



Energy Harvesting from Ambient Vibrations

Frédéric Giraud

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Energy Harvesting from Ambient Vibrations

Frédéric Giraud

L2EP – University Lille1

November 27, 2012

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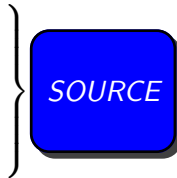
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We extract energy from an *ambient and free source*:

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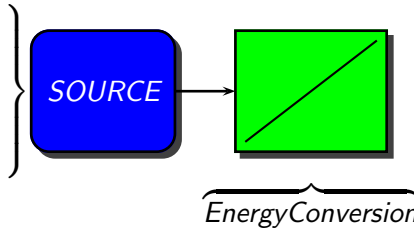
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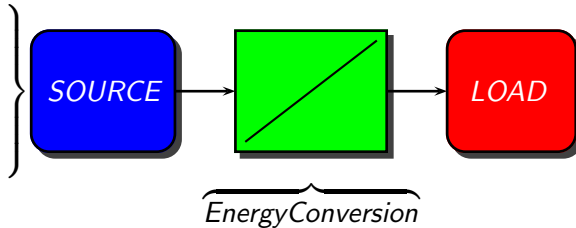
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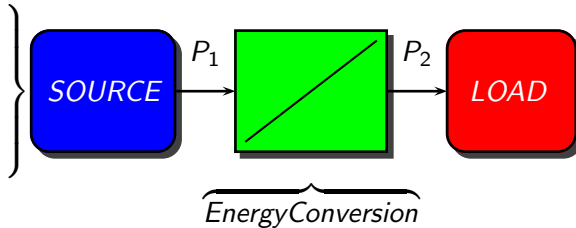
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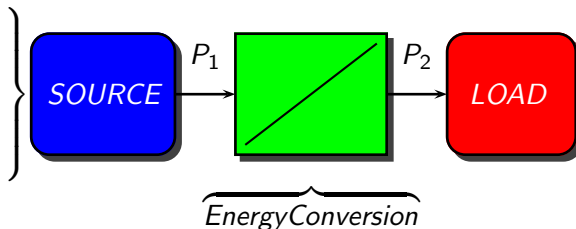
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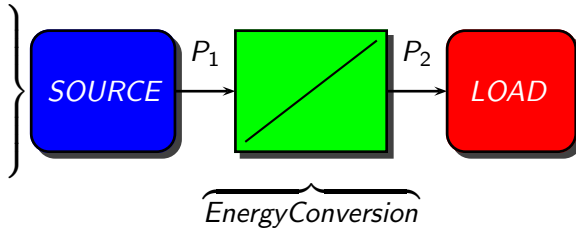


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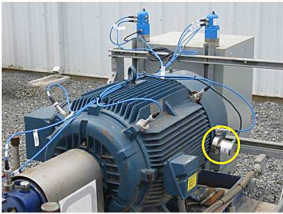
$\eta = \frac{P_2}{P_1} = 1 - \frac{P_1 - P_2}{P_1} \rightarrow$ Losses in the energy converter should be as small as possible.

Objectives: Sensors Network

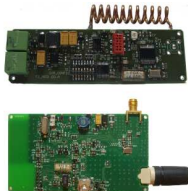


www.perpetuum.com

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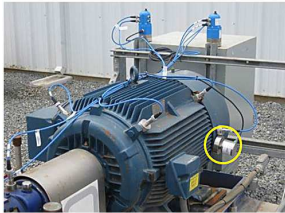


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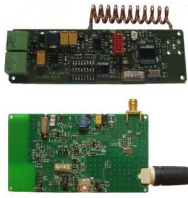


Gutiérrez, A Heterogeneous Wireless Identification Network for the Localization of Animals Based on Stochastic Movements

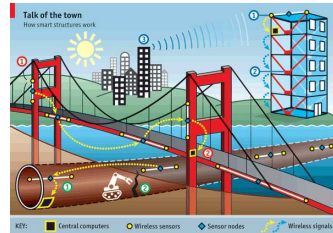
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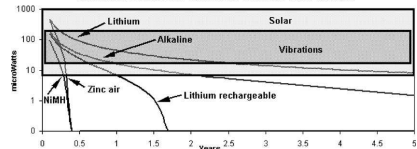


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<http://www.rfwirelessensors.com>, 2012

Continuous Power / cm² vs. Life for Several Power Sources



Roundy et al.: A study of low level vibrations as a power source for wireless sensor nodes.

Objectives: Power, just where you need it



<http://enocan.com>

- Wireless
- Reduce Cost, and is reconfigurable
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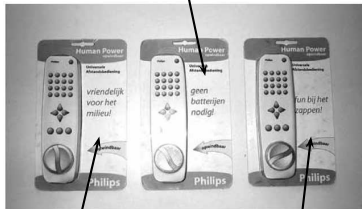


The economist – April 28th 2007

Objectives: Marketing Purpose

Experience: Same Remote controller energized by human power, but with different packaging.

"No need for Batteries"



"Green"

"Fun"

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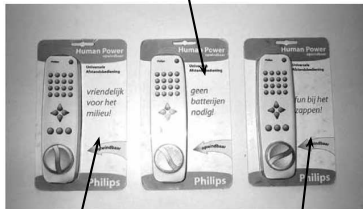
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54% of people Will choose the first one because it is eco-friendly.
(Jansen, Human power empirically explored)

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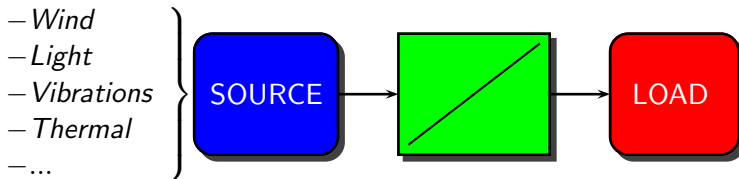
Several projects are born from this fact: **Metis** Produces energy from dancers' movements.



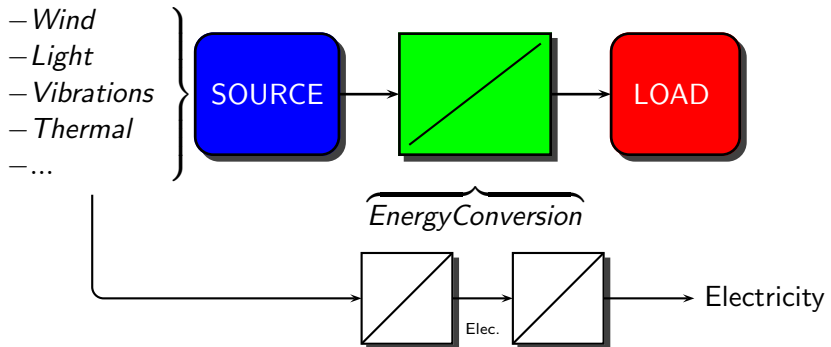
In Toulouse, the system **VIHA** proposes Smart Tiles to energise street lights.



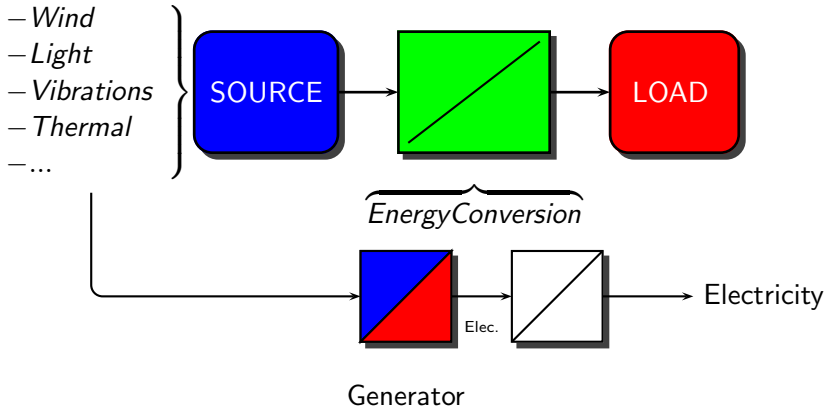
The energy converter



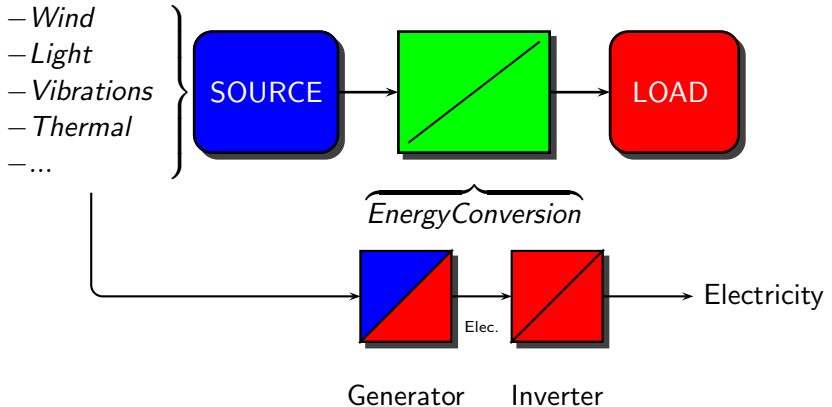
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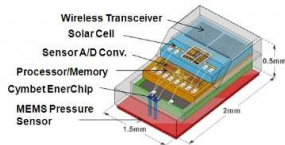


The energy converter



Solar Harvesters:

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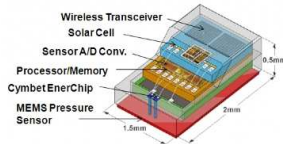


This sensor measures IntraOcular Pressure <http://cymbet.com>



Handbag to recharge electronic devices <http://www.neubers.de>

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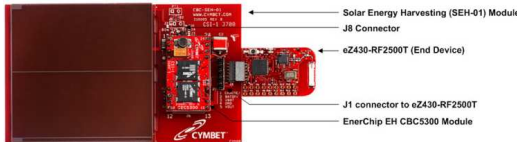
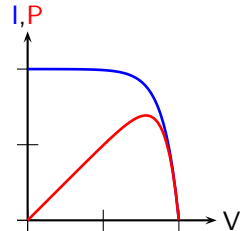


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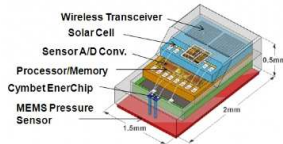
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Power and current as a function of voltage:



Solar Energy Harvester Evaluation Kit <http://ti.com>

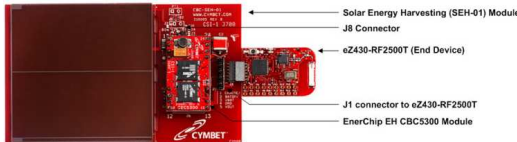
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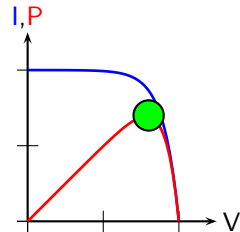


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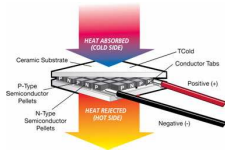
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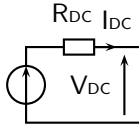


MPPT strategies, require Energy management of the system

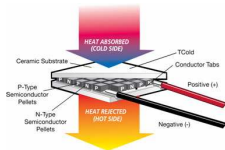
Thermoelectric



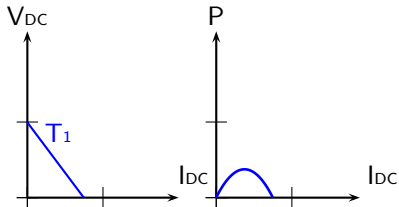
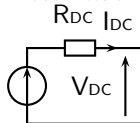
A Peltier module from <http://www.tellurex.com>



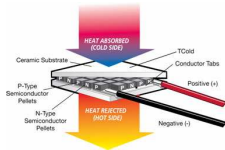
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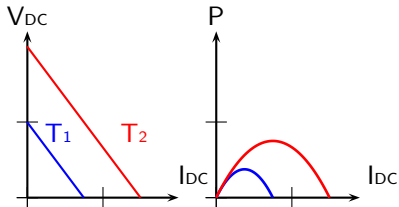
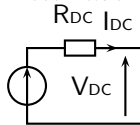
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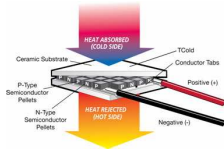
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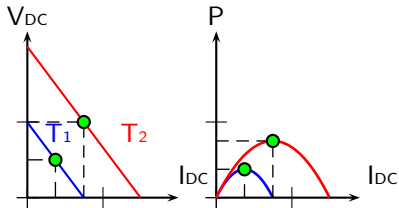
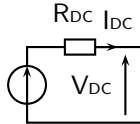
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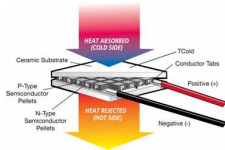
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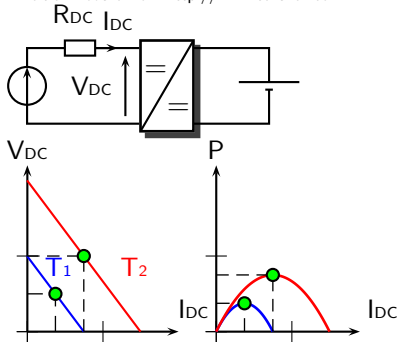
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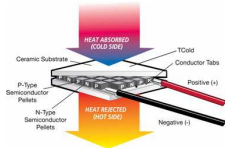
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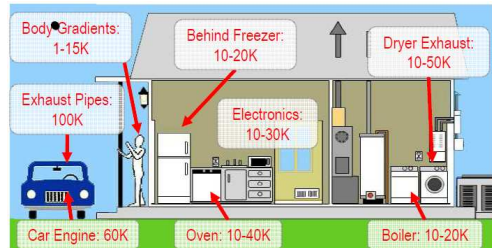
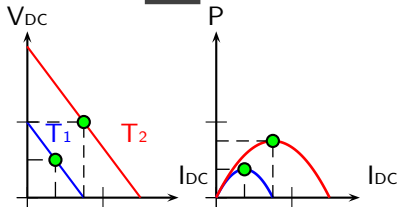
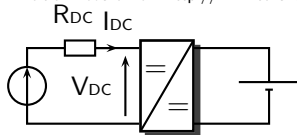
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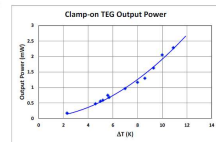


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Temperature Gradient (residential). Lindsay Miller,
<http://uc-ciee.org>

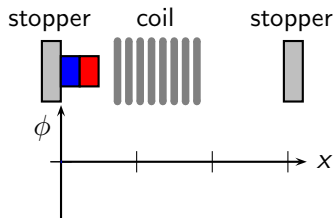
Clamp-on Drain Harvester



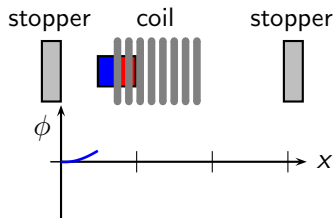
ΔT	V_{load} (mV)	P_{max} (mW)
5.0	38	0.56
8.0	55	1.16
10.0	73	2.05

plumbing application from <http://www.nextreme.com>

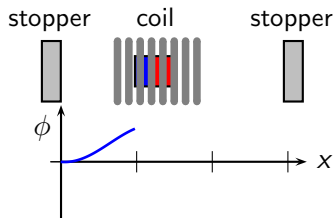
Magnetic



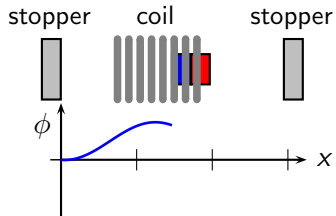
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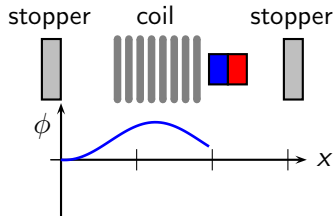
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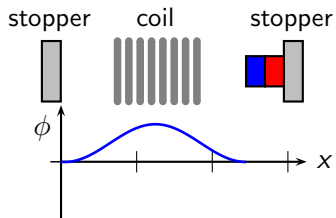
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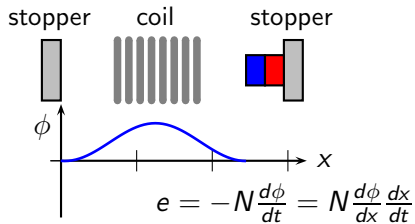
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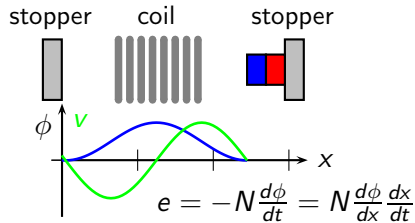
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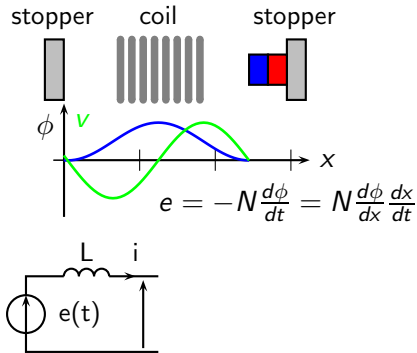
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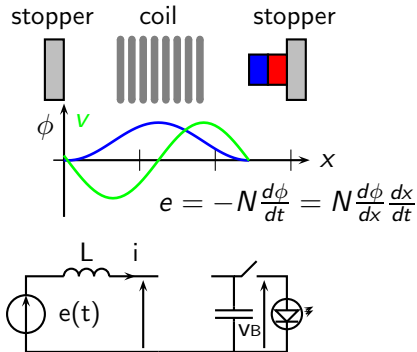
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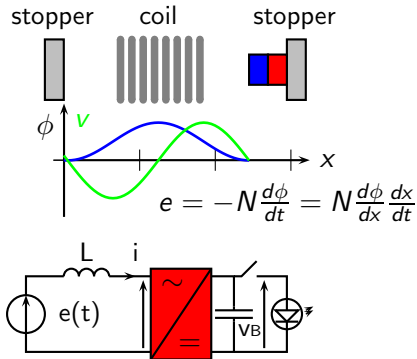
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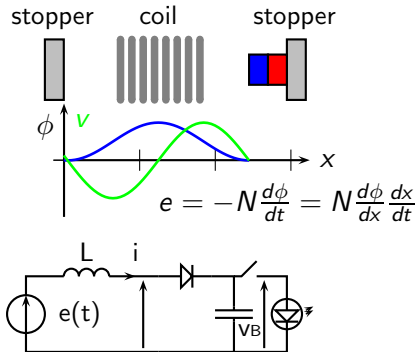
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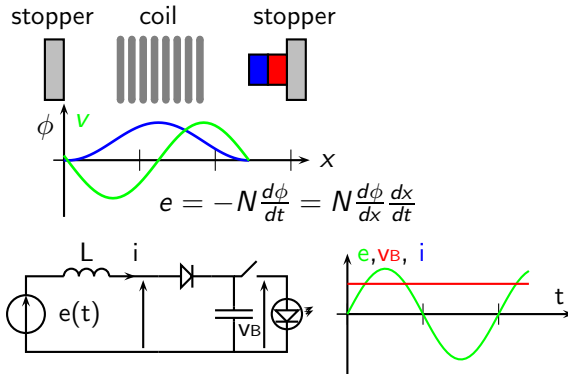
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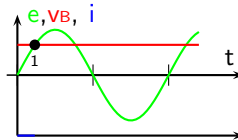
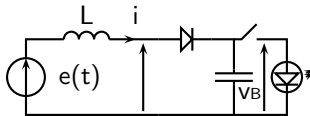
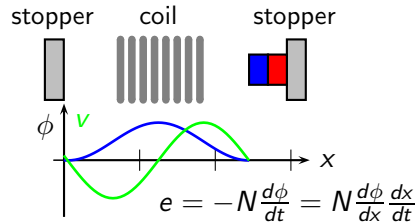
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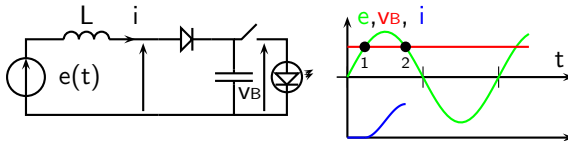
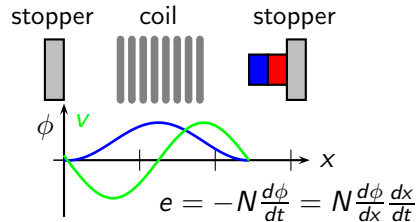


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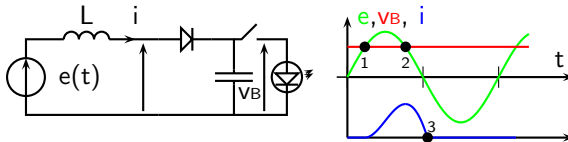
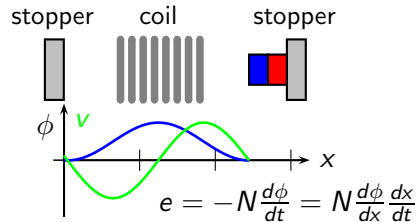
1) Diode turns ON

Magnetic



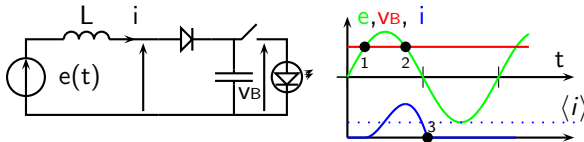
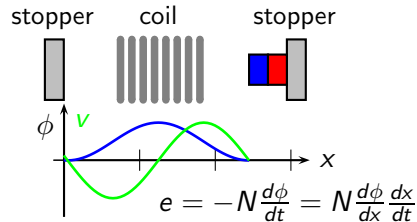
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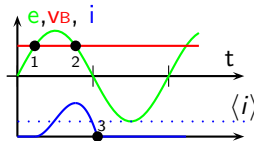
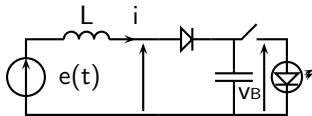
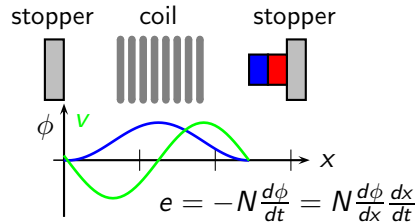
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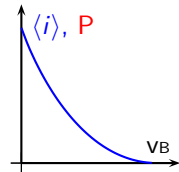


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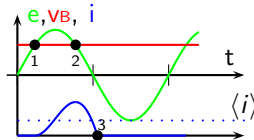
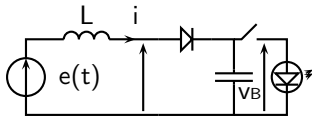
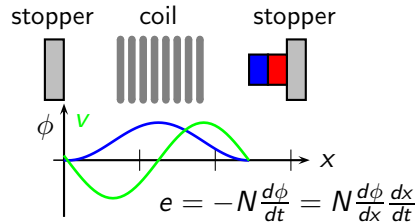
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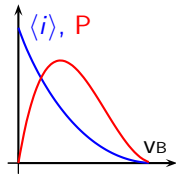
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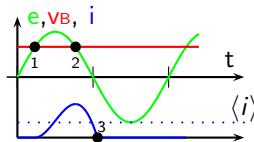
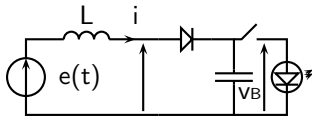
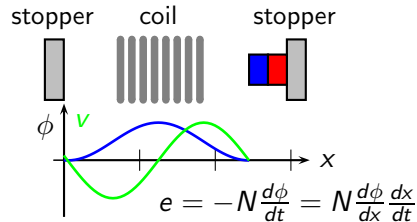


- 1) Diode turns ON
- 2) $\frac{di}{dt} = 0$ because $e - v_B = 0$
- 3) $i = 0$, diode turns OFF

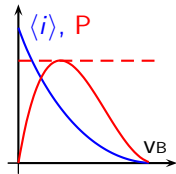


$$P = \langle v_B i \rangle = v_B \langle i \rangle$$

Magnetic

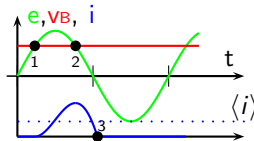
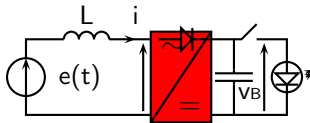
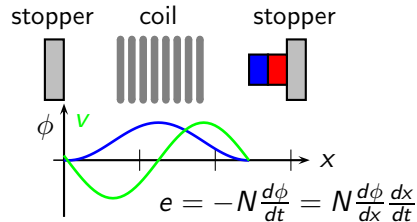


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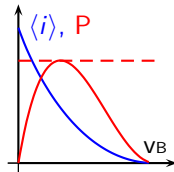


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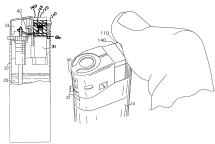


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Piezoelectric

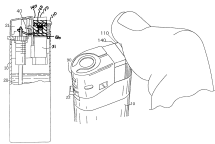


An electronic lighter,
<http://freepatentsonline.com>



Piezoelectric crystals

Piezoelectric

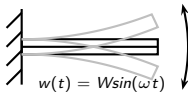


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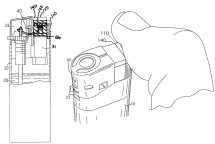


Piezoelectric crystals

Cantilever beam



Piezoelectric

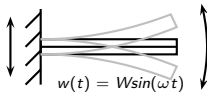


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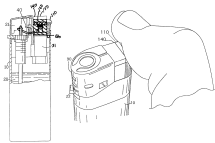


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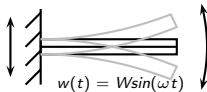


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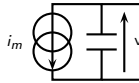


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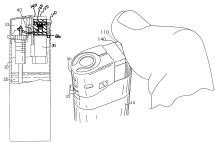


Equivalent electrical circuit



i_m is a current proportional
to the deformation speed

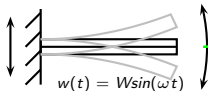
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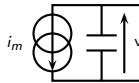
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Piezoelectric crystals
Cantilever beam

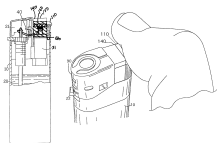


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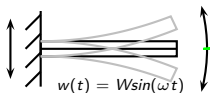
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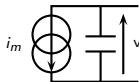
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Piezoelectric crystals
Cantilever beam



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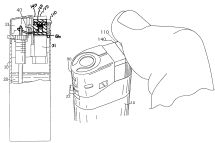


i_m is a current proportional
to the deformation speed

Comparison Magn. Piezo

Magnetic	Piezo.
Voltage source	Current source
Inductive	capacitive
Large Stroke	Small Stroke
Remote action	

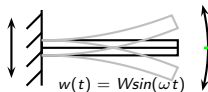
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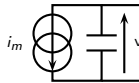
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Piezoelectric crystals
Cantilever beam



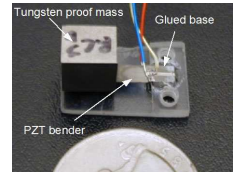
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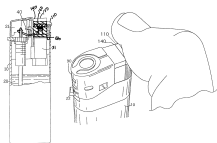
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Roundy, A piezoelectric vibration
based generator for wireless
electronics (2004)

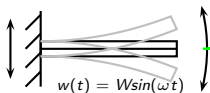
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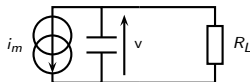
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Piezoelectric crystals
Cantilever beam



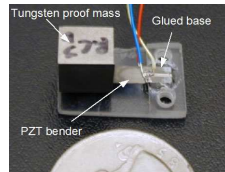
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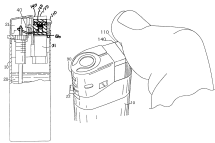
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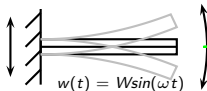
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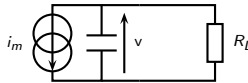
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Piezoelectric crystals
Cantilever beam

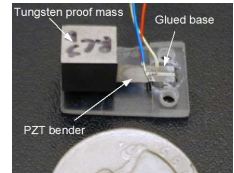


Equivalent electrical circuit

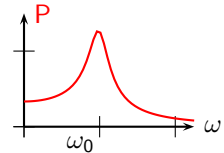


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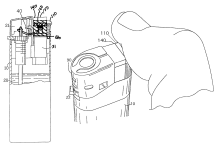
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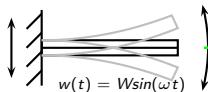
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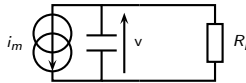
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Piezoelectric crystals
Cantilever beam

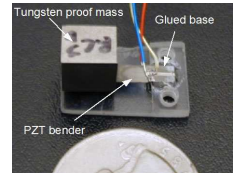


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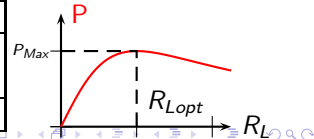
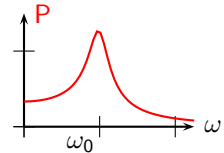


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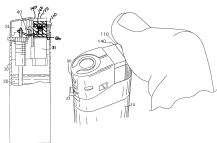
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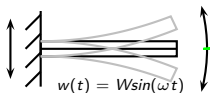
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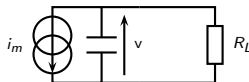
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Piezoelectric crystals
Cantilever beam



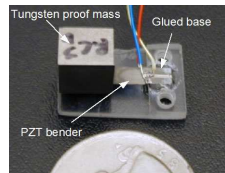
Energy Management Equivalent electrical circuit



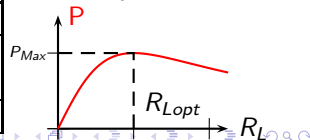
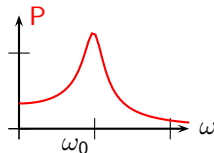
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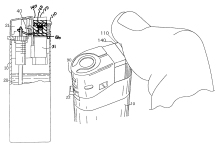
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Roundy, A piezoelectric vibration based generator for wireless electronics (2004)



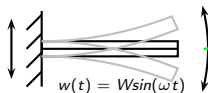
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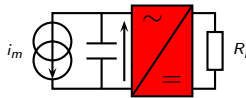
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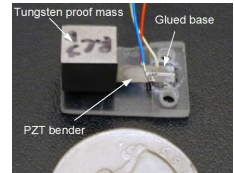


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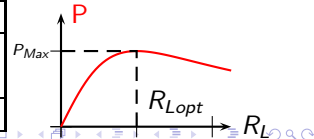
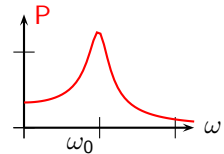


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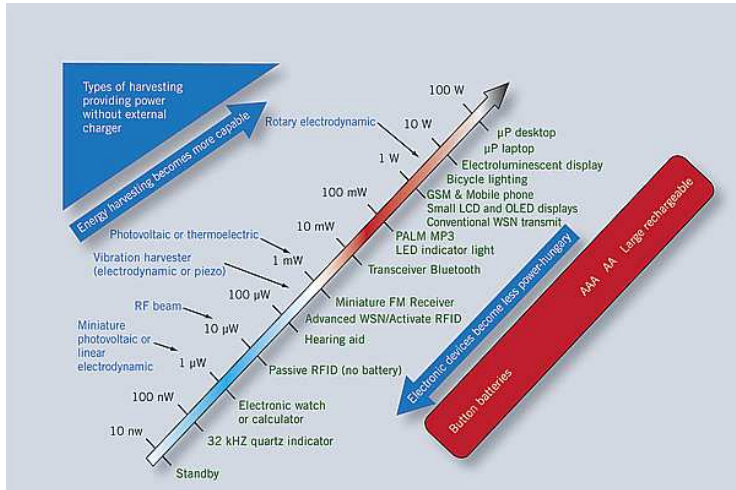
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There is no "one fit all" solution



Each solution may be efficient in a certain range of Power.

Meanwhile, shrinking Chips consumption come at a time when energy harvesting becomes efficient and practical
(source: IDtechex.com)

Table of contents

1 Introduction

- What is Energy Harvesting ?
- Generator Technologies
- Summary

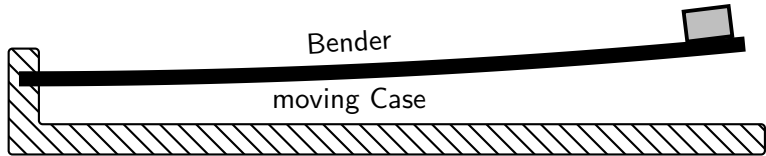
2 Modelling of a piezoelectric energy harvester

- Presentation of the system
- EMR of the system
- Power Extraction on a load resistor R_L from harmonic oscillation

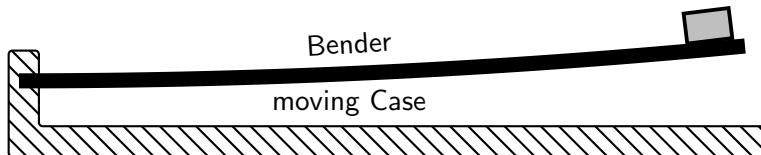
3 An Example of inverter

- Introduction
- SSHI: Synchronized Switch Harvesting on Inductor

Coordinates and assumptions

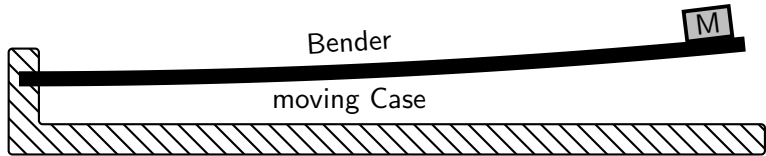


Coordinates and assumptions



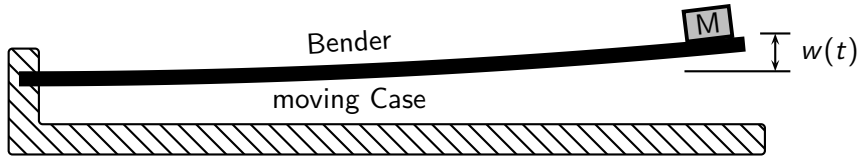
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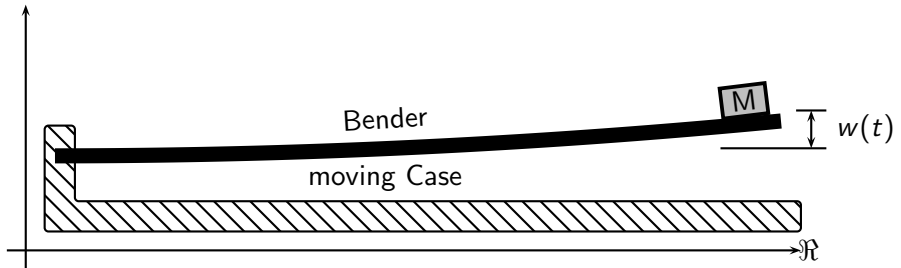
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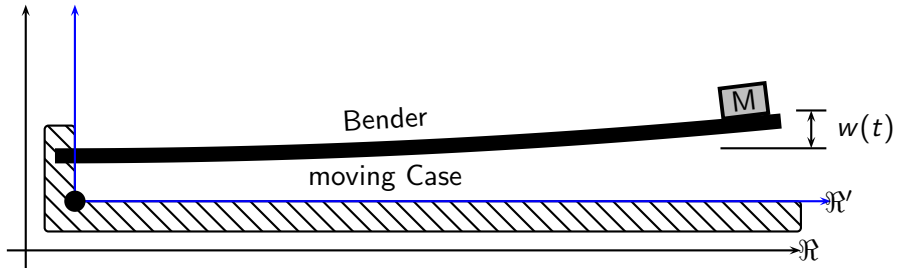
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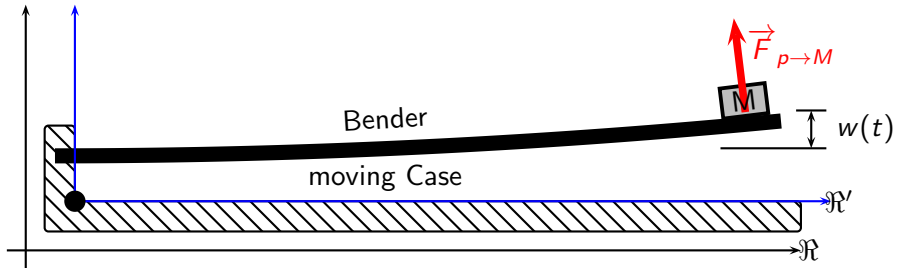
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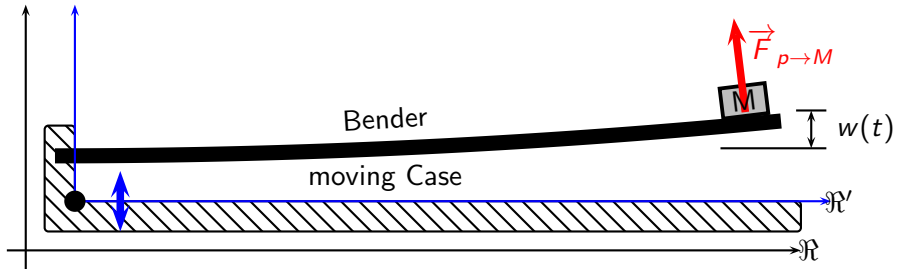
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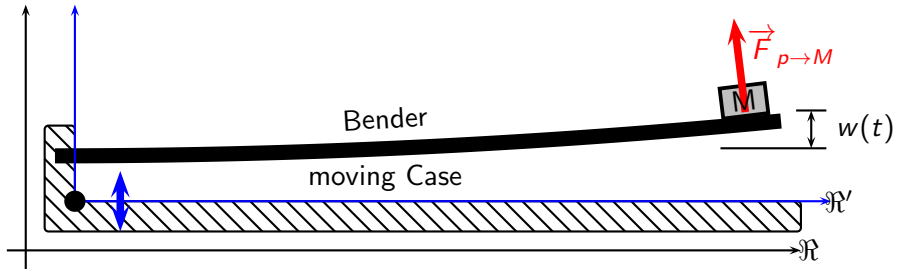
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Coordinates and assumptions



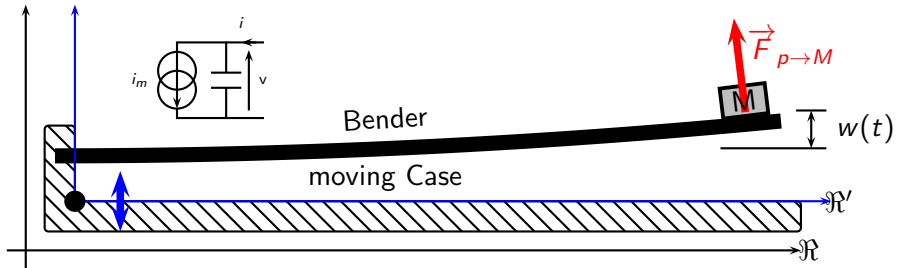
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Equations

- **Dynamic of the mass M :**

Glossary

- M the mass
- f , the force onto M
- A , vibration's amplitude
- ω , vibration's pulsation
- f_{acc} is the inertial force
- i current of the device (actuator convention)
- i_m motional current
- f_p inside piezo force
- N Piezoelectric force factor (depends on geometry)
- f_p Piezo internal force
- f_s Material internal elastic force
- K_s equivalent stiffness (depends on geometry)
- D_s Viscous coefficient

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- $M \frac{d^2}{dt^2} (w(t) + A \sin(\omega t)) = F_{p \rightarrow M} = f$

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• Electrical Behaviour: Capacitive

Glossary

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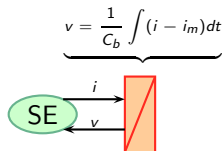
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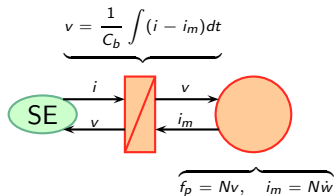
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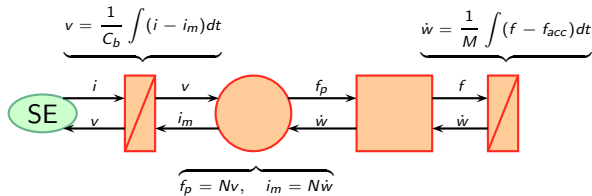
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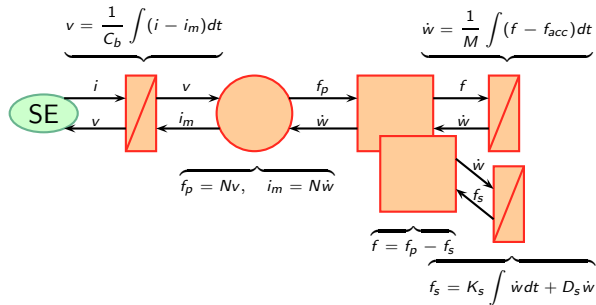
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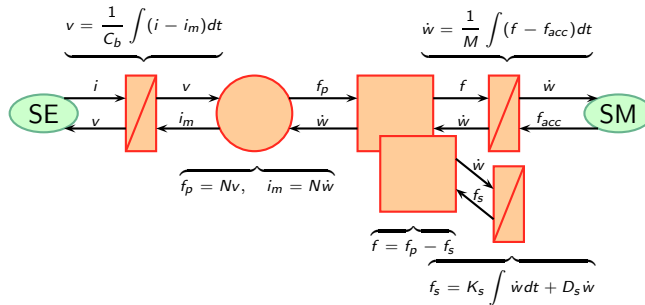
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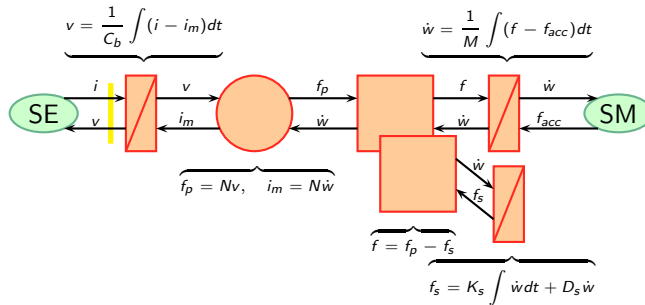
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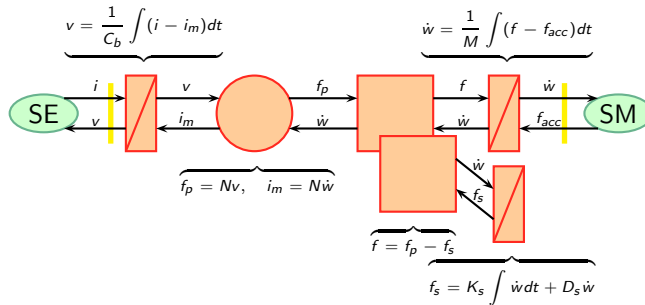


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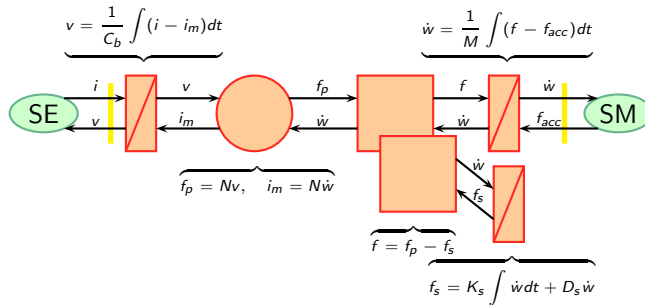
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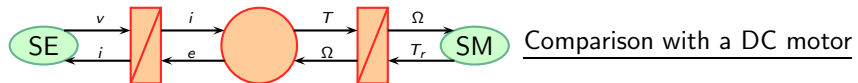


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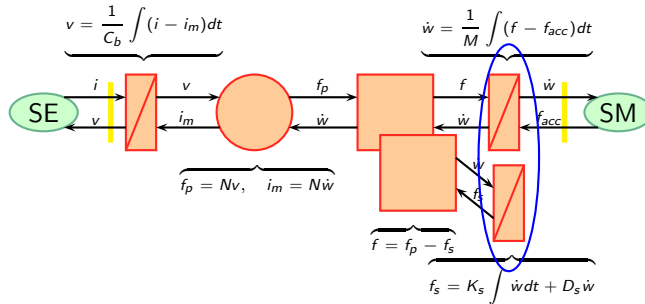


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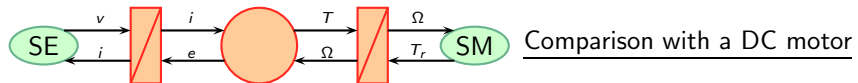


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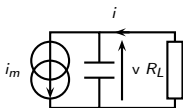
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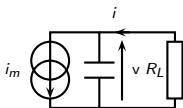
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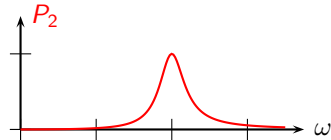
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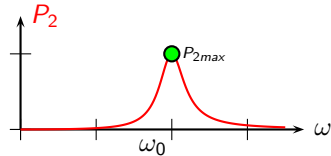
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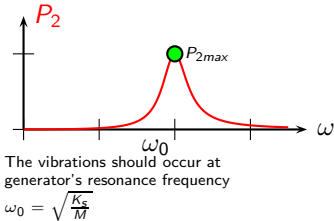
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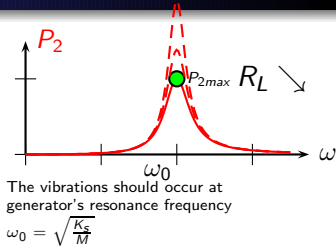
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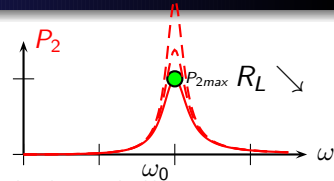
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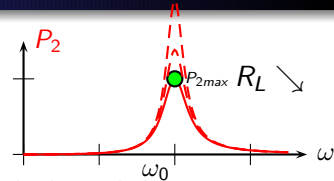


The vibrations should occur at generator's resonance frequency

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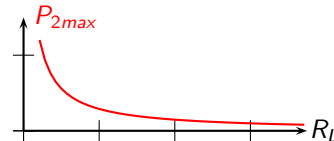
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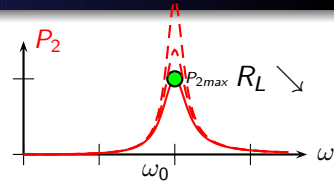
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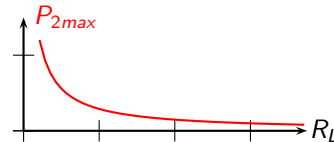
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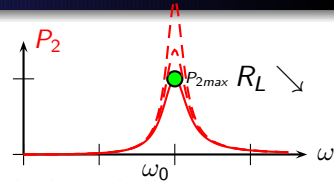
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We can harvest as much power as we want

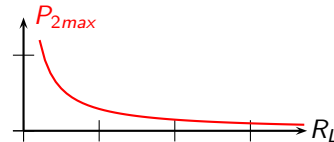
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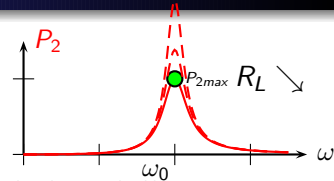
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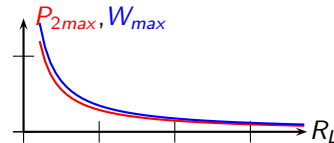
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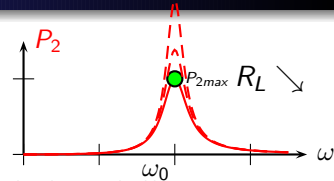
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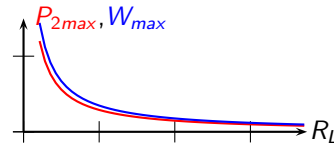
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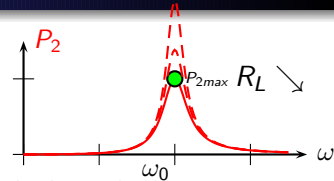
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We can harvest as much power as we want, at the expense of large displacement of the bender's tip

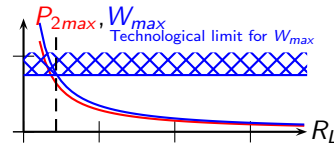
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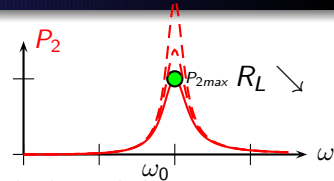
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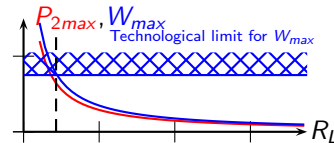
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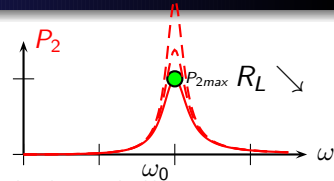
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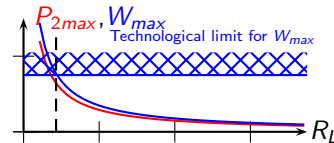
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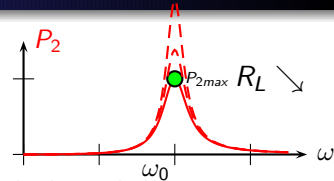
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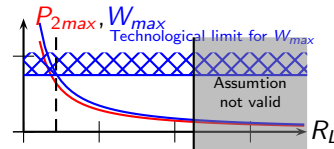
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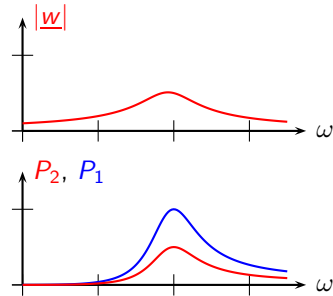
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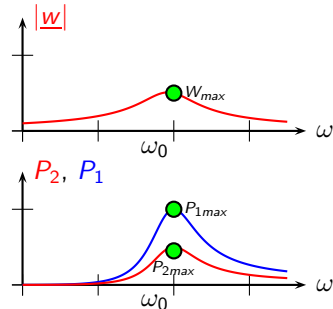
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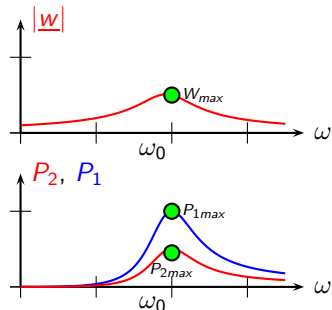
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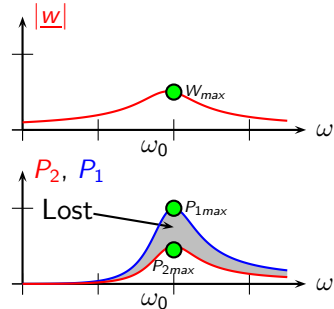
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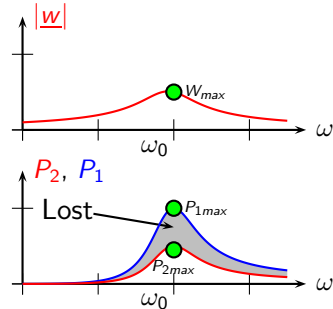
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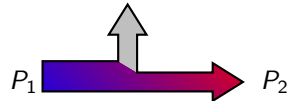
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Generator's Power Losses

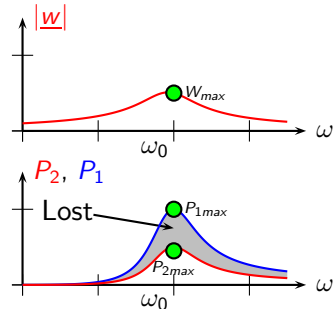


For a real generator ($D_s \neq 0$)

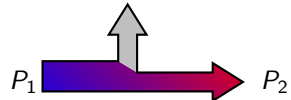
- $\underline{f} = N\underline{v} - K_s\underline{w} - D_s\dot{\underline{w}}$
- $\underline{w} = \frac{\underline{f}_{acc}}{(K_s - M\omega^2) + j\omega(D_s + N^2R_L)}$
- $P_2 = -\frac{1}{2}R_L|i_m|^2$
- $P_2 = -\frac{1}{2} \frac{R_L N^2 \omega^2 |\underline{f}_{acc}|^2}{(K_s - M\omega^2)^2 + \omega^2 (D_s + N^2R_L)^2}$
- $P_1 = -\frac{1}{2} \Re(\underline{f}_{acc} \cdot \dot{\underline{w}}^*)$
- $P_1 = -\frac{1}{2} \frac{(D_s + R_L N^2) \omega^2 |\underline{f}_{acc}|^2}{(K_s - M\omega^2)^2 + \omega^2 (D_s + N^2R_L)^2}$

ω_0 should still be the vibration's pulsation.

- $W_{max} = \frac{|\underline{f}_{acc}|}{(D_s + N^2R_L)\omega_0}, P_{1max} = \frac{1}{2} \frac{|\underline{f}_{acc}|^2}{D_s + N^2R_L},$
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Generator's Power Losses

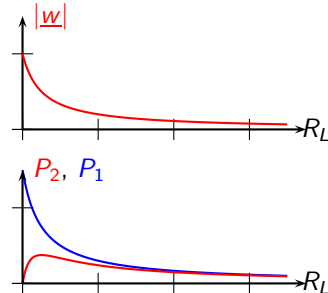


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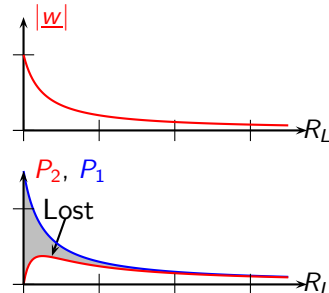
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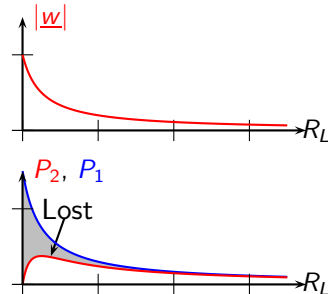


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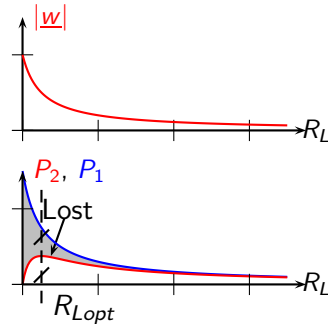


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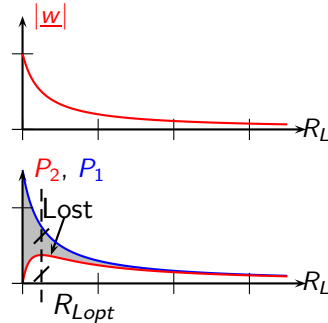


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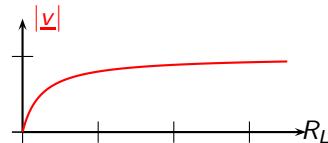
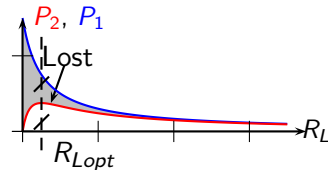
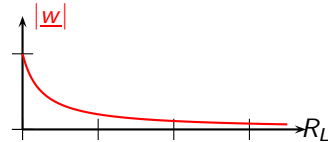


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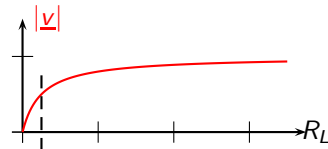
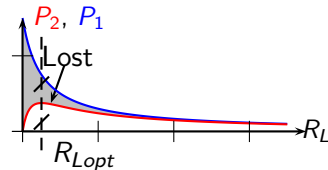
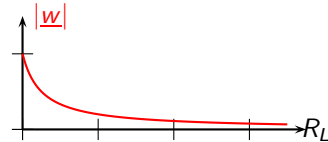


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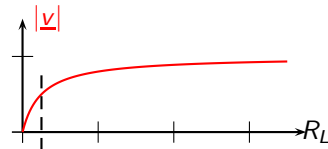
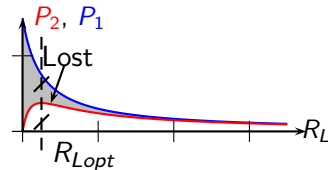
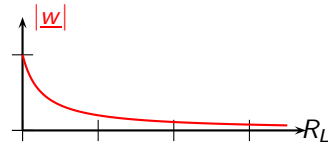


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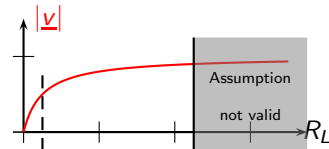
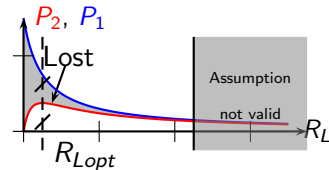
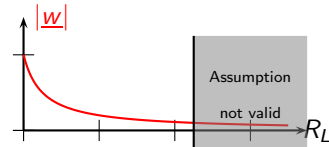


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Example

Device's properties

$N = 0.012 \text{ N/V}$, $K_s = 6300 \text{ N/m}$, $C_b = 300 \text{ nF}$, $D_s = 0.17 \text{ Ns/m}$,
 $M = 1 \text{ g}$

Calculate for $A = 0.1 \text{ mm}$

- the best working frequency,
- the harvested P_2 power in the best case,
- the power of the source P_1 in such best case,
- the optimal resistor R_L ,
- the deflection amplitude of the bender,
- the voltage for this working point.

Validation

Is $v = -R_L i_m$ a valid assumption?

Answers

- The best working frequency is given by

$$f_0 = \frac{1}{2\pi} \sqrt{\frac{K_s}{M}} = \frac{1}{2\pi} \sqrt{\frac{6300}{1.10^{-3}}} = \underline{400Hz}$$

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$$P_2 = \frac{|f_{acc}|^2}{8D_s} = \frac{(1.10^{-3} \cdot 1.10^{-4} \cdot (2\pi \cdot 400)^2)^2}{8.0,17} = \underline{464mW}$$

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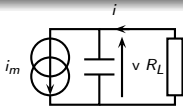
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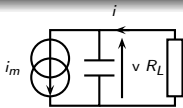
$$v_{max} = NR_L\omega_0|w| = 0,012.1180.2\pi.400.738.10^{-6} = \underline{26V}$$

- $Z_{Cb} = \frac{1}{C_b\omega} = \frac{1}{2\pi.400.300.10^{-9}} = \underline{1990\Omega} \approx R_L, \rightarrow$ **Calculations are NOT valid!**

For a real Generator, $D_s \neq 0$ and $v \neq -R_L i_m$



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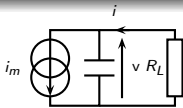


We can show:

$$\underline{v} = -\frac{R_L}{1+j\omega R_L C_b} \underline{i}_m = -j\omega N R_L \frac{1-j\omega R_L C_b}{1+(R_L C_b \omega)^2} \underline{w}_m$$

and we write: $\underline{v} = -j\omega N r_{Leq} \underline{w} - N k_{eq} \underline{w}$

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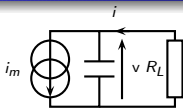
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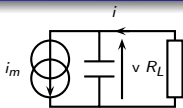
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$$\omega_0 \ll \frac{1}{R_L C_b}$$

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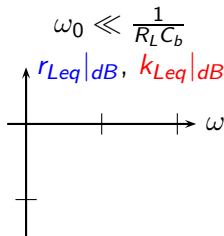


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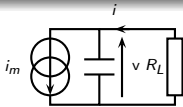
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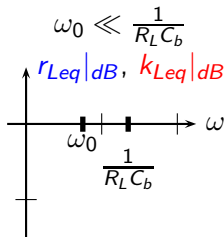


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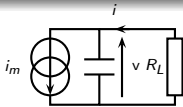
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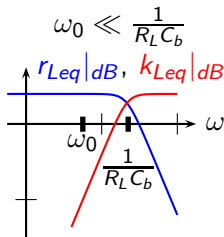


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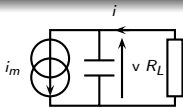
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For a real Generator, $D_s \neq 0$ and $v \neq -R_L i_m$

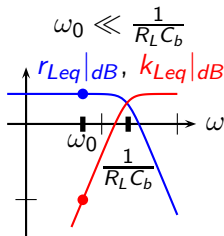


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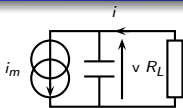
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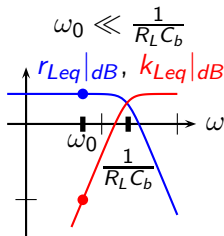


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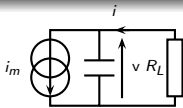
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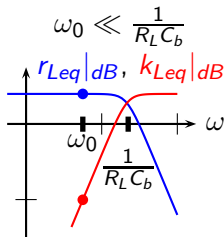


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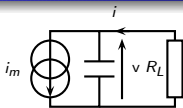
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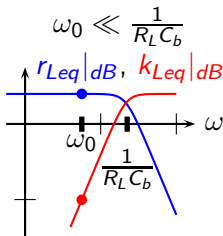


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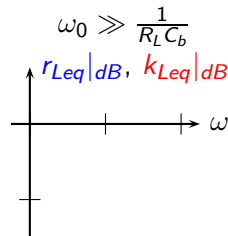
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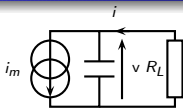
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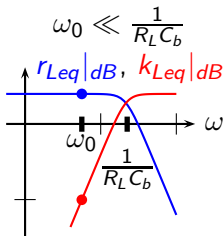


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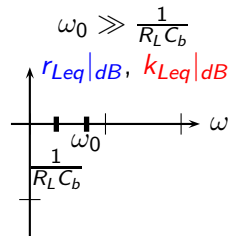
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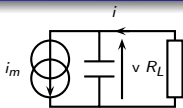
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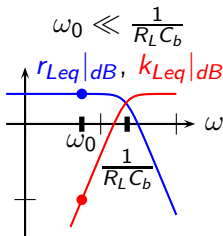


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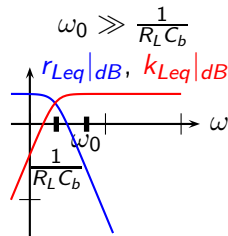
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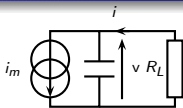
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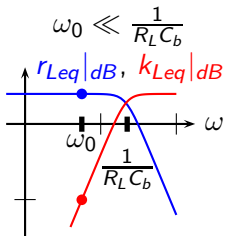


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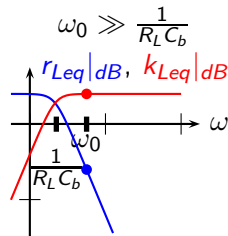
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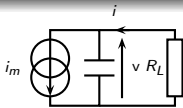
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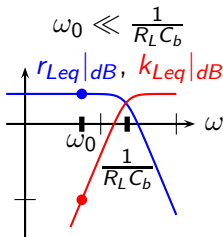


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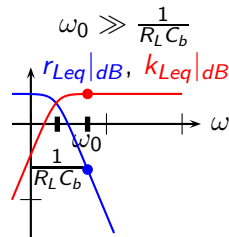
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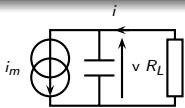


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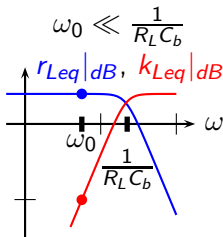


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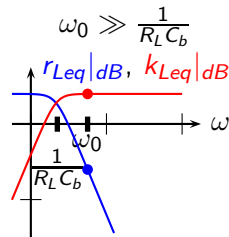
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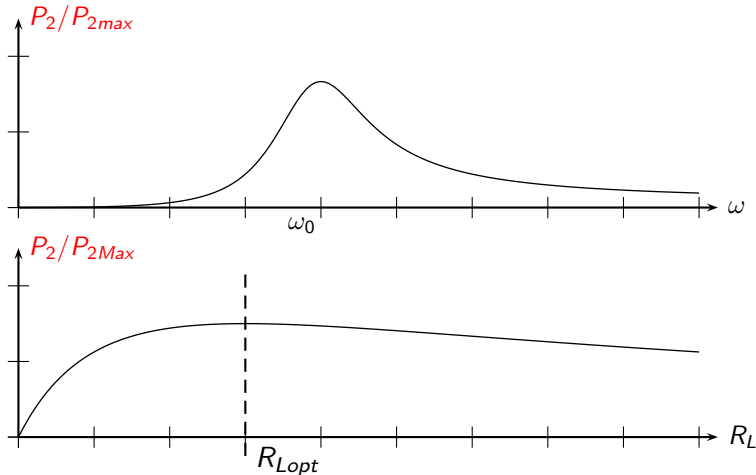


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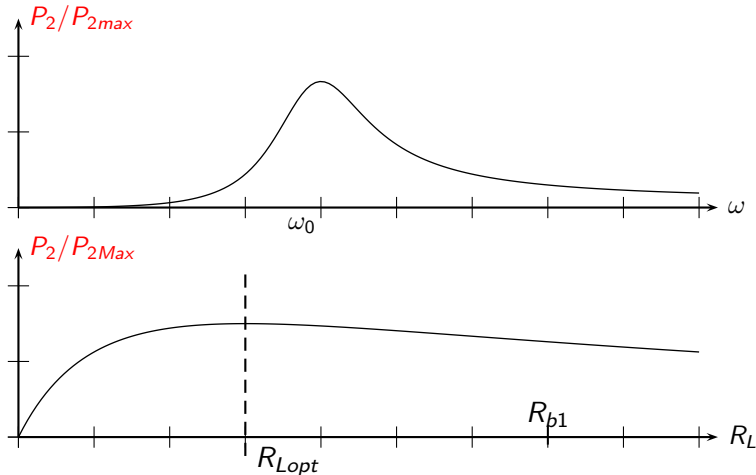
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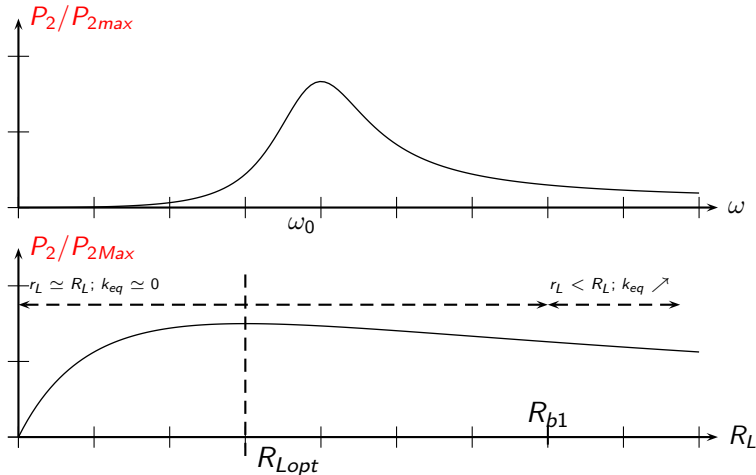
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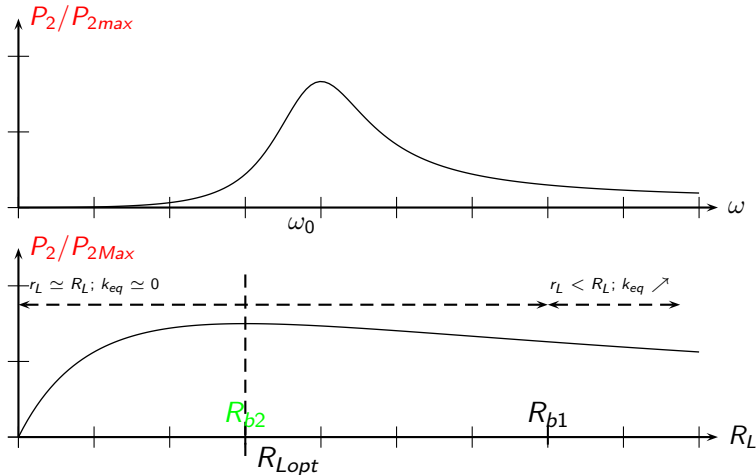
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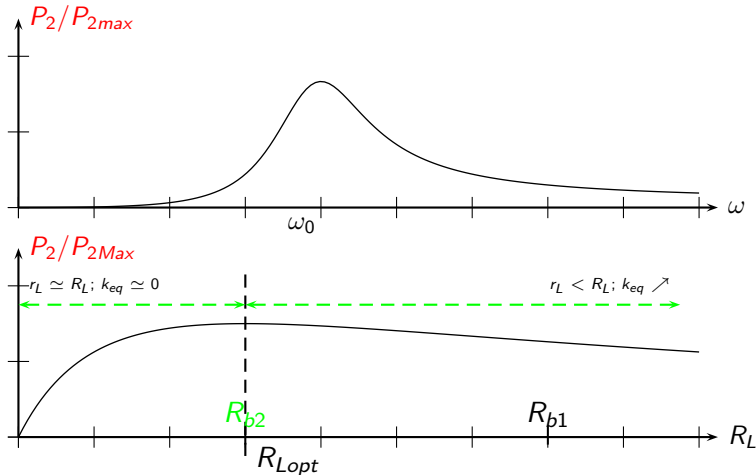
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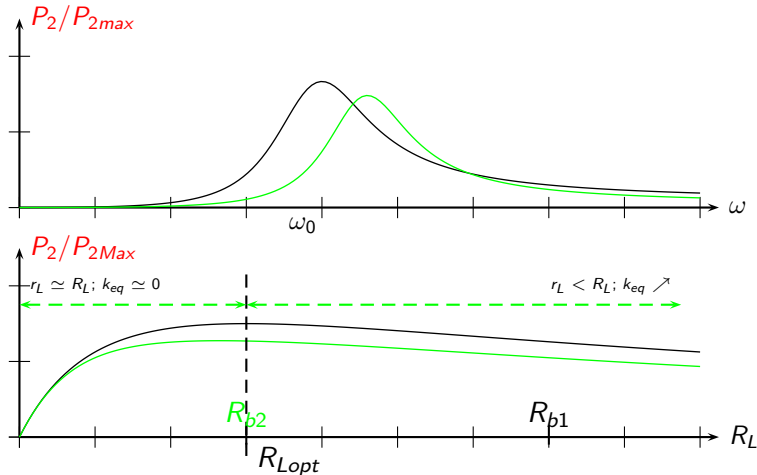
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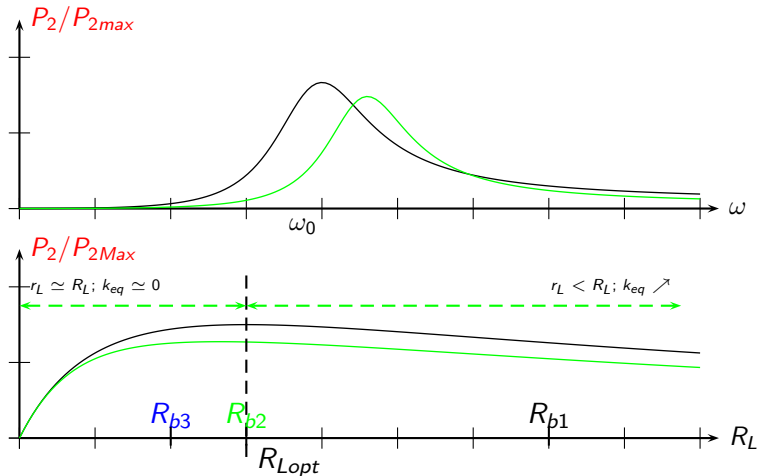
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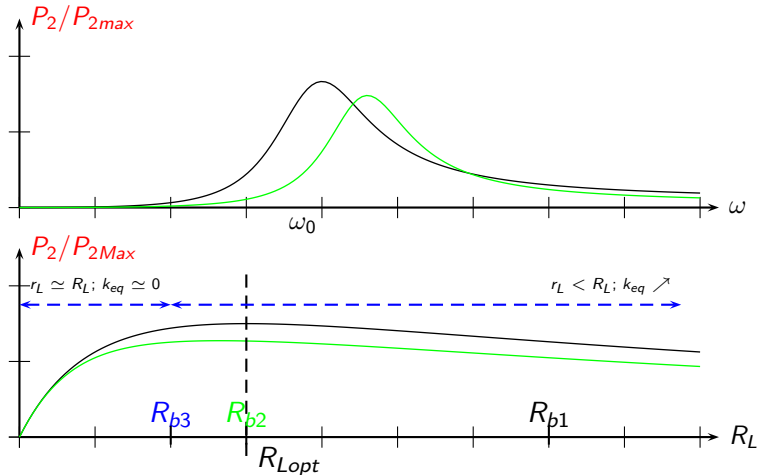
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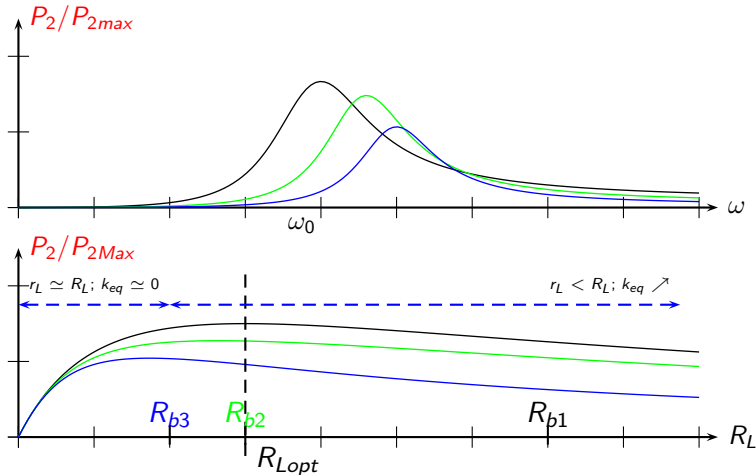
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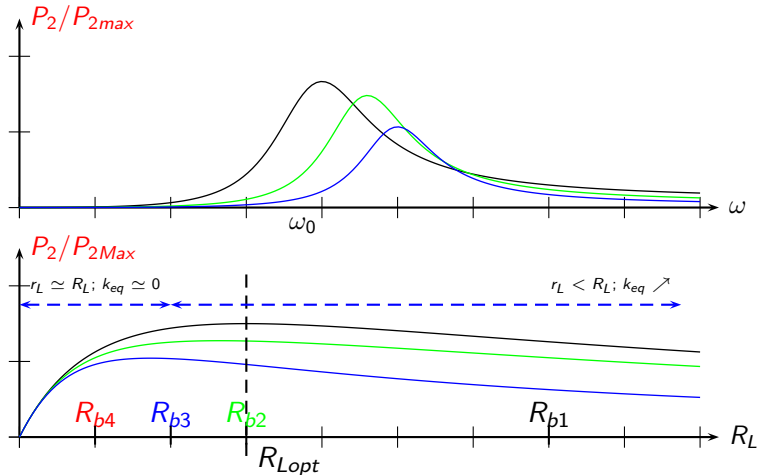
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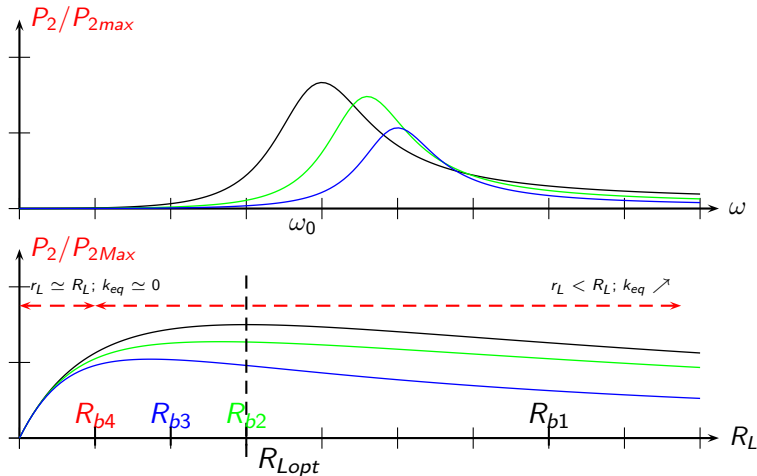
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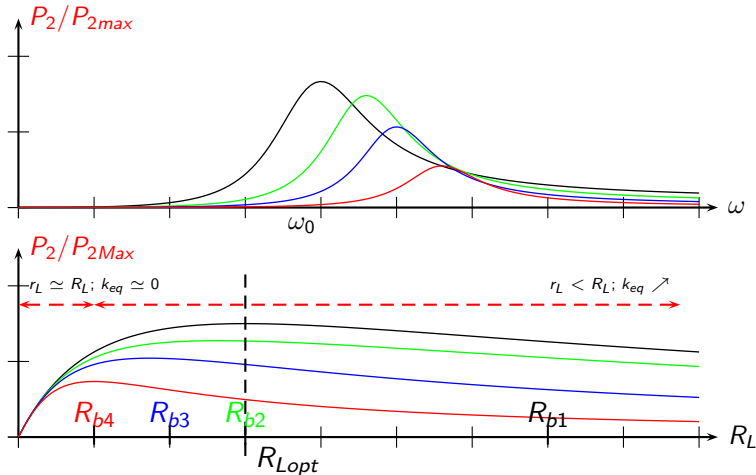
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If $R_{opt} > R_b$

The power is not well extracted because $P_2 < P_{2max}$. This happens for high damped mechanisms.

A simple resistor cannot recover the power optimally (and this is due to C_b).

What happens if $R_{opt} < R_b$ and $R_L \gg R_b$?

resonance is shifted

- $r_{Leq} = \frac{R_L}{1+(R_L C_b \omega)^2} \simeq \frac{R_L}{(R_L C_b \omega)^2}$
- $k_{eq} = R_L \frac{\omega^2 R_L C_b}{1+(R_L C_b \omega)^2} \simeq \frac{1}{C_b}$

$((K_s + N^2 k_{eq}) - M\omega^2) + j\omega(D_s + N^2 r_{Leq})\underline{w} = \underline{f}_{acc}$ leads to:

$$\omega'_0 = \sqrt{\frac{K_s + N^2 k_{eq}}{M}} = \sqrt{\frac{K_s + \frac{N^2}{C_b}}{M}}$$

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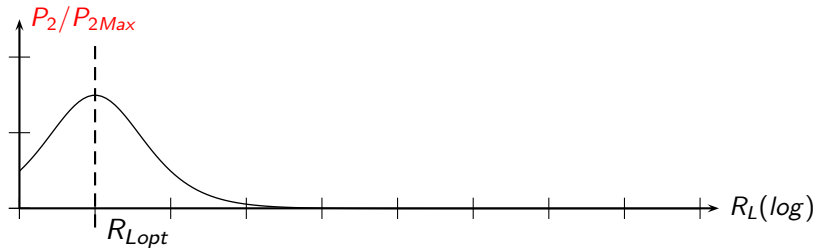
Another optimal resistor

The optimal power is harvested is $N^2 r_{Leq} = D_s$, leading to:

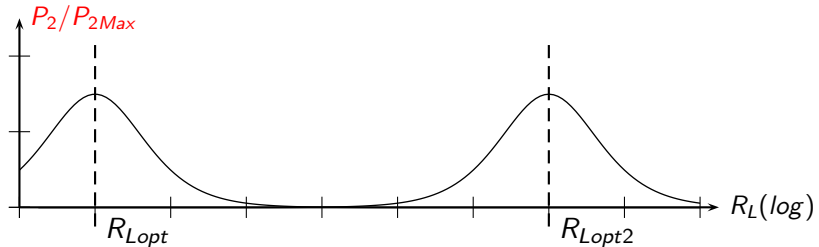
$$\frac{N^2 R_L}{(R_L C_b \omega'_0)^2} = \frac{N^2}{R_L C_b^2 \frac{K_s + \frac{N^2}{C_b}}{M}} = D_s$$

$$R_{Lopt2} = \frac{N^2}{D_s} \frac{1}{C_b^2 \frac{K_s}{M} (1 + \frac{N^2}{K_s C_b})} = \frac{R_b^2}{R_{Lopt}} \frac{1}{1 + \frac{N^2}{K_s C_b}}$$

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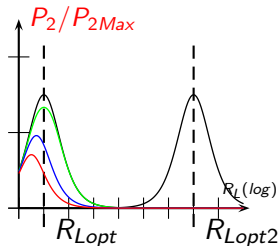


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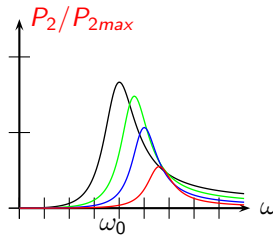
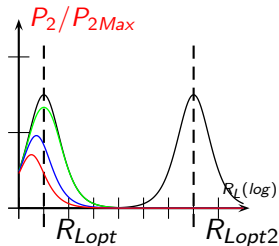
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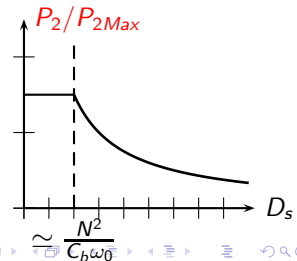
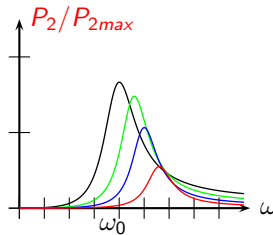
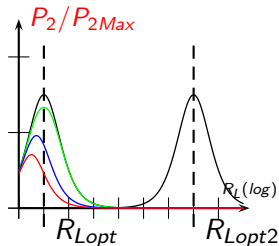
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Conclusion

- For the realistic case, there is one or two resistive loads which allow to extract the maximum of power,
 - There exist an optimal frequency which may vary if R_L is very high,
 - For highly damped structure, a simple resistor is not optimal.
- we always want to harvest the maximum of power!

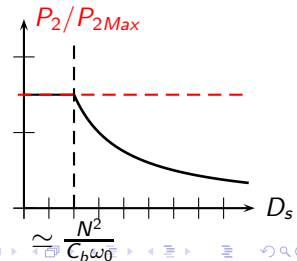
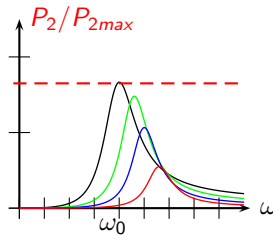
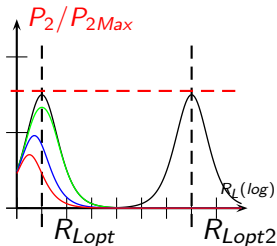


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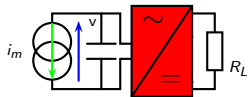
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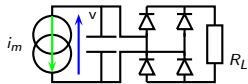
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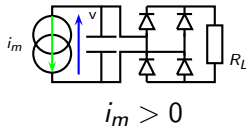
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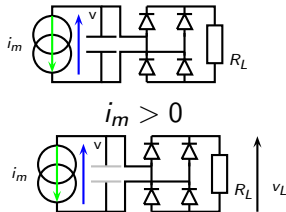
Introduction



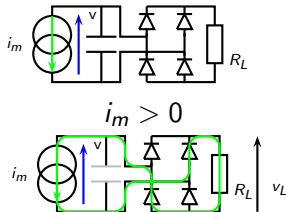
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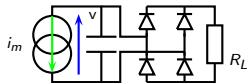
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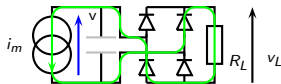
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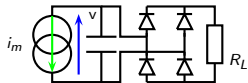
$$i_m > 0$$



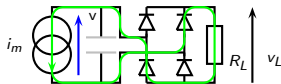
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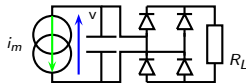


$$v_L = R_L i_m \text{ which is } > 0$$

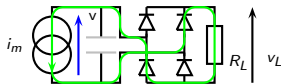
$$\text{and } v = -v_L = -R_L i_m$$

$$i_m < 0$$

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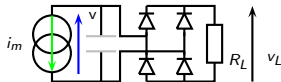
$$i_m > 0$$



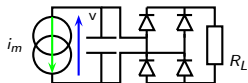
$$v_L = R_L i_m \text{ which is } > 0$$

$$\text{and } v = -v_L = -R_L i_m$$

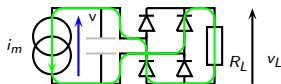
$$i_m < 0$$



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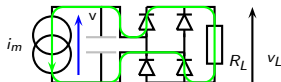
$$i_m > 0$$



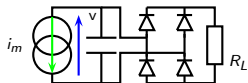
$$v_L = R_L i_m \text{ which is } > 0$$

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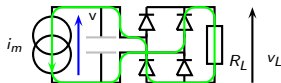
$$i_m < 0$$



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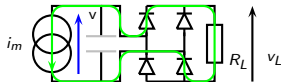
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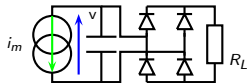


$$v_L = -R_L i_m \text{ which is } > 0$$

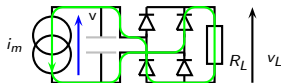
again, and

$$v = v_L = -R_L i_m$$

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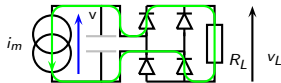
$$i_m > 0$$



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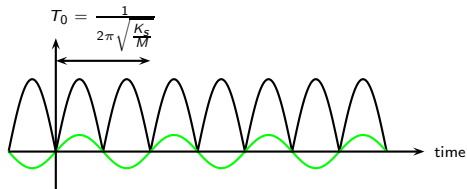
$$i_m < 0$$



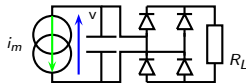
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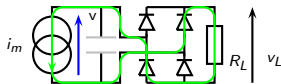
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Introduction



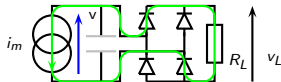
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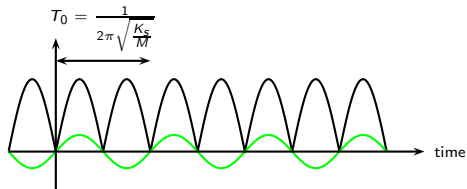
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$$v_L = -R_L i_m \text{ which is } > 0$$

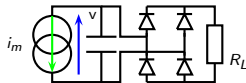
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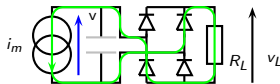


The current is rectified, but we still have $v = -R_L i_m$: Power harvesting depends on R_L

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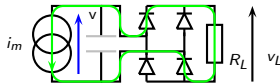
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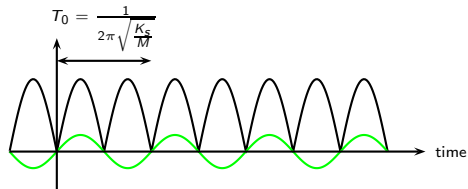
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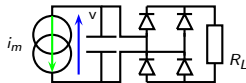
$$\text{again, and}$$

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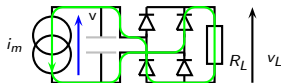


The current is rectified, but we still have $v = -R_L i_m$: Power harvesting depends on R_L . Moreover, it is still not optimal if $R_{opt} > R_b$.

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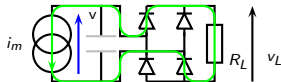
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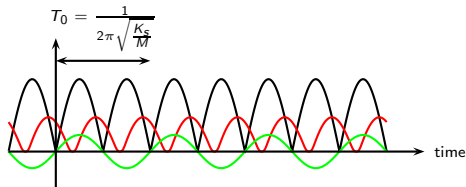
$$i_m < 0$$



$$v_L = -R_L i_m \text{ which is } > 0$$

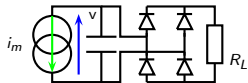
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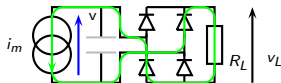


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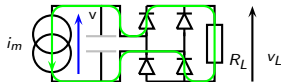
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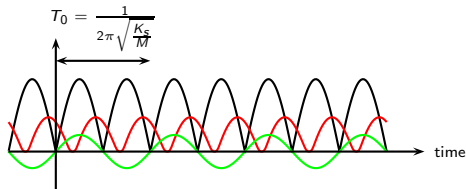
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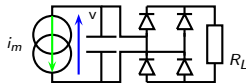
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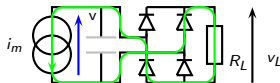


The current is rectified, but we still have $v = -R_L i_m$: Power harvesting depends on R_L .
Moreover, it is still not optimal if $R_{opt} > R_b$.
This is why, some want to compensate for C_b :

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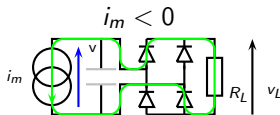


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$$v_L = R_L i_m \text{ which is } > 0$$

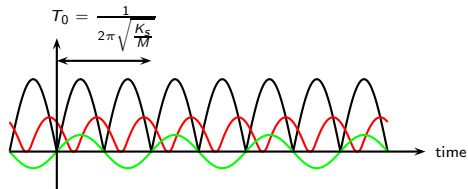
$$\text{and } v = -v_L = -R_L i_m$$



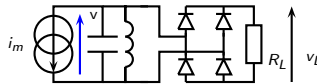
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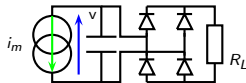
$$v = v_L = -R_L i_m$$



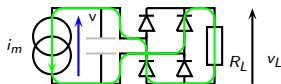
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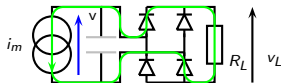
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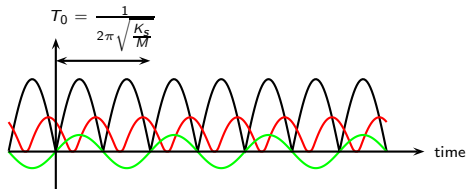
$$i_m < 0$$



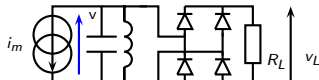
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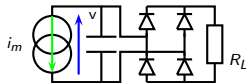


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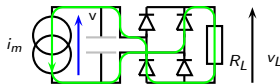


This is a bad solution because it works only for ω_0 (what if the frequency shifts?), and the Inductor is large (because ω_0 usually is small)

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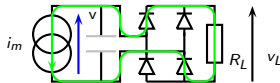
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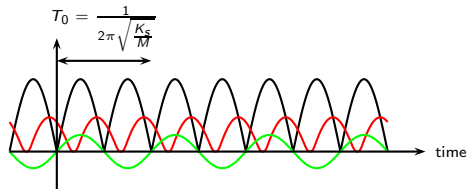
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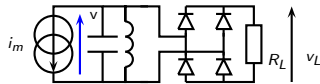
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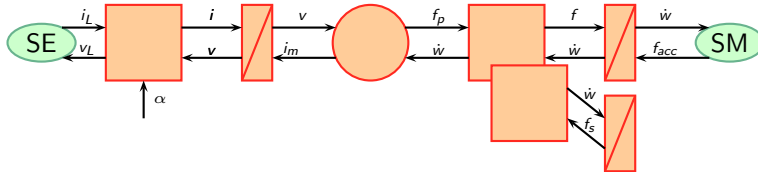
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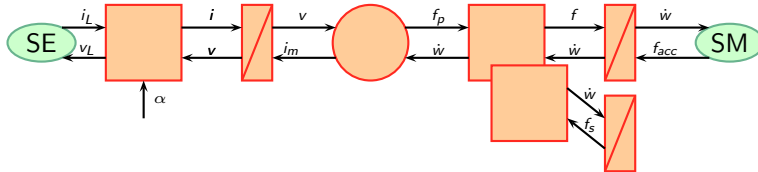
This is a bad solution because it works only for ω_0 (what if the frequency shifts?), and the Inductor is large (because ω_0 usually is small) \rightarrow Non linear Techniques

Why Synchronized

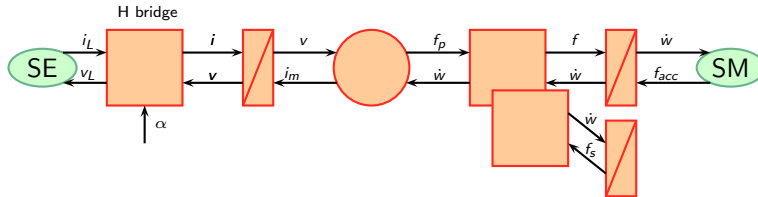
Why Synchronized



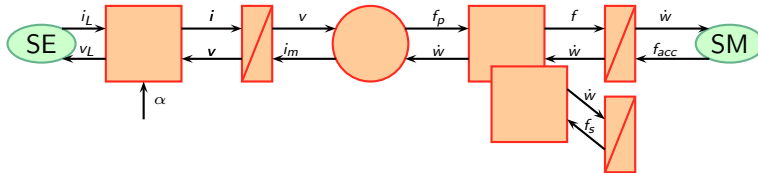
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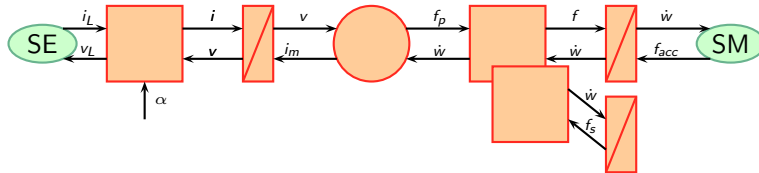


Why Synchronized



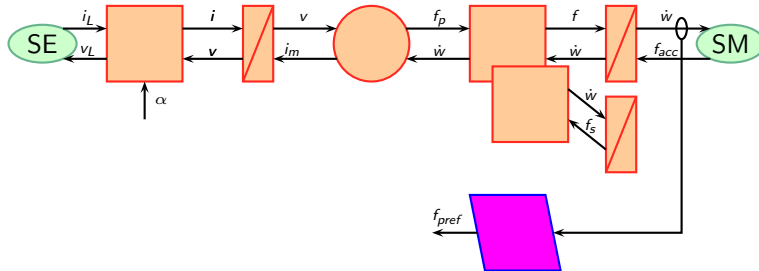
Strategy: $v = -R_{opt}i_m$ leads to $\frac{f_p}{N} = -\frac{D_s}{N^2}N\dot{w}$ or, $f_p = -D_s\dot{w}$.

Why Synchronized



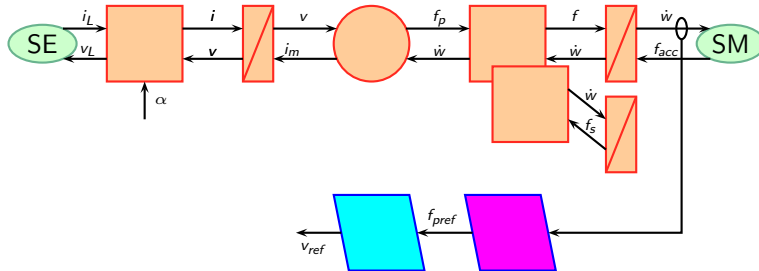
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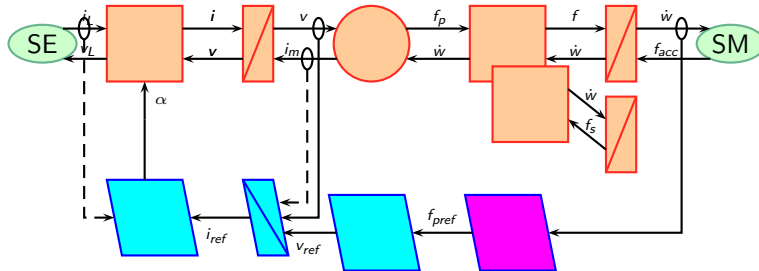
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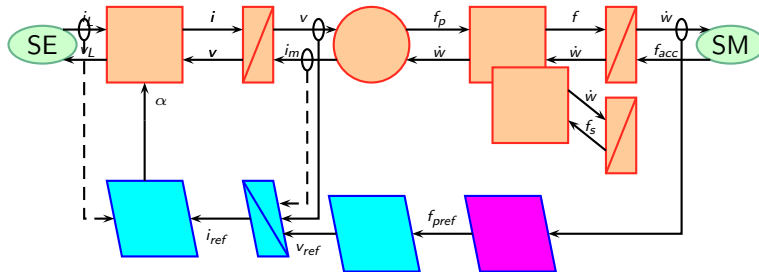
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Why Synchronized



Strategy: $v = -R_{opt}i_m$ leads to $\frac{f_p}{N} = -\frac{D_s}{N^2}N\dot{w}$ or, $f_p = -D_s\dot{w}$.
This shows that v should be controlled.

Why Synchronized



Strategy: $v = -R_{opt} i_m$ leads to $\frac{f_p}{N} = -\frac{D_s}{N^2} N \dot{w}$ or, $f_p = -D_s \dot{w}$. This shows that v should be controlled. SSHI does this with efficiency.

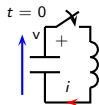
How it works

LC oscillations



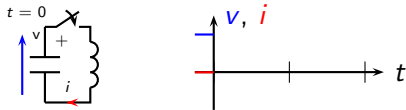
How it works

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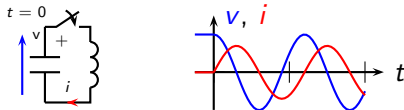
How it works

LC oscillations



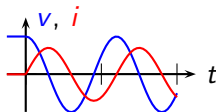
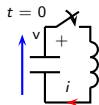
How it works

LC oscillations



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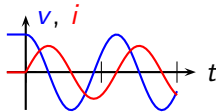
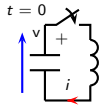
LC oscillations



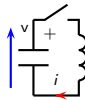
Switched inductor

How it works

LC oscillations

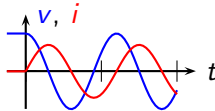
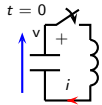


Switched inductor

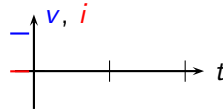
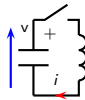


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LC oscillations

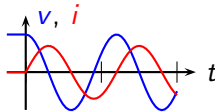
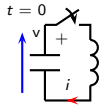


Switched inductor

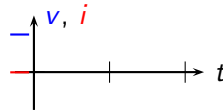


How it works

LC oscillations

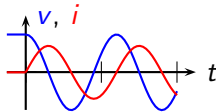
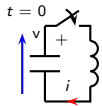


Switched inductor

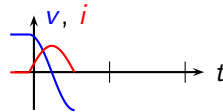


How it works

LC oscillations

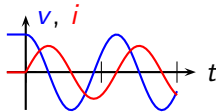
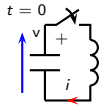


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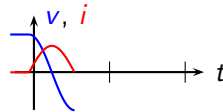
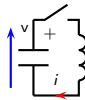


How it works

LC oscillations

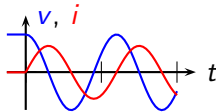
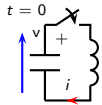


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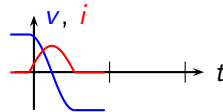
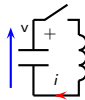


How it works

LC oscillations

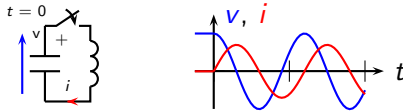


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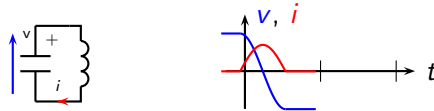


How it works

LC oscillations

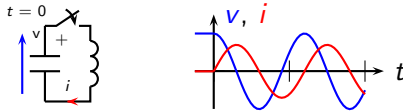


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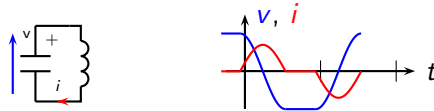


How it works

LC oscillations

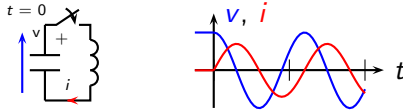


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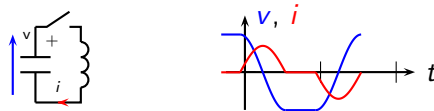


How it works

LC oscillations

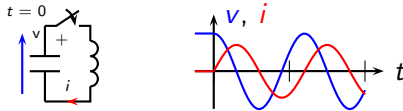


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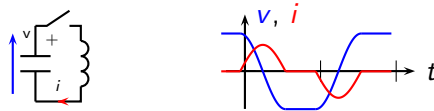


How it works

LC oscillations

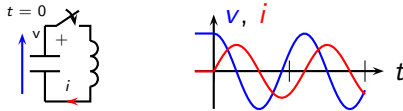


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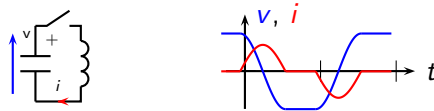


How it works

LC oscillations



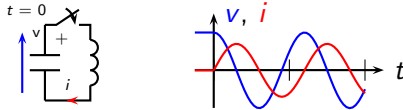
Switched inductor



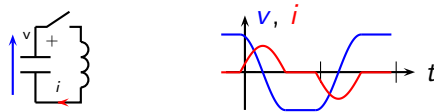
SSHI

How it works

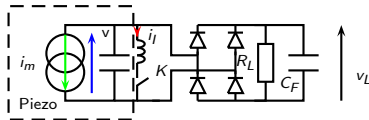
LC oscillations



Switched inductor

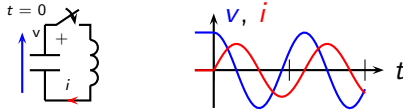


SSHI

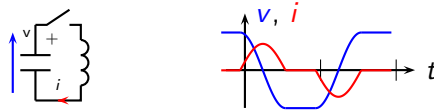


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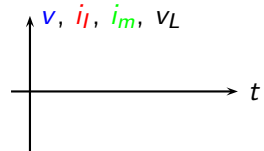
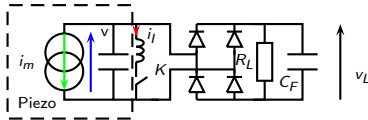
LC oscillations



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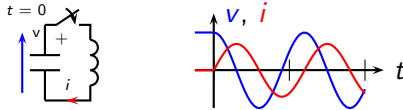


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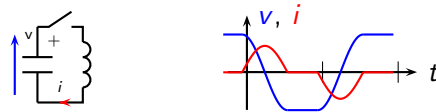


How it works

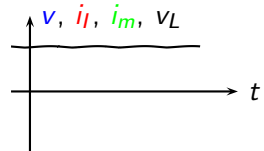
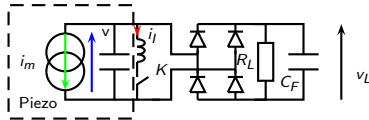
LC oscillations



Switched inductor

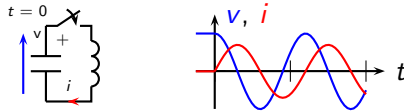


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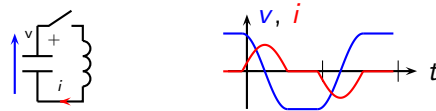


How it works

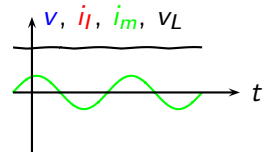
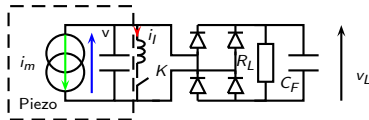
LC oscillations



Switched inductor

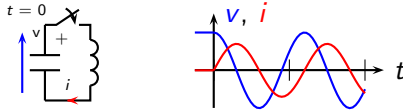


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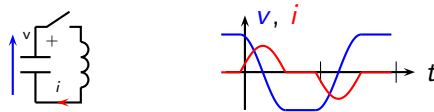


How it works

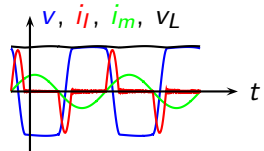
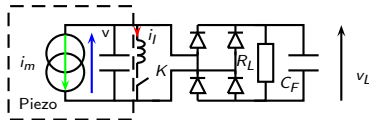
LC oscillations



Switched inductor

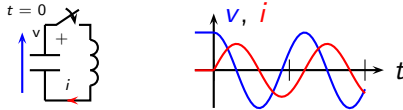


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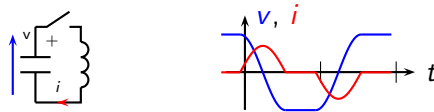


How it works

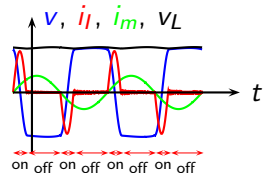
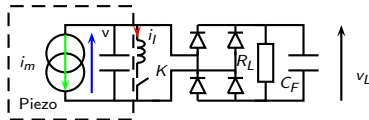
LC oscillations



Switched inductor



SSHI

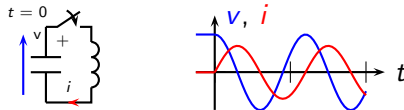


Switching is synchronized on \dot{w} , or

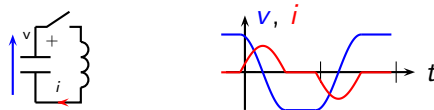
$$i_m = N\dot{w}.$$

How it works

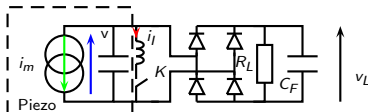
LC oscillations



Switched inductor

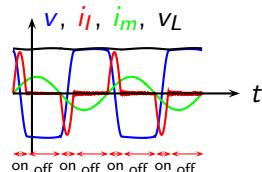


SSHI



Operating point: calculate v_L from i_m

1st Harmonic assumption: $P = \frac{1}{2} \frac{v_L^2}{R_L} \simeq \frac{1}{2} \frac{4v_L i_m}{\pi}$, $v_L = \frac{4}{\pi} R_L i_m$

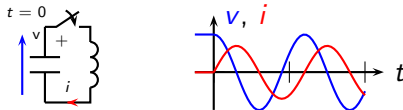


Switching is synchronized on \dot{w} , or

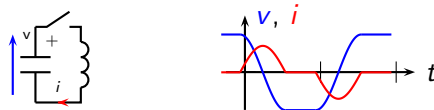
$$i_m = N\dot{w}.$$

How it works

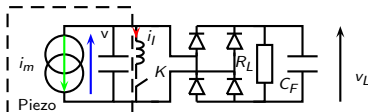
LC oscillations



Switched inductor



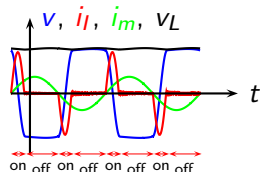
SSH



Operating point: calculate v_L from i_m

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SSH controls v and synchronises it, but doesn't impose $f_p = -D_s \dot{w}$.



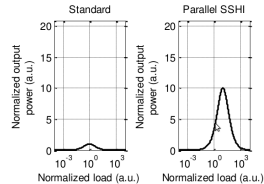
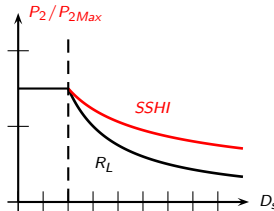
Switching is synchronized on \dot{w} , or

$$i_m = N \dot{w}.$$

Conclusion

Performances

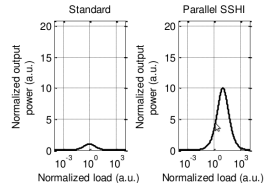
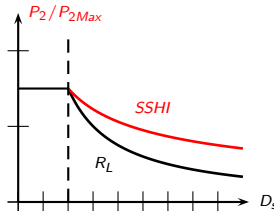
- SSHI can extract energy more efficiently than a resistor when damping is important,



Conclusion

Performances

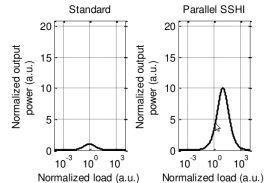
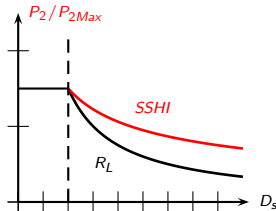
- SSHI can extract energy more efficiently than a resistor when damping is important,
- But Power extraction still depends on the load,



Conclusion

Performances

- SSHI can extract energy more efficiently than a resistor when damping is important,
- But Power extraction still depends on the load,
- Needs to measure bender's deflection $w(t)$.



General conclusion

In this presentation, applications of Energy Harvesting were shown. The modelling of a piezoelectric generator has shown that the power source needs an adaptation:

- in frequency,
- in load.

to maximize the harvested power.

The key energy management rules were presented through the analysis of the EMR of the system. A typical power electronic circuit was also presented, but the bibliography shows a lot of example.

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End of the presentation

Questions?