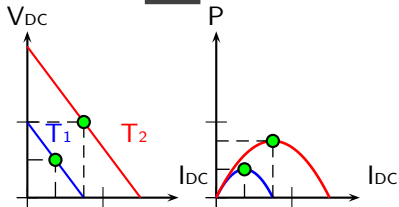
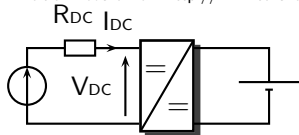




# Thermoelectric

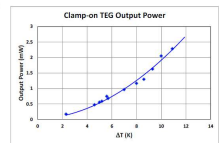


A Peltier module from <http://www.tellurex.com>



Temperature Gradient (residential). Lindsay Miller,  
<http://uc-ciee.org>

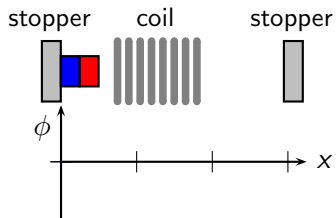
## Clamp-on Drain Harvester



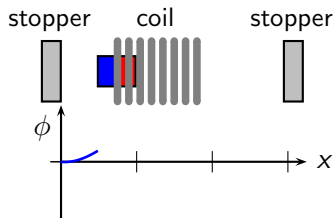
$\Delta T$	$V_{load}$ (mV)	$P_{max}$ (mW)
5.0	38	0.56
8.0	55	1.16
10.0	73	2.05

plumbing application from <http://www.nextreme.com>

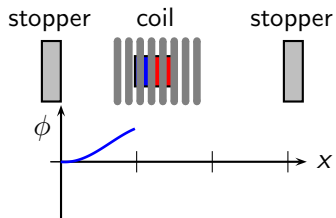
# Magnetic



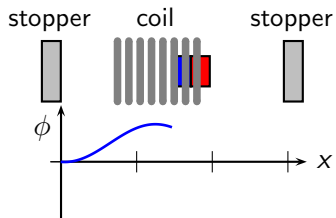
# Magnetic



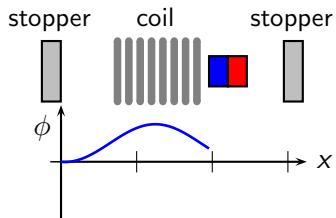
# Magnetic



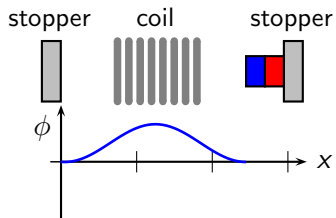
# Magnetic



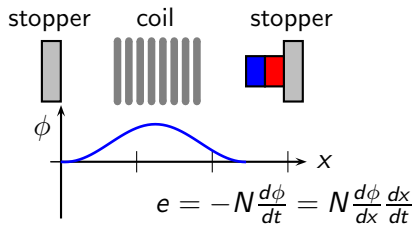
# Magnetic



# Magnetic

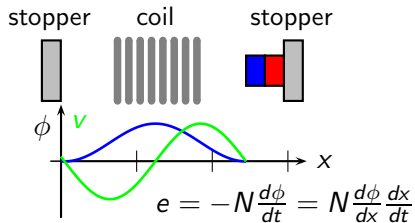


## Magnetic

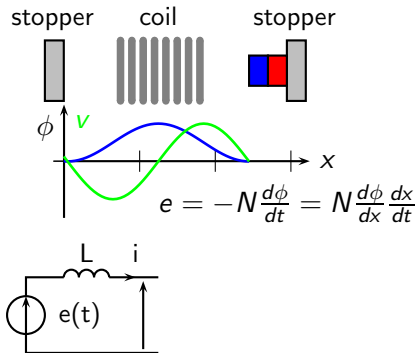




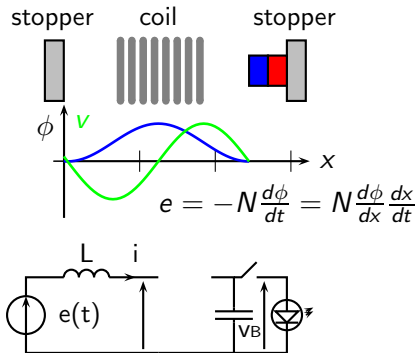
# Magnetic



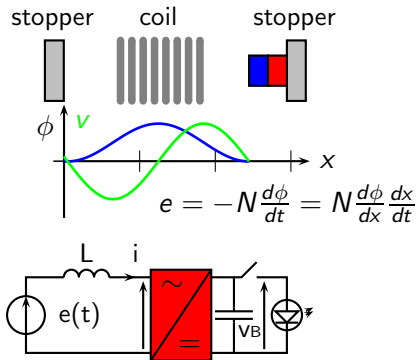
# Magnetic



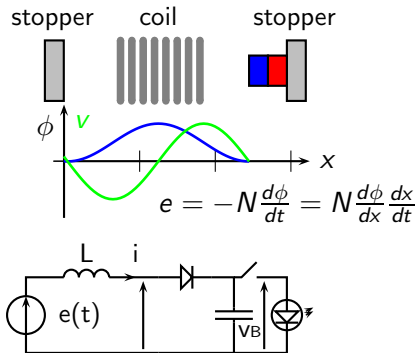
## Magnetic



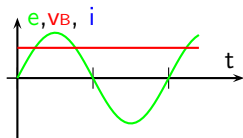
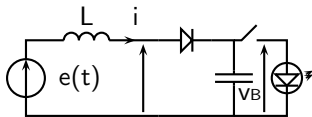
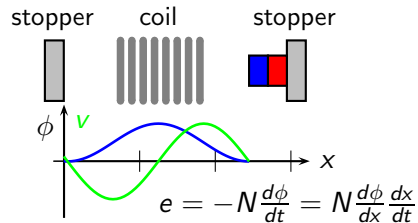
## Magnetic



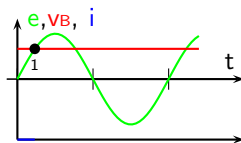
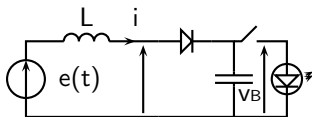
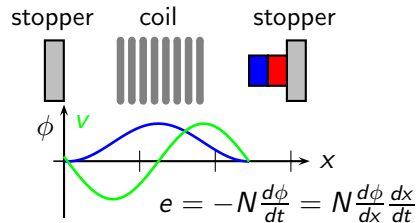
# Magnetic



## Magnetic

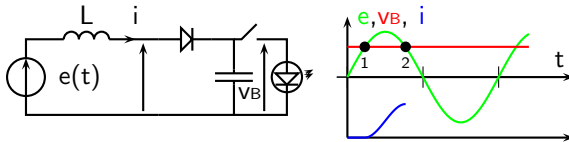
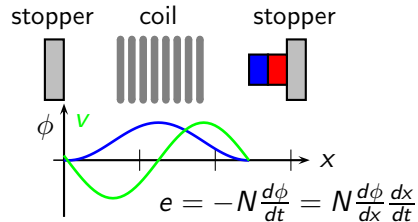


# Magnetic



1) Diode turns ON

## Magnetic

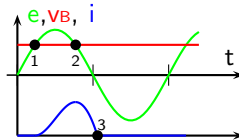
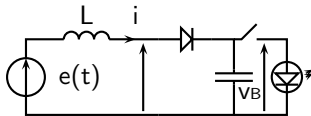
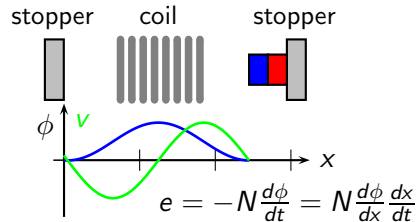


- 1) Diode turns ON
- 2)  $\frac{di}{dt} = 0$  because  $e - v_B = 0$



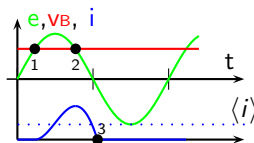
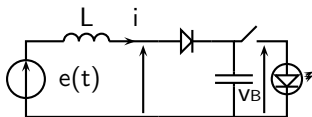
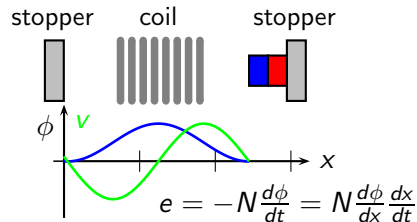


## Magnetic



- 1) Diode turns ON
- 2)  $\frac{di}{dt} = 0$  because  $e - v_B = 0$
- 3)  $i = 0$ , diode turns OFF

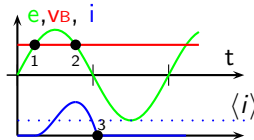
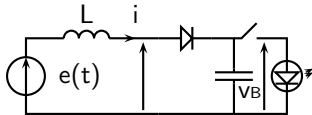
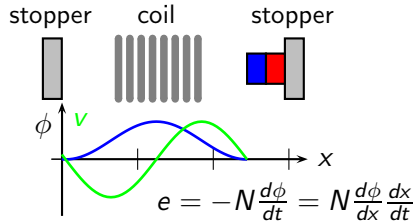
## Magnetic



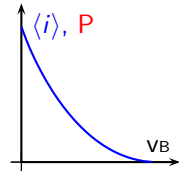
- 1) Diode turns ON
- 2)  $\frac{di}{dt} = 0$  because  $e - v_B = 0$
- 3)  $i = 0$ , diode turns OFF



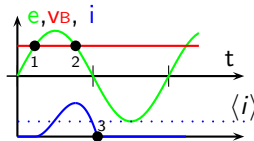
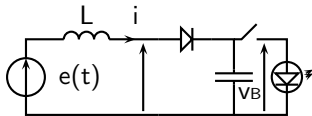
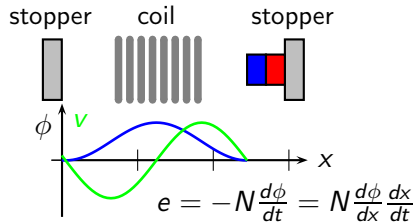
## Magnetic



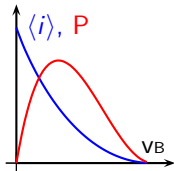
- 1) Diode turns ON
- 2)  $\frac{di}{dt} = 0$  because  $e - v_B = 0$
- 3)  $i = 0$ , diode turns OFF



## Magnetic

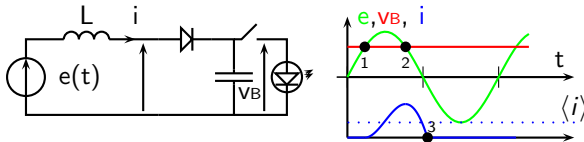
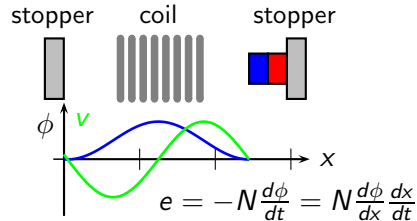


- 1) Diode turns ON
- 2)  $\frac{di}{dt} = 0$  because  $e - v_B = 0$
- 3)  $i = 0$ , diode turns OFF

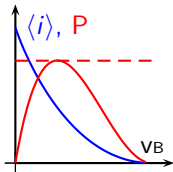


$$P = \langle v_B i \rangle = v_B \langle i \rangle$$

## Magnetic

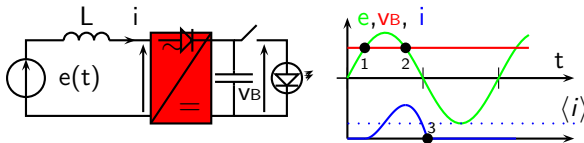
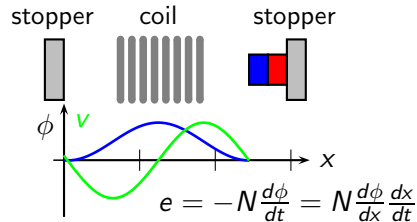


- 1) Diode turns ON
- 2)  $\frac{di}{dt} = 0$  because  $e - v_B = 0$
- 3)  $i = 0$ , diode turns OFF

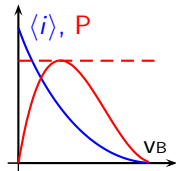


$$P = \langle v_B i \rangle = v_B \langle i \rangle$$

## Magnetic

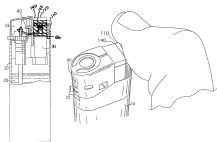


- 1) Diode turns ON
- 2)  $\frac{di}{dt} = 0$  because  $e - v_B = 0$
- 3)  $i = 0$ , diode turns OFF



$$P = \langle v_B i \rangle = v_B \langle i \rangle$$

# Piezoelectric

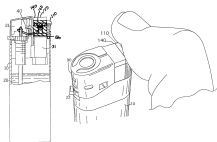


An electronic lighter,  
<http://freepatentsonline.com>



Piezoelectric crystals

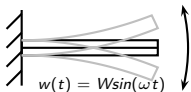
# Piezoelectric



An electronic lighter,  
<http://freepatentsonline.com>

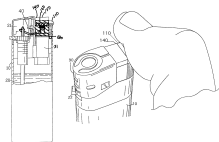


Piezoelectric crystals  
Cantilever beam





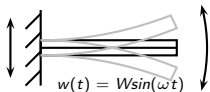
# Piezoelectric



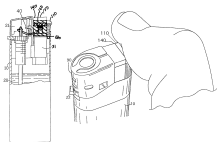
An electronic lighter,  
<http://freepatentsonline.com>



Piezoelectric crystals  
Cantilever beam



# Piezoelectric

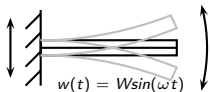


An electronic lighter,  
<http://freepatentsonline.com>

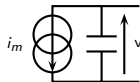


Piezoelectric crystals

## Cantilever beam

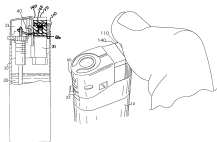


## Equivalent electrical circuit



$i_m$  is a current proportional  
to the deformation speed

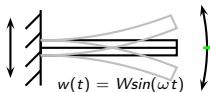
## Piezoelectric



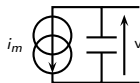
An electronic lighter,  
<http://freepatentsonline.com>



Piezoelectric crystals  
Cantilever beam

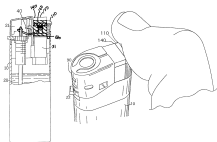


Equivalent electrical circuit



$i_m$  is a current proportional  
to the deformation speed

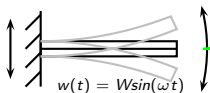
## Piezoelectric



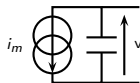
An electronic lighter,  
<http://freepatentsonline.com>



Piezoelectric crystals  
Cantilever beam



Equivalent electrical circuit

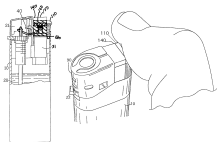


$i_m$  is a current proportional  
to the deformation speed

### Comparison Magn. Piezo

Magnetic	Piezo.
Voltage source	Current source
Inductive	capacitive
Large Stroke	Small Stroke
Remote action	

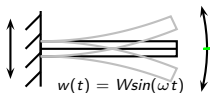
## Piezoelectric



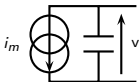
An electronic lighter,  
<http://freepatentsonline.com>



Piezoelectric crystals  
Cantilever beam



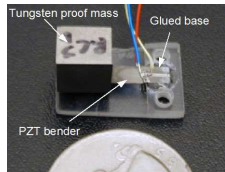
## Equivalent electrical circuit



$i_m$  is a current proportional  
to the deformation speed

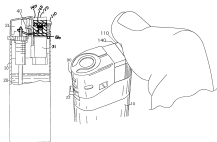
## Comparison Magn. Piezo

Magnetic	Piezo.
Voltage source	Current source
Inductive	capacitive
Large Stroke	Small Stroke
Remote action	



Roundy, A piezoelectric vibration based generator for wireless electronics (2004)

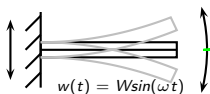
## Piezoelectric



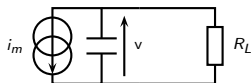
An electronic lighter,  
<http://freepatentsonline.com>



Piezoelectric crystals  
Cantilever beam

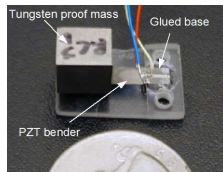


Equivalent electrical circuit



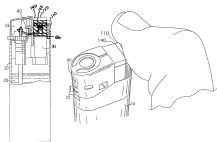
$i_m$  is a current proportional  
to the deformation speed  
**Comparison Magn. Piezo**

Magnetic	Piezo.
Voltage source	Current source
Inductive	capacitive
Large Stroke	Small Stroke
Remote action	



Roundy, A piezoelectric vibration based generator for wireless electronics (2004)

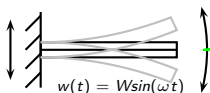
## Piezoelectric



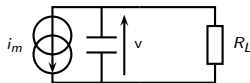
An electronic lighter,  
<http://freepatentsonline.com>



Piezoelectric crystals  
Cantilever beam

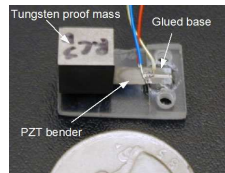


## Equivalent electrical circuit

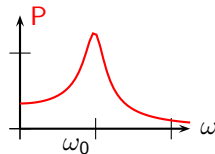


$i_m$  is a current proportional  
to the deformation speed  
**Comparison Magn. Piezo**

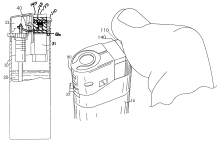
Magnetic	Piezo.
Voltage source	Current source
Inductive	capacitive
Large Stroke	Small Stroke
Remote action	



Roundy, A piezoelectric vibration based generator for wireless electronics (2004)



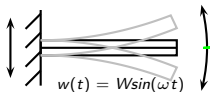
## Piezoelectric



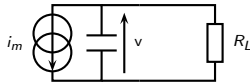
An electronic lighter,  
<http://freepatentsonline.com>



Piezoelectric crystals  
Cantilever beam

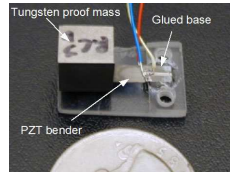


Equivalent electrical circuit

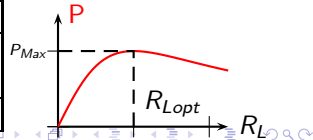
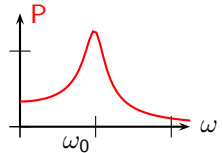


$i_m$  is a current proportional  
to the deformation speed  
**Comparison Magn. Piezo**

Magnetic	Piezo.
Voltage source	Current source
Inductive	capacitive
Large Stroke	Small Stroke
Remote action	

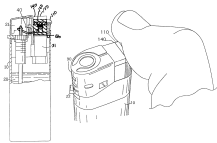


Roundy, A piezoelectric vibration based generator for wireless electronics (2004)





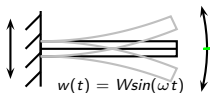
## Piezoelectric



An electronic lighter,  
<http://freepatentsonline.com>

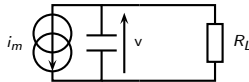


Piezoelectric crystals  
Cantilever beam



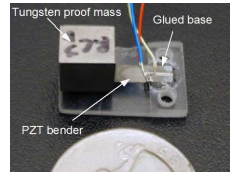
## Energy Management

### Equivalent electrical circuit

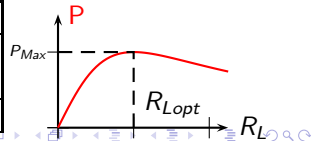
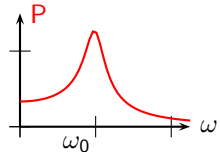


$i_m$  is a current proportional  
to the deformation speed  
**Comparison Magn. Piezo**

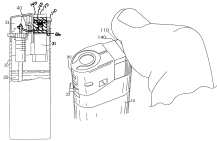
Magnetic	Piezo.
Voltage source	Current source
Inductive	capacitive
Large Stroke	Small Stroke
Remote action	



Roundy, A piezoelectric vibration based generator for wireless electronics (2004)



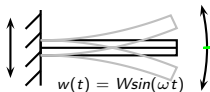
## Piezoelectric



An electronic lighter,  
<http://freepatentsonline.com>

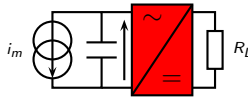


Piezoelectric crystals  
Cantilever beam



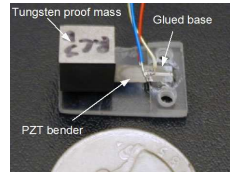
## Energy Management

### Equivalent electrical circuit

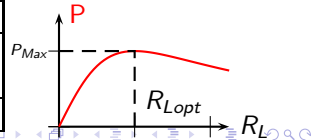
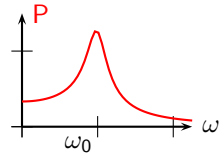


$i_m$  is a current proportional  
to the deformation speed  
**Comparison Magn. Piezo**

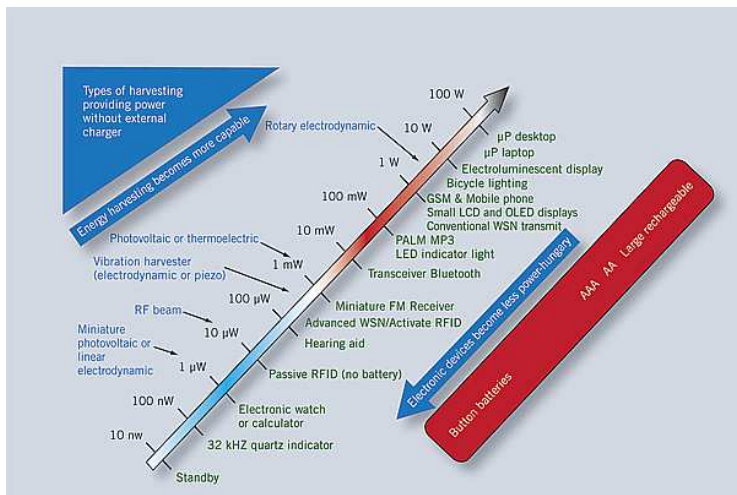
Magnetic	Piezo.
Voltage source	Current source
Inductive	capacitive
Large Stroke	Small Stroke
Remote action	



Roundy, A piezoelectric vibration based generator for wireless electronics (2004)



# There is no "one fit all" solution



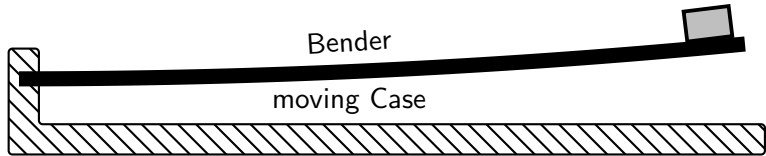
Each solution may be efficient in a certain range of Power.

Meanwhile, shrinking Chips consumption come at a time when energy harvesting becomes efficient and practical  
(source: IDtechex.com)

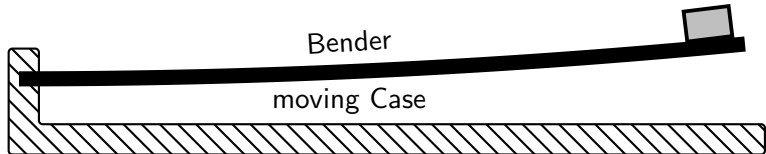
# Table of contents

- 1 Introduction
  - What is Energy Harvesting ?
  - Generator Technologies
  - Summary
- 2 Modelling of a piezoelectric energy harvester
  - Presentation of the system
  - EMR of the system
  - Power Extraction on a load resistor  $R_L$  from harmonic oscillation
- 3 An Example of inverter
  - Introduction
  - SSHI: Synchronized Switch Harvesting on Inductor

# Coordinates and assumptions

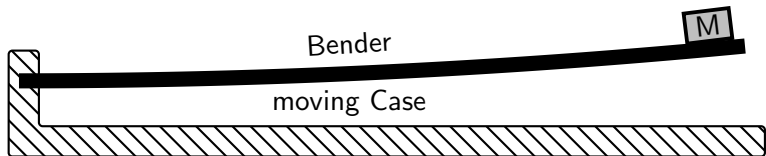


# Coordinates and assumptions



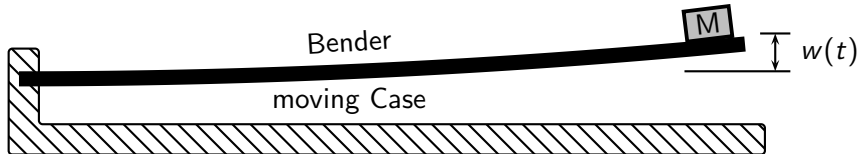
- The bender is attached to a vibrating and rigid case,

# Coordinates and assumptions



- The bender is attached to a vibrating and rigid case,
- A mass  $M$  is attached to increase power harvesting,

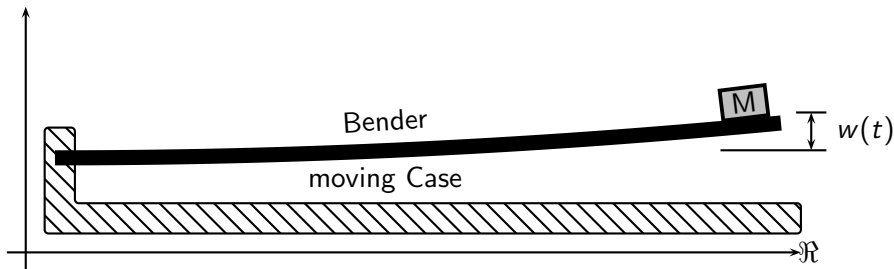
# Coordinates and assumptions



- The bender is attached to a vibrating and rigid case,
- A mass  $M$  is attached to increase power harvesting,
- $w(t)$  is the deflection of the beam,

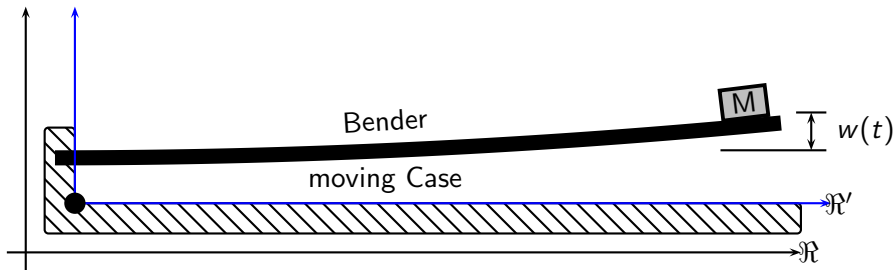


## Coordinates and assumptions



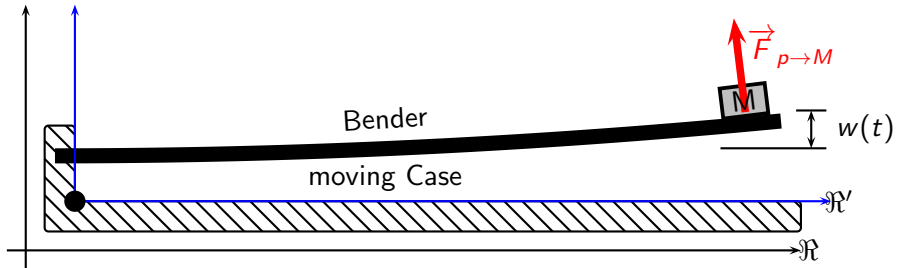
- The bender is attached to a vibrating and rigid case,
- A mass  $M$  is attached to increase power harvesting,
- $w(t)$  is the deflection of the beam,
- We define  $\mathcal{R}$  a fixed reference frame,

## Coordinates and assumptions



- The bender is attached to a vibrating and rigid case,
- A mass  $M$  is attached to increase power harvesting,
- $w(t)$  is the deflection of the beam,
- We define  $\mathcal{R}$  a fixed reference frame,
- And  $\mathcal{R}'$  a reference frame affixed to the vibrating case.

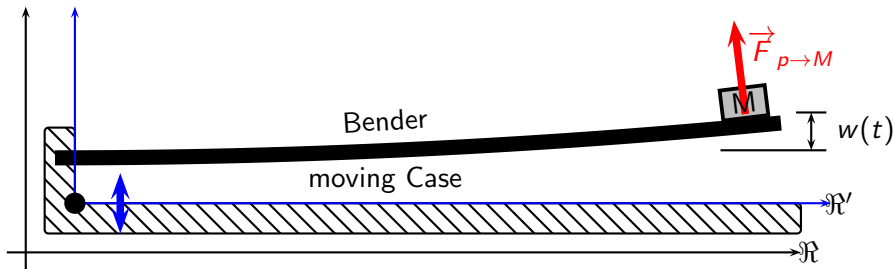
## Coordinates and assumptions



- The bender is attached to a vibrating and rigid case,
- A mass  $M$  is attached to increase power harvesting,
- $w(t)$  is the deflection of the beam,
- We define  $\mathcal{R}$  a fixed reference frame,
- And  $\mathcal{R}'$  a reference frame affixed to the vibrating case.

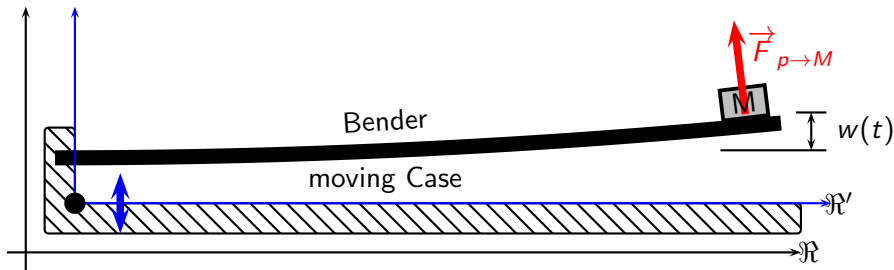
- $\vec{F}_{p \rightarrow m}$  is the force of the Bender onto the mass  $M$ .

## Coordinates and assumptions



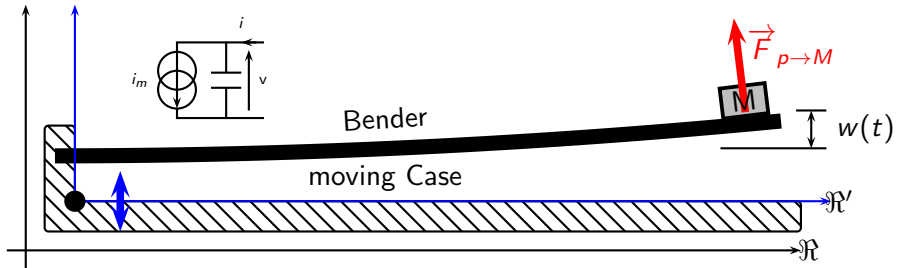
- The bender is attached to a vibrating and rigid case,
- A mass  $M$  is attached to increase power harvesting,
- $w(t)$  is the deflection of the beam,
- We define  $\mathcal{R}$  a fixed reference frame,
- And  $\mathcal{R}'$  a reference frame affixed to the vibrating case.
- $\vec{F}_{p \rightarrow m}$  is the force of the Bender onto the mass  $M$ .
- The case is supposed to be controlled in position, and we have  $y_c = A \sin(\omega t)$  the amplitude of the case's vibration.

## Coordinates and assumptions



- The bender is attached to a vibrating and rigid case,
- A mass  $M$  is attached to increase power harvesting,
- $w(t)$  is the deflection of the beam,
- We define  $\mathcal{R}$  a fixed reference frame,
- And  $\mathcal{R}'$  a reference frame affixed to the vibrating case.
- $\vec{F}_{p \rightarrow m}$  is the force of the Bender onto the mass  $M$ .
- The case is supposed to be controlled in position, and we have  $y_c = A \sin(\omega t)$  the amplitude of the case's vibration.
- Gravity is neglected, as well as Inertia momentum of the bender,

# Coordinates and assumptions



- The bender is attached to a vibrating and rigid case,
- A mass  $M$  is attached to increase power harvesting,
- $w(t)$  is the deflection of the beam,
- We define  $\mathcal{R}$  a fixed reference frame,
- And  $\mathcal{R}'$  a reference frame affixed to the vibrating case.
- $\vec{F}_{p \rightarrow m}$  is the force of the Bender onto the mass  $M$ .
- The case is supposed to be controlled in position, and we have  $y_c = A \sin(\omega t)$  the amplitude of the case's vibration.
- Gravity is neglected, as well as Inertia momentum of the bender,
- Actuator electrical convention.

# Equations

- **Dynamic of the mass  $M$ :**

## Glossary

---

- $M$  the mass
- $f$ , the force onto  $M$
- $A$ , vibration's amplitude
- $\omega$ , vibration's pulsation
- $f_{acc}$  is the inertial force
- $i$  current of the device (actuator convention)
- $i_m$  motional current
- $f_p$  inside piezo force
- $N$  Piezoelectric force factor (depends on geometry)
- $f_p$  Piezo internal force
- $f_s$  Material internal elastic force
- $K_s$  equivalent stiffness (depends on geometry)
- $D_s$  Viscous coefficient

# Equations

- **Dynamic of the mass  $M$ :**

- $M \frac{d^2}{dt^2} (w(t) + A \sin(\omega t)) = F_{p \rightarrow M} = f$

## Glossary

---

- $M$  the mass
- $f$ , the force onto  $M$
- $A$ , vibration's amplitude
- $\omega$ , vibration's pulsation
- $f_{acc}$  is the inertial force
- $i$  current of the device (actuator convention)
- $i_m$  motional current
- $f_p$  inside piezo force
- $N$  Piezoelectric force factor (depends on geometry)
- $f_p$  Piezo internal force
- $f_s$  Material internal elastic force
- $K_s$  equivalent stiffness (depends on geometry)
- $D_s$  Viscous coefficient



# Equations

## • Dynamic of the mass $M$ :

- $M \frac{d^2}{dt^2} (w(t) + A \sin(\omega t)) = F_{p \rightarrow M} = f$
- $M \ddot{w}(t) = f + MA\omega^2 \sin(\omega t) = f + f_{acc}$

## Glossary

---

- $M$  the mass
- $f$ , the force onto  $M$
- $A$ , vibration's amplitude
- $\omega$ , vibration's pulsation
- $f_{acc}$  is the inertial force
- $i$  current of the device (actuator convention)
- $i_m$  motional current
- $f_p$  inside piezo force
- $N$  Piezoelectric force factor (depends on geometry)
- $f_p$  Piezo internal force
- $f_s$  Material internal elastic force
- $K_s$  equivalent stiffness (depends on geometry)
- $D_s$  Viscous coefficient

# Equations

- **Dynamic of the mass  $M$ :**

- $M \frac{d^2}{dt^2} (w(t) + A \sin(\omega t)) = F_{p \rightarrow M} = f$
- $M \ddot{w}(t) = f + MA\omega^2 \sin(\omega t) = f + f_{acc}$

- **Electrical Behaviour:** Capacitive

## Glossary

---

- $M$  the mass
- $f$ , the force onto  $M$
- $A$ , vibration's amplitude
- $\omega$ , vibration's pulsation
- $f_{acc}$  is the inertial force
- $i$  current of the device (actuator convention)
- $i_m$  motional current
- $f_p$  inside piezo force
- $N$  Piezoelectric force factor (depends on geometry)
- $f_p$  Piezo internal force
- $f_s$  Material internal elastic force
- $K_s$  equivalent stiffness (depends on geometry)
- $D_s$  Viscous coefficient

# Equations

## • Dynamic of the mass $M$ :

- $M \frac{d^2}{dt^2} (w(t) + A \sin(\omega t)) = F_{p \rightarrow M} = f$
- $M \ddot{w}(t) = f + MA\omega^2 \sin(\omega t) = f + f_{acc}$

## • Electrical Behaviour: Capacitive

- $v = \frac{1}{C_b} \int (i - i_m) dt$

## Glossary

---

- $M$  the mass
- $f$ , the force onto  $M$
- $A$ , vibration's amplitude
- $\omega$ , vibration's pulsation
- $f_{acc}$  is the inertial force
- $i$  current of the device (actuator convention)
- $i_m$  motional current
- $f_p$  inside piezo force
- $N$  Piezoelectric force factor (depends on geometry)
- $f_p$  Piezo internal force
- $f_s$  Material internal elastic force
- $K_s$  equivalent stiffness (depends on geometry)
- $D_s$  Viscous coefficient

# Equations

- **Dynamic of the mass  $M$ :**

- $M \frac{d^2}{dt^2} (w(t) + A \sin(\omega t)) = F_{p \rightarrow M} = f$
- $M \ddot{w}(t) = f + MA\omega^2 \sin(\omega t) = f + f_{acc}$

- **Electrical Behaviour:** Capacitive

- $v = \frac{1}{C_b} \int (i - i_m) dt$

- **Piezoelectric effect**

## Glossary

---

- $M$  the mass
- $f$ , the force onto  $M$
- $A$ , vibration's amplitude
- $\omega$ , vibration's pulsation
- $f_{acc}$  is the inertial force
- $i$  current of the device (actuator convention)
- $i_m$  motional current
- $f_p$  inside piezo force
- $N$  Piezoelectric force factor (depends on geometry)
- $f_p$  Piezo internal force
- $f_s$  Material internal elastic force
- $K_s$  equivalent stiffness (depends on geometry)
- $D_s$  Viscous coefficient

# Equations

## • Dynamic of the mass $M$ :

- $M \frac{d^2}{dt^2} (w(t) + A \sin(\omega t)) = F_{p \rightarrow M} = f$
- $M \ddot{w}(t) = f + MA\omega^2 \sin(\omega t) = f + f_{acc}$

## • Electrical Behaviour: Capacitive

- $v = \frac{1}{C_b} \int (i - i_m) dt$

## • Piezoelectric effect

- $i_m$  derives from deflection:  $i_m = N \dot{w}$

## Glossary

---

- $M$  the mass
- $f$ , the force onto  $M$
- $A$ , vibration's amplitude
- $\omega$ , vibration's pulsation
- $f_{acc}$  is the inertial force
- $i$  current of the device (actuator convention)
- $i_m$  motional current
- $f_p$  inside piezo force
- $N$  Piezoelectric force factor (depends on geometry)
- $f_p$  Piezo internal force
- $f_s$  Material internal elastic force
- $K_s$  equivalent stiffness (depends on geometry)
- $D_s$  Viscous coefficient

# Equations

## • Dynamic of the mass $M$ :

- $M \frac{d^2}{dt^2} (w(t) + A \sin(\omega t)) = F_{p \rightarrow M} = f$
- $M \ddot{w}(t) = f + MA\omega^2 \sin(\omega t) = f + f_{acc}$

## • Electrical Behaviour: Capacitive

- $v = \frac{1}{C_b} \int (i - i_m) dt$

## • Piezoelectric effect

- $i_m$  derives from deflection:  $i_m = N \dot{w}$
- while  $v$  produces an internal force  
 $f_p = Nv$

## Glossary

---

- $M$  the mass
- $f$ , the force onto  $M$
- $A$ , vibration's amplitude
- $\omega$ , vibration's pulsation
- $f_{acc}$  is the inertial force
- $i$  current of the device (actuator convention)
- $i_m$  motional current
- $f_p$  inside piezo force
- $N$  Piezoelectric force factor (depends on geometry)
- $f_p$  Piezo internal force
- $f_s$  Material internal elastic force
- $K_s$  equivalent stiffness (depends on geometry)
- $D_s$  Viscous coefficient

# Equations

## • Dynamic of the mass $M$ :

- $M \frac{d^2}{dt^2} (w(t) + A \sin(\omega t)) = F_{p \rightarrow M} = f$
- $M \ddot{w}(t) = f + MA\omega^2 \sin(\omega t) = f + f_{acc}$

## • Electrical Behaviour: Capacitive

- $v = \frac{1}{C_b} \int (i - i_m) dt$

## • Piezoelectric effect

- $i_m$  derives from deflection:  $i_m = N \dot{w}$
- while  $v$  produces an internal force  
 $f_p = Nv$

## • Material's behaviour

## Glossary

---

- $M$  the mass
- $f$ , the force onto  $M$
- $A$ , vibration's amplitude
- $\omega$ , vibration's pulsation
- $f_{acc}$  is the inertial force
- $i$  current of the device (actuator convention)
- $i_m$  motional current
- $f_p$  inside piezo force
- $N$  Piezoelectric force factor (depends on geometry)
- $f_p$  Piezo internal force
- $f_s$  Material internal elastic force
- $K_s$  equivalent stiffness (depends on geometry)
- $D_s$  Viscous coefficient

# Equations

## • Dynamic of the mass $M$ :

- $M \frac{d^2}{dt^2} (w(t) + A \sin(\omega t)) = F_{p \rightarrow M} = f$
- $M \ddot{w}(t) = f + MA\omega^2 \sin(\omega t) = f + f_{acc}$

## • Electrical Behaviour: Capacitive

- $v = \frac{1}{C_b} \int (i - i_m) dt$

## • Piezoelectric effect

- $i_m$  derives from deflection:  $i_m = N \dot{w}$
- while  $v$  produces an internal force  
 $f_p = Nv$

## • Material's behaviour

- The material is elastic:  $f_s = K_s \int \dot{w} dt$

## Glossary

---

- $M$  the mass
- $f$ , the force onto  $M$
- $A$ , vibration's amplitude
- $\omega$ , vibration's pulsation
- $f_{acc}$  is the inertial force
- $i$  current of the device (actuator convention)
- $i_m$  motional current
- $f_p$  inside piezo force
- $N$  Piezoelectric force factor (depends on geometry)
- $f_p$  Piezo internal force
- $f_s$  Material internal elastic force
- $K_s$  equivalent stiffness (depends on geometry)
- $D_s$  Viscous coefficient



# Equations

## • Dynamic of the mass $M$ :

- $M \frac{d^2}{dt^2} (w(t) + A \sin(\omega t)) = F_{p \rightarrow M} = f$
- $M \ddot{w}(t) = f + MA\omega^2 \sin(\omega t) = f + f_{acc}$

## • Electrical Behaviour: Capacitive

- $v = \frac{1}{C_b} \int (i - i_m) dt$

## • Piezoelectric effect

- $i_m$  derives from deflection:  $i_m = N \dot{w}$
- while  $v$  produces an internal force  
 $f_p = Nv$

## • Material's behaviour

- The material is elastic:  $f_s = K_s \int \dot{w} dt$
- With some friction inside:  
 $f_s = K_s \int \dot{w} dt + D_s \dot{w}$

## Glossary

---

- $M$  the mass
- $f$ , the force onto  $M$
- $A$ , vibration's amplitude
- $\omega$ , vibration's pulsation
- $f_{acc}$  is the inertial force
- $i$  current of the device (actuator convention)
- $i_m$  motional current
- $f_p$  inside piezo force
- $N$  Piezoelectric force factor (depends on geometry)
- $f_p$  Piezo internal force
- $f_s$  Material internal elastic force
- $K_s$  equivalent stiffness (depends on geometry)
- $D_s$  Viscous coefficient

# Equations

## • Dynamic of the mass $M$ :

- $M \frac{d^2}{dt^2} (w(t) + A \sin(\omega t)) = F_{p \rightarrow M} = f$
- $M \ddot{w}(t) = f + MA\omega^2 \sin(\omega t) = f + f_{acc}$

## • Electrical Behaviour: Capacitive

- $v = \frac{1}{C_b} \int (i - i_m) dt$

## • Piezoelectric effect

- $i_m$  derives from deflection:  $i_m = N \dot{w}$
- while  $v$  produces an internal force  
 $f_p = Nv$

## • Material's behaviour

- The material is elastic:  $f_s = K_s \int \dot{w} dt$
- With some friction inside:  
 $f_s = K_s \int \dot{w} dt + D_s \dot{w}$
- These actions are opposite to the PE effect:  $f = f_p - f_s$

## Glossary

---

- $M$  the mass
- $f$ , the force onto  $M$
- $A$ , vibration's amplitude
- $\omega$ , vibration's pulsation
- $f_{acc}$  is the inertial force
- $i$  current of the device (actuator convention)
- $i_m$  motional current
- $f_p$  inside piezo force
- $N$  Piezoelectric force factor (depends on geometry)
- $f_p$  Piezo internal force
- $f_s$  Material internal elastic force
- $K_s$  equivalent stiffness (depends on geometry)
- $D_s$  Viscous coefficient

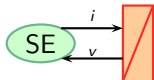
# EMR of the system

$$v = \frac{1}{C_b} \int (i - i_m) dt$$

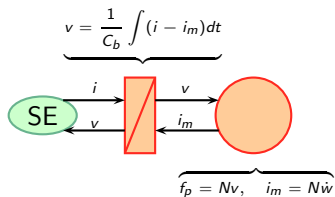


# EMR of the system

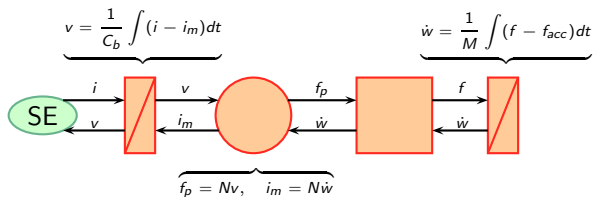
$$v = \frac{1}{C_b} \int (i - i_m) dt$$



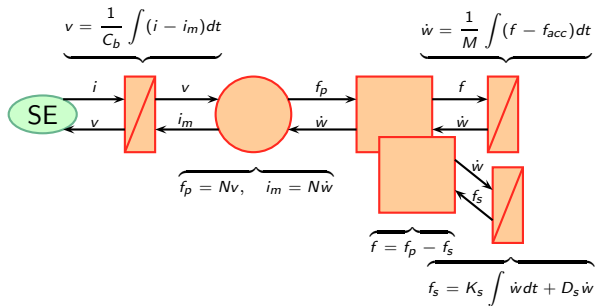
## EMR of the system



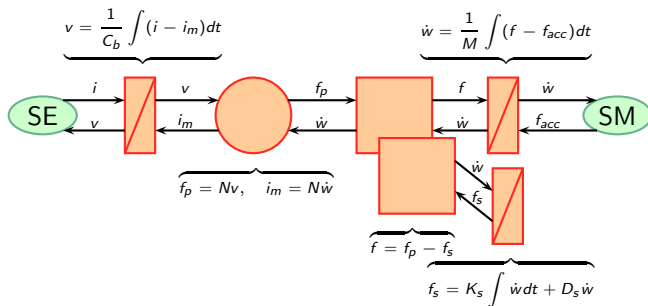
## EMR of the system



## EMR of the system

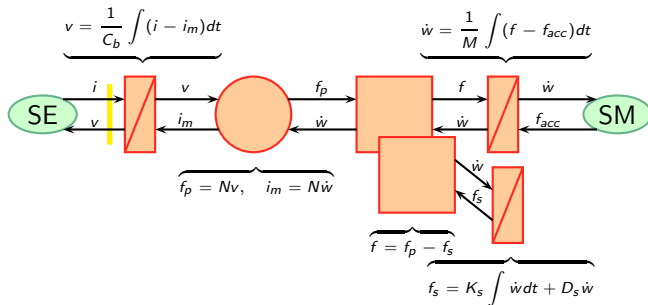


## EMR of the system



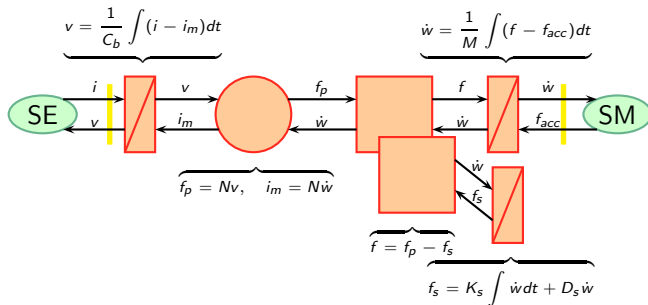


## EMR of the system



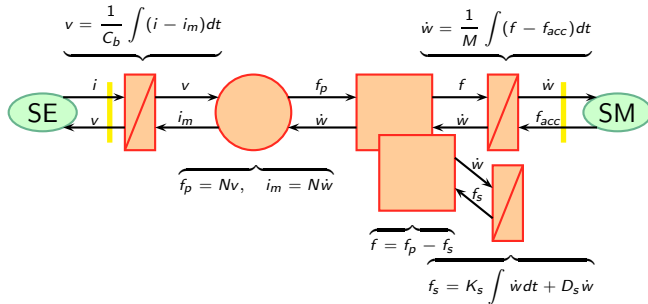
$p_e = v.i$  is the output power,

## EMR of the system

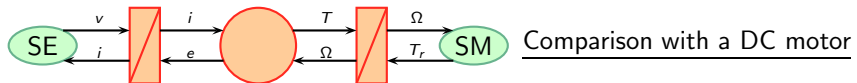


$p_e = v \cdot i$  is the output power,  $p_m = f_{acc} \dot{w}$  is the mechanical input power and both should be  $< 0$

## EMR of the system

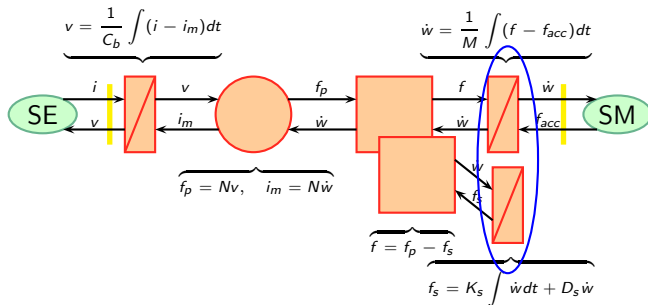


$p_e = v \cdot i$  is the output power,  $p_m = f_{acc} \dot{w}$  is the mechanical input power and both should be  $< 0$

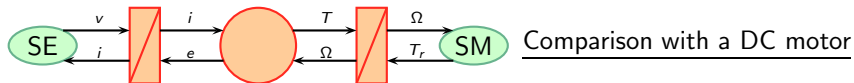


Things are not so different.

## EMR of the system



$p_e = v \cdot i$  is the output power,  $p_m = f_{acc} \dot{w}$  is the mechanical input power and both should be  $< 0$



Things are not so different.

# Asumption

- $f_{acc} = MA\omega^2 \sin(\omega t)$  for harmonic oscillations

## Asumption

- $f_{acc} = MA\omega^2 \sin(\omega t)$  for harmonic oscillations
- complex notation:  $\underline{x}$  is the complex phasor of  $x(t)$ , means  $x(t) = \Im(\underline{x})$

## Asumption

- $f_{acc} = MA\omega^2 \sin(\omega t)$  for harmonic oscillations
- complex notation:  $\underline{x}$  is the complex phasor of  $x(t)$ , means  $x(t) = \Im(\underline{x})$
- since oscillations are harmonic, we will write:  $\underline{x} = \underline{X}e^{j\omega t}$

## Asumption

- $f_{acc} = MA\omega^2 \sin(\omega t)$  for harmonic oscillations
- complex notation:  $\underline{x}$  is the complex phasor of  $x(t)$ , means  $x(t) = \Im(\underline{x})$
- since oscillations are harmonic, we will write:  $\underline{x} = \underline{X}e^{j\omega t}$
- for steady state operation,  $\underline{X}$  is constant, leading to  $\frac{dx}{dt} = j\omega \underline{X}e^{j\omega t}$



## Assumption

- $f_{acc} = MA\omega^2 \sin(\omega t)$  for harmonic oscillations
- complex notation:  $\underline{x}$  is the complex phasor of  $x(t)$ , means  $x(t) = \Im(\underline{x})$
- since oscillations are harmonic, we will write:  $\underline{x} = \underline{X}e^{j\omega t}$
- for steady state operation,  $\underline{X}$  is constant, leading to  $\frac{dx}{dt} = j\omega \underline{X}e^{j\omega t}$
- $|\underline{x}| = |\underline{X}| = X$

## Assumption

- $f_{acc} = MA\omega^2 \sin(\omega t)$  for harmonic oscillations
- complex notation:  $\underline{x}$  is the complex phasor of  $x(t)$ , means  $x(t) = \Im(\underline{x})$
- since oscillations are harmonic, we will write:  $\underline{x} = \underline{X}e^{j\omega t}$
- for steady state operation,  $\underline{X}$  is constant, leading to  $\frac{dx}{dt} = j\omega \underline{X}e^{j\omega t}$
- $|\underline{x}| = |\underline{X}| = X$
- for example,  $\underline{f}_{acc} = MA\omega^2 e^{j\omega t}$  and  $|\underline{f}_{acc}| = MA\omega^2$ .

## Assumption

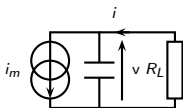
- $f_{acc} = MA\omega^2 \sin(\omega t)$  for harmonic oscillations
- complex notation:  $\underline{x}$  is the complex phasor of  $x(t)$ , means  $x(t) = \Im(\underline{x})$
- since oscillations are harmonic, we will write:  $\underline{x} = \underline{X}e^{j\omega t}$
- for steady state operation,  $\underline{X}$  is constant, leading to  $\frac{dx}{dt} = j\omega \underline{X}e^{j\omega t}$
- $|\underline{x}| = |\underline{X}| = X$
- for example,  $\underline{f}_{acc} = MA\omega^2 e^{j\omega t}$  and  $|\underline{f}_{acc}| = MA\omega^2$ .
- $v = -R_L i$

## Assumption

- $f_{acc} = MA\omega^2 \sin(\omega t)$  for harmonic oscillations
- complex notation:  $\underline{x}$  is the complex phasor of  $x(t)$ , means  $x(t) = \Im(\underline{x})$
- since oscillations are harmonic, we will write:  $\underline{x} = \underline{X}e^{j\omega t}$
- for steady state operation,  $\underline{X}$  is constant, leading to  $\frac{dx}{dt} = j\omega \underline{X}e^{j\omega t}$
- $|\underline{x}| = |\underline{X}| = X$
- for example,  $\underline{f}_{acc} = MA\omega^2 e^{j\omega t}$  and  $|\underline{f}_{acc}| = MA\omega^2$ .
- $v = -R_L i$

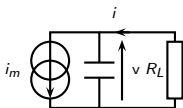
# Assumption

- $f_{acc} = MA\omega^2 \sin(\omega t)$  for harmonic oscillations
- complex notation:  $\underline{x}$  is the complex phasor of  $x(t)$ , means  $x(t) = \Im(\underline{x})$
- since oscillations are harmonic, we will write:  $\underline{x} = \underline{X}e^{j\omega t}$
- for steady state operation,  $\underline{X}$  is constant, leading to  $\frac{d\underline{x}}{dt} = j\omega \underline{X}e^{j\omega t}$
- $|\underline{x}| = |\underline{X}| = X$
- for example,  $\underline{f}_{acc} = MA\omega^2 e^{j\omega t}$  and  $|\underline{f}_{acc}| = MA\omega^2$ .
- $v = -R_L i$



## Assumption

- $f_{acc} = MA\omega^2 \sin(\omega t)$  for harmonic oscillations
- complex notation:  $\underline{x}$  is the complex phasor of  $x(t)$ , means  $x(t) = \Im(\underline{x})$
- since oscillations are harmonic, we will write:  $\underline{x} = \underline{X}e^{j\omega t}$
- for steady state operation,  $\underline{X}$  is constant, leading to  $\frac{dx}{dt} = j\omega \underline{X}e^{j\omega t}$
- $|\underline{x}| = |\underline{X}| = X$
- for example,  $\underline{f}_{acc} = MA\omega^2 e^{j\omega t}$  and  $|\underline{f}_{acc}| = MA\omega^2$ .
- $v = -R_L i$



And  $R_L \ll \frac{1}{C_b\omega}$ , yields  
 $v \simeq -R_L i_m$

# For an ideal generator ( $D_s = 0$ )

- $M\ddot{w} = \underline{f} + \underline{f}_{acc}$

# For an ideal generator ( $D_s = 0$ )

- $M\ddot{\underline{w}} = \underline{f} + \underline{f}_{acc}$
- $\underline{f} = N\underline{v} - K_s\underline{w}$



# For an ideal generator ( $D_s = 0$ )

- $M\ddot{\underline{w}} = \underline{f} + \underline{f}_{acc}$
- $\underline{f} = N\underline{v} - K_s\underline{w}$
- $\underline{v} = -R_L\underline{i}_m = -R_L N\dot{\underline{w}}$

## For an ideal generator ( $D_s = 0$ )

- $M\ddot{w} = \underline{f} + \underline{f}_{acc}$
- $\underline{f} = N\underline{v} - K_s\underline{w}$
- $\underline{v} = -R_L\underline{i}_m = -R_L N\dot{w}$
- $M\ddot{w} + N^2 R_L \dot{w} + K_s w = \underline{f}_{acc}$

## For an ideal generator ( $D_s = 0$ )

- $M\ddot{w} = \underline{f} + \underline{f}_{acc}$
- $\underline{f} = N\underline{v} - K_s\underline{w}$
- $\underline{v} = -R_L\underline{i}_m = -R_L N\dot{w}$
- $M\ddot{w} + N^2 R_L \dot{w} + K_s w = \underline{f}_{acc}$   
→  $R_L$  acts as a damping
- $P_2 = -\frac{1}{2}R_L|i_m|^2$

## For an ideal generator ( $D_s = 0$ )

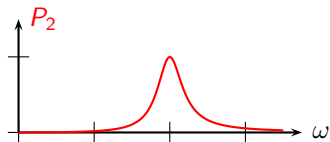
- $M\ddot{w} = \underline{f} + \underline{f}_{acc}$
- $\underline{f} = N\underline{v} - K_s\underline{w}$
- $\underline{v} = -R_L\underline{i}_m = -R_L N\dot{w}$
- $M\ddot{w} + N^2 R_L \dot{w} + K_s w = \underline{f}_{acc}$   
→  $R_L$  acts as a damping
- $P_2 = -\frac{1}{2}R_L|i_m|^2$
- $|i_m| = N|\dot{w}| = \frac{\omega \cdot N \cdot |f_{acc}|}{\sqrt{(K_s - M\omega^2)^2 + (N^2 R_L \omega)^2}}$

# For an ideal generator ( $D_s = 0$ )

- $M\ddot{w} = \underline{f} + \underline{f}_{acc}$
- $\underline{f} = N\underline{v} - K_s\underline{w}$
- $\underline{v} = -R_L\underline{i}_m = -R_L N\dot{w}$
- $M\ddot{w} + N^2 R_L \dot{w} + K_s w = \underline{f}_{acc}$   
→  $R_L$  acts as a damping
- $P_2 = -\frac{1}{2} R_L |i_m|^2$
- $|i_m| = N|\dot{w}| = \frac{\omega \cdot N \cdot |f_{acc}|}{\sqrt{(K_s - M\omega^2)^2 + (N^2 R_L \omega)^2}}$
- $P_2 = -\frac{1}{2} \frac{N^2 R_L \omega^2 |f_{acc}|^2}{(K_s - M\omega^2)^2 + (N^2 R_L \omega)^2}$

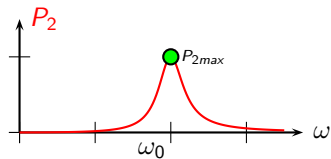
## For an ideal generator ( $D_s = 0$ )

- $M\ddot{w} = \underline{f} + \underline{f}_{acc}$
- $\underline{f} = N\underline{v} - K_s\underline{w}$
- $\underline{v} = -R_L\underline{i}_m = -R_L N\dot{w}$
- $M\ddot{w} + N^2 R_L \dot{w} + K_s w = \underline{f}_{acc}$   
 $\rightarrow R_L$  acts as a damping
- $P_2 = -\frac{1}{2} R_L |i_m|^2$
- $|i_m| = N|\dot{w}| = \frac{\omega \cdot N \cdot |f_{acc}|}{\sqrt{(K_s - M\omega^2)^2 + (N^2 R_L \omega)^2}}$
- $P_2 = -\frac{1}{2} \frac{N^2 R_L \omega^2 |f_{acc}|^2}{(K_s - M\omega^2)^2 + (N^2 R_L \omega)^2}$



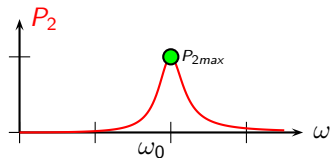
## For an ideal generator ( $D_s = 0$ )

- $M\ddot{w} = \underline{f} + \underline{f}_{acc}$
- $\underline{f} = N\underline{v} - K_s\underline{w}$
- $\underline{v} = -R_L\underline{i}_m = -R_L N\dot{w}$
- $M\ddot{w} + N^2 R_L \dot{w} + K_s w = \underline{f}_{acc}$   
 $\rightarrow R_L$  acts as a damping
- $P_2 = -\frac{1}{2} R_L |i_m|^2$
- $|i_m| = N|\dot{w}| = \frac{\omega \cdot N \cdot |f_{acc}|}{\sqrt{(K_s - M\omega^2)^2 + (N^2 R_L \omega)^2}}$
- $P_2 = -\frac{1}{2} \frac{N^2 R_L \omega^2 |f_{acc}|^2}{(K_s - M\omega^2)^2 + (N^2 R_L \omega)^2}$



## For an ideal generator ( $D_s = 0$ )

- $M\ddot{w} = \underline{f} + \underline{f}_{acc}$
- $\underline{f} = N\underline{v} - K_s\underline{w}$
- $\underline{v} = -R_L\underline{i}_m = -R_L N\dot{w}$
- $M\ddot{w} + N^2 R_L \dot{w} + K_s w = \underline{f}_{acc}$   
 $\rightarrow R_L$  acts as a damping
- $P_2 = -\frac{1}{2} R_L |i_m|^2$
- $|i_m| = N|\dot{w}| = \frac{\omega \cdot N \cdot |f_{acc}|}{\sqrt{(K_s - M\omega^2)^2 + (N^2 R_L \omega)^2}}$
- $P_2 = -\frac{1}{2} \frac{N^2 R_L \omega^2 |f_{acc}|^2}{(K_s - M\omega^2)^2 + (N^2 R_L \omega)^2}$



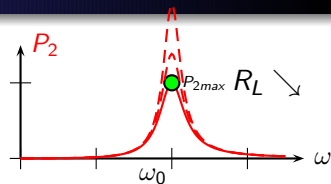
The vibrations should occur at generator's resonance frequency

$$\omega_0 = \sqrt{\frac{K_s}{M}}$$



## For an ideal generator ( $D_s = 0$ )

- $M\ddot{w} = \underline{f} + \underline{f}_{acc}$
- $\underline{f} = N\underline{v} - K_s\underline{w}$
- $\underline{v} = -R_L\underline{i}_m = -R_L N\dot{w}$
- $M\ddot{w} + N^2 R_L \dot{w} + K_s w = \underline{f}_{acc}$   
 $\rightarrow R_L$  acts as a damping
- $P_2 = -\frac{1}{2} R_L |i_m|^2$
- $|i_m| = N|\dot{w}| = \frac{\omega \cdot N \cdot |f_{acc}|}{\sqrt{(K_s - M\omega^2)^2 + (N^2 R_L \omega)^2}}$
- $P_2 = -\frac{1}{2} \frac{N^2 R_L \omega^2 |f_{acc}|^2}{(K_s - M\omega^2)^2 + (N^2 R_L \omega)^2}$

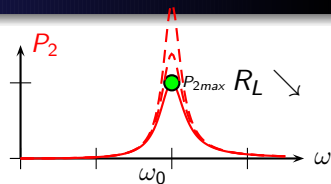


The vibrations should occur at generator's resonance frequency

$$\omega_0 = \sqrt{\frac{K_s}{M}}$$

For an ideal generator ( $D_s = 0$ )

- $M\ddot{w} = \underline{f} + \underline{f}_{acc}$
- $\underline{f} = N\underline{v} - K_s\underline{w}$
- $\underline{v} = -R_L\underline{i}_m = -R_L N\dot{w}$
- $M\ddot{w} + N^2 R_L \dot{w} + K_s w = \underline{f}_{acc}$   
→  $R_L$  acts as a damping
- $P_2 = -\frac{1}{2} R_L |i_m|^2$
- $|i_m| = N|\dot{w}| = \frac{\omega \cdot N \cdot |f_{acc}|}{\sqrt{(K_s - M\omega^2)^2 + (N^2 R_L \omega)^2}}$
- $P_2 = -\frac{1}{2} \frac{N^2 R_L \omega^2 |f_{acc}|^2}{(K_s - M\omega^2)^2 + (N^2 R_L \omega)^2}$
- $P_{2Max} = \frac{(MA\omega^2)^2}{2N^2 R_L}$

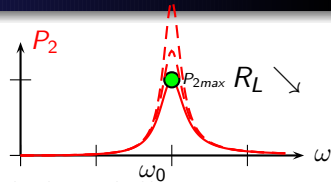


The vibrations should occur at generator's resonance frequency

$$\omega_0 = \sqrt{\frac{K_s}{M}}$$

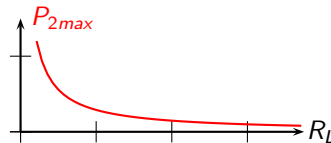
For an ideal generator ( $D_s = 0$ )

- $M\ddot{w} = \underline{f} + \underline{f}_{acc}$
- $\underline{f} = N\underline{v} - K_s\underline{w}$
- $\underline{v} = -R_L\underline{i}_m = -R_L N\dot{w}$
- $M\ddot{w} + N^2 R_L \dot{w} + K_s w = \underline{f}_{acc}$   
→  $R_L$  acts as a damping
- $P_2 = -\frac{1}{2} R_L |i_m|^2$
- $|i_m| = N|\dot{w}| = \frac{\omega \cdot N \cdot |f_{acc}|}{\sqrt{(K_s - M\omega^2)^2 + (N^2 R_L \omega)^2}}$
- $P_2 = -\frac{1}{2} \frac{N^2 R_L \omega^2 |f_{acc}|^2}{(K_s - M\omega^2)^2 + (N^2 R_L \omega)^2}$
- $P_{2Max} = \frac{(MA\omega^2)^2}{2N^2 R_L}$



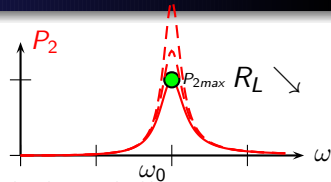
The vibrations should occur at generator's resonance frequency

$$\omega_0 = \sqrt{\frac{K_s}{M}}$$



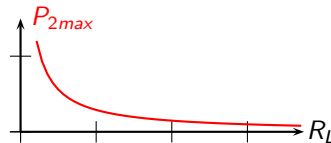
For an ideal generator ( $D_s = 0$ )

- $M\ddot{w} = \underline{f} + \underline{f}_{acc}$
- $\underline{f} = N\underline{v} - K_s\underline{w}$
- $\underline{v} = -R_L\underline{i}_m = -R_L N\dot{w}$
- $M\ddot{w} + N^2 R_L \dot{w} + K_s w = \underline{f}_{acc}$   
→  $R_L$  acts as a damping
- $P_2 = -\frac{1}{2} R_L |i_m|^2$
- $|i_m| = N|\dot{w}| = \frac{\omega \cdot N \cdot |f_{acc}|}{\sqrt{(K_s - M\omega^2)^2 + (N^2 R_L \omega)^2}}$
- $P_2 = -\frac{1}{2} \frac{N^2 R_L \omega^2 |f_{acc}|^2}{(K_s - M\omega^2)^2 + (N^2 R_L \omega)^2}$
- $P_{2Max} = \frac{(MA\omega^2)^2}{2N^2 R_L}$



The vibrations should occur at generator's resonance frequency

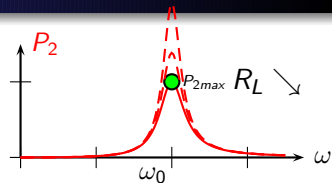
$$\omega_0 = \sqrt{\frac{K_s}{M}}$$



We can harvest as much power as we want

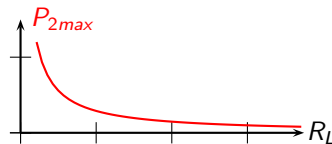
## For an ideal generator ( $D_s = 0$ )

- $M\ddot{w} = \underline{f} + \underline{f}_{acc}$
- $\underline{f} = N\underline{v} - K_s\underline{w}$
- $\underline{v} = -R_L\underline{i}_m = -R_L N\dot{w}$
- $M\ddot{w} + N^2 R_L \dot{w} + K_s w = \underline{f}_{acc}$   
 $\rightarrow R_L$  acts as a damping
- $P_2 = -\frac{1}{2} R_L |i_m|^2$
- $|i_m| = N|\dot{w}| = \frac{\omega \cdot N \cdot |f_{acc}|}{\sqrt{(K_s - M\omega^2)^2 + (N^2 R_L \omega)^2}}$
- $P_2 = -\frac{1}{2} \frac{N^2 R_L \omega^2 |f_{acc}|^2}{(K_s - M\omega^2)^2 + (N^2 R_L \omega)^2}$
- $P_{2Max} = \frac{(MA\omega^2)^2}{2N^2 R_L}$ ,  $W_{max} = \frac{MA\omega}{N^2 R_L}$



The vibrations should occur at generator's resonance frequency

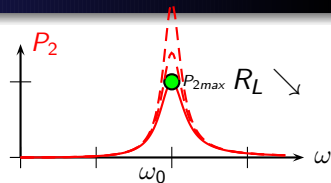
$$\omega_0 = \sqrt{\frac{K_s}{M}}$$



We can harvest as much power as we want

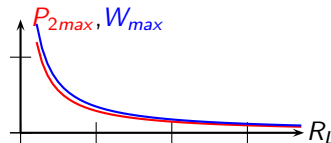
For an ideal generator ( $D_s = 0$ )

- $M\ddot{w} = \underline{f} + \underline{f}_{acc}$
- $\underline{f} = N\underline{v} - K_s\underline{w}$
- $\underline{v} = -R_L\underline{i}_m = -R_L N\dot{w}$
- $M\ddot{w} + N^2 R_L \dot{w} + K_s w = \underline{f}_{acc}$   
→  $R_L$  acts as a damping
- $P_2 = -\frac{1}{2} R_L |i_m|^2$
- $|i_m| = N|\dot{w}| = \frac{\omega \cdot N \cdot |f_{acc}|}{\sqrt{(K_s - M\omega^2)^2 + (N^2 R_L \omega)^2}}$
- $P_2 = -\frac{1}{2} \frac{N^2 R_L \omega^2 |f_{acc}|^2}{(K_s - M\omega^2)^2 + (N^2 R_L \omega)^2}$
- $P_{2Max} = \frac{(MA\omega^2)^2}{2N^2 R_L}$ ,  $W_{max} = \frac{MA\omega}{N^2 R_L}$



The vibrations should occur at generator's resonance frequency

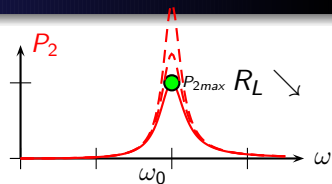
$$\omega_0 = \sqrt{\frac{K_s}{M}}$$



We can harvest as much power as we want

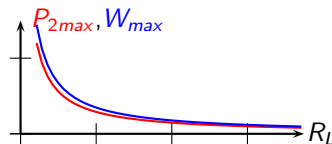
## For an ideal generator ( $D_s = 0$ )

- $M\ddot{w} = \underline{f} + \underline{f}_{acc}$
- $\underline{f} = N\underline{v} - K_s\underline{w}$
- $\underline{v} = -R_L\underline{i}_m = -R_L N\dot{w}$
- $M\ddot{w} + N^2 R_L \dot{w} + K_s w = \underline{f}_{acc}$   
 $\rightarrow R_L$  acts as a damping
- $P_2 = -\frac{1}{2} R_L |i_m|^2$
- $|i_m| = N|\dot{w}| = \frac{\omega \cdot N \cdot |f_{acc}|}{\sqrt{(K_s - M\omega^2)^2 + (N^2 R_L \omega)^2}}$
- $P_2 = -\frac{1}{2} \frac{N^2 R_L \omega^2 |f_{acc}|^2}{(K_s - M\omega^2)^2 + (N^2 R_L \omega)^2}$
- $P_{2Max} = \frac{(MA\omega^2)^2}{2N^2 R_L}$ ,  $W_{max} = \frac{MA\omega}{N^2 R_L}$



The vibrations should occur at generator's resonance frequency

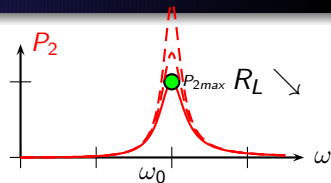
$$\omega_0 = \sqrt{\frac{K_s}{M}}$$



We can harvest as much power as we want, at the expense of large displacement of the bender's tip

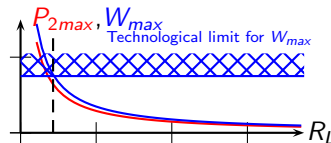
For an ideal generator ( $D_s = 0$ )

- $M\ddot{w} = \underline{f} + \underline{f}_{acc}$
- $\underline{f} = N\underline{v} - K_s\underline{w}$
- $\underline{v} = -R_L\underline{i}_m = -R_L N\dot{w}$
- $M\ddot{w} + N^2 R_L \dot{w} + K_s w = \underline{f}_{acc}$   
→  $R_L$  acts as a damping
- $P_2 = -\frac{1}{2} R_L |i_m|^2$
- $|i_m| = N|\dot{w}| = \frac{\omega \cdot N \cdot |f_{acc}|}{\sqrt{(K_s - M\omega^2)^2 + (N^2 R_L \omega)^2}}$
- $P_2 = -\frac{1}{2} \frac{N^2 R_L \omega^2 |f_{acc}|^2}{(K_s - M\omega^2)^2 + (N^2 R_L \omega)^2}$
- $P_{2Max} = \frac{(MA\omega^2)^2}{2N^2 R_L}$ ,  $W_{max} = \frac{MA\omega}{N^2 R_L}$



The vibrations should occur at generator's resonance frequency

$$\omega_0 = \sqrt{\frac{K_s}{M}}$$

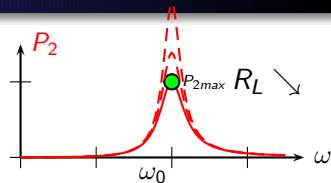


We can harvest as much power as we want, at the expense of large displacement of the bender's tip



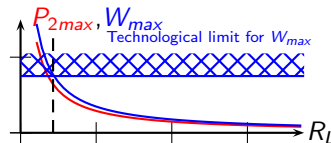
## For an ideal generator ( $D_s = 0$ )

- $M\ddot{w} = \underline{f} + \underline{f}_{acc}$
- $\underline{f} = N\underline{v} - K_s\underline{w}$
- $\underline{v} = -R_L\underline{i}_m = -R_L N\dot{w}$
- $M\ddot{w} + N^2 R_L \dot{w} + K_s w = \underline{f}_{acc}$   
 $\rightarrow R_L$  acts as a damping
- $P_2 = -\frac{1}{2} R_L |i_m|^2$
- $|i_m| = N|\dot{w}| = \frac{\omega \cdot N \cdot |f_{acc}|}{\sqrt{(K_s - M\omega^2)^2 + (N^2 R_L \omega)^2}}$
- $P_2 = -\frac{1}{2} \frac{N^2 R_L \omega^2 |f_{acc}|^2}{(K_s - M\omega^2)^2 + (N^2 R_L \omega)^2}$
- $P_{2Max} = \frac{(MA\omega^2)^2}{2N^2 R_L}$ ,  $W_{max} = \frac{MA\omega}{N^2 R_L}$ ,  
 $v_{max} = \frac{MA\omega^2}{N}$



The vibrations should occur at generator's resonance frequency

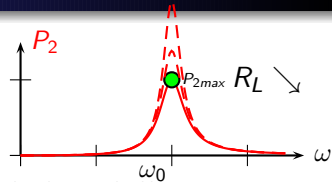
$$\omega_0 = \sqrt{\frac{K_s}{M}}$$



We can harvest as much power as we want, at the expense of large displacement of the bender's tip

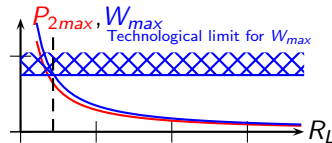
## For an ideal generator ( $D_s = 0$ )

- $M\ddot{w} = \underline{f} + \underline{f}_{acc}$
- $\underline{f} = N\underline{v} - K_s\underline{w}$
- $\underline{v} = -R_L\underline{i}_m = -R_L N\dot{w}$
- $M\ddot{w} + N^2 R_L \dot{w} + K_s w = \underline{f}_{acc}$   
→  $R_L$  acts as a damping
- $P_2 = -\frac{1}{2} R_L |i_m|^2$
- $|i_m| = N|\dot{w}| = \frac{\omega \cdot N \cdot |f_{acc}|}{\sqrt{(K_s - M\omega^2)^2 + (N^2 R_L \omega)^2}}$
- $P_2 = -\frac{1}{2} \frac{N^2 R_L \omega^2 |f_{acc}|^2}{(K_s - M\omega^2)^2 + (N^2 R_L \omega)^2}$
- $P_{2Max} = \frac{(MA\omega^2)^2}{2N^2 R_L}$ ,  $W_{max} = \frac{MA\omega}{N^2 R_L}$ ,  
 $v_{max} = \frac{MA\omega^2}{N}$
- $P_{1Max} = \frac{1}{2} f_{acc} \omega W_{Max} = \frac{(MA\omega^2)^2}{N^2 R_L} = P_{2Max}$



The vibrations should occur at generator's resonance frequency

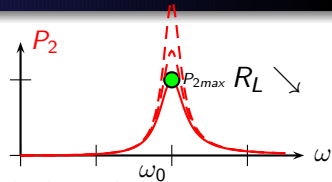
$$\omega_0 = \sqrt{\frac{K_s}{M}}$$



We can harvest as much power as we want, at the expense of large displacement of the bender's tip

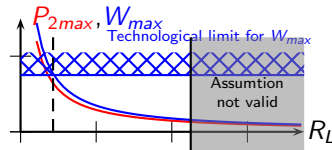
# For an ideal generator ( $D_s = 0$ )

- $M\ddot{w} = \underline{f} + \underline{f}_{acc}$
- $\underline{f} = N\underline{v} - K_s\underline{w}$
- $\underline{v} = -R_L\underline{i}_m = -R_L N\dot{w}$
- $M\ddot{w} + N^2 R_L \dot{w} + K_s w = \underline{f}_{acc}$   
→  $R_L$  acts as a damping
- $P_2 = -\frac{1}{2} R_L |i_m|^2$
- $|i_m| = N|\dot{w}| = \frac{\omega \cdot N \cdot |f_{acc}|}{\sqrt{(K_s - M\omega^2)^2 + (N^2 R_L \omega)^2}}$
- $P_2 = -\frac{1}{2} \frac{N^2 R_L \omega^2 |f_{acc}|^2}{(K_s - M\omega^2)^2 + (N^2 R_L \omega)^2}$
- $P_{2Max} = \frac{(MA\omega^2)^2}{2N^2 R_L}$ ,  $W_{max} = \frac{MA\omega}{N^2 R_L}$ ,  
 $v_{max} = \frac{MA\omega^2}{N}$
- $P_{1Max} = \frac{1}{2} f_{acc} \omega W_{Max} = \frac{(MA\omega^2)^2}{N^2 R_L} = P_{2Max}$



The vibrations should occur at generator's resonance frequency

$$\omega_0 = \sqrt{\frac{K_s}{M}}$$



We can harvest as much power as we want, at the expense of large displacement of the bender's tip

# For a real generator ( $D_s \neq 0$ )

- $\underline{f} = N\underline{v} - K_s\underline{w} - D_s\underline{\dot{w}}$

# For a real generator ( $D_s \neq 0$ )

- $\underline{f} = N\underline{v} - K_s\underline{w} - D_s\dot{\underline{w}}$
- $\underline{w} = \frac{\underline{f}_{acc}}{(K_s - M\omega^2) + j\omega(D_s + N^2R_L)}$

## For a real generator ( $D_s \neq 0$ )

- $\underline{f} = N\underline{v} - K_s\underline{w} - D_s\dot{\underline{w}}$
- $\underline{w} = \frac{\underline{f}_{acc}}{(K_s - M\omega^2) + j\omega(D_s + N^2R_L)}$
- $P_2 = -\frac{1}{2}R_L|i_m|^2$

## For a real generator ( $D_s \neq 0$ )

- $\underline{f} = N\underline{v} - K_s\underline{w} - D_s\dot{\underline{w}}$
- $\underline{w} = \frac{\underline{f}_{acc}}{(K_s - M\omega^2) + j\omega(D_s + N^2R_L)}$
- $P_2 = -\frac{1}{2}R_L|i_m|^2$
- $P_2 = -\frac{1}{2} \frac{R_L N^2 \omega^2 |\underline{f}_{acc}|^2}{(K_s - M\omega^2)^2 + \omega^2 (D_s + N^2 R_L)^2}$

## For a real generator ( $D_s \neq 0$ )

- $\underline{f} = N\underline{v} - K_s\underline{w} - D_s\dot{\underline{w}}$
- $\underline{w} = \frac{\underline{f}_{acc}}{(K_s - M\omega^2) + j\omega(D_s + N^2R_L)}$
- $P_2 = -\frac{1}{2}R_L|i_m|^2$
- $P_2 = -\frac{1}{2} \frac{R_L N^2 \omega^2 |\underline{f}_{acc}|^2}{(K_s - M\omega^2)^2 + \omega^2 (D_s + N^2 R_L)^2}$
- $P_1 = -\frac{1}{2} \Re(\underline{f}_{acc} \cdot \dot{\underline{w}}^*)$

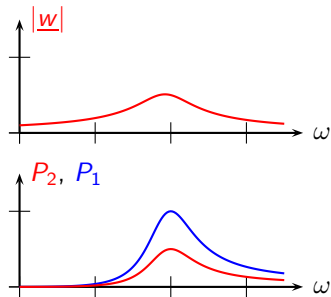


## For a real generator ( $D_s \neq 0$ )

- $\underline{f} = N\underline{v} - K_s\underline{w} - D_s\dot{\underline{w}}$
- $\underline{w} = \frac{\underline{f}_{acc}}{(K_s - M\omega^2) + j\omega(D_s + N^2R_L)}$
- $P_2 = -\frac{1}{2}R_L|i_m|^2$
- $P_2 = -\frac{1}{2} \frac{R_L N^2 \omega^2 |\underline{f}_{acc}|^2}{(K_s - M\omega^2)^2 + \omega^2 (D_s + N^2 R_L)^2}$
- $P_1 = -\frac{1}{2} \Re(\underline{f}_{acc} \cdot \dot{\underline{w}}^*)$
- $P_1 = -\frac{1}{2} \frac{(D_s + R_L N^2) \omega^2 |\underline{f}_{acc}|^2}{(K_s - M\omega^2)^2 + \omega^2 (D_s + N^2 R_L)^2}$

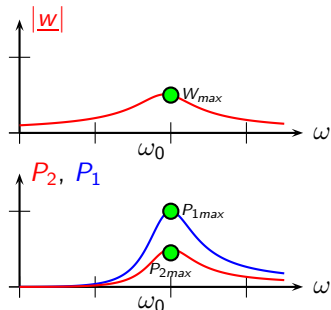
## For a real generator ( $D_s \neq 0$ )

- $\underline{f} = N\underline{v} - K_s\underline{w} - D_s\dot{\underline{w}}$
- $\underline{w} = \frac{\underline{f}_{acc}}{(K_s - M\omega^2) + j\omega(D_s + N^2R_L)}$
- $P_2 = -\frac{1}{2}R_L|i_m|^2$
- $P_2 = -\frac{1}{2} \frac{R_L N^2 \omega^2 |\underline{f}_{acc}|^2}{(K_s - M\omega^2)^2 + \omega^2 (D_s + N^2 R_L)^2}$
- $P_1 = -\frac{1}{2} \Re(\underline{f}_{acc} \cdot \dot{\underline{w}}^*)$
- $P_1 = -\frac{1}{2} \frac{(D_s + R_L N^2) \omega^2 |\underline{f}_{acc}|^2}{(K_s - M\omega^2)^2 + \omega^2 (D_s + N^2 R_L)^2}$



# For a real generator ( $D_s \neq 0$ )

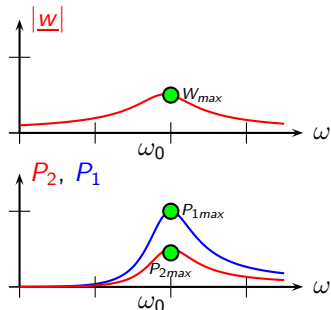
- $\underline{f} = N\underline{v} - K_s\underline{w} - D_s\dot{\underline{w}}$
- $\underline{w} = \frac{\underline{f}_{acc}}{(K_s - M\omega^2) + j\omega(D_s + N^2R_L)}$
- $P_2 = -\frac{1}{2}R_L|i_m|^2$
- $P_2 = -\frac{1}{2} \frac{R_L N^2 \omega^2 |\underline{f}_{acc}|^2}{(K_s - M\omega^2)^2 + \omega^2 (D_s + N^2 R_L)^2}$
- $P_1 = -\frac{1}{2} \Re(\underline{f}_{acc} \cdot \dot{\underline{w}}^*)$
- $P_1 = -\frac{1}{2} \frac{(D_s + R_L N^2) \omega^2 |\underline{f}_{acc}|^2}{(K_s - M\omega^2)^2 + \omega^2 (D_s + N^2 R_L)^2}$



## For a real generator ( $D_s \neq 0$ )

- $\underline{f} = N\underline{v} - K_s\underline{w} - D_s\dot{\underline{w}}$
- $\underline{w} = \frac{\underline{f}_{acc}}{(K_s - M\omega^2) + j\omega(D_s + N^2R_L)}$
- $P_2 = -\frac{1}{2}R_L|i_m|^2$
- $P_2 = -\frac{1}{2} \frac{R_L N^2 \omega^2 |\underline{f}_{acc}|^2}{(K_s - M\omega^2)^2 + \omega^2 (D_s + N^2 R_L)^2}$
- $P_1 = -\frac{1}{2} \Re(\underline{f}_{acc} \cdot \dot{\underline{w}}^*)$
- $P_1 = -\frac{1}{2} \frac{(D_s + R_L N^2) \omega^2 |\underline{f}_{acc}|^2}{(K_s - M\omega^2)^2 + \omega^2 (D_s + N^2 R_L)^2}$

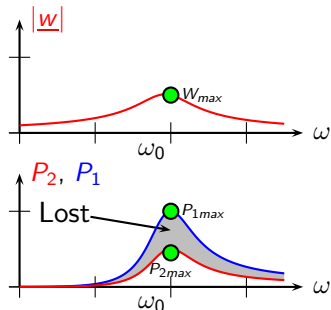
$\omega_0$  should still be the vibration's pulsation.



## For a real generator ( $D_s \neq 0$ )

- $\underline{f} = N\underline{v} - K_s\underline{w} - D_s\dot{\underline{w}}$
- $\underline{w} = \frac{\underline{f}_{acc}}{(K_s - M\omega^2) + j\omega(D_s + N^2R_L)}$
- $P_2 = -\frac{1}{2}R_L|i_m|^2$
- $P_2 = -\frac{1}{2} \frac{R_L N^2 \omega^2 |f_{acc}|^2}{(K_s - M\omega^2)^2 + \omega^2 (D_s + N^2 R_L)^2}$
- $P_1 = -\frac{1}{2} \Re(\underline{f}_{acc} \cdot \dot{\underline{w}}^*)$
- $P_1 = -\frac{1}{2} \frac{(D_s + R_L N^2) \omega^2 |f_{acc}|^2}{(K_s - M\omega^2)^2 + \omega^2 (D_s + N^2 R_L)^2}$

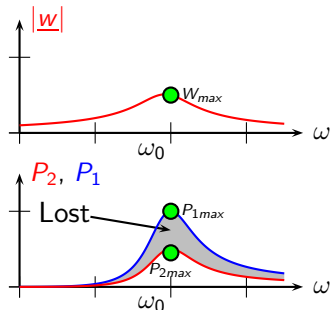
$\omega_0$  should still be the vibration's pulsation.



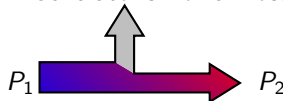
## For a real generator ( $D_s \neq 0$ )

- $\underline{f} = N\underline{v} - K_s\underline{w} - D_s\dot{\underline{w}}$
- $\underline{w} = \frac{\underline{f}_{acc}}{(K_s - M\omega^2) + j\omega(D_s + N^2R_L)}$
- $P_2 = -\frac{1}{2}R_L|i_m|^2$
- $P_2 = -\frac{1}{2} \frac{R_L N^2 \omega^2 |\underline{f}_{acc}|^2}{(K_s - M\omega^2)^2 + \omega^2 (D_s + N^2 R_L)^2}$
- $P_1 = -\frac{1}{2} \Re(\underline{f}_{acc} \cdot \dot{\underline{w}}^*)$
- $P_1 = -\frac{1}{2} \frac{(D_s + R_L N^2) \omega^2 |\underline{f}_{acc}|^2}{(K_s - M\omega^2)^2 + \omega^2 (D_s + N^2 R_L)^2}$

$\omega_0$  should still be the vibration's pulsation.



Generator's Power Losses

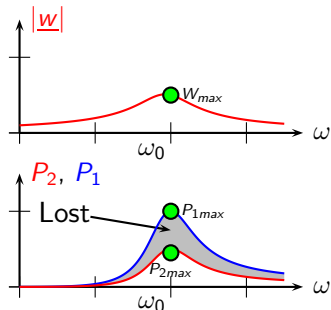


## For a real generator ( $D_s \neq 0$ )

- $\underline{f} = N\underline{v} - K_s\underline{w} - D_s\dot{\underline{w}}$
- $\underline{w} = \frac{\underline{f}_{acc}}{(K_s - M\omega^2) + j\omega(D_s + N^2R_L)}$
- $P_2 = -\frac{1}{2}R_L|i_m|^2$
- $P_2 = -\frac{1}{2} \frac{R_L N^2 \omega^2 |f_{acc}|^2}{(K_s - M\omega^2)^2 + \omega^2 (D_s + N^2 R_L)^2}$
- $P_1 = -\frac{1}{2} \Re(\underline{f}_{acc} \cdot \dot{\underline{w}}^*)$
- $P_1 = -\frac{1}{2} \frac{(D_s + R_L N^2) \omega^2 |f_{acc}|^2}{(K_s - M\omega^2)^2 + \omega^2 (D_s + N^2 R_L)^2}$

$\omega_0$  should still be the vibration's pulsation.

- $W_{max} = \frac{|f_{acc}|}{(D_s + N^2 R_L) \omega_0}$ ,  $P_{1max} = \frac{1}{2} \frac{|f_{acc}|^2}{D_s + N^2 R_L}$ ,  
 $P_{2max} = \frac{1}{2} \frac{N^2 R_L |f_{acc}|^2}{(D_s + N^2 R_L)^2}$



Generator's Power Losses



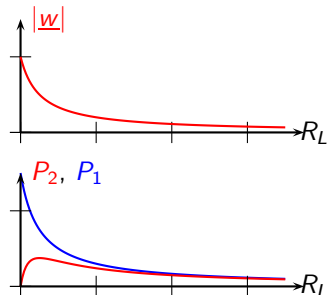
## For a real generator ( $D_s \neq 0$ )

- $W_{max} = \frac{|f_{acc}|}{(D_s + N^2 R_L) \omega_0}, P_{1max} = \frac{1}{2} \frac{|f_{acc}|^2}{D_s + N^2 R_L}$
- $P_{2max} = \frac{1}{2} \frac{N^2 R_L |f_{acc}|^2}{(D_s + N^2 R_L)^2}$



# For a real generator ( $D_s \neq 0$ )

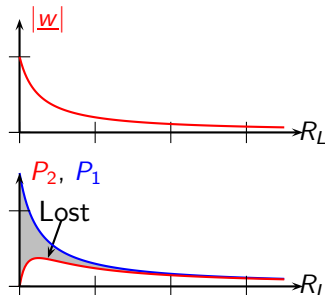
- $W_{max} = \frac{|f_{acc}|}{(D_s + N^2 R_L) \omega_0}, P_{1max} = \frac{1}{2} \frac{|f_{acc}|^2}{D_s + N^2 R_L}$
- $P_{2max} = \frac{1}{2} \frac{N^2 R_L |f_{acc}|^2}{(D_s + N^2 R_L)^2}$



## For a real generator ( $D_s \neq 0$ )

- $W_{max} = \frac{|f_{acc}|}{(D_s + N^2 R_L) \omega_0}$ ,  $P_{1max} = \frac{1}{2} \frac{|f_{acc}|^2}{D_s + N^2 R_L}$
- $P_{2max} = \frac{1}{2} \frac{N^2 R_L |f_{acc}|^2}{(D_s + N^2 R_L)^2}$

When  $R_L \searrow$ ,  $P_1 \nearrow$ , which is good, but the losses increase as well: it exists an optimal value of  $R_L$  which optimizes  $P_2$  for a given vibration amplitude.

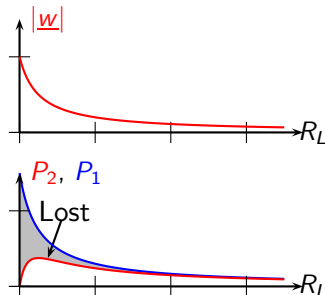


## For a real generator ( $D_s \neq 0$ )

- $W_{max} = \frac{|f_{acc}|}{(D_s + N^2 R_L) \omega_0}$ ,  $P_{1max} = \frac{1}{2} \frac{|f_{acc}|^2}{D_s + N^2 R_L}$
- $P_{2max} = \frac{1}{2} \frac{N^2 R_L |f_{acc}|^2}{(D_s + N^2 R_L)^2}$

When  $R_L \searrow$ ,  $P_1 \nearrow$ , which is good, but the losses increase as well: it exists an optimal value of  $R_L$  which optimizes  $P_2$  for a given vibration amplitude. It can be shown:

- $R_{Lopt} = \frac{D_s}{N^2}$ ,
- $P_{2opt} = \frac{|f_{acc}|^2}{8D_s}$
- $P_{1opt} = \frac{|f_{acc}|^2}{4D_s} = 2 \cdot P_{2opt}$

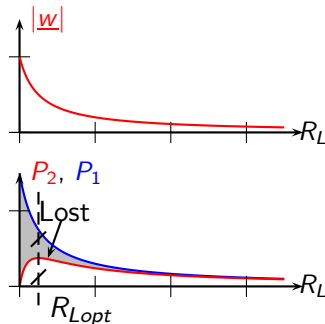


## For a real generator ( $D_s \neq 0$ )

- $W_{max} = \frac{|f_{acc}|}{(D_s + N^2 R_L) \omega_0}$ ,  $P_{1max} = \frac{1}{2} \frac{|f_{acc}|^2}{D_s + N^2 R_L}$
- $P_{2max} = \frac{1}{2} \frac{N^2 R_L |f_{acc}|^2}{(D_s + N^2 R_L)^2}$

When  $R_L \searrow$ ,  $P_1 \nearrow$ , which is good, but the losses increase as well: it exists an optimal value of  $R_L$  which optimizes  $P_2$  for a given vibration amplitude. It can be shown:

- $R_{Lopt} = \frac{D_s}{N^2}$ ,
- $P_{2opt} = \frac{|f_{acc}|^2}{8D_s}$
- $P_{1opt} = \frac{|f_{acc}|^2}{4D_s} = 2 \cdot P_{2opt}$

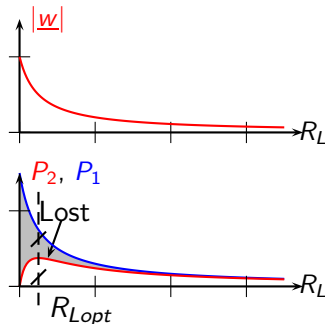


## For a real generator ( $D_s \neq 0$ )

- $W_{max} = \frac{|f_{acc}|}{(D_s + N^2 R_L) \omega_0}$ ,  $P_{1max} = \frac{1}{2} \frac{|f_{acc}|^2}{D_s + N^2 R_L}$
- $P_{2max} = \frac{1}{2} \frac{N^2 R_L |f_{acc}|^2}{(D_s + N^2 R_L)^2}$

When  $R_L \searrow$ ,  $P_1 \nearrow$ , which is good, but the losses increase as well: it exists an optimal value of  $R_L$  which optimizes  $P_2$  for a given vibration amplitude. It can be shown:

- $R_{Lopt} = \frac{D_s}{N^2}$ ,
- $P_{2opt} = \frac{|f_{acc}|^2}{8D_s}$
- $P_{1opt} = \frac{|f_{acc}|^2}{4D_s} = 2 \cdot P_{2opt}$
- $|\underline{v}_{max}| = NR_L \omega_0 |\underline{w}| = \frac{NR_L |f_{acc}|}{(D_s + N^2 R_L)}$

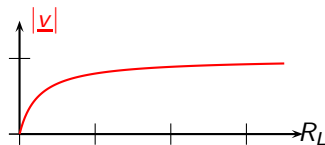
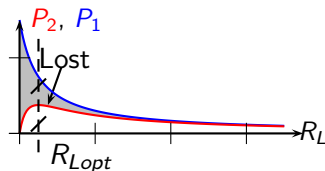
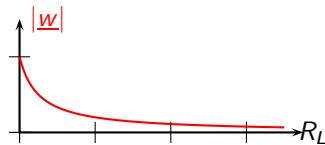


## For a real generator ( $D_s \neq 0$ )

- $W_{max} = \frac{|f_{acc}|}{(D_s + N^2 R_L) \omega_0}$ ,  $P_{1max} = \frac{1}{2} \frac{|f_{acc}|^2}{D_s + N^2 R_L}$
- $P_{2max} = \frac{1}{2} \frac{N^2 R_L |f_{acc}|^2}{(D_s + N^2 R_L)^2}$

When  $R_L \searrow$ ,  $P_1 \nearrow$ , which is good, but the losses increase as well: it exists an optimal value of  $R_L$  which optimizes  $P_2$  for a given vibration amplitude. It can be shown:

- $R_{Lopt} = \frac{D_s}{N^2}$ ,
- $P_{2opt} = \frac{|f_{acc}|^2}{8D_s}$
- $P_{1opt} = \frac{|f_{acc}|^2}{4D_s} = 2 \cdot P_{2opt}$
- $|v_{max}| = NR_L \omega_0 |w| = \frac{NR_L |f_{acc}|}{(D_s + N^2 R_L)}$

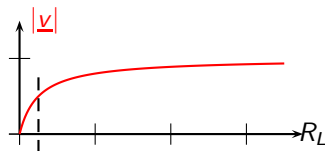
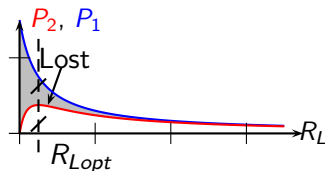
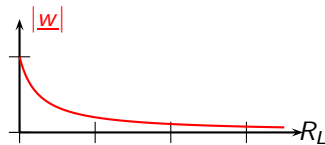


## For a real generator ( $D_s \neq 0$ )

- $W_{max} = \frac{|f_{acc}|}{(D_s + N^2 R_L) \omega_0}$ ,  $P_{1max} = \frac{1}{2} \frac{|f_{acc}|^2}{D_s + N^2 R_L}$
- $P_{2max} = \frac{1}{2} \frac{N^2 R_L |f_{acc}|^2}{(D_s + N^2 R_L)^2}$

When  $R_L \searrow$ ,  $P_1 \nearrow$ , which is good, but the losses increase as well: it exists an optimal value of  $R_L$  which optimizes  $P_2$  for a given vibration amplitude. It can be shown:

- $R_{Lopt} = \frac{D_s}{N^2}$ ,
- $P_{2opt} = \frac{|f_{acc}|^2}{8D_s}$
- $P_{1opt} = \frac{|f_{acc}|^2}{4D_s} = 2 \cdot P_{2opt}$
- $|v_{max}| = NR_L \omega_0 |w| = \frac{NR_L |f_{acc}|}{(D_s + N^2 R_L)}$

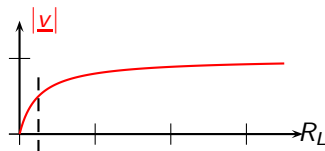
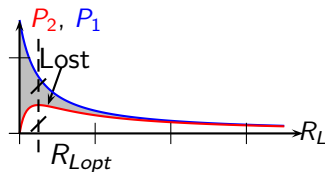
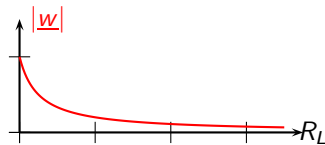


## For a real generator ( $D_s \neq 0$ )

- $W_{max} = \frac{|f_{acc}|}{(D_s + N^2 R_L) \omega_0}$ ,  $P_{1max} = \frac{1}{2} \frac{|f_{acc}|^2}{D_s + N^2 R_L}$
- $P_{2max} = \frac{1}{2} \frac{N^2 R_L |f_{acc}|^2}{(D_s + N^2 R_L)^2}$

When  $R_L \searrow$ ,  $P_1 \nearrow$ , which is good, but the losses increase as well: it exists an optimal value of  $R_L$  which optimizes  $P_2$  for a given vibration amplitude. It can be shown:

- $R_{Lopt} = \frac{D_s}{N^2}$ ,
- $P_{2opt} = \frac{|f_{acc}|^2}{8D_s}$
- $P_{1opt} = \frac{|f_{acc}|^2}{4D_s} = 2 \cdot P_{2opt}$
- $|v_{max}| = NR_L \omega_0 |w| = \frac{NR_L |f_{acc}|}{(D_s + N^2 R_L)}$
- voltage is not so high.



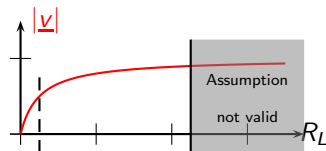
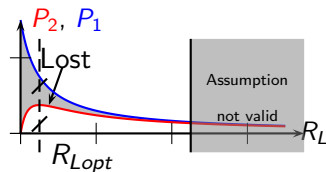
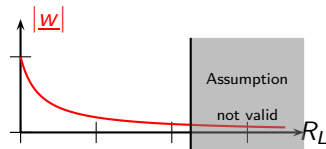


## For a real generator ( $D_s \neq 0$ )

- $W_{max} = \frac{|f_{acc}|}{(D_s + N^2 R_L \omega_0)}, P_{1max} = \frac{1}{2} \frac{|f_{acc}|^2}{D_s + N^2 R_L}$
- $P_{2max} = \frac{1}{2} \frac{N^2 R_L |f_{acc}|^2}{(D_s + N^2 R_L)^2}$

When  $R_L \searrow$ ,  $P_1 \nearrow$ , which is good, but the losses increase as well: it exists an optimal value of  $R_L$  which optimizes  $P_2$  for a given vibration amplitude. It can be shown:

- $R_{Lopt} = \frac{D_s}{N^2},$
- $P_{2opt} = \frac{|f_{acc}|^2}{8D_s}$
- $P_{1opt} = \frac{|f_{acc}|^2}{4D_s} = 2 \cdot P_{2opt}$
- $|v_{max}| = NR_L \omega_0 |w| = \frac{NR_L |f_{acc}|}{(D_s + N^2 R_L)}$
- voltage is not so high.



## Example

### Device's properties

$$N = 0.012 \text{ N/V}, K_s = 6300 \text{ N/m}, C_b = 300 \text{ nF}, D_s = 0.17 \text{ Ns/m}, \\ M = 1 \text{ g}$$

### Calculate for $A = 0.1 \text{ mm}$

- the best working frequency,
- the harvested  $P_2$  power in the best case,
- the power of the source  $P_1$  in such best case,
- the optimal resistor  $R_L$ ,
- the deflection amplitude of the bender,
- the voltage for this working point.

### Validation

Is  $v = -R_L i_m$  a valid assumption?

## Answers

- The best working frequency is given by

$$f_0 = \frac{1}{2\pi} \sqrt{\frac{K_s}{M}} = \frac{1}{2\pi} \sqrt{\frac{6300}{1.10^{-3}}} = \underline{400Hz}$$

## Answers

- The best working frequency is given by

$$f_0 = \frac{1}{2\pi} \sqrt{\frac{K_s}{M}} = \frac{1}{2\pi} \sqrt{\frac{6300}{1 \cdot 10^{-3}}} = \underline{400\text{Hz}}$$

- For the best case,  $P_2$  is given by

$$P_2 = \frac{|f_{acc}|^2}{8D_s} = \frac{(1 \cdot 10^{-3} \cdot 1 \cdot 10^{-4} \cdot (2\pi \cdot 400)^2)^2}{8 \cdot 0,17} = \underline{464\text{mW}}$$

## Answers

- The best working frequency is given by

$$f_0 = \frac{1}{2\pi} \sqrt{\frac{K_s}{M}} = \frac{1}{2\pi} \sqrt{\frac{6300}{1 \cdot 10^{-3}}} = \underline{400\text{Hz}}$$

- For the best case,  $P_2$  is given by

$$P_2 = \frac{|f_{acc}|^2}{8D_s} = \frac{(1 \cdot 10^{-3} \cdot 1 \cdot 10^{-4} \cdot (2\pi \cdot 400)^2)^2}{8 \cdot 0,17} = \underline{464\text{mW}}$$

- For the best case  $P_1 = 2 \cdot P_2 = \underline{928\text{mW}}$

## Answers

- The best working frequency is given by

$$f_0 = \frac{1}{2\pi} \sqrt{\frac{K_s}{M}} = \frac{1}{2\pi} \sqrt{\frac{6300}{1.10^{-3}}} = \underline{400\text{Hz}}$$

- For the best case,  $P_2$  is given by

$$P_2 = \frac{|f_{\text{acc}}|^2}{8D_s} = \frac{(1.10^{-3} \cdot 1.10^{-4} \cdot (2\pi \cdot 400)^2)^2}{8.0,17} = \underline{464\text{mW}}$$

- For the best case  $P_1 = 2 \cdot P_2 = \underline{928\text{mW}}$

- The optimal resistor  $R_{Lopt}$  is given by

$$R_{Lopt} = \frac{D_s}{N^2} = \frac{0,17}{0,012^2} = \underline{1180\Omega}$$

## Answers

- The best working frequency is given by

$$f_0 = \frac{1}{2\pi} \sqrt{\frac{K_s}{M}} = \frac{1}{2\pi} \sqrt{\frac{6300}{1.10^{-3}}} = \underline{400\text{Hz}}$$

- For the best case,  $P_2$  is given by

$$P_2 = \frac{|f_{acc}|^2}{8D_s} = \frac{(1.10^{-3} \cdot 1.10^{-4} \cdot (2\pi \cdot 400)^2)^2}{8 \cdot 0,17} = \underline{464\text{mW}}$$

- For the best case  $P_1 = 2 \cdot P_2 = \underline{928\text{mW}}$

- The optimal resistor  $R_{Lopt}$  is given by

$$R_{Lopt} = \frac{D_s}{N^2} = \frac{0,17}{0,012^2} = \underline{1180\Omega}$$

- The deflection  $W_{max}$  is given by

$$W_{max} = \frac{|f_{acc}|}{(D_s + N^2 R_L)\omega_0} = \frac{(1.10^{-3} \cdot 1.10^{-4} \cdot (2\pi \cdot 400)^2)}{2 \cdot 0,17 \cdot 2\pi \cdot 400} = \underline{738\mu\text{m!}}$$

## Answers

- The best working frequency is given by

$$f_0 = \frac{1}{2\pi} \sqrt{\frac{K_s}{M}} = \frac{1}{2\pi} \sqrt{\frac{6300}{1.10^{-3}}} = \underline{400\text{Hz}}$$

- For the best case,  $P_2$  is given by

$$P_2 = \frac{|f_{acc}|^2}{8D_s} = \frac{(1.10^{-3} \cdot 1.10^{-4} \cdot (2\pi \cdot 400)^2)^2}{8 \cdot 0,17} = \underline{464\text{mW}}$$

- For the best case  $P_1 = 2 \cdot P_2 = \underline{928\text{mW}}$

- The optimal resistor  $R_{Lopt}$  is given by

$$R_{Lopt} = \frac{D_s}{N^2} = \frac{0,17}{0,012^2} = \underline{1180\Omega}$$

- The deflection  $W_{max}$  is given by

$$W_{max} = \frac{|f_{acc}|}{(D_s + N^2 R_L) \omega_0} = \frac{(1.10^{-3} \cdot 1.10^{-4} \cdot (2\pi \cdot 400)^2)}{2 \cdot 0,17 \cdot 2\pi \cdot 400} = \underline{738\mu\text{m!}}$$

- The voltage is then given by

$$v_{max} = N R_L \omega_0 |w| = 0,012 \cdot 1180 \cdot 2\pi \cdot 400 \cdot 738 \cdot 10^{-6} = \underline{26\text{V}}$$



# Answers

- The best working frequency is given by

$$f_0 = \frac{1}{2\pi} \sqrt{\frac{K_s}{M}} = \frac{1}{2\pi} \sqrt{\frac{6300}{1.10^{-3}}} = \underline{400\text{Hz}}$$

- For the best case,  $P_2$  is given by

$$P_2 = \frac{|f_{acc}|^2}{8D_s} = \frac{(1.10^{-3} \cdot 1.10^{-4} \cdot (2\pi \cdot 400)^2)^2}{8 \cdot 0,17} = \underline{464\text{mW}}$$

- For the best case  $P_1 = 2 \cdot P_2 = \underline{928\text{mW}}$

- The optimal resistor  $R_{Lopt}$  is given by

$$R_{Lopt} = \frac{D_s}{N^2} = \frac{0,17}{0,012^2} = \underline{1180\Omega}$$

- The deflection  $W_{max}$  is given by

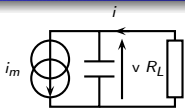
$$W_{max} = \frac{|f_{acc}|}{(D_s + N^2 R_L) \omega_0} = \frac{(1.10^{-3} \cdot 1.10^{-4} \cdot (2\pi \cdot 400)^2)}{2 \cdot 0,17 \cdot 2\pi \cdot 400} = \underline{738\mu\text{m}}$$

- The voltage is then given by

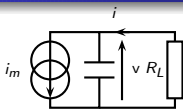
$$v_{max} = N R_L \omega_0 |w| = 0,012 \cdot 1180 \cdot 2\pi \cdot 400 \cdot 738 \cdot 10^{-6} = \underline{26\text{V}}$$

- $Z_{Cb} = \frac{1}{C_b \omega} = \frac{1}{2\pi \cdot 400 \cdot 300 \cdot 10^{-9}} = \underline{1990\Omega} \approx R_L, \rightarrow$  **Calculations are NOT valid!**

For a real Generator,  $D_s \neq 0$  and  $v \neq -R_L i_m$



For a real Generator,  $D_s \neq 0$  and  $v \neq -R_L i_m$

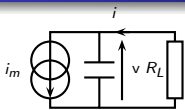


We can show:

$$\underline{v} = -\frac{R_L}{1+j\omega R_L C_b} \underline{i}_m = -j\omega N R_L \frac{1-j\omega R_L C_b}{1+(R_L C_b \omega)^2} \underline{w}_m$$

and we write:  $\underline{v} = -j\omega N r_{Leq} \underline{w} - N k_{eq} \underline{w}$

# For a real Generator, $D_s \neq 0$ and $v \neq -R_L i_m$



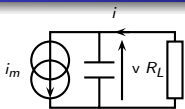
We can show:

$$\underline{v} = -\frac{R_L}{1+j\omega R_L C_b} \underline{i}_m = -j\omega N R_L \frac{1-j\omega R_L C_b}{1+(R_L C_b \omega)^2} \underline{\omega}_m$$

and we write:  $\underline{v} = -j\omega N r_{Leq} \underline{\omega} - N k_{eq} \underline{\omega}$

2 Cases to consider, since  $\omega$  should be  $\omega_0$ :

## For a real Generator, $D_s \neq 0$ and $v \neq -R_L i_m$



We can show:

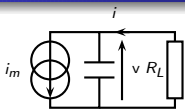
$$\underline{v} = -\frac{R_L}{1+j\omega R_L C_b} \underline{i}_m = -j\omega N R_L \frac{1-j\omega R_L C_b}{1+(R_L C_b \omega)^2} \underline{w}_m$$

and we write:  $\underline{v} = -j\omega N r_{Leq} \underline{w} - N k_{eq} \underline{w}$

2 Cases to consider, since  $\omega$  should be  $\omega_0$ :

$$\omega_0 \ll \frac{1}{R_L C_b}$$

# For a real Generator, $D_s \neq 0$ and $v \neq -R_L i_m$

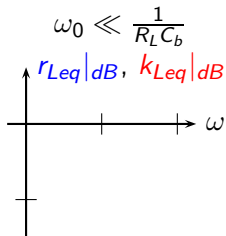


We can show:

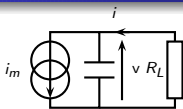
$$\underline{v} = -\frac{R_L}{1+j\omega R_L C_b} \underline{i}_m = -j\omega N R_L \frac{1-j\omega R_L C_b}{1+(R_L C_b \omega)^2} \underline{w}_m$$

and we write:  $\underline{v} = -j\omega N r_{Leq} \underline{w} - N k_{eq} \underline{w}$

2 Cases to consider, since  $\omega$  should be  $\omega_0$ :



# For a real Generator, $D_s \neq 0$ and $v \neq -R_L i_m$

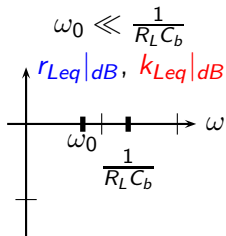


We can show:

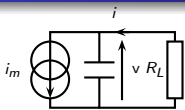
$$\underline{v} = -\frac{R_L}{1+j\omega R_L C_b} \underline{i}_m = -j\omega N R_L \frac{1-j\omega R_L C_b}{1+(R_L C_b \omega)^2} \underline{w}_m$$

and we write:  $\underline{v} = -j\omega N r_{Leq} \underline{w} - N k_{eq} \underline{w}$

2 Cases to consider, since  $\omega$  should be  $\omega_0$ :



For a real Generator,  $D_s \neq 0$  and  $v \neq -R_L i_m$

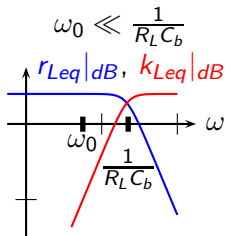


We can show:

$$\underline{v} = -\frac{R_L}{1+j\omega R_L C_b} \underline{i}_m = -j\omega N R_L \frac{1-j\omega R_L C_b}{1+(R_L C_b \omega)^2} \underline{w}_m$$

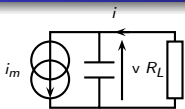
and we write:  $\underline{v} = -j\omega N r_{Leq} \underline{w} - N k_{eq} \underline{w}$

2 Cases to consider, since  $\omega$  should be  $\omega_0$ :





For a real Generator,  $D_s \neq 0$  and  $v \neq -R_L i_m$

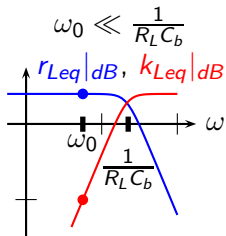


We can show:

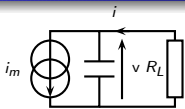
$$\underline{v} = -\frac{R_L}{1+j\omega R_L C_b} \underline{i}_m = -j\omega N R_L \frac{1-j\omega R_L C_b}{1+(R_L C_b \omega)^2} \underline{w}_m$$

and we write:  $\underline{v} = -j\omega N r_{Leq} \underline{w} - N k_{eq} \underline{w}$

2 Cases to consider, since  $\omega$  should be  $\omega_0$ :



For a real Generator,  $D_s \neq 0$  and  $v \neq -R_L i_m$

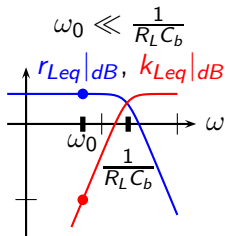


We can show:

$$\underline{v} = -\frac{R_L}{1+j\omega R_L C_b} \underline{i}_m = -j\omega N R_L \frac{1-j\omega R_L C_b}{1+(R_L C_b \omega)^2} \underline{w}_m$$

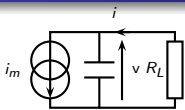
and we write:  $\underline{v} = -j\omega N r_{Leq} \underline{w} - N k_{eq} \underline{w}$

2 Cases to consider, since  $\omega$  should be  $\omega_0$ :



$r_{Leq} \simeq R_L$  and  $k_{Leq} \simeq 0$ , no problem,  
just use the assumption

For a real Generator,  $D_s \neq 0$  and  $v \neq -R_L i_m$



We can show:

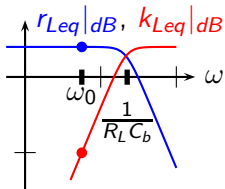
$$\underline{v} = -\frac{R_L}{1+j\omega R_L C_b} \underline{i}_m = -j\omega N R_L \frac{1-j\omega R_L C_b}{1+(R_L C_b \omega)^2} \underline{w}_m$$

and we write:  $\underline{v} = -j\omega N r_{Leq} \underline{w} - N k_{eq} \underline{w}$

2 Cases to consider, since  $\omega$  should be  $\omega_0$ :

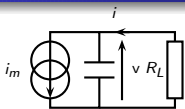
$$\omega_0 \ll \frac{1}{R_L C_b}$$

$$\omega_0 \gg \frac{1}{R_L C_b}$$



$r_{Leq} \simeq R_L$  and  $k_{Leq} \simeq 0$ , no problem,  
just use the assumption

For a real Generator,  $D_s \neq 0$  and  $v \neq -R_L i_m$

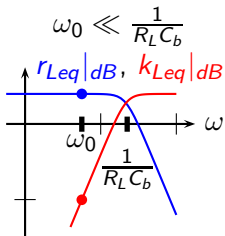


We can show:

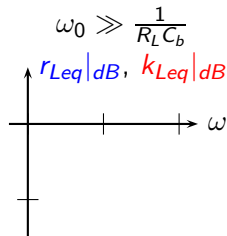
$$\underline{v} = -\frac{R_L}{1+j\omega R_L C_b} \underline{i}_m = -j\omega N R_L \frac{1-j\omega R_L C_b}{1+(R_L C_b \omega)^2} \underline{w}_m$$

and we write:  $\underline{v} = -j\omega N r_{Leq} \underline{w} - N k_{eq} \underline{w}$

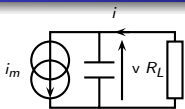
2 Cases to consider, since  $\omega$  should be  $\omega_0$ :



$r_{Leq} \simeq R_L$  and  $k_{Leq} \simeq 0$ , no problem,  
 just use the assumption



For a real Generator,  $D_s \neq 0$  and  $v \neq -R_L i_m$

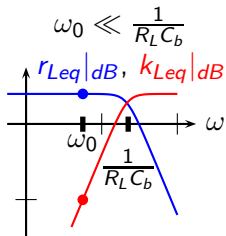


We can show:

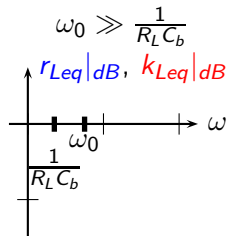
$$\underline{v} = -\frac{R_L}{1+j\omega R_L C_b} \underline{i}_m = -j\omega N R_L \frac{1-j\omega R_L C_b}{1+(R_L C_b \omega)^2} \underline{w}_m$$

and we write:  $\underline{v} = -j\omega N r_{Leq} \underline{w} - N k_{eq} \underline{w}$

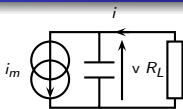
2 Cases to consider, since  $\omega$  should be  $\omega_0$ :



$r_{Leq} \simeq R_L$  and  $k_{Leq} \simeq 0$ , no problem,  
 just use the assumption



# For a real Generator, $D_s \neq 0$ and $v \neq -R_L i_m$

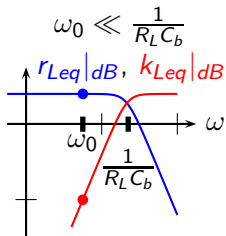


We can show:

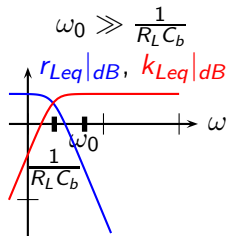
$$\underline{v} = -\frac{R_L}{1+j\omega R_L C_b} \underline{i}_m = -j\omega N R_L \frac{1-j\omega R_L C_b}{1+(R_L C_b \omega)^2} \underline{w}_m$$

and we write:  $\underline{v} = -j\omega N r_{Leq} \underline{w} - N k_{eq} \underline{w}$

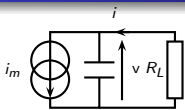
2 Cases to consider, since  $\omega$  should be  $\omega_0$ :



$r_{Leq} \simeq R_L$  and  $k_{Leq} \simeq 0$ , no problem,  
 just use the assumption



For a real Generator,  $D_s \neq 0$  and  $v \neq -R_L i_m$

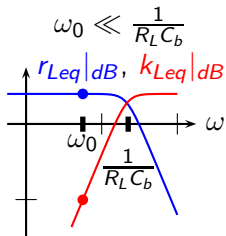


We can show:

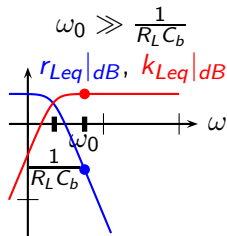
$$\underline{v} = -\frac{R_L}{1+j\omega R_L C_b} \underline{i}_m = -j\omega N R_L \frac{1-j\omega R_L C_b}{1+(R_L C_b \omega)^2} \underline{w}_m$$

and we write:  $\underline{v} = -j\omega N r_{Leq} \underline{w} - N k_{eq} \underline{w}$

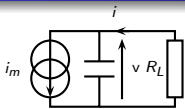
2 Cases to consider, since  $\omega$  should be  $\omega_0$ :



$r_{Leq} \simeq R_L$  and  $k_{Leq} \simeq 0$ , no problem,  
 just use the assumption



For a real Generator,  $D_s \neq 0$  and  $v \neq -R_L i_m$



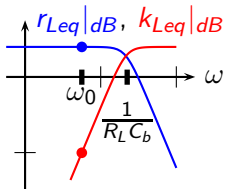
We can show:

$$\underline{v} = -\frac{R_L}{1+j\omega R_L C_b} \underline{i}_m = -j\omega N R_L \frac{1-j\omega R_L C_b}{1+(R_L C_b \omega)^2} \underline{w}_m$$

$$\text{and we write: } \underline{v} = -j\omega N r_{Leq} \underline{w} - N k_{eq} \underline{w}$$

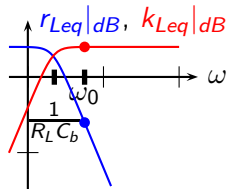
2 Cases to consider, since  $\omega$  should be  $\omega_0$ :

$$\omega_0 \ll \frac{1}{R_L C_b}$$



$r_{Leq} \simeq R_L$  and  $k_{Leq} \simeq 0$ , no problem,  
just use the assumption

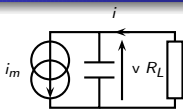
$$\omega_0 \gg \frac{1}{R_L C_b}$$



$r_{Leq} \ll R_L$  and  $k_{Leq}$  is high, an other  
study is needed



# For a real Generator, $D_s \neq 0$ and $v \neq -R_L i_m$

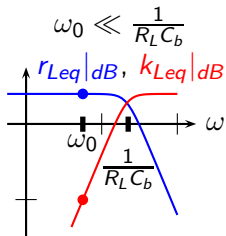


We can show:

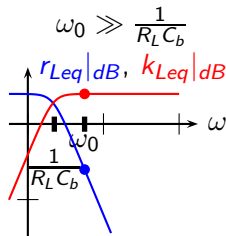
$$\underline{v} = -\frac{R_L}{1+j\omega R_L C_b} \underline{i}_m = -j\omega N R_L \frac{1-j\omega R_L C_b}{1+(R_L C_b \omega)^2} \underline{w}_m$$

and we write:  $\underline{v} = -j\omega N r_{Leq} \underline{w} - N k_{eq} \underline{w}$

2 Cases to consider, since  $\omega$  should be  $\omega_0$ :



$r_{Leq} \simeq R_L$  and  $k_{Leq} \simeq 0$ , no problem,  
 just use the assumption

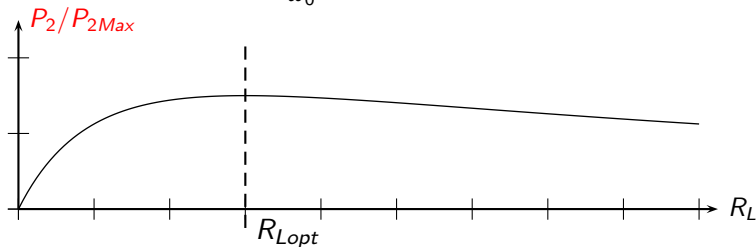
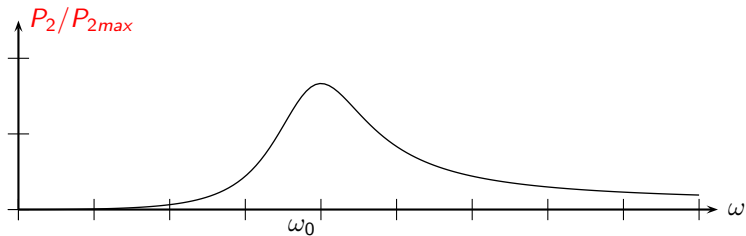


$r_{Leq} \ll R_L$  and  $k_{Leq}$  is high, an other  
 study is needed

$$((K_s + N^2 k_{eq}) - M\omega^2) + j\omega(D_s + N^2 r_{Leq}) \underline{w} = \underline{f}_{acc}$$

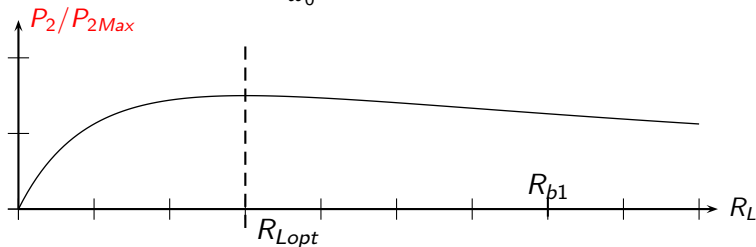
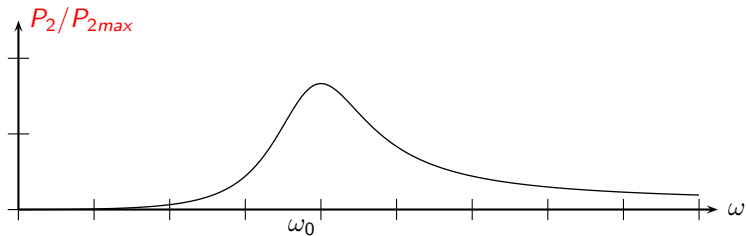
## Rule of the Thumb

Let  $R_b$  defined as  $R_b C_b \omega_0 = 1$ ,  $\omega_0 < \frac{1}{R_L C_b} \rightarrow R_L < R_b$



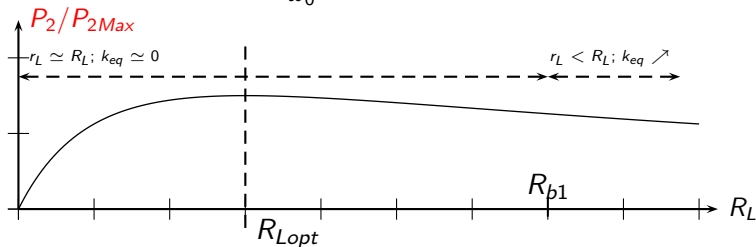
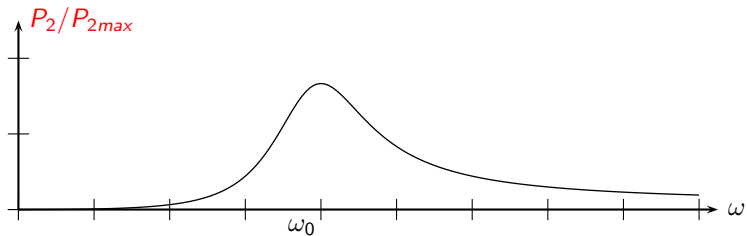
## Rule of the Thumb

Let  $R_b$  defined as  $R_b C_b \omega_0 = 1$ ,  $\omega_0 < \frac{1}{R_L C_b} \rightarrow R_L < R_b$



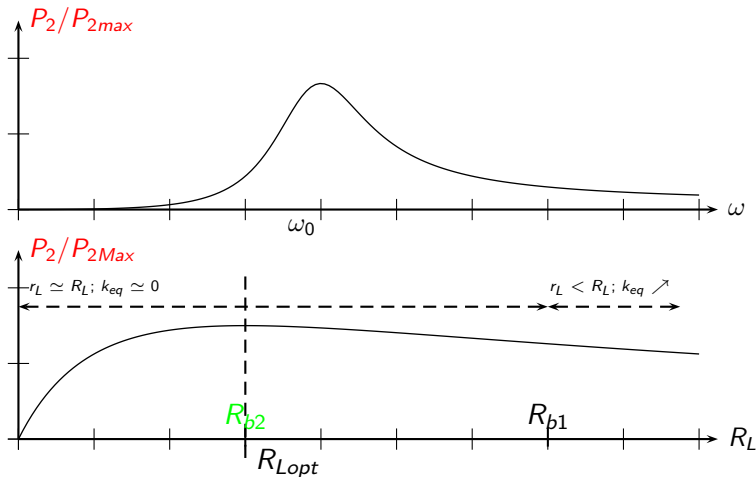
# Rule of the Thumb

Let  $R_b$  defined as  $R_b C_b \omega_0 = 1$ ,  $\omega_0 < \frac{1}{R_L C_b} \rightarrow R_L < R_b$



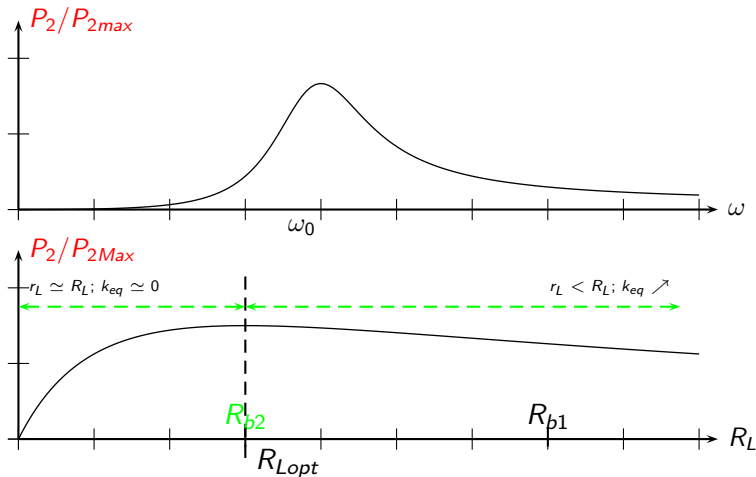
# Rule of the Thumb

Let  $R_b$  defined as  $R_b C_b \omega_0 = 1$ ,  $\omega_0 < \frac{1}{R_L C_b} \rightarrow R_L < R_b$



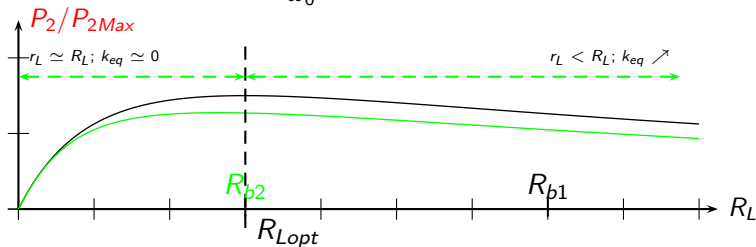
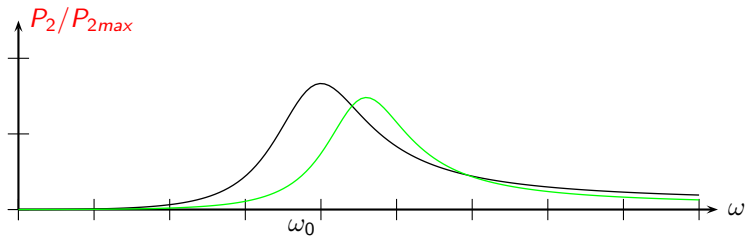
# Rule of the Thumb

Let  $R_b$  defined as  $R_b C_b \omega_0 = 1$ ,  $\omega_0 < \frac{1}{R_L C_b} \rightarrow R_L < R_b$



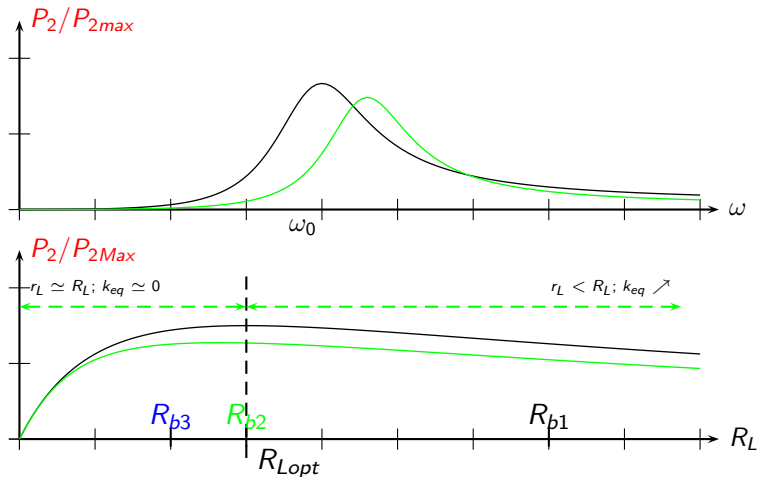
# Rule of the Thumb

Let  $R_b$  defined as  $R_b C_b \omega_0 = 1$ ,  $\omega_0 < \frac{1}{R_L C_b} \rightarrow R_L < R_b$



# Rule of the Thumb

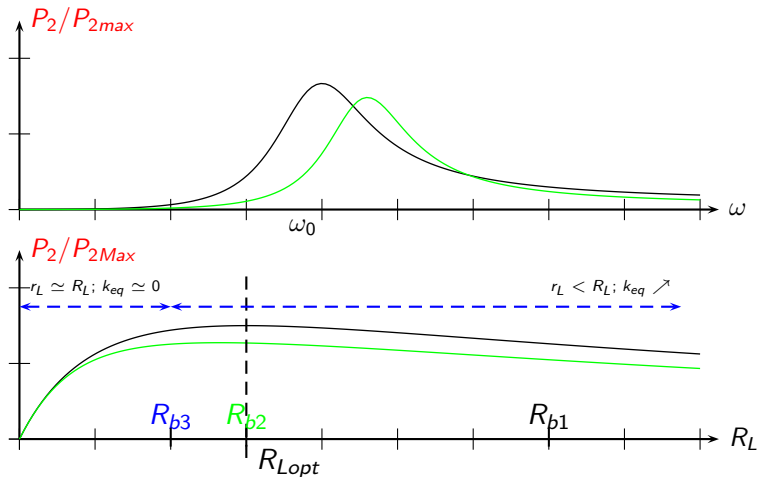
Let  $R_b$  defined as  $R_b C_b \omega_0 = 1$ ,  $\omega_0 < \frac{1}{R_L C_b} \rightarrow R_L < R_b$





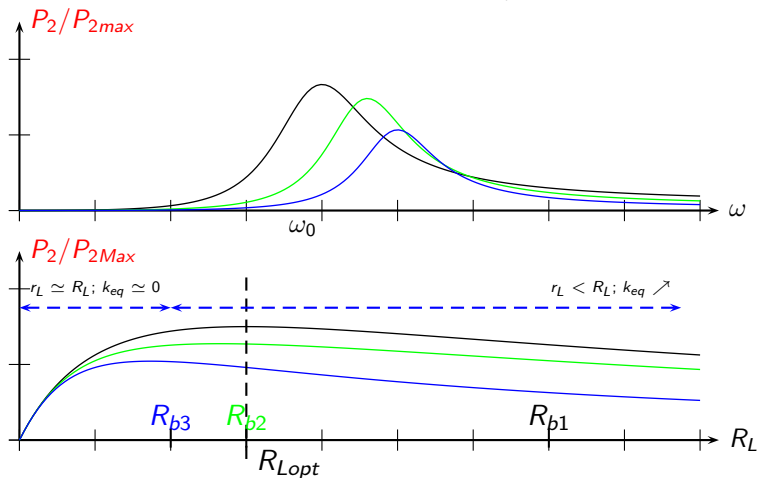
# Rule of the Thumb

Let  $R_b$  defined as  $R_b C_b \omega_0 = 1$ ,  $\omega_0 < \frac{1}{R_L C_b} \rightarrow R_L < R_b$



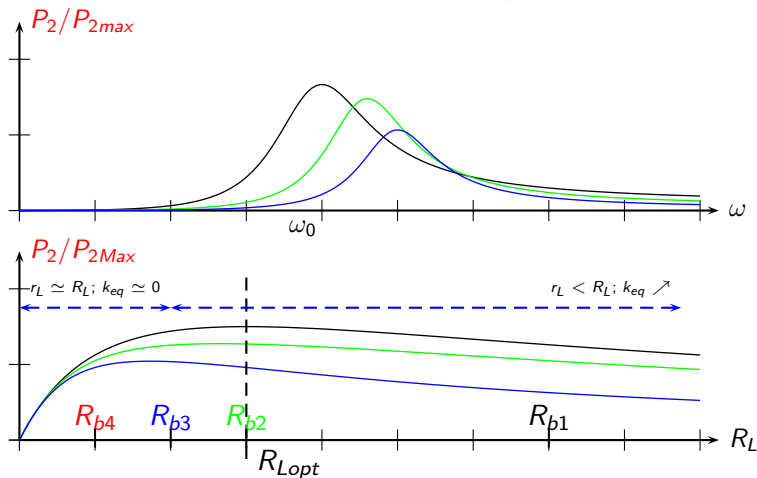
# Rule of the Thumb

Let  $R_b$  defined as  $R_b C_b \omega_0 = 1$ ,  $\omega_0 < \frac{1}{R_L C_b} \rightarrow R_L < R_b$



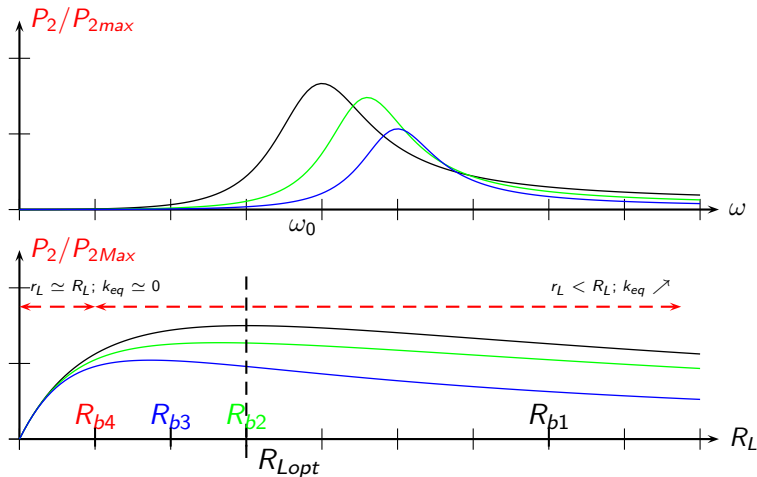
# Rule of the Thumb

Let  $R_b$  defined as  $R_b C_b \omega_0 = 1$ ,  $\omega_0 < \frac{1}{R_L C_b} \rightarrow R_L < R_b$



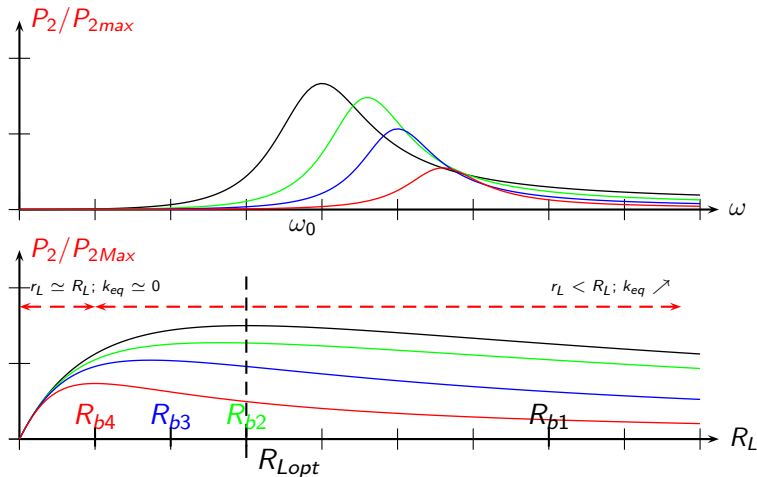
# Rule of the Thumb

Let  $R_b$  defined as  $R_b C_b \omega_0 = 1$ ,  $\omega_0 < \frac{1}{R_L C_b} \rightarrow R_L < R_b$



# Rule of the Thumb

Let  $R_b$  defined as  $R_b C_b \omega_0 = 1$ ,  $\omega_0 < \frac{1}{R_L C_b} \rightarrow R_L < R_b$



## Partial Conclusion

If  $R_{opt} < R_b$

A resistor can recover the power optimally. But the study was not valid for high  $R_L$ . What happens for  $R_L \gg R_b$ ?

## Partial Conclusion

If  $R_{opt} < R_b$

A resistor can recover the power optimally. But the study was not valid for high  $R_L$ . What happens for  $R_L \gg R_b$ ?

If  $R_{opt} > R_b$

The power is not well extracted because  $P_2 < P_{2max}$ . This happens for high damped mechanisms.

A simple resistor cannot recover the power optimally (and this is due to  $C_b$ ).

## What happens if $R_{opt} < R_b$ and $R_L \gg R_b$ ?

resonance is shifted

- $r_{Leq} = \frac{R_L}{1+(R_L C_b \omega)^2} \simeq \frac{R_L}{(R_L C_b \omega)^2}$

- $k_{eq} = R_L \frac{\omega^2 R_L C_b}{1+(R_L C_b \omega)^2} \simeq \frac{1}{C_b}$

$((K_s + N^2 k_{eq}) - M\omega^2) + j\omega(D_s + N^2 r_{Leq})\underline{w} = \underline{f}_{acc}$  leads to:

$$\omega'_0 = \sqrt{\frac{K_s + N^2 k_{eq}}{M}} = \sqrt{\frac{K_s + \frac{N^2}{C_b}}{M}}$$



## What happens if $R_{opt} < R_b$ and $R_L \gg R_b$ ?

resonance is shifted

- $r_{Leq} = \frac{R_L}{1+(R_L C_b \omega)^2} \simeq \frac{R_L}{(R_L C_b \omega)^2}$

- $k_{eq} = R_L \frac{\omega^2 R_L C_b}{1+(R_L C_b \omega)^2} \simeq \frac{1}{C_b}$

$((K_s + N^2 k_{eq}) - M\omega^2) + j\omega(D_s + N^2 r_{Leq})\underline{w} = \underline{f}_{acc}$  leads to:

$$\omega'_0 = \sqrt{\frac{K_s + N^2 k_{eq}}{M}} = \sqrt{\frac{K_s + \frac{N^2}{C_b}}{M}}$$

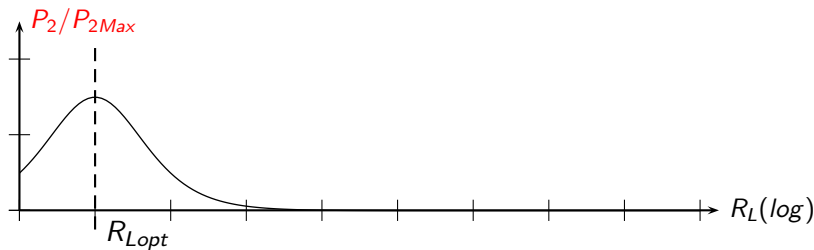
Another optimal resistor

The optimal power is harvested is  $N^2 r_{Leq} = D_s$ , leading to:

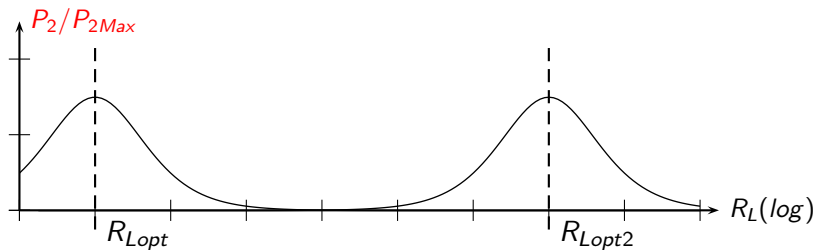
$$\frac{N^2 R_L}{(R_L C_b \omega'_0)^2} = \frac{N^2}{R_L C_b^2 \frac{K_s + \frac{N^2}{C_b}}{M}} = D_s$$

$$R_{Lopt2} = \frac{N^2}{D_s} \frac{1}{C_b^2 \frac{K_s}{M} (1 + \frac{N^2}{K_s C_b})} = \frac{R_b^2}{R_{Lopt}} \frac{1}{1 + \frac{N^2}{K_s C_b}}$$

# What happens if $R_{opt} < R_b$ and $R_L \gg R_b$ ?

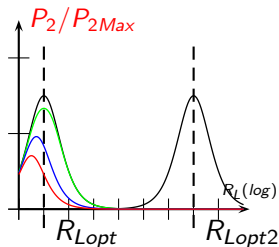


# What happens if $R_{opt} < R_b$ and $R_L \gg R_b$ ?



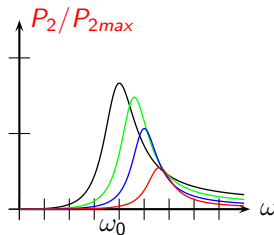
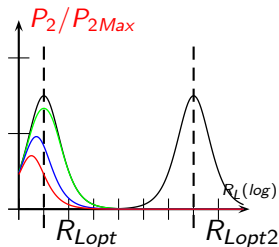
# Conclusion

- For the realistic case, there is one or two resistive loads which allow to extract the maximum of power,



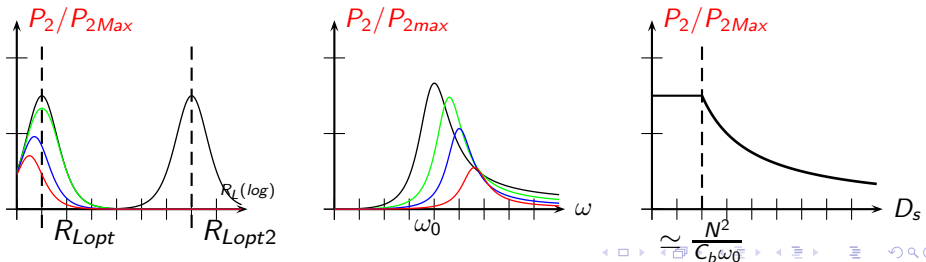
# Conclusion

- For the realistic case, there is one or two resistive loads which allow to extract the maximum of power,
- There exist an optimal frequency which may vary if  $R_L$  is very high,



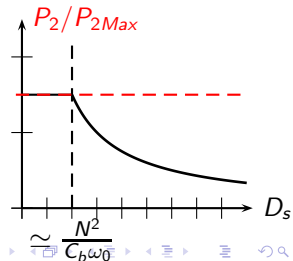
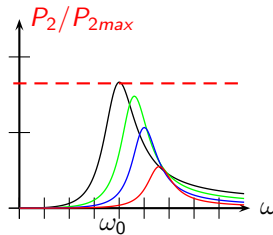
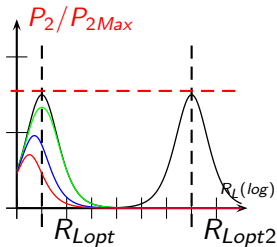
# Conclusion

- For the realistic case, there is one or two resistive loads which allow to extract the maximum of power,
- There exist an optimal frequency which may vary if  $R_L$  is very high,
- For highly damped structure, a simple resistor is not optimal.



# Conclusion

- For the realistic case, there is one or two resistive loads which allow to extract the maximum of power,
  - There exist an optimal frequency which may vary if  $R_L$  is very high,
  - For highly damped structure, a simple resistor is not optimal.
- we always want to harvest the maximum of power!

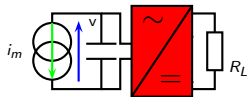


# Table of contents

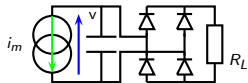
- 1 Introduction
  - What is Energy Harvesting ?
  - Generator Technologies
  - Summary
- 2 Modelling of a piezoelectric energy harvester
  - Presentation of the system
  - EMR of the system
  - Power Extraction on a load resistor  $R_L$  from harmonic oscillation
- 3 An Example of inverter
  - Introduction
  - SSHI: Synchronized Switch Harvesting on Inductor



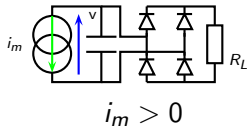
# Introduction



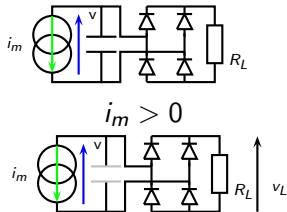
# Introduction



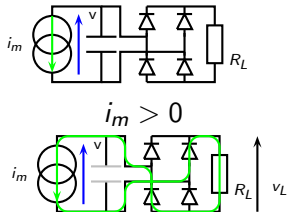
# Introduction



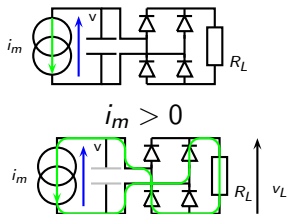
# Introduction



# Introduction



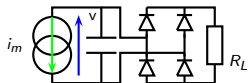
# Introduction



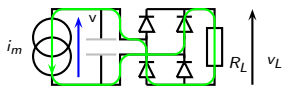
$$v_L = R_L i_m \text{ which is } > 0$$

$$\text{and } v = -v_L = -R_L i_m$$

## Introduction



$$i_m > 0$$

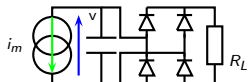


$$v_L = R_L i_m \text{ which is } > 0$$

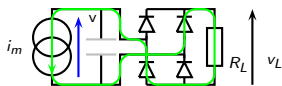
$$\text{and } v = -v_L = -R_L i_m$$

$$i_m < 0$$

## Introduction



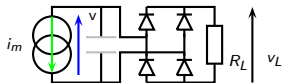
$$i_m > 0$$



$$v_L = R_L i_m \text{ which is } > 0$$

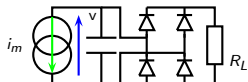
$$\text{and } v = -v_L = -R_L i_m$$

$$i_m < 0$$

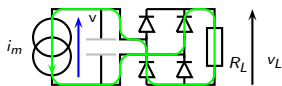




# Introduction



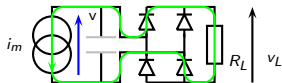
$$i_m > 0$$



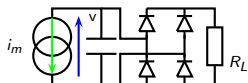
$$v_L = R_L i_m \text{ which is } > 0$$

$$\text{and } v = -v_L = -R_L i_m$$

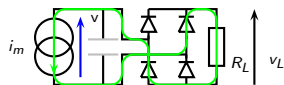
$$i_m < 0$$



## Introduction



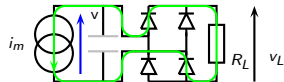
$$i_m > 0$$



$$v_L = R_L i_m \text{ which is } > 0$$

$$\text{and } v = -v_L = -R_L i_m$$

$$i_m < 0$$

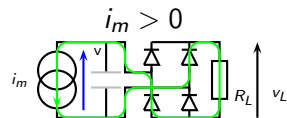
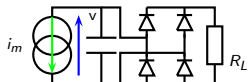


$$v_L = -R_L i_m \text{ which is } > 0$$

$$\text{again, and}$$

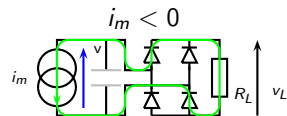
$$v = v_L = -R_L i_m$$

## Introduction



$$v_L = R_L i_m \text{ which is } > 0$$

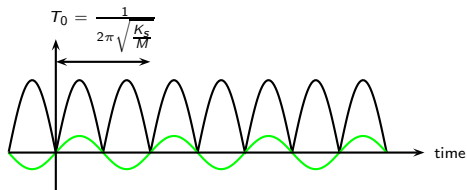
$$\text{and } v = -v_L = -R_L i_m$$



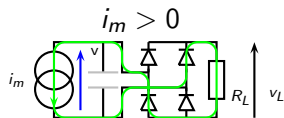
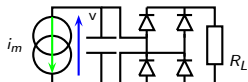
$$v_L = -R_L i_m \text{ which is } > 0$$

again, and

$$v = v_L = -R_L i_m$$

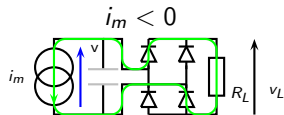


## Introduction



$$v_L = R_L i_m \text{ which is } > 0$$

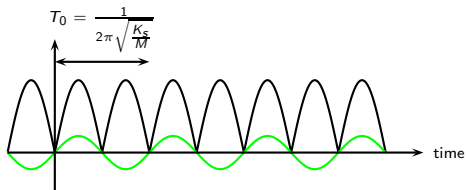
$$\text{and } v = -v_L = -R_L i_m$$



$$v_L = -R_L i_m \text{ which is } > 0$$

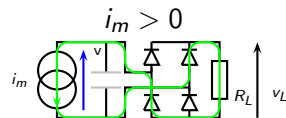
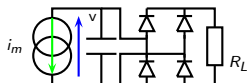
again, and

$$v = v_L = -R_L i_m$$



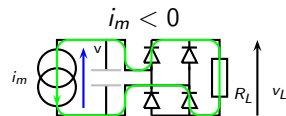
The current is rectified, but we still have  $v = -R_L i_m$ : Power harvesting depends on  $R_L$

## Introduction



$$v_L = R_L i_m \text{ which is } > 0$$

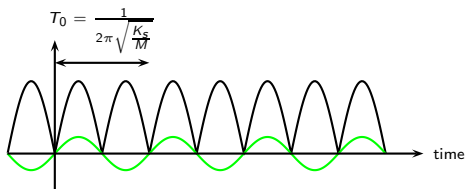
$$\text{and } v = -v_L = -R_L i_m$$



$$v_L = -R_L i_m \text{ which is } > 0$$

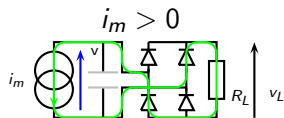
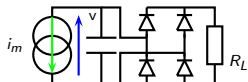
again, and

$$v = v_L = -R_L i_m$$



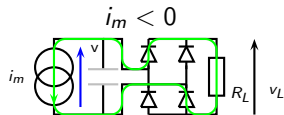
The current is rectified, but we still have  $v = -R_L i_m$ : Power harvesting depends on  $R_L$   
Moreover, it is still not optimal if  $R_{opt} > R_b$ .

## Introduction



$$v_L = R_L i_m \text{ which is } > 0$$

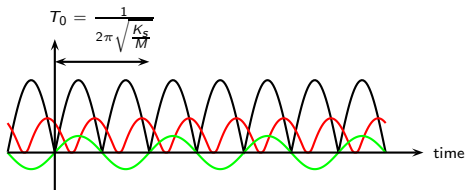
$$\text{and } v = -v_L = -R_L i_m$$



$$v_L = -R_L i_m \text{ which is } > 0$$

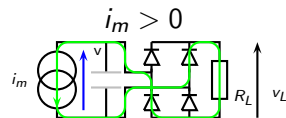
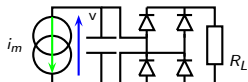
again, and

$$v = v_L = -R_L i_m$$



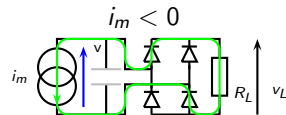
The current is rectified, but we still have  $v = -R_L i_m$ : Power harvesting depends on  $R_L$   
Moreover, it is still not optimal if  $R_{opt} > R_b$ .

## Introduction



$$v_L = R_L i_m \text{ which is } > 0$$

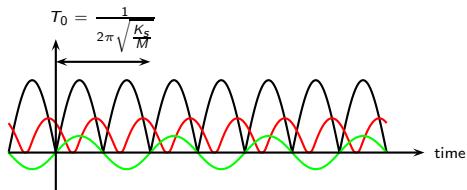
$$\text{and } v = -v_L = -R_L i_m$$



$$v_L = -R_L i_m \text{ which is } > 0$$

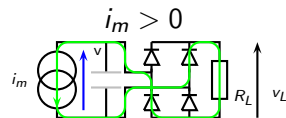
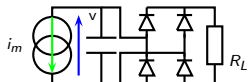
$$\text{again, and}$$

$$v = v_L = -R_L i_m$$

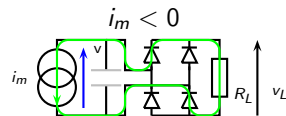


The current is rectified, but we still have  $v = -R_L i_m$ : Power harvesting depends on  $R_L$   
Moreover, it is still not optimal if  $R_{opt} > R_b$ .  
This is why, some want to compensate for  $C_b$ :

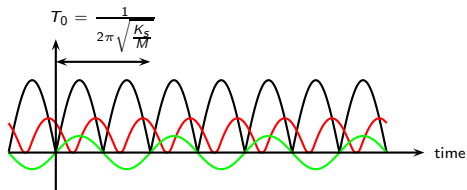
## Introduction



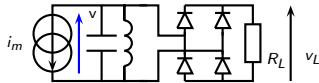
$v_L = R_L i_m$  which is  $> 0$   
and  $v = -v_L = -R_L i_m$



$v_L = -R_L i_m$  which is  $> 0$   
again, and  
 $v = v_L = -R_L i_m$

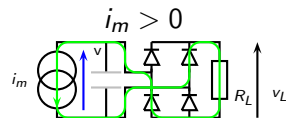
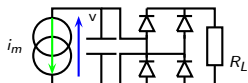


The current is rectified, but we still have  $v = -R_L i_m$ : Power harvesting depends on  $R_L$   
Moreover, it is still not optimal if  $R_{opt} > R_b$ .  
This is why, some want to compensate for  $C_b$ :

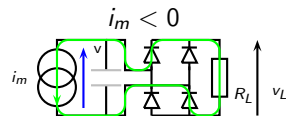




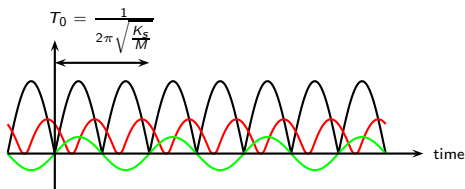
## Introduction



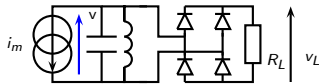
$v_L = R_L i_m$  which is  $> 0$   
and  $v = -v_L = -R_L i_m$



$v_L = -R_L i_m$  which is  $> 0$   
again, and  
 $v = v_L = -R_L i_m$

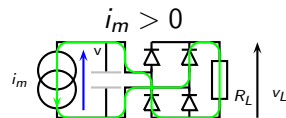
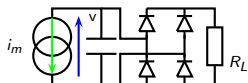


The current is rectified, but we still have  
 $v = -R_L i_m$ : Power harvesting depends on  $R_L$   
Moreover, it is still not optimal if  $R_{opt} > R_b$ .  
This is why, some want to compensate for  $C_b$ :

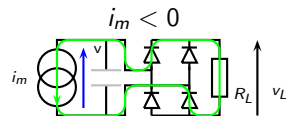


This is a bad solution because it works only for  $\omega_0$  (what if the frequency shifts?), and the Inductor is large (because  $\omega_0$  usually is small)

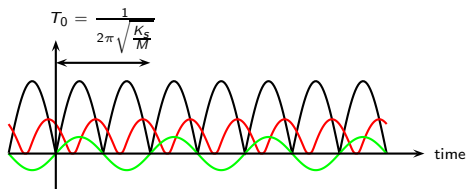
## Introduction



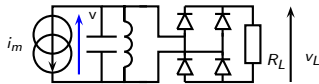
$v_L = R_L i_m$  which is  $> 0$   
and  $v = -v_L = -R_L i_m$



$v_L = -R_L i_m$  which is  $> 0$   
again, and  
 $v = v_L = -R_L i_m$



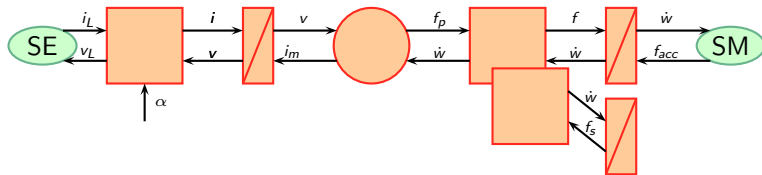
The current is rectified, but we still have  $v = -R_L i_m$ : Power harvesting depends on  $R_L$   
Moreover, it is still not optimal if  $R_{opt} > R_b$ .  
This is why, some want to compensate for  $C_b$ :



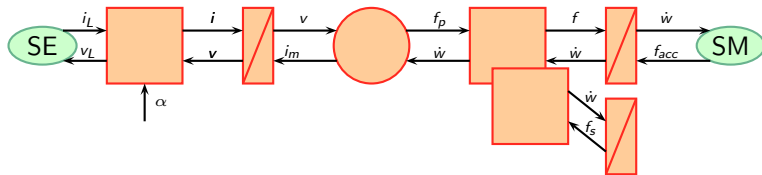
This is a bad solution because it works only for  $\omega_0$  (what if the frequency shifts?), and the Inductor is large (because  $\omega_0$  usually is small)  
→ Non linear Techniques

# Why Synchronized

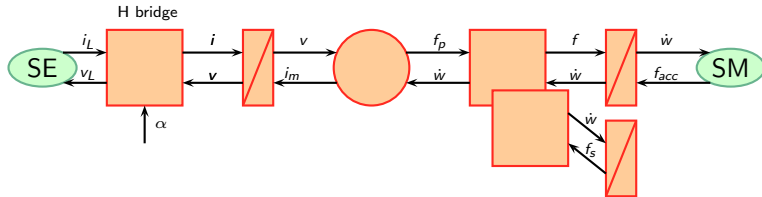
# Why Synchronized



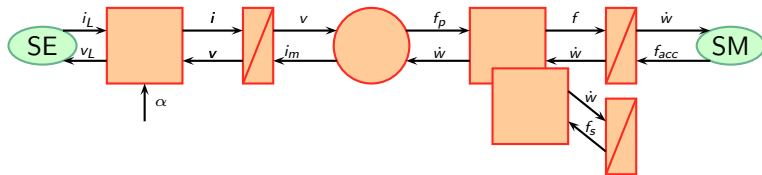
# Why Synchronized



# Why Synchronized

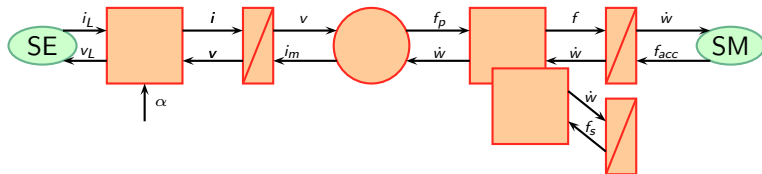


# Why Synchronized



Strategy:  $v = -R_{opt}i_m$  leads to  $\frac{f_p}{N} = -\frac{D_s}{N^2}N\dot{w}$  or,  $f_p = -D_s\dot{w}$ .

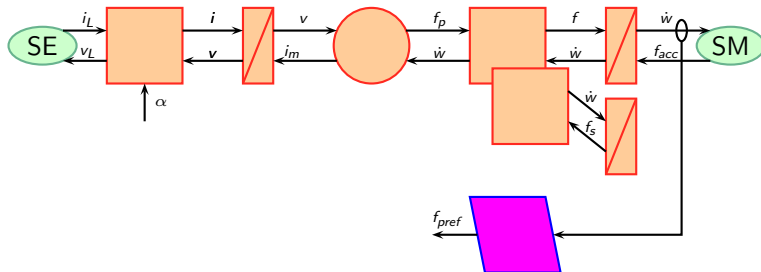
# Why Synchronized



Strategy:  $v = -R_{opt}i_m$  leads to  $\frac{f_p}{N} = -\frac{D_s}{N^2}N\dot{w}$  or,  $f_p = -D_s\dot{w}$ .

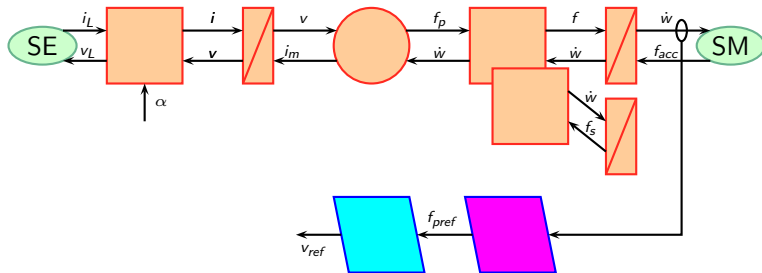


# Why Synchronized



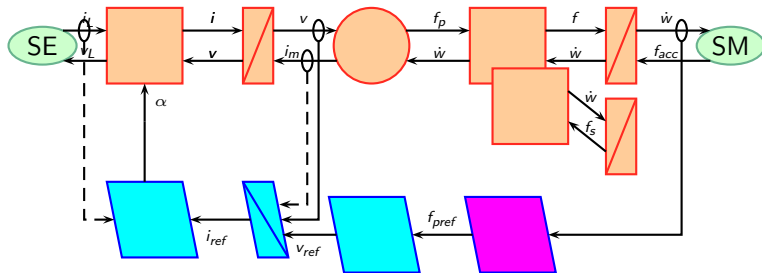
Strategy:  $v = -R_{opt}i_m$  leads to  $\frac{f_p}{N} = -\frac{D_s}{N^2}N\dot{w}$  or,  $f_p = -D_s\dot{w}$ .

# Why Synchronized



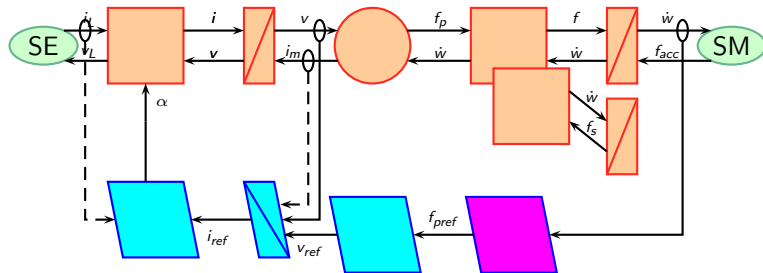
Strategy:  $v = -R_{opt}i_m$  leads to  $\frac{f_p}{N} = -\frac{D_s}{N^2}N\dot{w}$  or,  $f_p = -D_s\dot{w}$ .

# Why Synchronized



Strategy:  $v = -R_{opt} i_m$  leads to  $\frac{f_p}{N} = -\frac{D_s}{N^2} N \dot{w}$  or,  $f_p = -D_s \dot{w}$ .  
 This shows that  $v$  should be controlled.

# Why Synchronized



Strategy:  $v = -R_{opt} i_m$  leads to  $\frac{f_p}{N} = -\frac{D_s}{N^2} N \dot{w}$  or,  $f_p = -D_s \dot{w}$ .  
 This shows that  $v$  should be controlled. SSHI does this with efficiency.

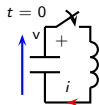
# How it works

*LC* oscillations



# How it works

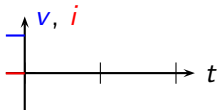
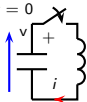
LC oscillations



# How it works

LC oscillations

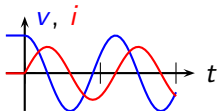
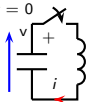
$t = 0$



# How it works

LC oscillations

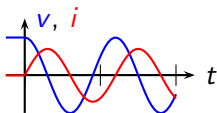
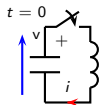
$t = 0$





# How it works

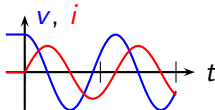
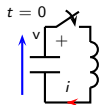
LC oscillations



Switched inductor

# How it works

LC oscillations

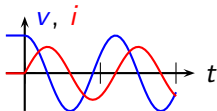
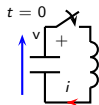


Switched inductor

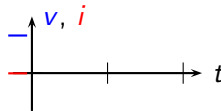
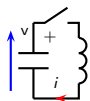


# How it works

LC oscillations

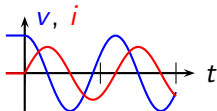
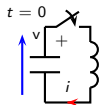


Switched inductor

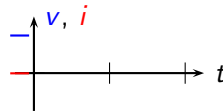


# How it works

LC oscillations

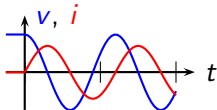
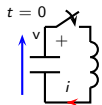


Switched inductor

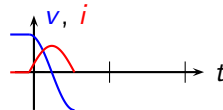


# How it works

LC oscillations

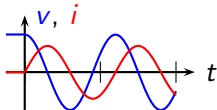
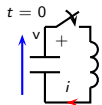


Switched inductor

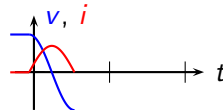
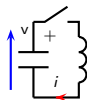


# How it works

LC oscillations

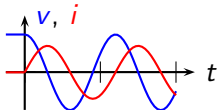
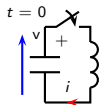


Switched inductor

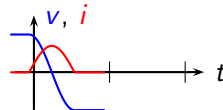
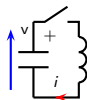


# How it works

LC oscillations

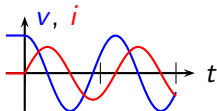
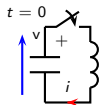


Switched inductor

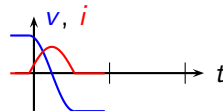


# How it works

LC oscillations



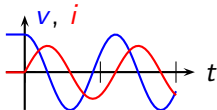
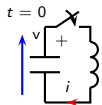
Switched inductor



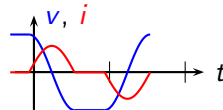


# How it works

LC oscillations

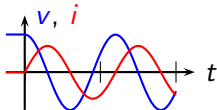
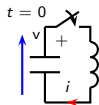


Switched inductor

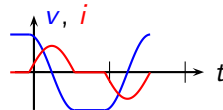
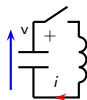


# How it works

LC oscillations

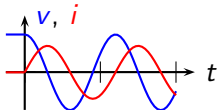
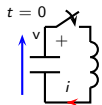


Switched inductor

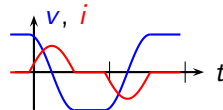
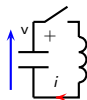


# How it works

LC oscillations

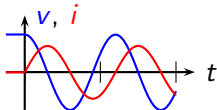
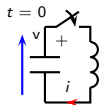


Switched inductor

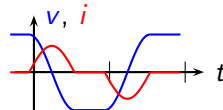
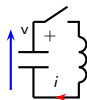


# How it works

LC oscillations



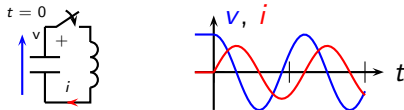
Switched inductor



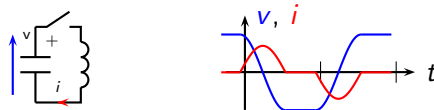
SSHI

# How it works

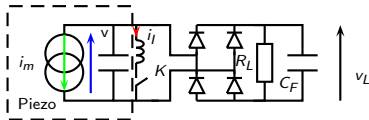
LC oscillations



Switched inductor

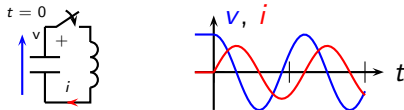


SSHI

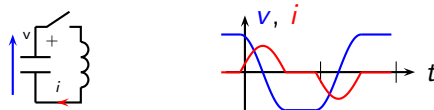


# How it works

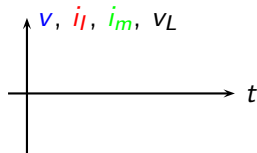
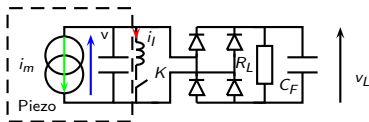
LC oscillations



Switched inductor

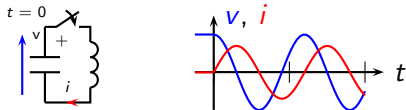


SSHI

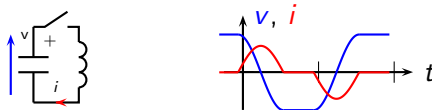


# How it works

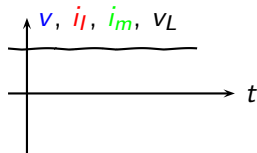
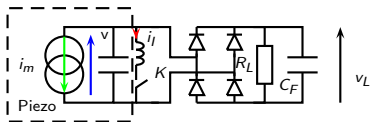
## LC oscillations



## Switched inductor

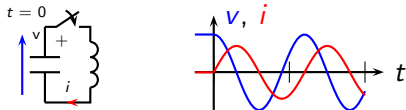


## SSHI

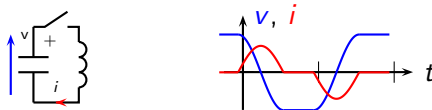


# How it works

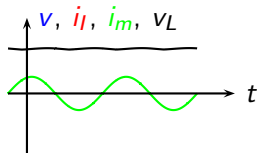
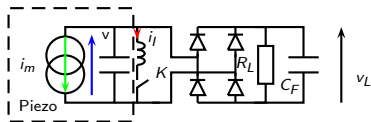
## LC oscillations



## Switched inductor



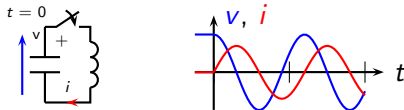
## SSHI



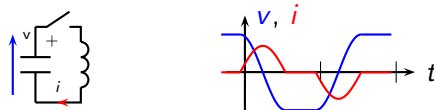


# How it works

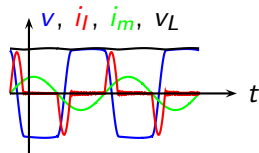
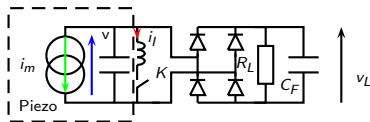
## LC oscillations



## Switched inductor

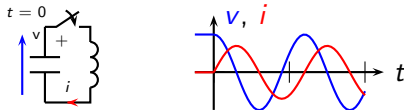


## SSHI

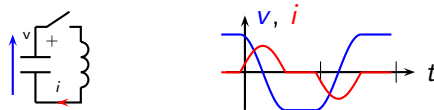


# How it works

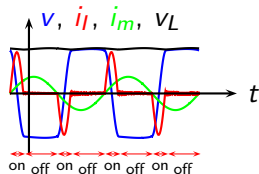
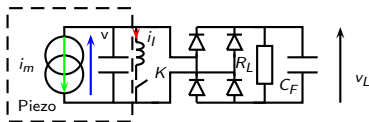
## LC oscillations



## Switched inductor



## SSHI

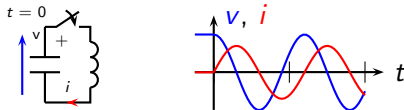


Switching is synchronized on  $\dot{w}$ , or

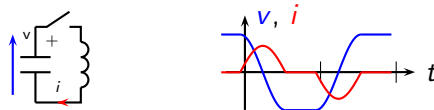
$$i_m = N\dot{w}$$

# How it works

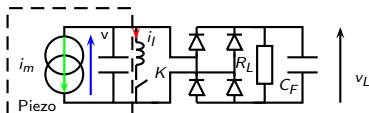
## LC oscillations



## Switched inductor

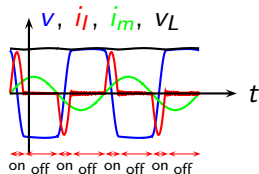


## SSHI



Operating point: calculate  $v_L$  from  $i_m$

1<sup>st</sup> Harmonic assumption:  $P = \frac{1}{2} \frac{V_L^2}{R_L} \simeq \frac{1}{2} \frac{4V_L I_m}{\pi}$ ,  $V_L = \frac{4}{\pi} R_L I_m$

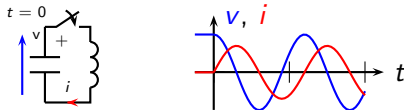


Switching is synchronized on  $\dot{w}$ , or

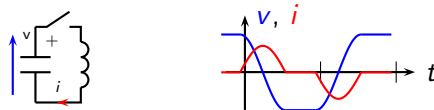
$$i_m = N\dot{w}$$

# How it works

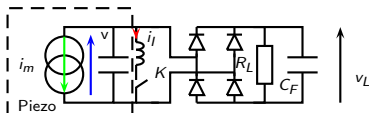
## LC oscillations



## Switched inductor



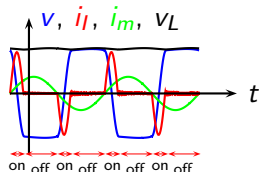
## SSHI



Operating point: calculate  $v_L$  from  $i_m$

1<sup>st</sup> Harmonic assumption:  $P = \frac{1}{2} \frac{V_L^2}{R_L} \simeq \frac{1}{2} \frac{4V_L I_m}{\pi}$ ,  $V_L = \frac{4}{\pi} R_L I_m$

SSHI controls  $v$  and synchronises it, but doesn't impose  $f_p = -D_s \dot{w}$ .



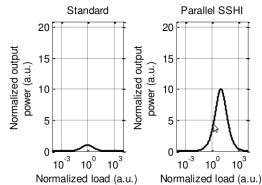
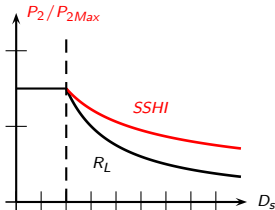
Switching is synchronized on  $\dot{w}$ , or

$$i_m = N \dot{w}.$$

# Conclusion

## Performances

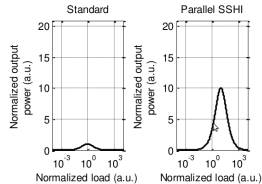
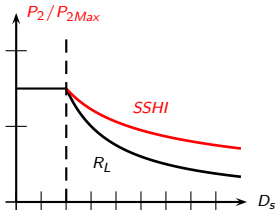
- SSHI can extract energy more efficiently than a resistor when damping is important,



# Conclusion

## Performances

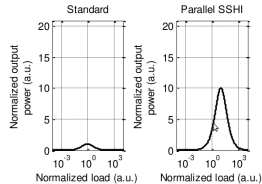
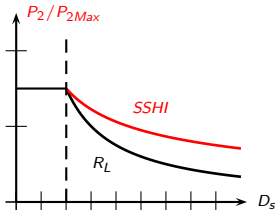
- SSHI can extract energy more efficiently than a resistor when damping is important,
- But Power extraction still depends on the load,



# Conclusion

## Performances

- SSHI can extract energy more efficiently than a resistor when damping is important,
- But Power extraction still depends on the load,
- Needs to measure bender's deflection  $w(t)$ .



## General conclusion

In this presentation, applications of Energy Harvesting were shown. The modelling of a piezoelectric generator has shown that the power source needs an adaptation:




- in frequency,
- in load.

to maximize the harvested power.

The key energy management rules were presented through the analysis of the EMR of the system. A typical power electronic circuit was also presented, but the bibliography shows a lot of example.



# References I

-  S Adhikari, M I Friswell, and D J Inman, *Piezoelectric energy harvesting from broadband random vibrations*, Smart Materials and Structures **18** (2009), no. 11, 115005.
-  R. G Ballas, H. F Schlaak, and A. J Schmid, *Closed form analysis of piezoelectric multilayer bending actuators using constituent equations*, The 13th International Conference on Solid-State Sensors, Actuators and Microsystems, 2005. Digest of Technical Papers. TRANSDUCERS '05, vol. 1, IEEE, June 2005, pp. 788–791 Vol. 1.
-  R. Djugum, P. Trivailo, and K. Graves, *A study of energy harvesting from piezoelectrics using impact forces*, The European Physical Journal Applied Physics **48** (2009), no. 1, 11101.
-  Daniel Guyomar, Gal Sebald, Sbastien Pruvost, Mickal Lallart, Akram Khodayari, and Claude Richard, *Energy harvesting from ambient vibrations and heat*, Journal of Intelligent Material Systems and Structures **20** (2009), no. 5, 609–624.
-  Aman Kansal, Jason Hsu, Sadaf Zahedi, and Mani B. Srivastava, *Power management in energy harvesting sensor networks*, ACM Transactions on Embedded Computing Systems **6** (2007), 32–es.
-  Elie Lefeuvre, Adrien Badel, Claude Richard, and Daniel Guyomar, *Piezoelectric energy harvesting device optimization by synchronous electric charge extraction*, Journal of Intelligent Material Systems and Structures **16** (2005), no. 10, 865–876.
-  H. Lhermet, C. Condemine, M. Plissonnier, R. Salot, P. Audebert, and M. Rosset, *Efficient power management circuit: From thermal energy harvesting to above-IC microbattery energy storage*, IEEE Journal of Solid-State Circuits **43** (2008), no. 1, 246–255.

# References II



G. A. Lesieutre, G. K. Ottman, and H. F. Hofmann, *Damping as a result of piezoelectric energy harvesting*, Journal of Sound and Vibration **269** (2004), no. 3-5, 991–1001.



D. Niyato, E. Hossain, M. M Rashid, and V. K Bhargava, *Wireless sensor networks with energy harvesting technologies: a game-theoretic approach to optimal energy management*, IEEE Wireless Communications **14** (2007), no. 4, 90–96.



Shad Roundy, Paul Kenneth Wright, and Jan M. Rabaey, *Energy scavenging for wireless sensor networks: with special focus on vibrations*, Springer, 2003.



G. Sebald, E. Lefeuvre, and D. Guyomar, *Pyroelectric energy conversion: Optimization principles*, IEEE Transactions on Ultrasonics, Ferroelectrics and Frequency Control **55** (2008), no. 3, 538–551.



Qing-Ming Wang and L. E Cross, *Constitutive equations of symmetrical triple layer piezoelectric benders*, IEEE Transactions on Ultrasonics, Ferroelectrics and Frequency Control **46** (1999), no. 6, 1343–1351.

# End of the presentation

Questions?