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Energy Harvesting from Ambient Vibrations

Frédéric Giraud

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Energy Harvesting from Ambient Vibrations

Frédéric Giraud

L2EP – University Lille1

November 27, 2012

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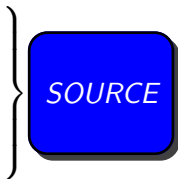
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We extract energy from an *ambient and free source*:

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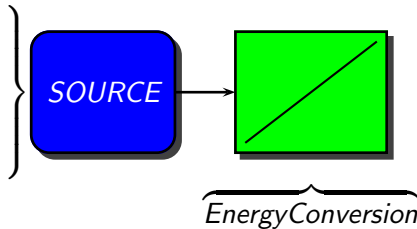
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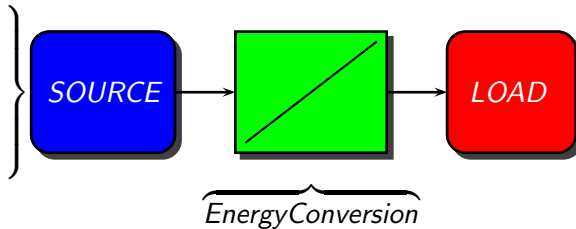
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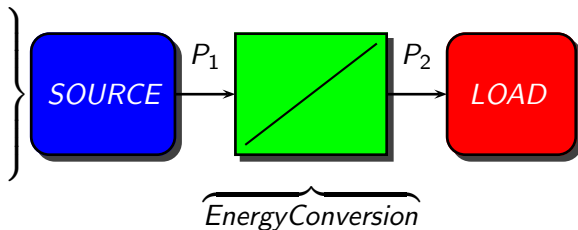
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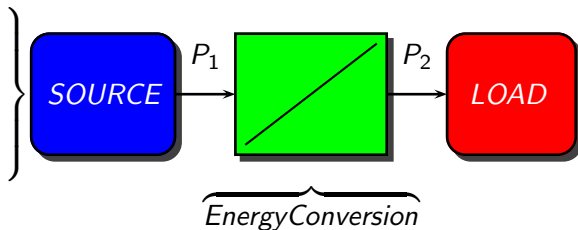
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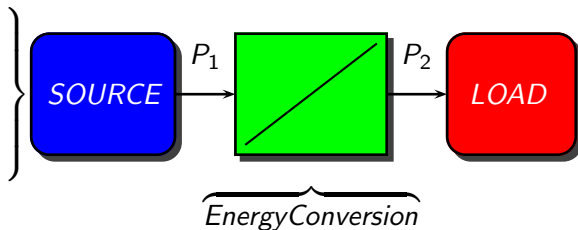


We talk about **Energy Harvesting** or also **energy scavenging** when the converted power is small, typically less than 1W.

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We talk about **Energy Harvesting** or also **energy scavenging** when the converted power is small, typically less than 1W.

$\eta = \frac{P_2}{P_1} = 1 - \frac{P_1 - P_2}{P_1} \rightarrow$ Losses in the energy converter should be as small as possible.

Objectives: Sensors Network



www.perpetuum.com

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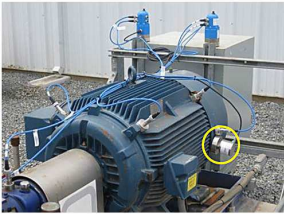


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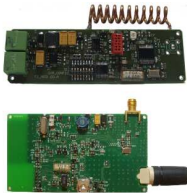


Gutiérrez, A Heterogeneous Wireless Identification Network for the Localization of Animals Based on Stochastic Movements

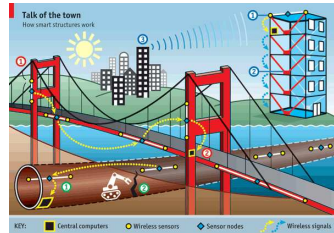
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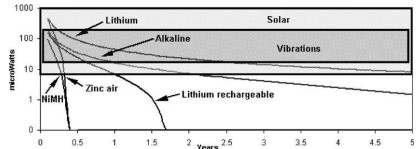


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<http://www.rfwirelessensors.com>, 2012

Continuous Power / cm² vs. Life for Several Power Sources



Roundy et Al.: A study of low level vibrations as a power source for wireless sensor nodes.

Objectives: Power, just where you need it



<http://enocean.com>

- Wireless
- Reduce Cost, and is reconfigurable
- Better Waste Cycle (Information from EnOcean)

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Innowattech's systems produces power with vehicles



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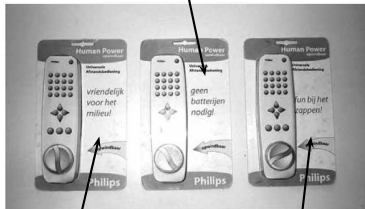


The economist – April 28th 2007

Objectives: Marketing Purpose

Experience: Same Remote controller energized by human power, but with different packaging.

"No need for Batteries"



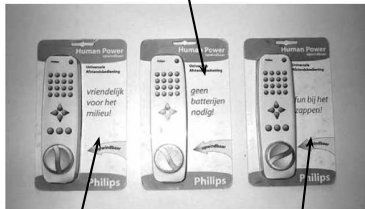
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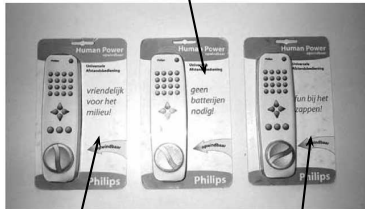
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54% of people Will choose the first one because it is eco-friendly.
(Jansen, Human power empirically explored)

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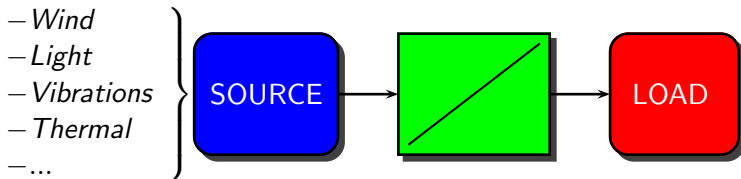
Several projects are born from this fact: **Metis** Produces energy from dancers' movements.



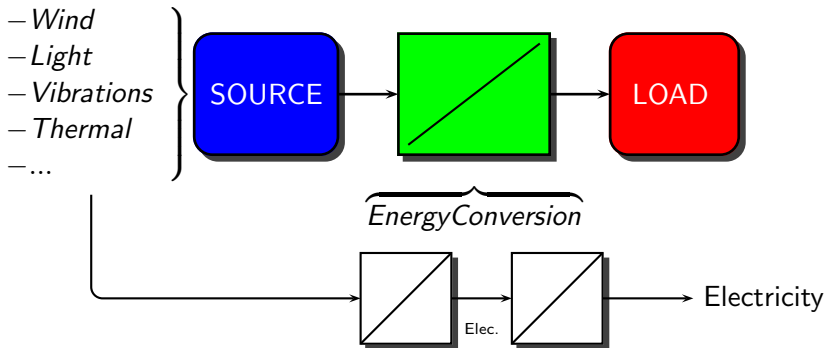
In Toulouse, the system **VIHA** proposes Smart Tiles to energies street lights.



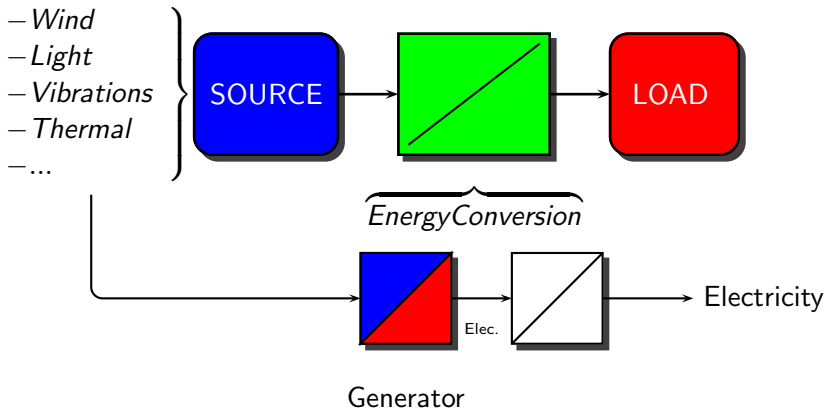
The energy converter



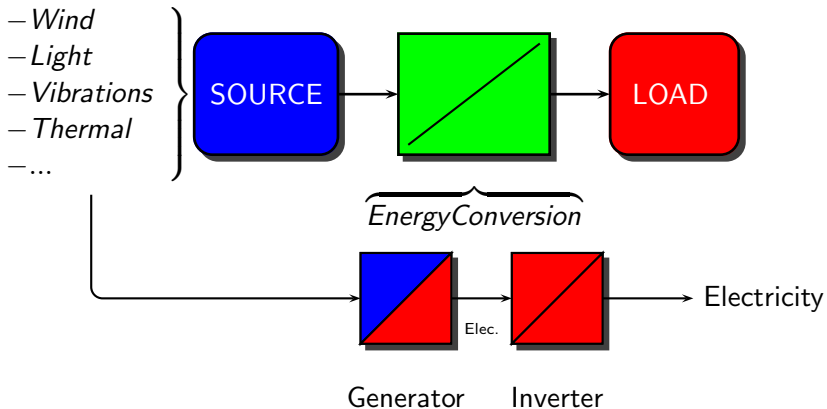
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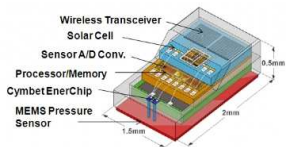


The energy converter



Solar Harvesters:

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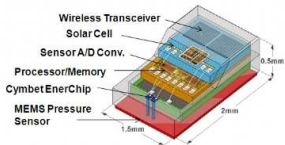


This sensor measures IntraOcular Pressure <http://cymbet.com>



Handbag to recharge electronic devices <http://www.neubers.de>

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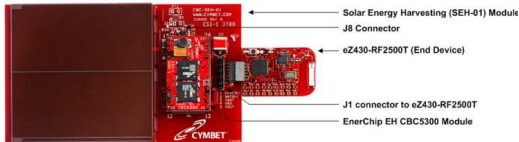
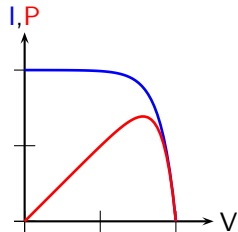


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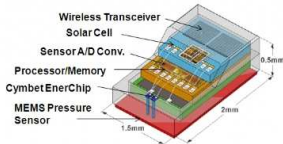
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Power and current as a function of voltage:



Solar Energy Harvester Evaluation Kit <http://ti.com>

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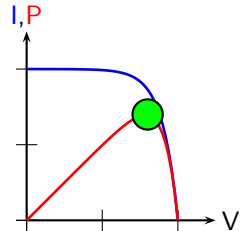


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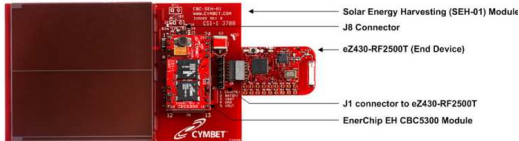


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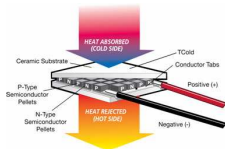


MPPT strategies, require Energy management of the system

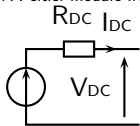


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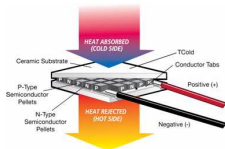
Thermoelectric



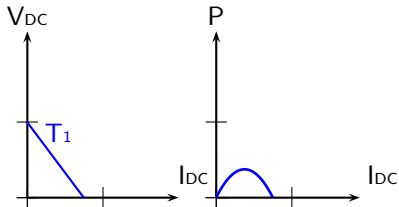
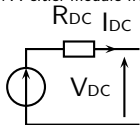
A Peltier module from <http://www.tellurex.com>



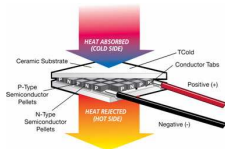
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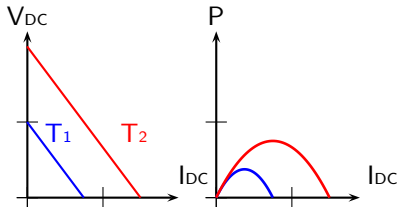
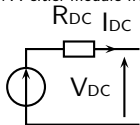
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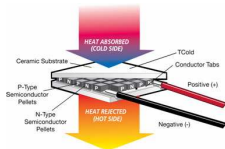
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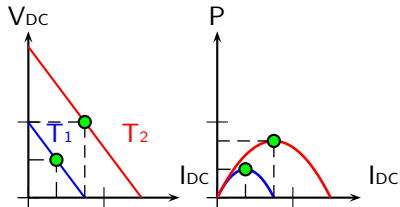
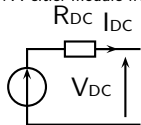
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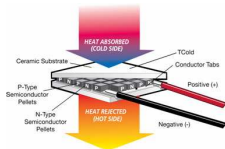
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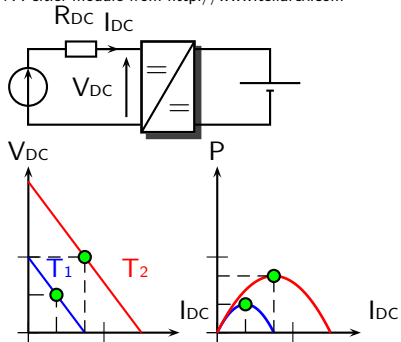
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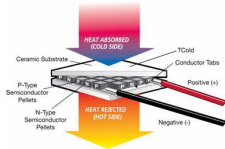
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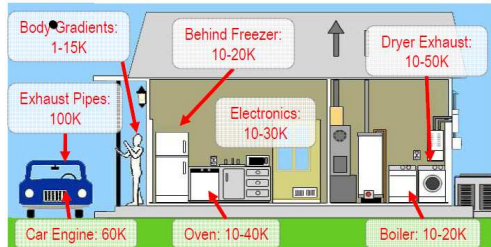
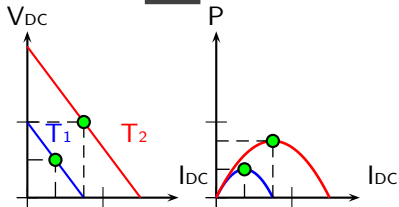
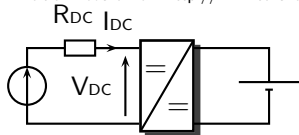
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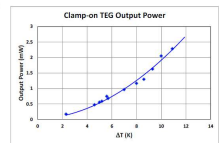


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Temperature Gradient (residential). Lindsay Miller,
<http://uc-ciee.org>

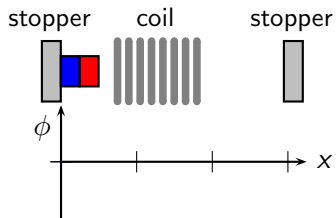
Clamp-on Drain Harvester



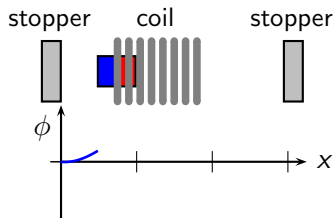
| ΔT | V_{load} (mV) | P_{max} (mW) |
|------------|-----------------|----------------|
| 5.0 | 38 | 0.56 |
| 8.0 | 55 | 1.16 |
| 10.0 | 73 | 2.05 |

plumbing application from <http://www.nextreme.com>

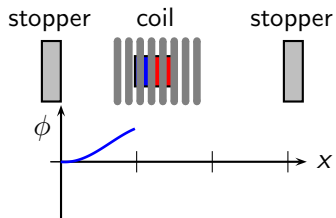
Magnetic



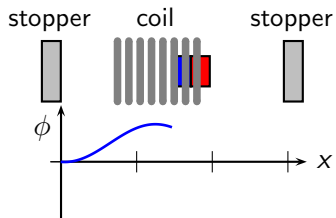
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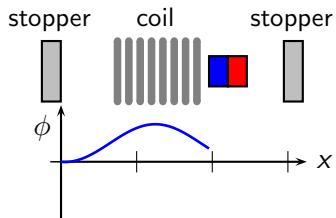
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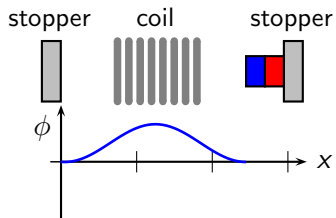
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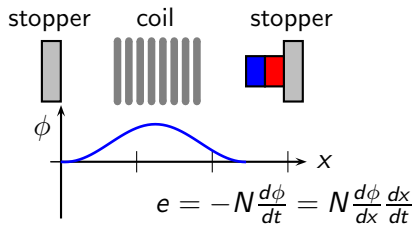
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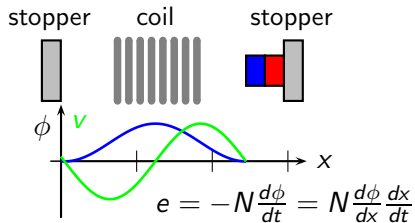
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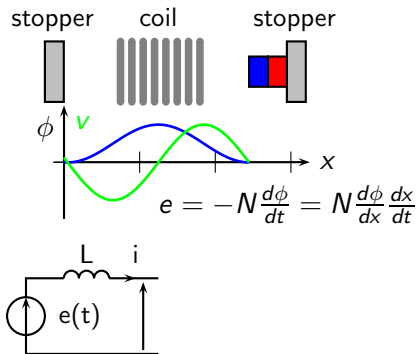
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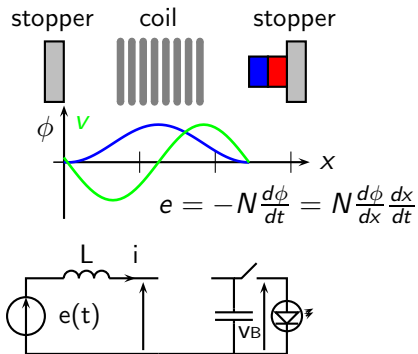
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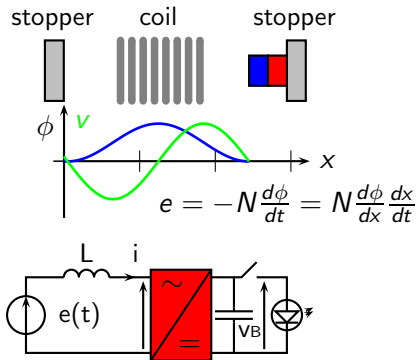
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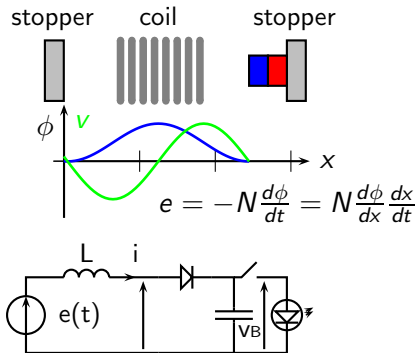
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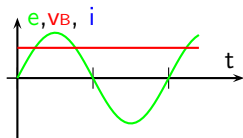
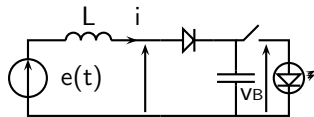
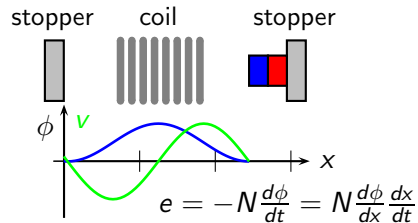
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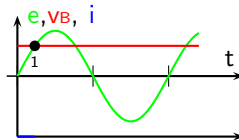
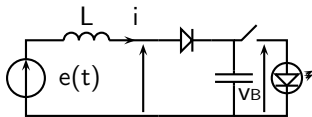
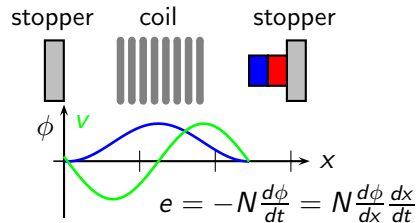
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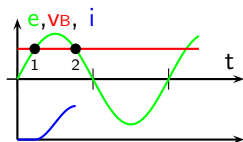
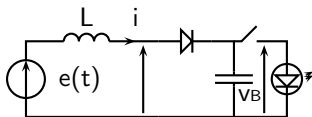
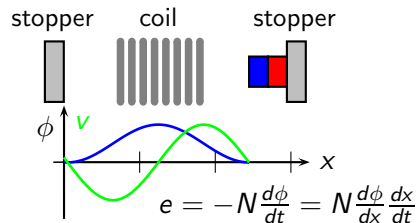


Magnetic



1) Diode turns ON

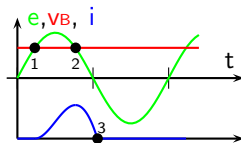
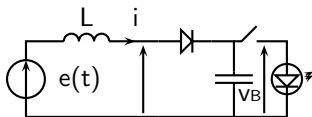
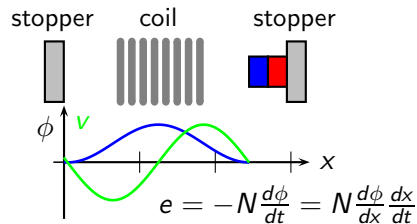
Magnetic



- 1) Diode turns ON
- 2) $\frac{di}{dt} = 0$ because $e - v_B = 0$



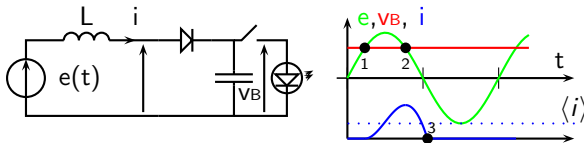
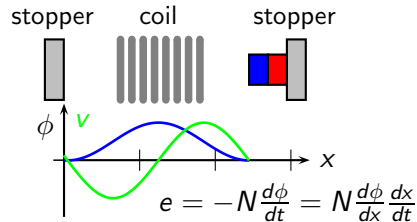
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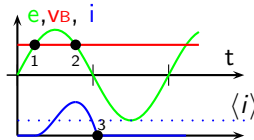
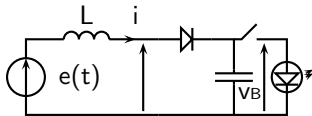
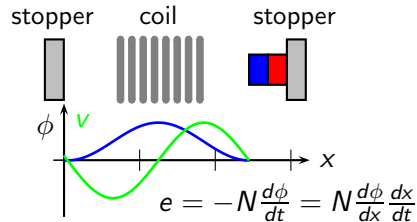
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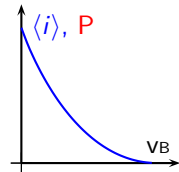
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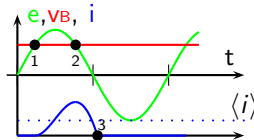
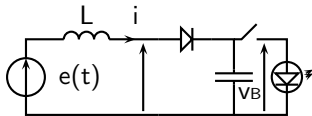
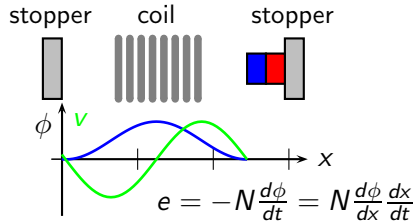
Magnetic



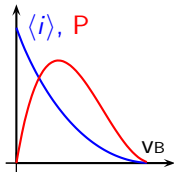
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Magnetic

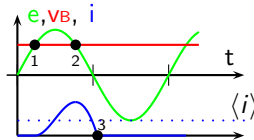
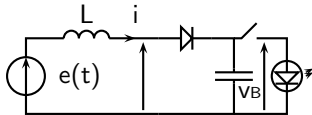
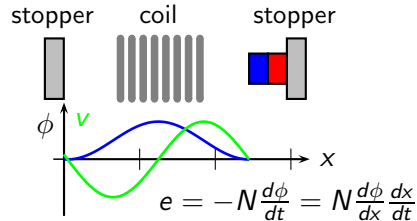


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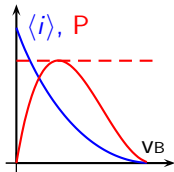


$$P = \langle v_B i \rangle = v_B \langle i \rangle$$

Magnetic

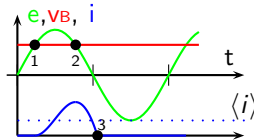
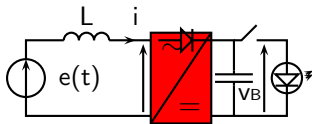
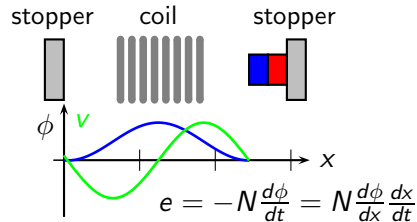


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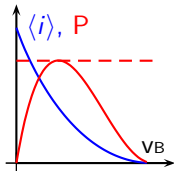


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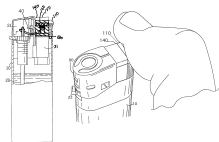


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Piezoelectric

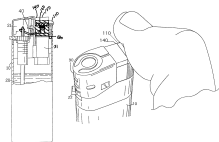


An electronic lighter,
<http://freepatentsonline.com>



Piezoelectric crystals

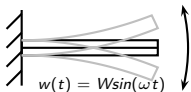
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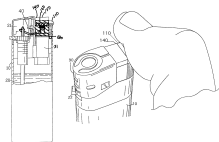
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Piezoelectric crystals
Cantilever beam



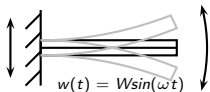
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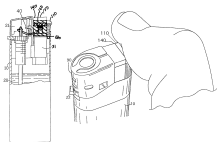
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Piezoelectric crystals
Cantilever beam



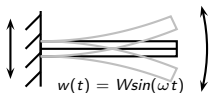
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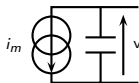
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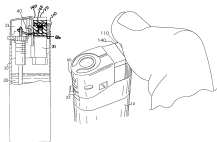


Equivalent electrical circuit



i_m is a current proportional
to the deformation speed

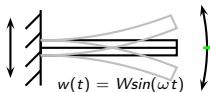
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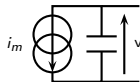
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Piezoelectric crystals
Cantilever beam

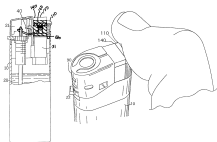


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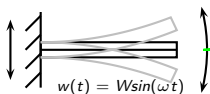
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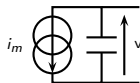
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Piezoelectric crystals
Cantilever beam



Equivalent electrical circuit

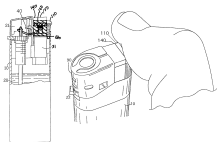


i_m is a current proportional
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Comparison Magn. Piezo

| | |
|----------------|----------------|
| Magnetic | Piezo. |
| Voltage source | Current source |
| Inductive | capacitive |
| Large Stroke | Small Stroke |
| Remote action | |

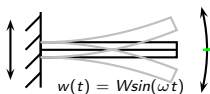
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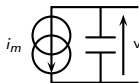
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Piezoelectric crystals
Cantilever beam

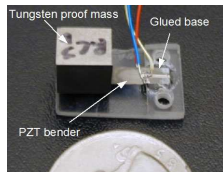


Equivalent electrical circuit



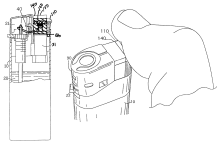
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Roundy, A piezoelectric vibration based generator for wireless electronics (2004)

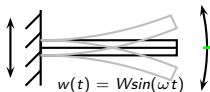
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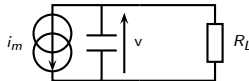
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Piezoelectric crystals
Cantilever beam

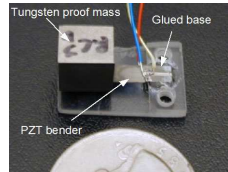


Equivalent electrical circuit



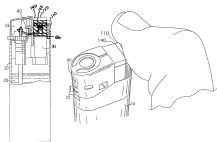
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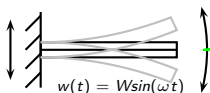
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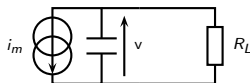
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Piezoelectric crystals
Cantilever beam

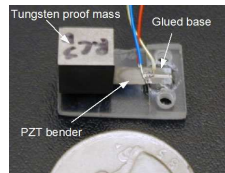


Equivalent electrical circuit

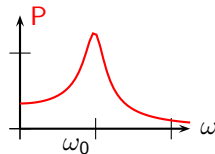


i_m is a current proportional
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Comparison Magn. Piezo

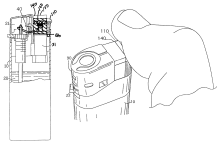
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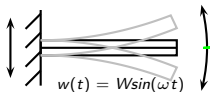
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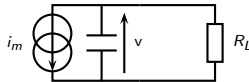
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Piezoelectric crystals
Cantilever beam

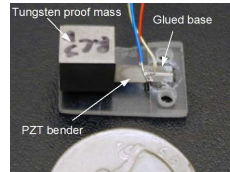


Equivalent electrical circuit

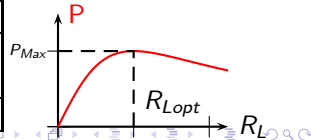
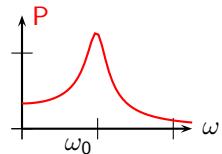


i_m is a current proportional
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Comparison Magn. Piezo

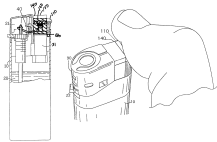
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Roundy, A piezoelectric vibration based generator for wireless electronics (2004)



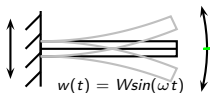
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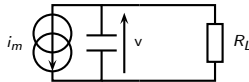


Piezoelectric crystals
Cantilever beam



Energy Management

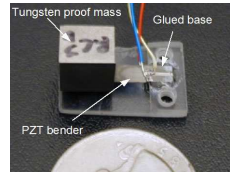
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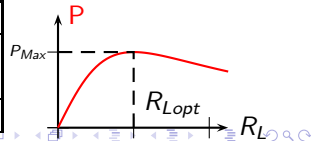
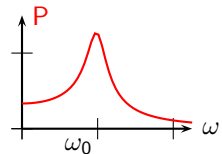
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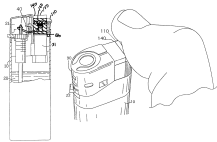
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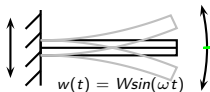
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An electronic lighter,
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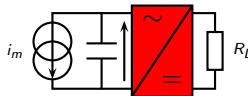


Piezoelectric crystals
Cantilever beam



Energy Management

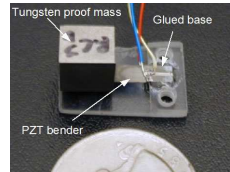
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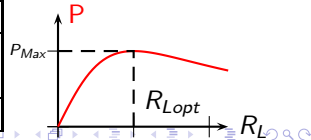
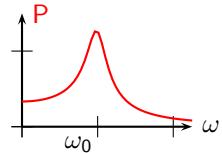
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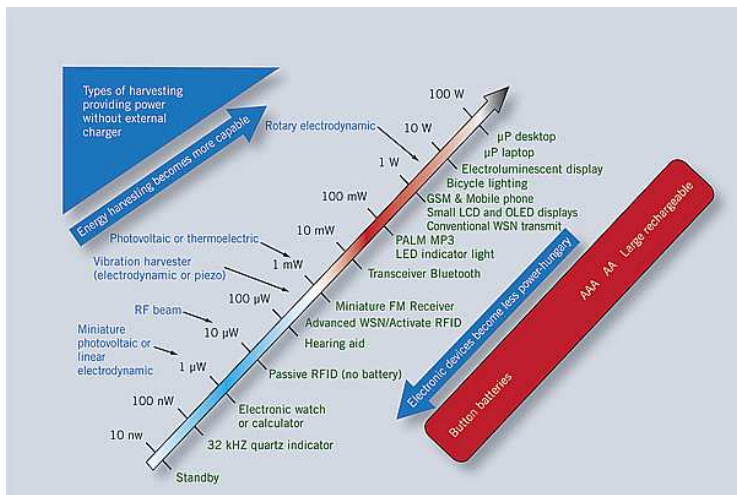
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Roundy, A piezoelectric vibration based generator for wireless electronics (2004)



There is no "one fit all" solution



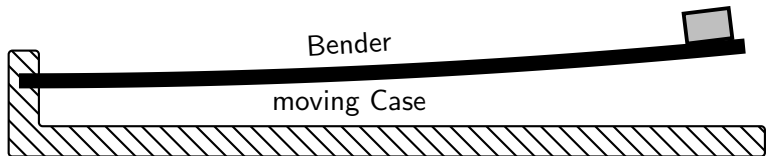
Each solution may be efficient in a certain range of Power.

Meanwhile, shrinking Chips consumption come at a time when energy harvesting becomes efficient and practical (source: IDtechex.com)

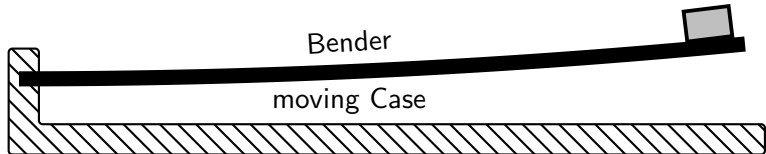
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 - Power Extraction on a load resistor R_L from harmonic oscillation
- 3 An Example of inverter
 - Introduction
 - SSHI: Synchronized Switch Harvesting on Inductor

Coordinates and assumptions

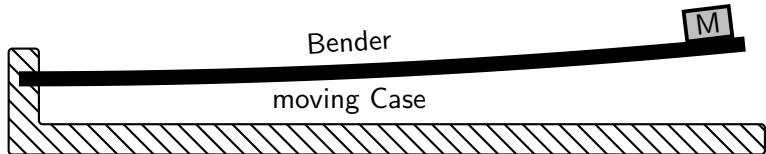


Coordinates and assumptions



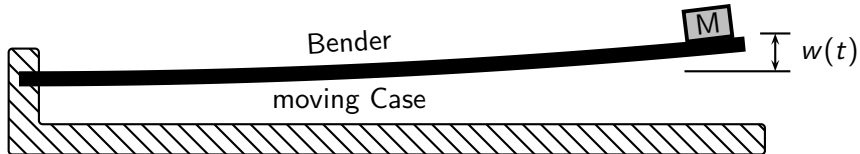
- The bender is attached to a vibrating and rigid case,

Coordinates and assumptions



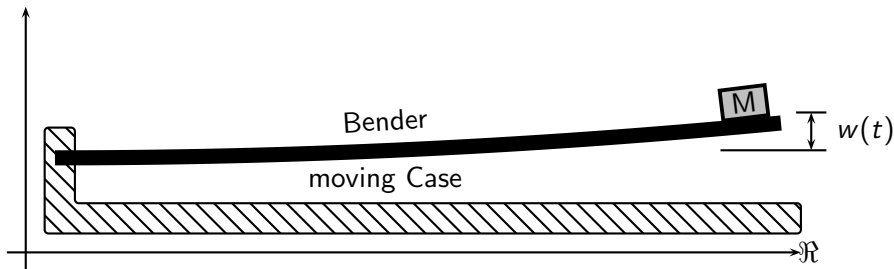
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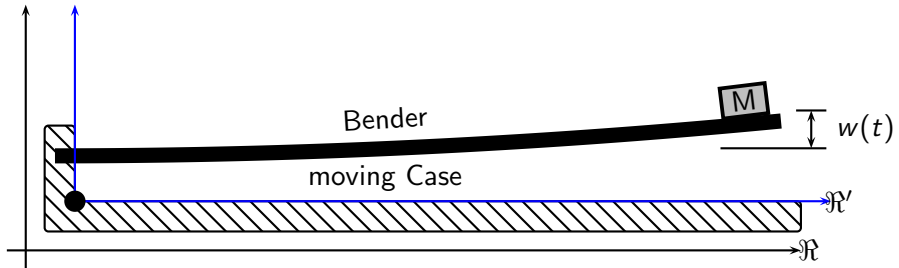
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Coordinates and assumptions



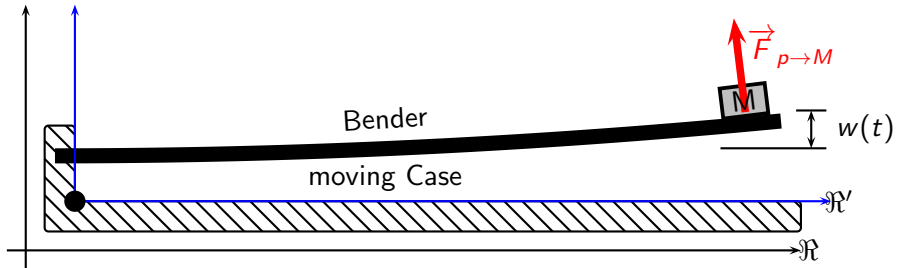
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Coordinates and assumptions



- The bender is attached to a vibrating and rigid case,
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- And \mathcal{R}' a reference frame affixed to the vibrating case.

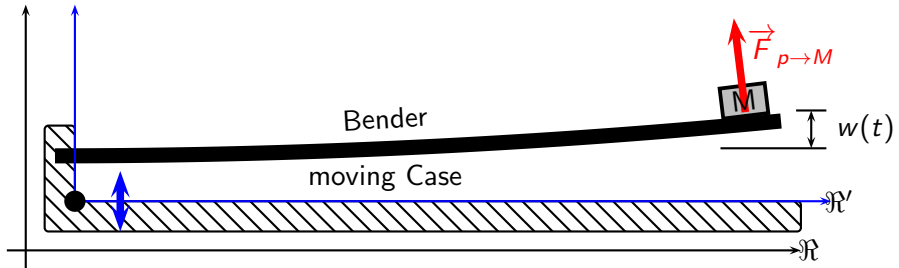
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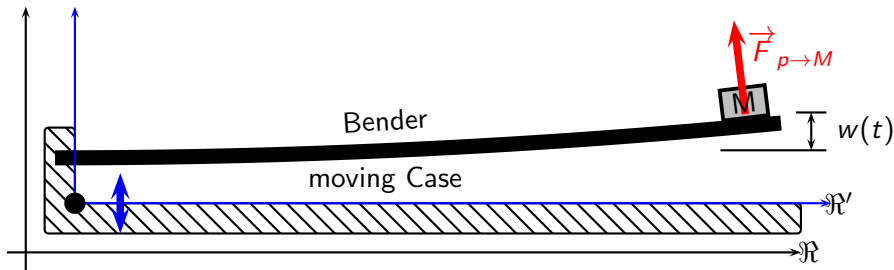
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Coordinates and assumptions



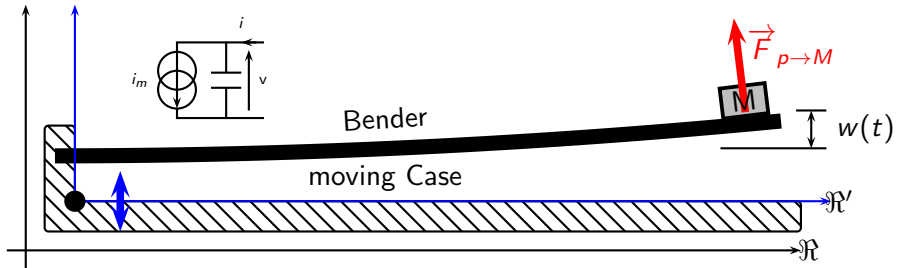
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- And \mathcal{R}' a reference frame affixed to the vibrating case.
- $\vec{F}_{p \rightarrow m}$ is the force of the Bender onto the mass M .
- The case is supposed to be controlled in position, and we have $y_c = A \sin(\omega t)$ the amplitude of the case's vibration.
- Gravity is neglected, as well as Inertia momentum of the bender,
- Actuator electrical convention.

Equations

- **Dynamic of the mass M :**

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- M the mass
- f , the force onto M
- A , vibration's amplitude
- ω , vibration's pulsation
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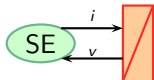
EMR of the system

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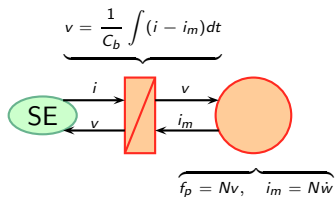


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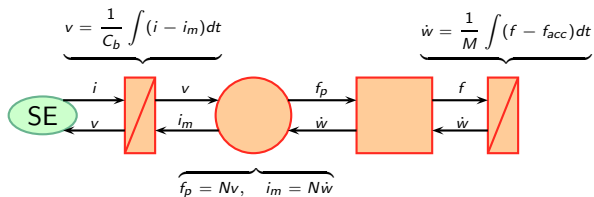
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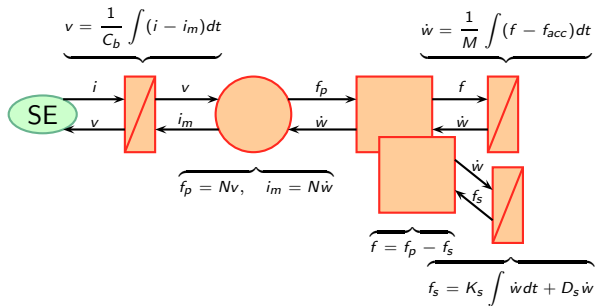
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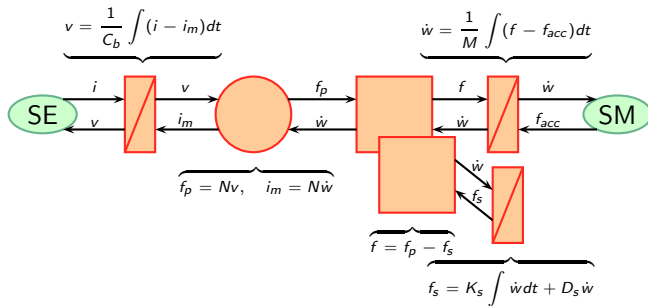
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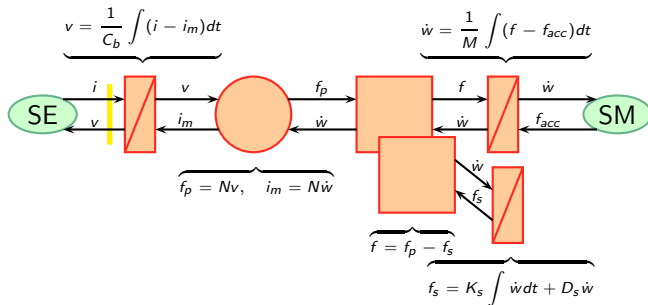
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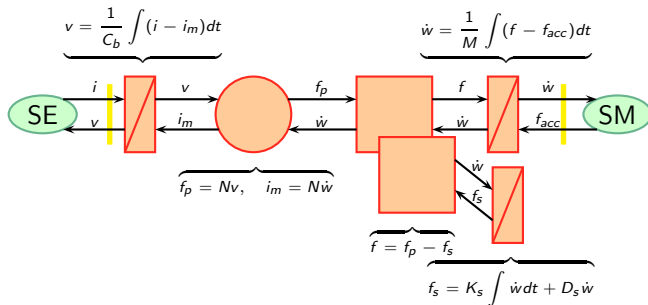


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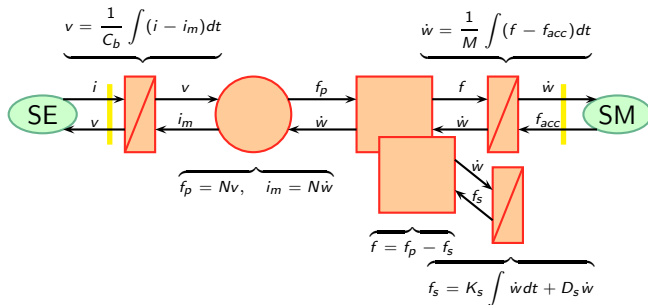
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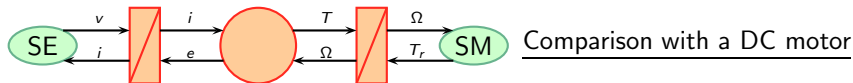


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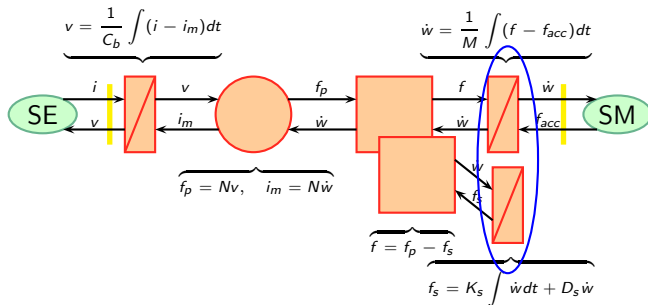


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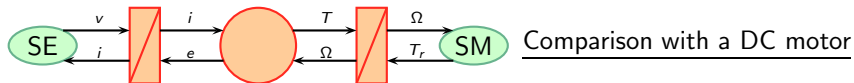


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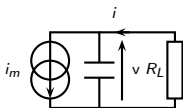
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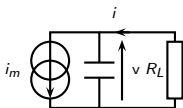
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And $R_L \ll \frac{1}{C_b\omega}$, yields
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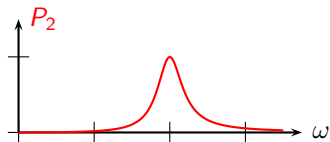
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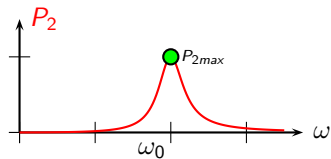
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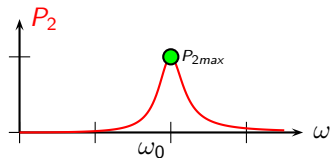
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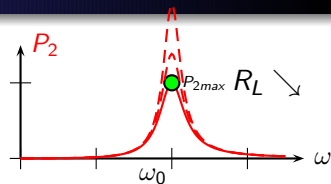


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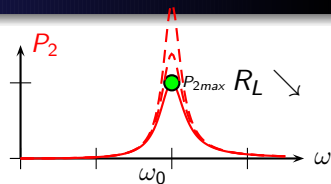


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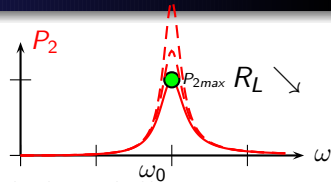


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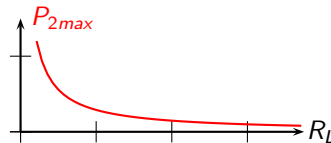
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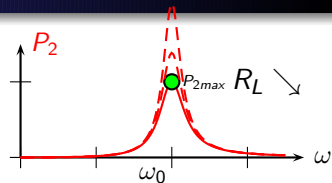
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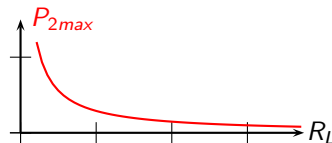
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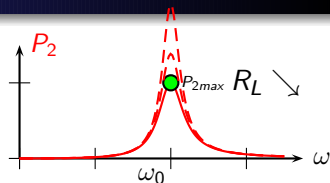
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We can harvest as much power as we want

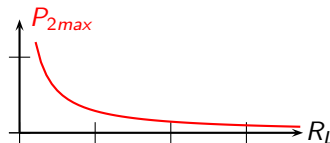
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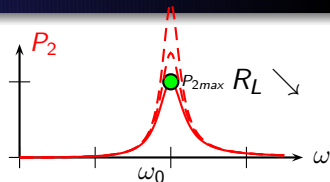
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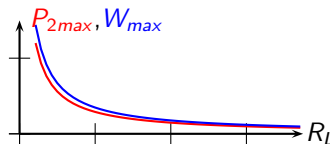
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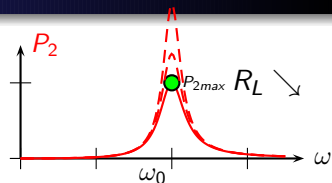
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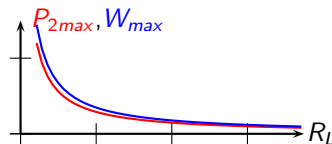
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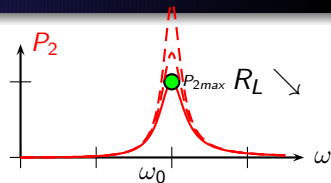
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We can harvest as much power as we want, at the expense of large displacement of the bender's tip

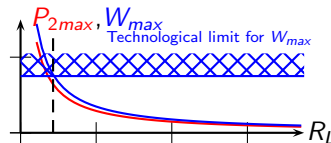
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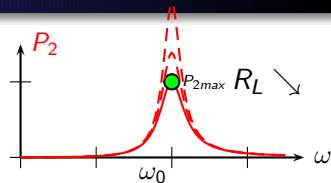
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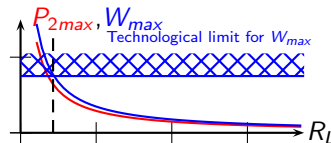
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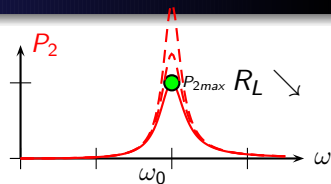
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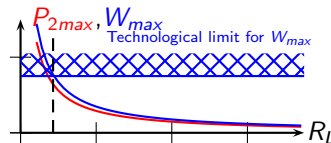
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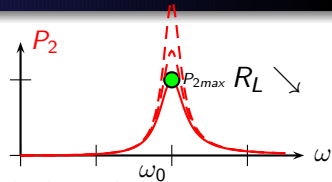
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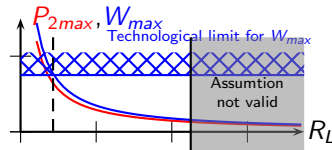
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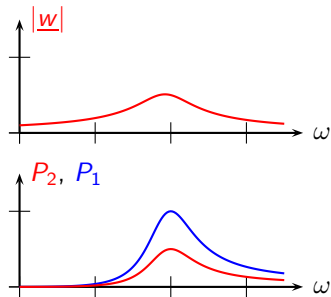
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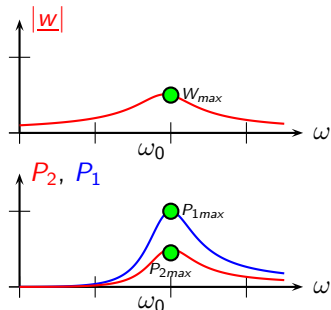
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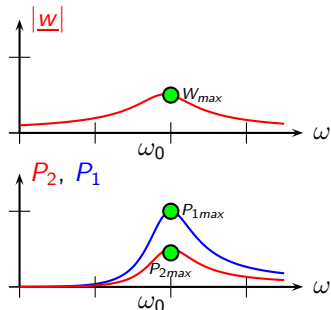
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- $P_2 = -\frac{1}{2}R_L|i_m|^2$
- $P_2 = -\frac{1}{2} \frac{R_L N^2 \omega^2 |\underline{f}_{acc}|^2}{(K_s - M\omega^2)^2 + \omega^2 (D_s + N^2 R_L)^2}$
- $P_1 = -\frac{1}{2} \Re(\underline{f}_{acc} \cdot \dot{\underline{w}}^*)$
- $P_1 = -\frac{1}{2} \frac{(D_s + R_L N^2) \omega^2 |\underline{f}_{acc}|^2}{(K_s - M\omega^2)^2 + \omega^2 (D_s + N^2 R_L)^2}$

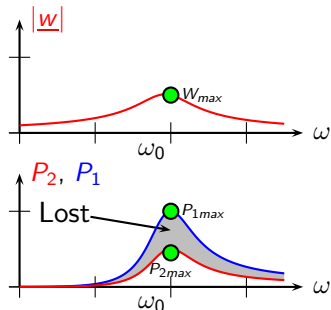
ω_0 should still be the vibration's pulsation.



For a real generator ($D_s \neq 0$)

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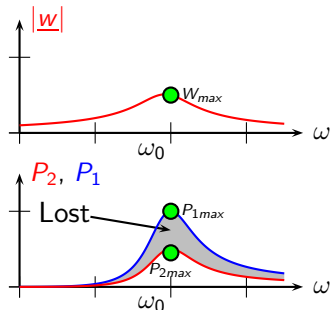
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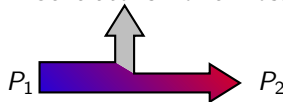
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Generator's Power Losses

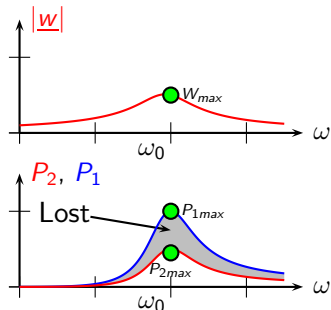


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Generator's Power Losses

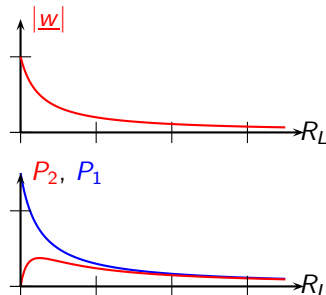


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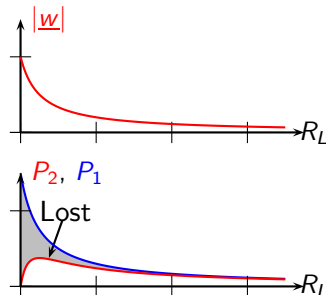
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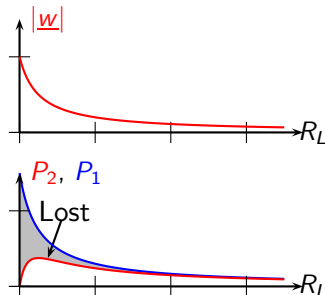


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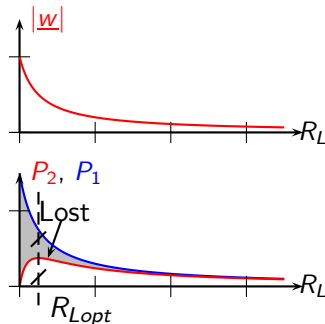


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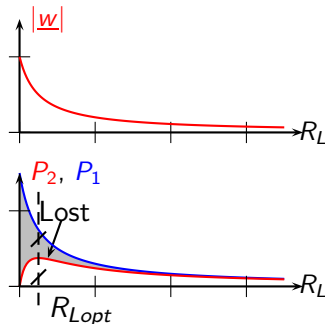


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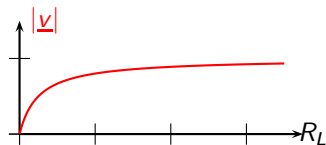
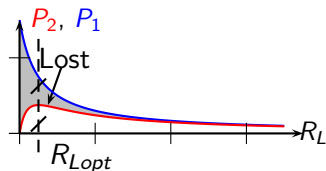
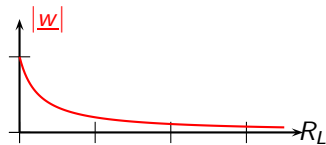


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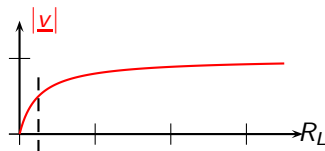
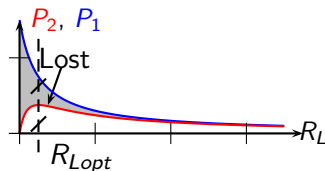
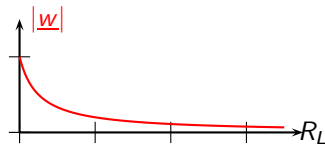


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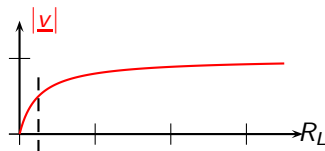
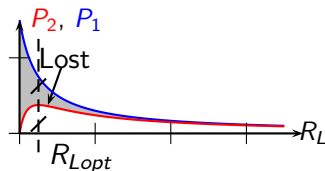
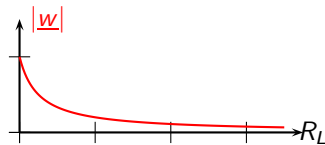


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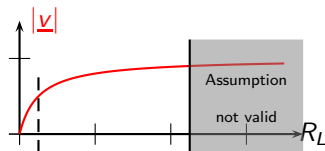
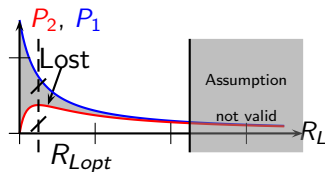
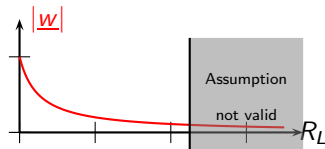


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Example

Device's properties

$$N = 0.012 \text{ N/V}, K_s = 6300 \text{ N/m}, C_b = 300 \text{ nF}, D_s = 0.17 \text{ Ns/m}, \\ M = 1 \text{ g}$$

Calculate for $A = 0.1 \text{ mm}$

- the best working frequency,
- the harvested P_2 power in the best case,
- the power of the source P_1 in such best case,
- the optimal resistor R_L ,
- the deflection amplitude of the bender,
- the voltage for this working point.

Validation

Is $v = -R_L i_m$ a valid assumption?

Answers

- The best working frequency is given by

$$f_0 = \frac{1}{2\pi} \sqrt{\frac{K_s}{M}} = \frac{1}{2\pi} \sqrt{\frac{6300}{1.10^{-3}}} = \underline{400Hz}$$

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- For the best case, P_2 is given by

$$P_2 = \frac{|f_{acc}|^2}{8D_s} = \frac{(1 \cdot 10^{-3} \cdot 1 \cdot 10^{-4} \cdot (2\pi \cdot 400)^2)^2}{8 \cdot 0,17} = \underline{464\text{mW}}$$

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- For the best case $P_1 = 2 \cdot P_2 = \underline{928\text{mW}}$

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$$R_{Lopt} = \frac{D_s}{N^2} = \frac{0,17}{0,012^2} = \underline{1180\Omega}$$

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$$W_{max} = \frac{|f_{acc}|}{(D_s + N^2 R_L)\omega_0} = \frac{(1.10^{-3} \cdot 1.10^{-4} \cdot (2\pi \cdot 400)^2)}{2 \cdot 0,17 \cdot 2\pi \cdot 400} = \underline{738\mu\text{m!}}$$

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- The voltage is then given by

$$v_{max} = N R_L \omega_0 |w| = 0,012 \cdot 1180 \cdot 2\pi \cdot 400 \cdot 738 \cdot 10^{-6} = \underline{26\text{V}}$$

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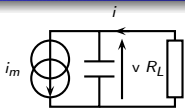
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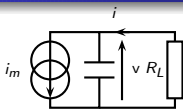
$$v_{max} = N R_L \omega_0 |w| = 0,012 \cdot 1180 \cdot 2\pi \cdot 400 \cdot 738 \cdot 10^{-6} = \underline{26\text{V}}$$

- $Z_{Cb} = \frac{1}{C_b \omega} = \frac{1}{2\pi \cdot 400 \cdot 300 \cdot 10^{-9}} = \underline{1990\Omega} \approx R_L, \rightarrow$ **Calculations are NOT valid!**

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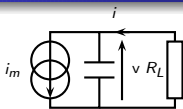


We can show:

$$\underline{v} = -\frac{R_L}{1+j\omega R_L C_b} \underline{i}_m = -j\omega N R_L \frac{1-j\omega R_L C_b}{1+(R_L C_b \omega)^2} \underline{w}_m$$

and we write: $\underline{v} = -j\omega N r_{Leq} \underline{w} - N k_{eq} \underline{w}$

For a real Generator, $D_s \neq 0$ and $v \neq -R_L i_m$



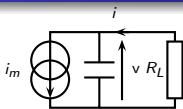
We can show:

$$\underline{v} = -\frac{R_L}{1+j\omega R_L C_b} \underline{i}_m = -j\omega N R_L \frac{1-j\omega R_L C_b}{1+(R_L C_b \omega)^2} \underline{\omega}_m$$

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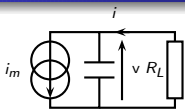
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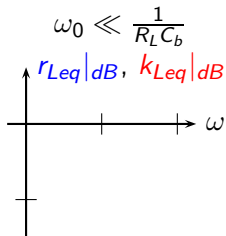


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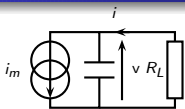
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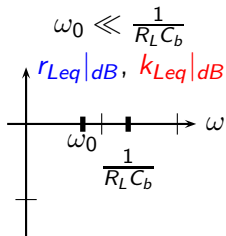


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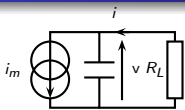
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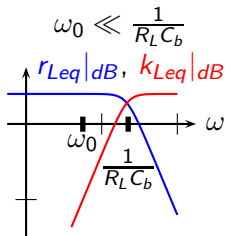


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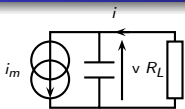
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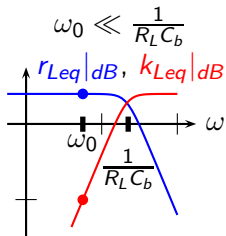


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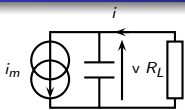
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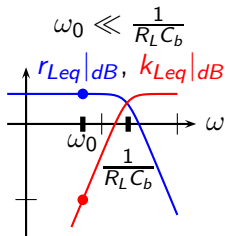


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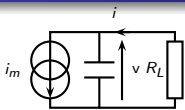
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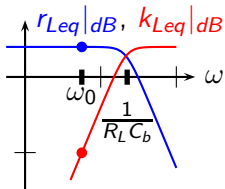
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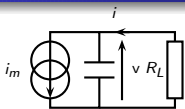
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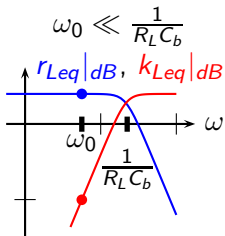


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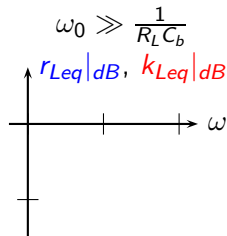
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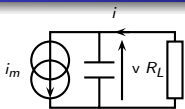
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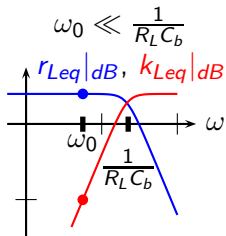


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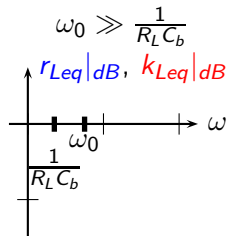
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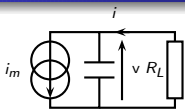
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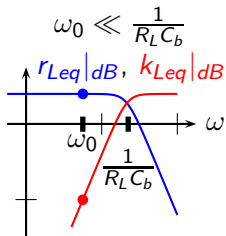


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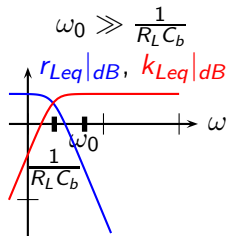
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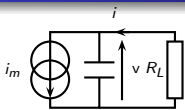
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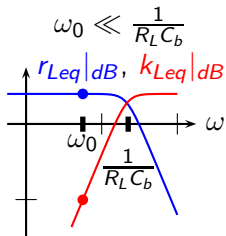


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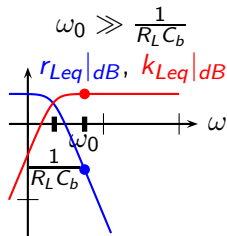
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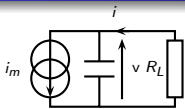
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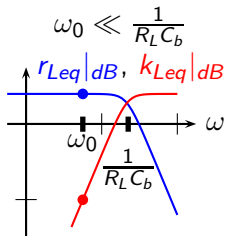


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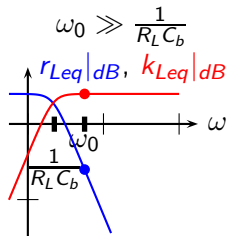
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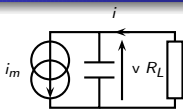


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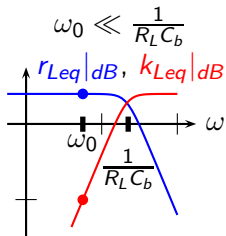


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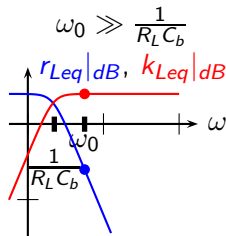
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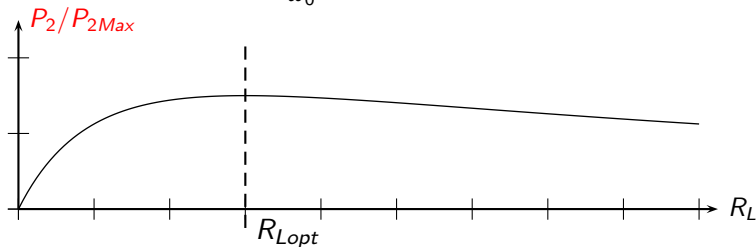
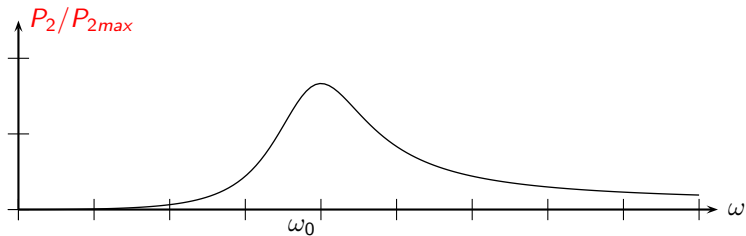


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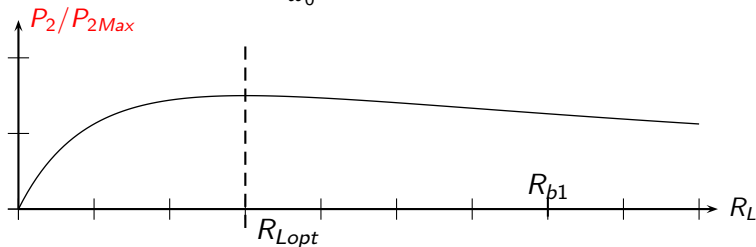
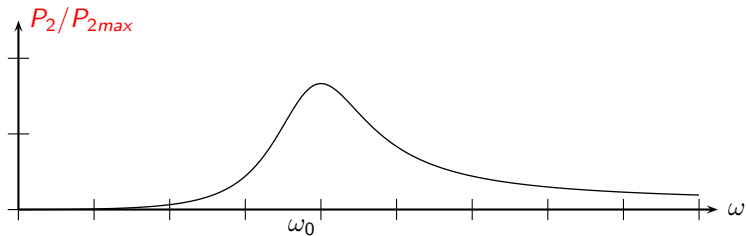
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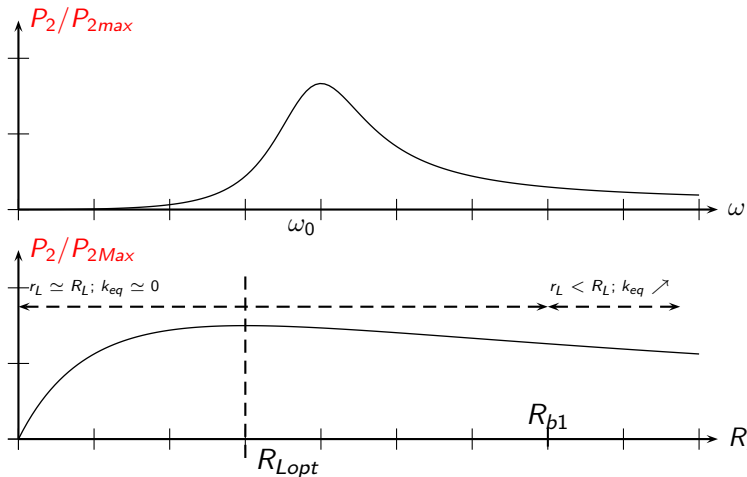
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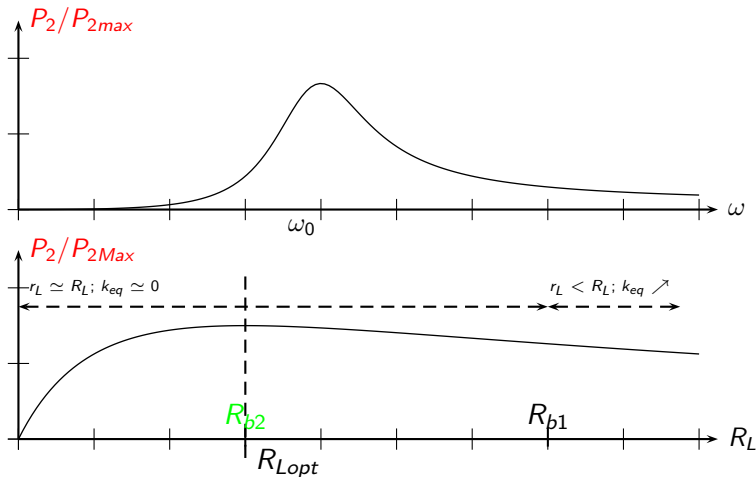
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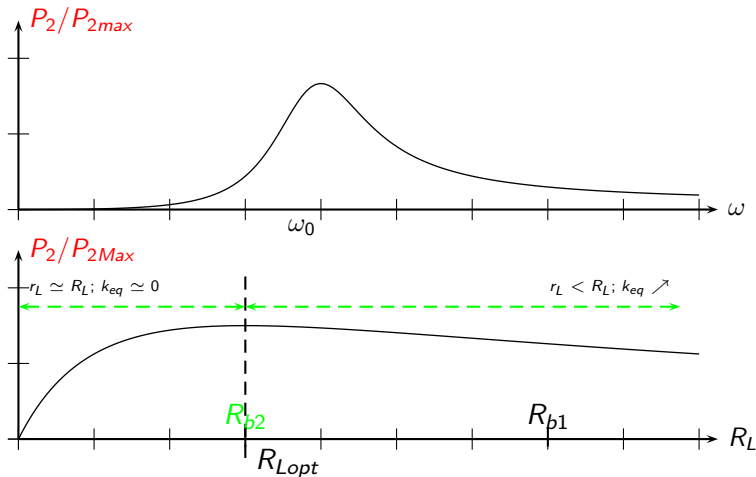
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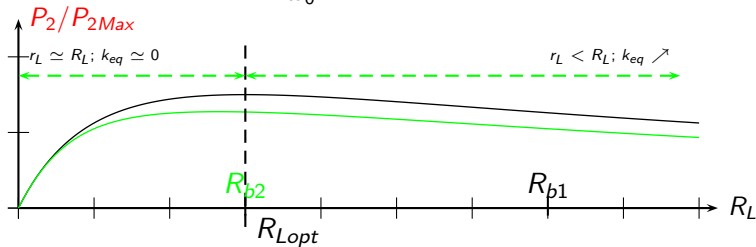
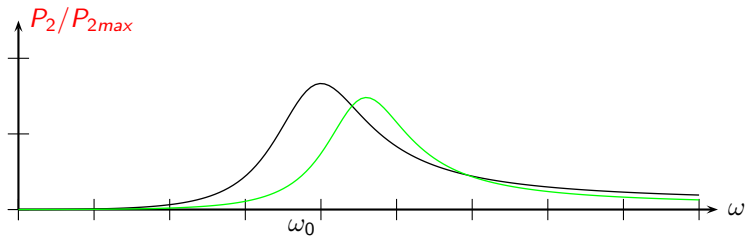
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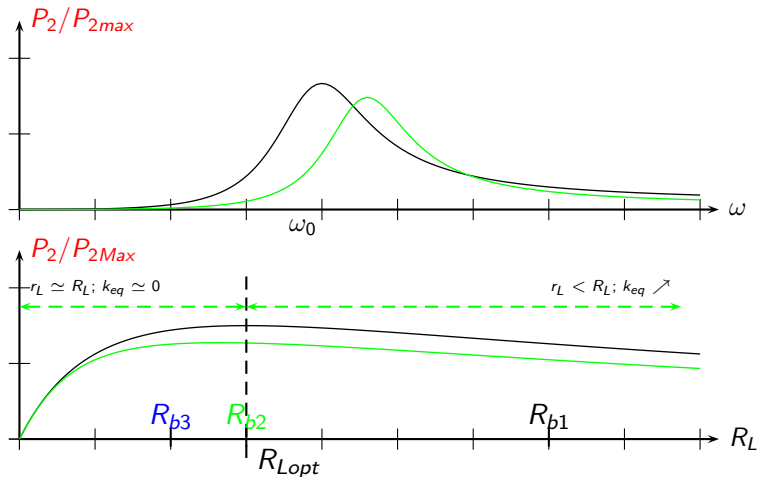
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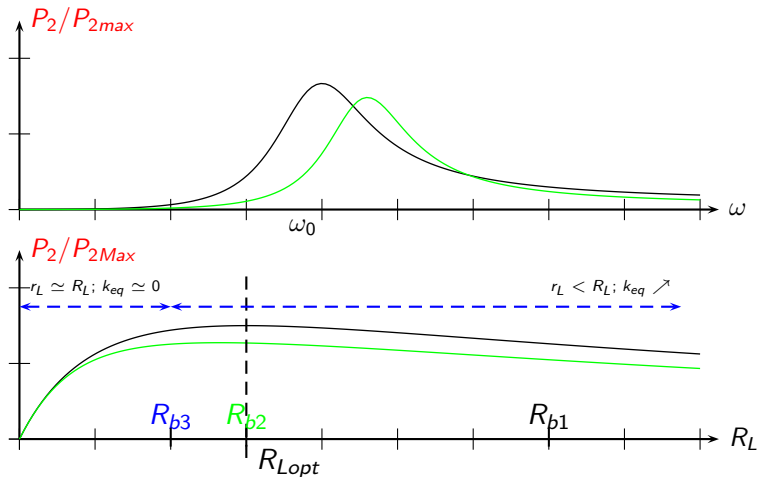
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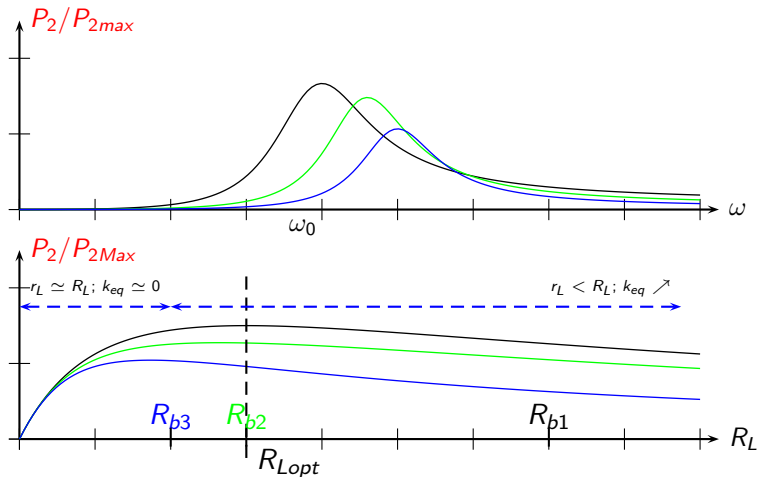
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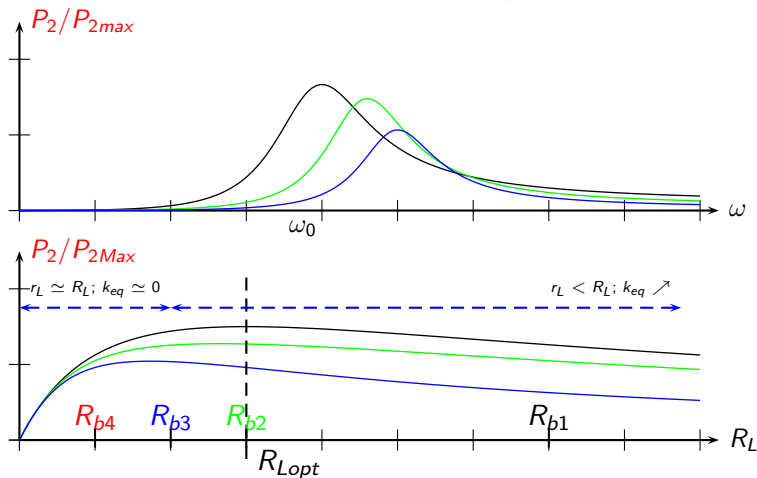
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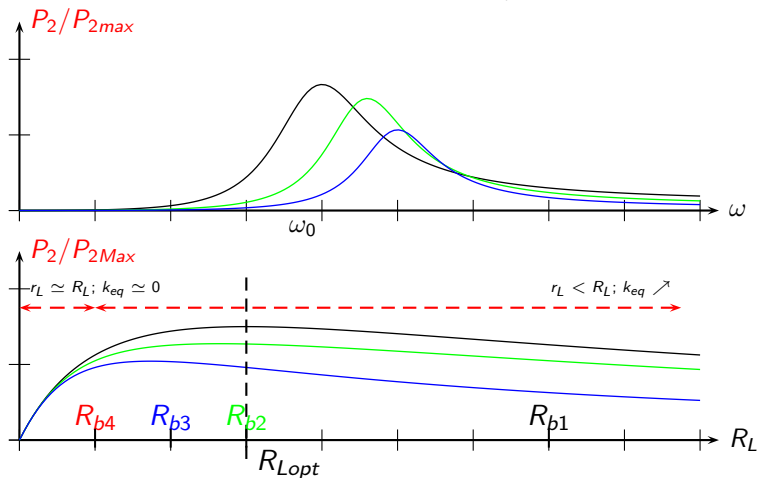
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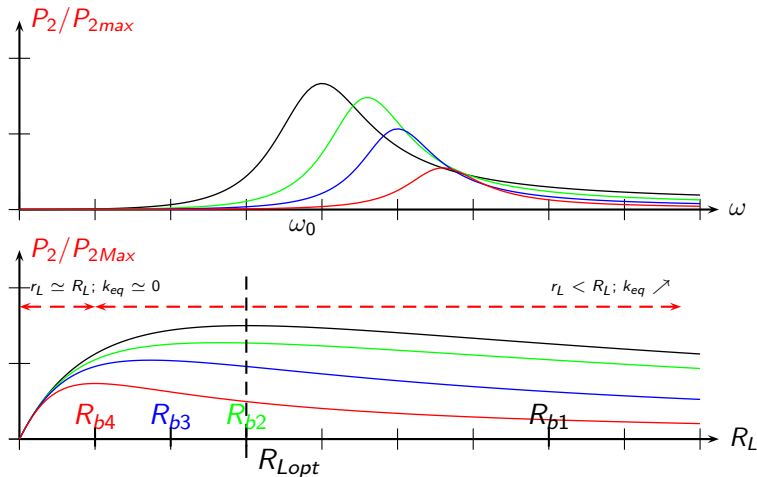
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Partial Conclusion

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If $R_{opt} > R_b$

The power is not well extracted because $P_2 < P_{2max}$. This happens for high damped mechanisms.

A simple resistor cannot recover the power optimally (and this is due to C_b).

What happens if $R_{opt} < R_b$ and $R_L \gg R_b$?

resonance is shifted

- $r_{Leq} = \frac{R_L}{1+(R_L C_b \omega)^2} \simeq \frac{R_L}{(R_L C_b \omega)^2}$

- $k_{eq} = R_L \frac{\omega^2 R_L C_b}{1+(R_L C_b \omega)^2} \simeq \frac{1}{C_b}$

$((K_s + N^2 k_{eq}) - M\omega^2) + j\omega(D_s + N^2 r_{Leq})\underline{w} = \underline{f}_{acc}$ leads to:

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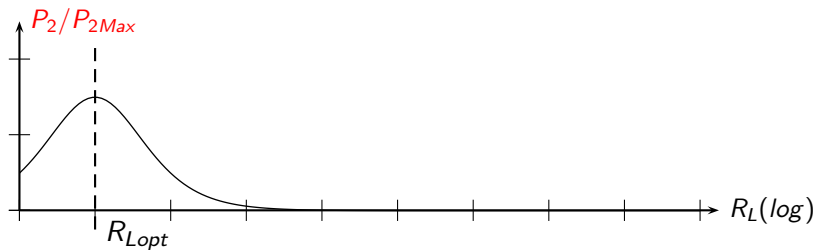
Another optimal resistor

The optimal power is harvested is $N^2 r_{Leq} = D_s$, leading to:

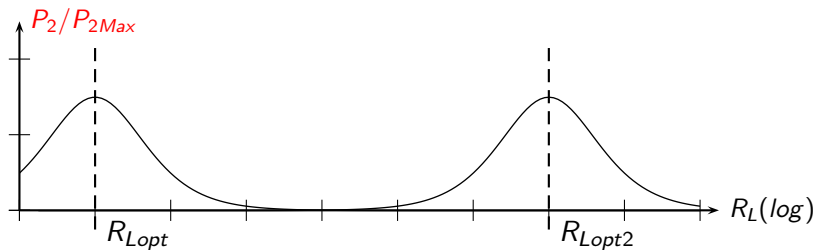
$$\frac{N^2 R_L}{(R_L C_b \omega'_0)^2} = \frac{N^2}{R_L C_b^2 \frac{K_s + \frac{N^2}{C_b}}{M}} = D_s$$

$$R_{Lopt2} = \frac{N^2}{D_s} \frac{1}{C_b^2 \frac{K_s}{M} (1 + \frac{N^2}{K_s C_b})} = \frac{R_b^2}{R_{Lopt}} \frac{1}{1 + \frac{N^2}{K_s C_b}}$$

What happens if $R_{opt} < R_b$ and $R_L \gg R_b$?

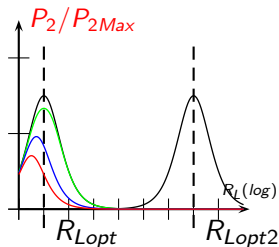


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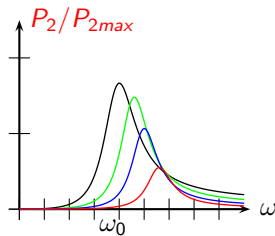
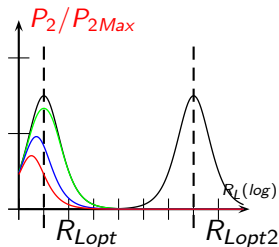
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- For the realistic case, there is one or two resistive loads which allow to extract the maximum of power,



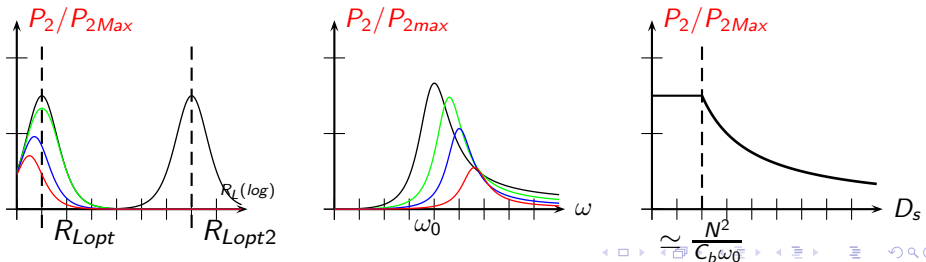
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Conclusion

- For the realistic case, there is one or two resistive loads which allow to extract the maximum of power,
 - There exist an optimal frequency which may vary if R_L is very high,
 - For highly damped structure, a simple resistor is not optimal.
- we always want to harvest the maximum of power!

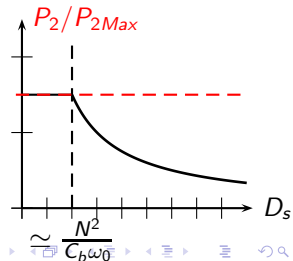
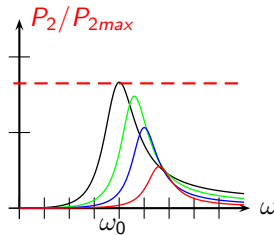
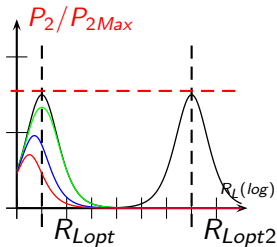
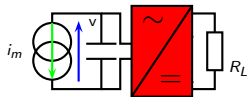


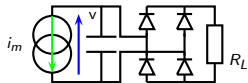
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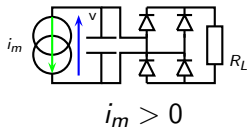
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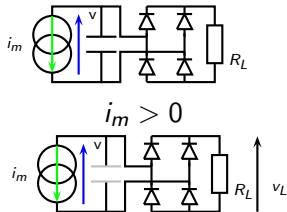
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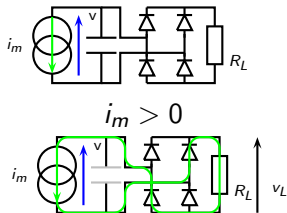
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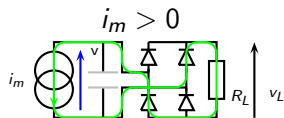
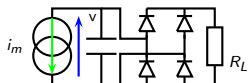
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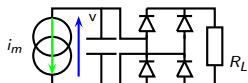
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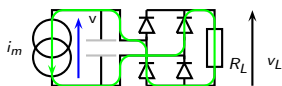
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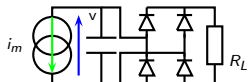


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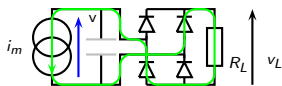
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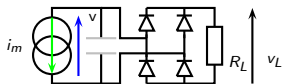
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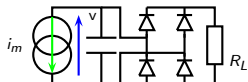
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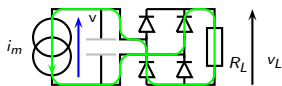
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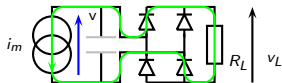
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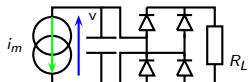
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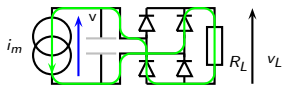
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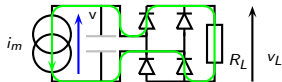
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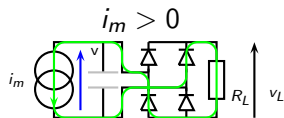
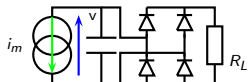


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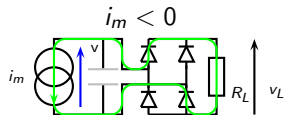
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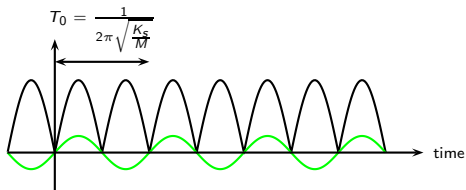
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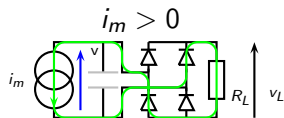
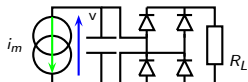
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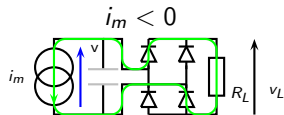
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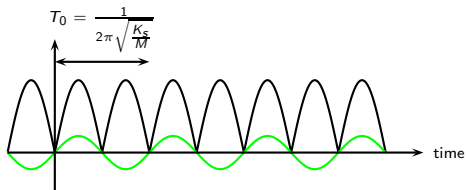
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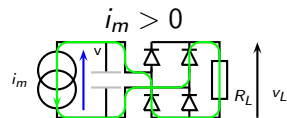
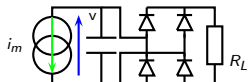


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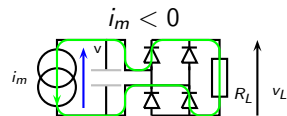
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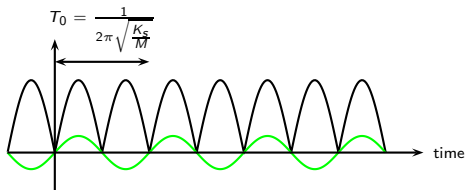
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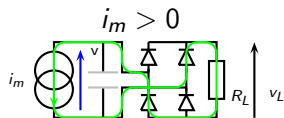
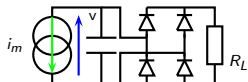
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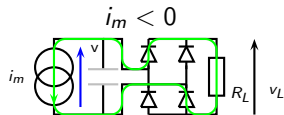
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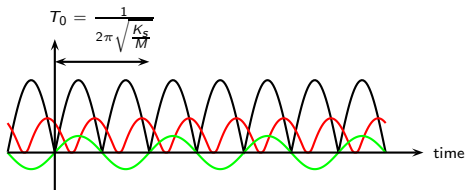
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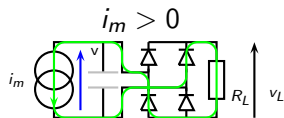
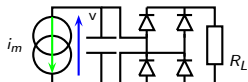
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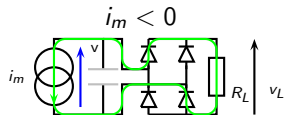


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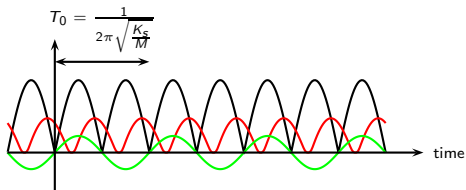
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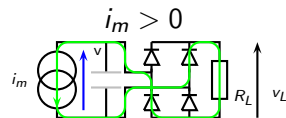
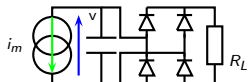


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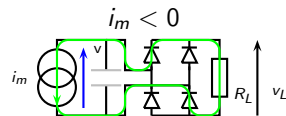


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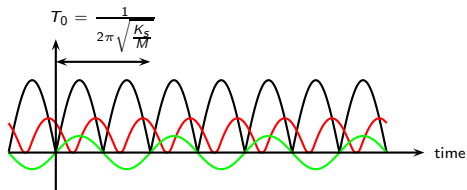
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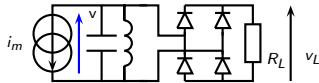
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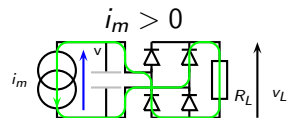
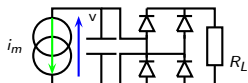
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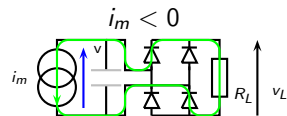
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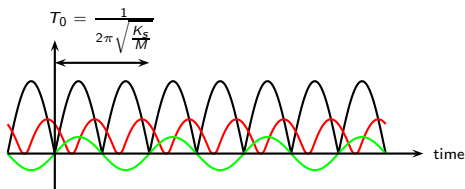
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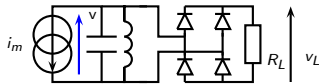
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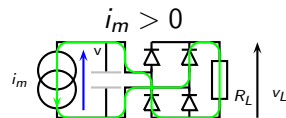
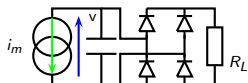


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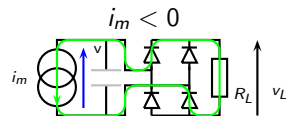


This is a bad solution because it works only for ω_0 (what if the frequency shifts?), and the Inductor is large (because ω_0 usually is small)

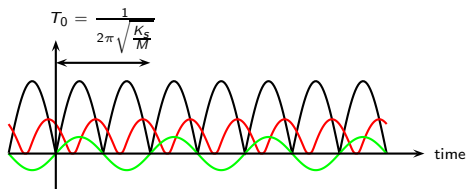
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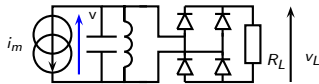
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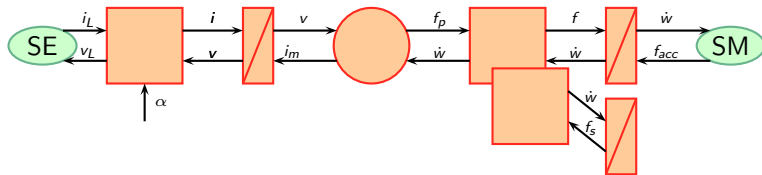
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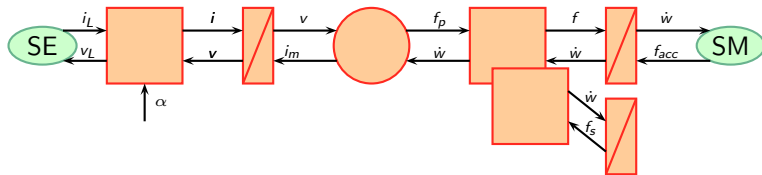
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→ Non linear Techniques

Why Synchronized

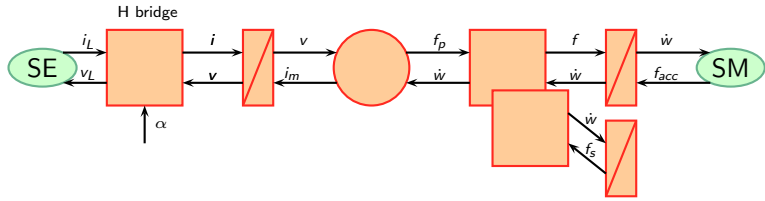
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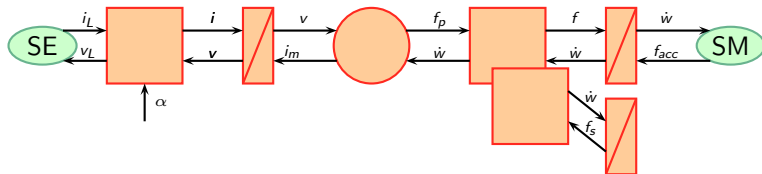
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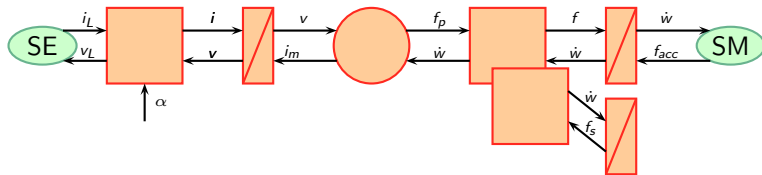


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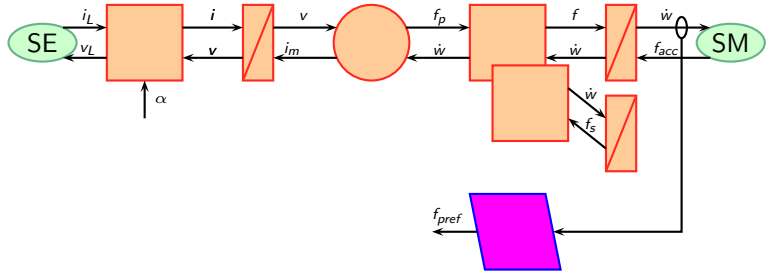
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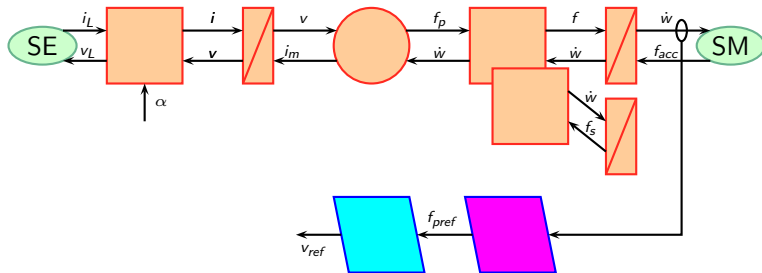
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Why Synchronized



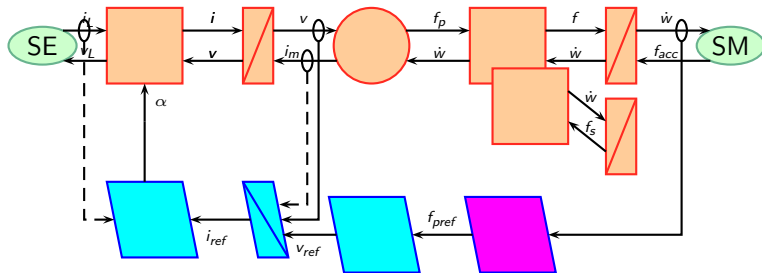
Strategy: $v = -R_{opt}i_m$ leads to $\frac{f_p}{N} = -\frac{D_s}{N^2}N\dot{w}$ or, $f_p = -D_s\dot{w}$.

Why Synchronized



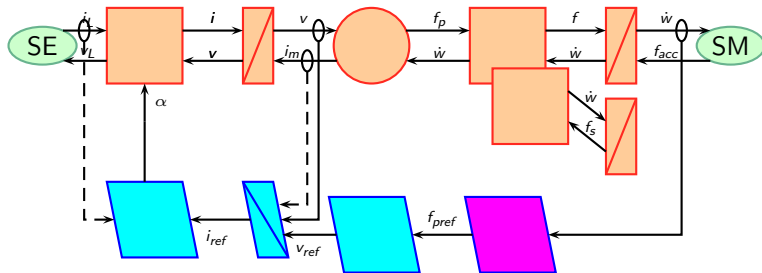
Strategy: $v = -R_{opt}i_m$ leads to $\frac{f_p}{N} = -\frac{D_s}{N^2}N\dot{w}$ or, $f_p = -D_s\dot{w}$.

Why Synchronized



Strategy: $v = -R_{opt} i_m$ leads to $\frac{f_p}{N} = -\frac{D_s}{N^2} N \dot{w}$ or, $f_p = -D_s \dot{w}$.
 This shows that v should be controlled.

Why Synchronized



Strategy: $v = -R_{opt} i_m$ leads to $\frac{f_p}{N} = -\frac{D_s}{N^2} N \dot{w}$ or, $f_p = -D_s \dot{w}$.
 This shows that v should be controlled. SSHI does this with efficiency.

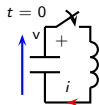
How it works

LC oscillations



How it works

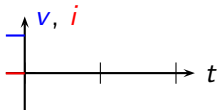
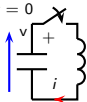
LC oscillations



How it works

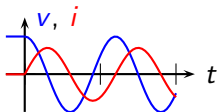
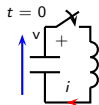
LC oscillations

$t = 0$



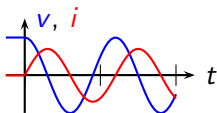
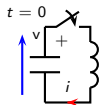
How it works

LC oscillations



How it works

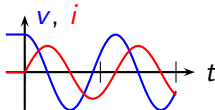
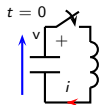
LC oscillations



Switched inductor

How it works

LC oscillations

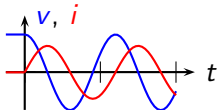
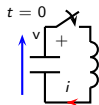


Switched inductor

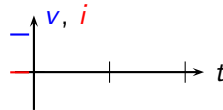
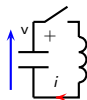


How it works

LC oscillations

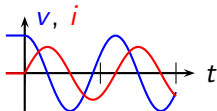
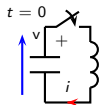


Switched inductor

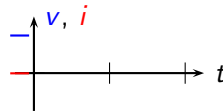


How it works

LC oscillations

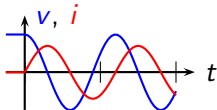
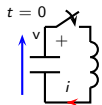


Switched inductor

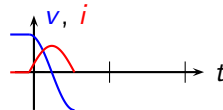


How it works

LC oscillations

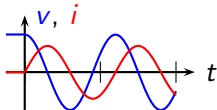
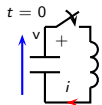


Switched inductor

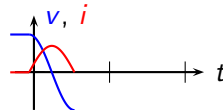
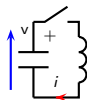


How it works

LC oscillations

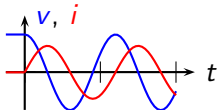
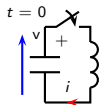


Switched inductor

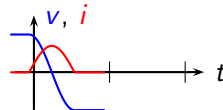
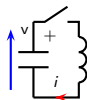


How it works

LC oscillations

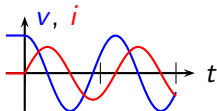
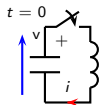


Switched inductor

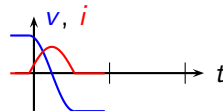


How it works

LC oscillations

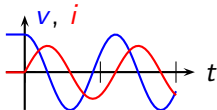
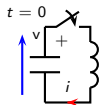


Switched inductor

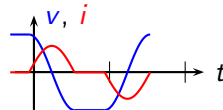


How it works

LC oscillations

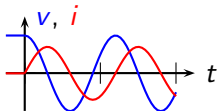
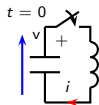


Switched inductor

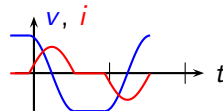
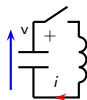


How it works

LC oscillations

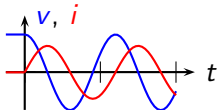
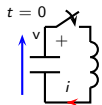


Switched inductor

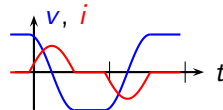
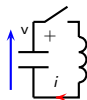


How it works

LC oscillations

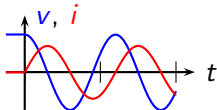
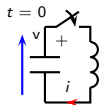


Switched inductor

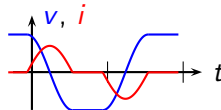
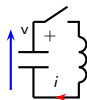


How it works

LC oscillations



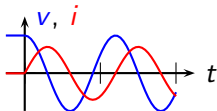
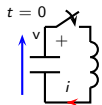
Switched inductor



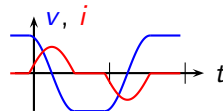
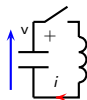
SSHI

How it works

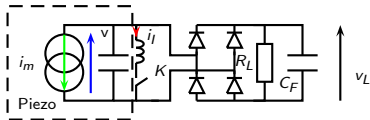
LC oscillations



Switched inductor

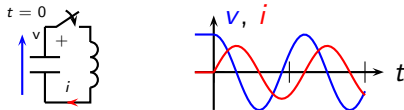


SSHI

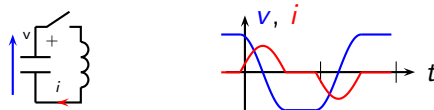


How it works

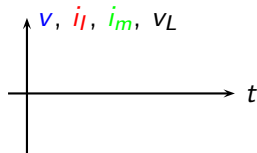
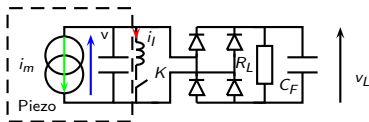
LC oscillations



Switched inductor

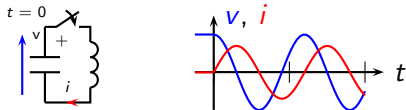


SSHI

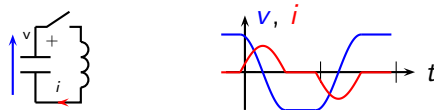


How it works

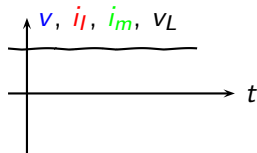
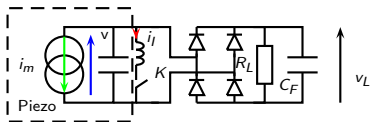
LC oscillations



Switched inductor

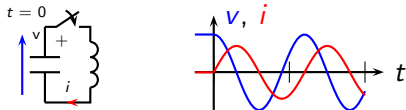


SSHI

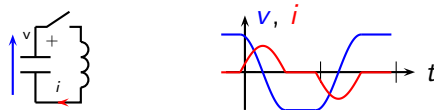


How it works

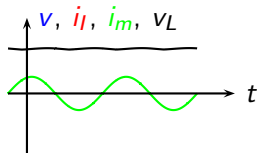
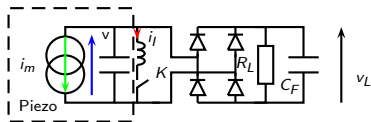
LC oscillations



Switched inductor

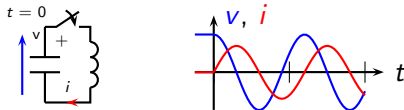


SSHI

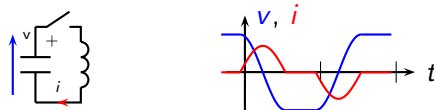


How it works

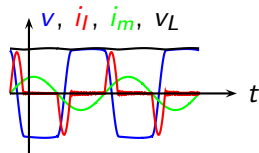
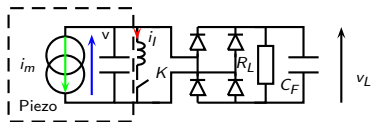
LC oscillations



Switched inductor

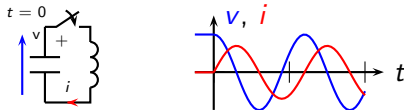


SSHI

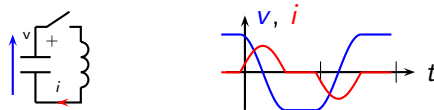


How it works

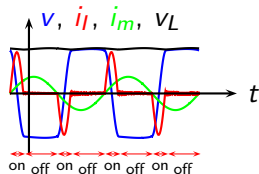
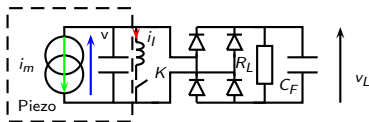
LC oscillations



Switched inductor



SSHI

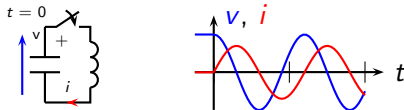


Switching is synchronized on \dot{w} , or

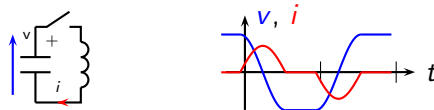
$$i_m = N\dot{w}$$

How it works

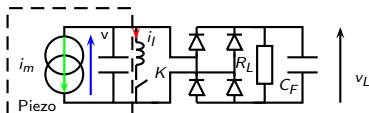
LC oscillations



Switched inductor

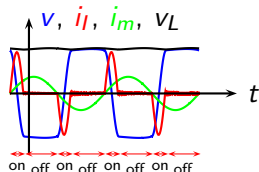


SSHI



Operating point: calculate v_L from i_m

1st Harmonic assumption: $P = \frac{1}{2} \frac{V_L^2}{R_L} \simeq \frac{1}{2} \frac{4V_L I_m}{\pi}$, $V_L = \frac{4}{\pi} R_L I_m$

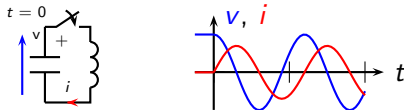


Switching is synchronized on \dot{w} , or

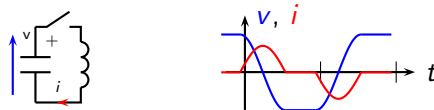
$$i_m = N\dot{w}$$

How it works

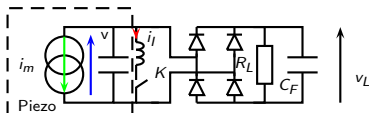
LC oscillations



Switched inductor



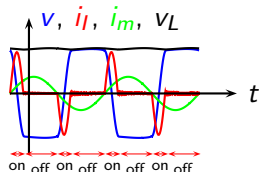
SSHI



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SSHI controls v and synchronises it, but doesn't impose $f_p = -D_s \dot{w}$.



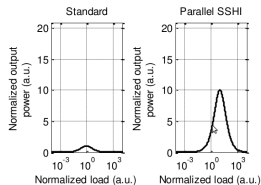
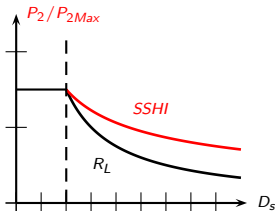
Switching is synchronized on \dot{w} , or

$$i_m = N \dot{w}$$

Conclusion

Performances

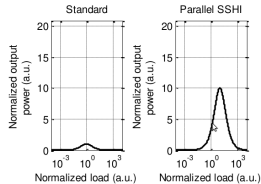
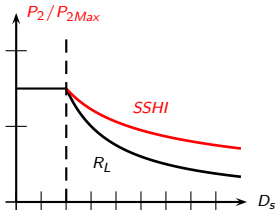
- SSHI can extract energy more efficiently than a resistor when damping is important,



Conclusion

Performances

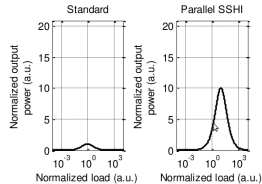
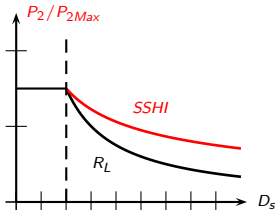
- SSHI can extract energy more efficiently than a resistor when damping is important,
- But Power extraction still depends on the load,



Conclusion

Performances

- SSHI can extract energy more efficiently than a resistor when damping is important,
- But Power extraction still depends on the load,
- Needs to measure bender's deflection $w(t)$.



General conclusion




In this presentation, applications of Energy Harvesting were shown. The modelling of a piezoelectric generator has shown that the power source needs an adaptation:

- in frequency,
- in load.

to maximize the harvested power.

The key energy management rules were presented through the analysis of the EMR of the system. A typical power electronic circuit was also presented, but the bibliography shows a lot of example.

References I

-  S Adhikari, M I Friswell, and D J Inman, *Piezoelectric energy harvesting from broadband random vibrations*, Smart Materials and Structures **18** (2009), no. 11, 115005.
-  R. G Ballas, H. F Schlaak, and A. J Schmid, *Closed form analysis of piezoelectric multilayer bending actuators using constituent equations*, The 13th International Conference on Solid-State Sensors, Actuators and Microsystems, 2005. Digest of Technical Papers. TRANSDUCERS '05, vol. 1, IEEE, June 2005, pp. 788–791 Vol. 1.
-  R. Djugum, P. Trivailo, and K. Graves, *A study of energy harvesting from piezoelectrics using impact forces*, The European Physical Journal Applied Physics **48** (2009), no. 1, 11101.
-  Daniel Guyomar, Gal Sebald, Sbastien Pruvost, Mickal Lallart, Akram Khodayari, and Claude Richard, *Energy harvesting from ambient vibrations and heat*, Journal of Intelligent Material Systems and Structures **20** (2009), no. 5, 609–624.
-  Aman Kansal, Jason Hsu, Sadaf Zahedi, and Mani B. Srivastava, *Power management in energy harvesting sensor networks*, ACM Transactions on Embedded Computing Systems **6** (2007), 32–es.
-  Elie Lefeuvre, Adrien Badel, Claude Richard, and Daniel Guyomar, *Piezoelectric energy harvesting device optimization by synchronous electric charge extraction*, Journal of Intelligent Material Systems and Structures **16** (2005), no. 10, 865–876.
-  H. Lhermet, C. Condemine, M. Plissonnier, R. Salot, P. Audebert, and M. Rosset, *Efficient power management circuit: From thermal energy harvesting to above-IC microbattery energy storage*, IEEE Journal of Solid-State Circuits **43** (2008), no. 1, 246–255.

References II



G. A. Lesieutre, G. K. Ottman, and H. F. Hofmann, *Damping as a result of piezoelectric energy harvesting*, Journal of Sound and Vibration **269** (2004), no. 3-5, 991–1001.



D. Niyato, E. Hossain, M. M Rashid, and V. K Bhargava, *Wireless sensor networks with energy harvesting technologies: a game-theoretic approach to optimal energy management*, IEEE Wireless Communications **14** (2007), no. 4, 90–96.



Shad Roundy, Paul Kenneth Wright, and Jan M. Rabaey, *Energy scavenging for wireless sensor networks: with special focus on vibrations*, Springer, 2003.



G. Sebald, E. Lefeuvre, and D. Guyomar, *Pyroelectric energy conversion: Optimization principles*, IEEE Transactions on Ultrasonics, Ferroelectrics and Frequency Control **55** (2008), no. 3, 538–551.



Qing-Ming Wang and L. E Cross, *Constitutive equations of symmetrical triple layer piezoelectric benders*, IEEE Transactions on Ultrasonics, Ferroelectrics and Frequency Control **46** (1999), no. 6, 1343–1351.

End of the presentation

Questions?