Bond Graphs - A graphical language for the analysis of multiphysical systems

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Bond Graphs

A graphical language for the analysis of multiphysical systems

Pierre Haessig, CentraleSupélec (campus of Rennes)
Slide deck 1: the language
Course outline

- Bond graph objectives
- The bond graph language
  - Bonds and power variables: the physical analogy
  - Elements
- Practice: reading & creating bond graphs
- Causality and derivation of mathematical models
Bond graph objectives

- a simple unified graphical language for many physical domains
- acausal models
  - to preserve the physical structure of the real physical system
  - which highlight energy exchange
- but with the (optional) superposition of a *computational causality* information
Bond graph model structure is *hybrid*

Two extremal structures of model:

1. Block diagrams, with very clear computational structure
   - but lost physical structure
2. Physical network-type diagrams (electrical, mechanical)
   - but no computational information

\[ U : \text{Se} \quad \rightarrow \quad 1 \quad \rightarrow \quad 0 \quad \rightarrow \quad C : \text{C} \]

\[ \downarrow \quad \downarrow \]

\[ I : \text{L} \quad \quad R : \text{R} \]
Model comparisons

BG, Circuit (acausal), Block diagram (causal)

\[ \text{U : Se} \rightarrow 1 \rightarrow 0 \rightarrow \text{C : C} \]

\[ \text{I : L} \rightarrow \text{R : R} \]
The bond graph language

- Bonds & power variables
- Elements
Bonds

Bonds model the physical interaction of two elements which exchange energy.

The interaction happens through two generalized physical variables: effort & flow, collectively named the “power variables”.

\[ \text{Effort} \quad \text{Flow} \]
Physical analogy in bond graphs

Each physical domain has a specific choice for the generalized *effort* and *flow* variables of each bond:

<table>
<thead>
<tr>
<th>Physical Domain</th>
<th>Effort $e$</th>
<th>Flow $f$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Translational mechanics</td>
<td>Force $F$ (N)</td>
<td>Velocity $v$ (m/s)</td>
</tr>
<tr>
<td>Rotational mechanics</td>
<td>Torque $\Gamma$ (N.m)</td>
<td>Angular velocity $\omega$ (rad/s)</td>
</tr>
<tr>
<td>Electricity</td>
<td>Voltage $u$ (V)</td>
<td>Current $i$ (A)</td>
</tr>
<tr>
<td>Thermal transfers</td>
<td>Temperature $T$ (K)</td>
<td>Entropy flow rate $\dot{S}$ (J/K/s)</td>
</tr>
<tr>
<td>Hydraulic</td>
<td>Pressure $P$ (N/m²)</td>
<td>Volume flow rate $Q_v$ (m³/s)</td>
</tr>
</tbody>
</table>

**Property:** the two variables of each bond are chosen such that:

$$Effort \times Flow = Power \ (\text{Watt})$$
## Reminder: Modelica's physical analogy

Modelica's analogy is based on the port's *connection* behavior

<table>
<thead>
<tr>
<th>Potential</th>
<th>Flow</th>
</tr>
</thead>
<tbody>
<tr>
<td>Translat. mech.</td>
<td><strong>Position</strong> $s$ (m)</td>
</tr>
<tr>
<td>Rotational mech.</td>
<td>Angular <strong>position</strong> $\varphi$ (rad)</td>
</tr>
<tr>
<td>Electricity</td>
<td>Voltage $u$ (V)</td>
</tr>
<tr>
<td>Thermal transfers</td>
<td>Temperature $T$ (K)</td>
</tr>
<tr>
<td>Hydraulic</td>
<td>Pressure $P$ (N/m²)</td>
</tr>
</tbody>
</table>

⚠ Differences with bond graph:

- Force and Torque are **switched**: BG's Effort $\rightarrow$ Modelica's Flow
- Different vocabulary in **bold**: in particular position vs speed
Comparison with Modelica's analogy

BG & Modelica:
- both introduce an analogy between variables across different physical domains
- but using a different classification & vocabulary, because each is built on different foundations

BG's analogy: group variables as “effort” or “flow”
- by preserving common physical sense (ex.: voltage ↔ force, current ↔ speed)
- with constraint $e \times f = Power$

Modelica's analogy: group variables as “potential” or “flow”
- by preserving the connection topology of graphical diagrams (ex.: voltage ↔ position because both are equal at interconnection of ports)
- (with no constraint on the product $e \times f$)
# Relation between BG's and Modelica's analogies

<table>
<thead>
<tr>
<th>Domain</th>
<th>Mechanics</th>
<th>Others (e.g. electricity)</th>
</tr>
</thead>
<tbody>
<tr>
<td>BG effort</td>
<td>Mod flow</td>
<td>Mod potential</td>
</tr>
<tr>
<td>BG flow</td>
<td>Mod der(potential)</td>
<td>Mod flow (*)</td>
</tr>
</tbody>
</table>

**Observations:**

- for all domains except mechanics, we have:
  - BG effort = Modelica potential
  - BG flow = Modelica flow (*)
- but for mechanics:
  - it's reversed
  - and we have an extra derivative: speed = der(position)
- (*) thermal domain is an exception: heat flow (J/s) vs entropy flow (J/K/s)
Bonds

- Exchange of energy between elements
- Half arrow = direction of positive power flow
- Drawing conventions:
  - e above/left, f below/right
  - half arrow on the side of the flow (i.e. below/right)
Bond graph elements

Energy stores
- I store
- C store

Dissipators
- R (resistor)

Sources
- Se (effort source)
- Sf (flow source)

Junctions
- 0-junction
- 1-junction
- Transformer
- Gyrator

conserve power
Junctions

Unlike in network-type diagrams, the connection of elements is not achieved with the topology of links (e.g. loops of wires), but using explicitly one of the two junction elements:

- “0” junction: common effort
- “1” junction: common flow

Also in the junction category: transformers and gyrators

Common property: **instantaneous power is conserved**
0 junction

“Common effort” junction:

\[ e_1 = e_2 = \ldots = e_n \]

Flows are distributed (incoming sum = outgoing sum), according to the orientation of the bonds. On the example:

\[ f_1 = f_2 + \ldots + f_n \]
1 junction

“Common flow” junction:

\[ f_1 = f_2 = \ldots = f_n \]

Efforts are distributed (incoming sum = outgoing sum), according to the orientation of the bonds. On the example:

\[ e_1 = e_2 + \ldots + e_n \]
Two-port junctions: transformers & gyrators

Transmit power with a scaling of efforts & flows:

- in the same domain (unitless scaling)
- between two domains (scaling with a physical unit)
Transformer (TF)

\[ f_2 = m.f_1 \]
\[ m.e_2 = e_1 \]

\( m \) is the transformer ratio

Examples:

- Mechanical gear pair: \( \omega_2 = (r_2/r_1).\omega_1 \)
- Cable — Pulley: \( v = r.\omega \)
- Electrical transformer: \( v_1 = m.v_2 \) (⚠ inverted definition of the transformer ratio)
**Gyrator (GY)**

\[ e_2 = r \cdot f_1 \]
\[ e_1 = r \cdot f_2 \]

\( r \) is the gyrator ratio

**Example:**
- EMF of a DC motor: \( e = K \cdot \Omega \) and \( C = K \cdot i \)
Modulation of Transformers (Gyrators)

The transformer (gyrator) can be modulated by a signal (using a signal arrow $\rightarrow$):

$\begin{align*}
  &m(t) & &r(t) \\
  \frac{e_1}{f_1} \xrightarrow{MTF} &\frac{e_2}{f_2} \\
  \frac{e_1}{f_1} \xrightarrow{MGY} &\frac{e_2}{f_2}
\end{align*}$

Examples: crank-slider mechanism, averaged DC-DC converter.
One-port elements

There are 3 basic energy consuming/storing devices:

- **Dissipator**: resistor
- **Energy stores**:
  - **C** store, also called **Compliance** or **Capacitor**
  - **I** store, also called **Inertia**

In addition, there are two sources: **Se** (effort) and **Sf** (flow source)

Remark: in electricity, *one-port* element = device with *two* electrical pins
Resistor (R)

Relation (linear case):

\[ e = R \cdot f \]

Property: power is irreversibly dissipated (as heat)

Examples:
- Electrical resistor: \( u = R \cdot i \)
- Mechanical damper: \( f = d \cdot v \)
C energy store (also called Compliance or Capacitor)

Using the “generalized displacement” $q$ (an “energy” variable):

$$q = \int f \, dt$$

C store relation:

$$q = \Phi_C(e)$$

Linear C store:

$$q = C \cdot e$$

→ consequence: $f = C \cdot \frac{de}{dt}$
C store examples (linear)

Mechanics:
- displacement = (kinematic) displacement $x = \int v \cdot dt$
- relation: $x = (1/k).f$
- C store = spring

Electricity:
- displacement = charge $q = \int i \cdot dt$ (Coulomb)
- relation $q = C.u$
- C store = (electrical) capacitor
I energy store (also called Inertia)

Using the “generalized momentum” \( p \) (an “energy” variable):

\[
p = \int e \, dt
\]

I store relation:

\[
p = \Phi_I(f)
\]

Linear I store:

\[
p = I \cdot f
\]

→ consequence: \( e = I \cdot \frac{df}{dt} \)
I store examples (linear)

Mechanics:
• momentum = mechanical momentum \( p = \int f \, dt \)
• relation: \( p = m.v \), that is \( f = m.\frac{dv}{dt} \) (inertial force)
• I store = mechanical \textbf{inertia}

Electrity:
• momentum = magnetic flux linkage \( \lambda = \int u \, dt \) (\( V.s = Wb \))
• relation: \( \lambda = L.i \), that is \( v = L.\frac{di}{dt} \)
• I store = \textbf{inductor}
Parametrization of R, C, I elements

The value of a linear R/C/I element is appended with the notation “: x”.

Same notation is used for the value of a source (next slide).
Sources

Sources either impose the effort (Se) or the flow (Sf).

Se examples:
- Electricity: voltage source
- Mechanics: imposed torque or force

Sf examples:
- Electricity: current source
- Mechanics: imposed speed
Bond Graphs practice

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Slide deck 2: practice
Course outline

- Bond graph objectives
- The bond graph language
  - Bonds and power variables: the physical analogy
  - Elements
- Practice: reading & creating bond graphs
- Causality and derivation of mathematical models
Practice

Objectives: being able to

1. **Read** BGs
   - recognize classical structure

2. **Create** BGs from network diagrams (electrical, mechanical)
M1: 1 junctions represent “velocity points”

Questions:
- Fill the blanks on the BG
- Represent the physical system
- Derive the overall mechanical equation of the system
M1: 1 junctions represent “velocity points” (S)

This BG represents the free fall of a mass $m$ with equation:

$$m \cdot \frac{dv_m}{dt} = -m \cdot g$$
M2: Springs and dampers typically on 0 junctions

\[ v_a \quad 0 \quad v_b \]

\[ 1 \quad 0 \quad 1 \]

\[ F_k \]

\[ C : 1/k \]

Questions:

- Fill the blanks on the BG
- Represent the physical system
- Give the expression of \( F_k \)
M2: Springs and dampers typically on 0 junctions (Sol.)

This BG represents a spring of stiffness $k$ between two points A and B.

$$F_k = k \int (v_a - v_b) \, dt = k(x_a - x_b)$$
Mechanical BGs: main structures

- velocity points represented by 1 junctions
  - an Inertia can attach directly to its corresponding 1 junction
- components which react on a velocity/position differences (spring and dampers) are typically connected to a 0 junction placed between two 1 junctions
E1: 0 junctions represent voltage nodes

Questions:
- Fill the blanks on the BG
- Represent the physical system
E1: 0 junctions represent voltage nodes (Sol.)
E2: 1 junction for voltage drop on a RLC component

Questions:

- Fill the blanks on the BG
- Represent the physical system
- Compute $i_r$
E2: 1 junction for voltage drop on a RLC component (Sol.)

\[ i_r = \frac{e_a - e_b}{r} \]
E3: Voltage source

...
Bond Graphs practice

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Slide deck 3: causality
Course outline

- Bond graph objectives
- The bond graph language
  - Bonds and power variables: the physical analogy
  - Elements
- Practice: reading & creating bond graphs
- Causality and derivation of mathematical models
  - Principles
  - Practice
Computational causality & derivation of mathematical models
Causally completed bonds

Reminder: bonds express acausal physical links between components

Optionnaly, bonds can be completed with "causality stroke/arrow" which indicate the direction of the computation flow.

Remark: this computational causality doesn't express a physical cause → consequence relationship
Meaning and significance of causality in BGs

BG to block diagrams fragments

A \[\begin{array}{c}
\text{e} \\
\text{f}
\end{array}\] B

A

\[\begin{array}{c}
\text{e} \\
\text{f}
\end{array}\]

B

Mnemonic: the causality arrow is the direction of the *effort signal arrow*

To be continued...
Rules for each class of BG components

- sources
- junctions
- energy stores & dissipators
Causality for Sources

For sources, causality orientation is **compulsory**:

- Se imposes the effort \( \rightarrow \) outgoing effort
- Sf imposes the flow \( \rightarrow \) outgoing flow
Causality for junctions: 0 junction

Compulsory rule

For 0 junction (share effort): “One and only one incoming effort”

Remark: causality arrows unrelated to bond orientations
Causality for junctions: 1 junction

Compulsory rule

For 1 junction (share flow): “One and only one incoming flow

Remark: causality arrows unrelated to bond orientations
Causality for junctions: TF

\[ e_1 \quad f_1 \quad m \quad f_2 \quad e_2 \]

[Compulsory] Transformers \textbf{preserve} the direction of the e&f signals

- one incoming effort
- one incoming flow
Causality for junctions: GY

[Compulsory] Gyrators reverse the direction of the e&f signals

• either two incoming efforts
• or two incoming flows
Causality for Energy stores

C and I energy stores can accept the two possible causalities:

- **Integral causality** is the one yielding an integrator block
- **Derivative causality** is the one yielding an derivator block

For computational easiness, integrators are prefered, so

Integral causality = **Preferred** causality
Causality for Energy stores: C

Integral causality: receives a flow, imposes an effort

- Capacitor: current → voltage
- Spring: speed/position → force
Causality for Energy stores: I

Integral causality: receives a flow, imposes an effort

- Inductor: voltage $\rightarrow$ current
- Intertia: force $\rightarrow$ speed
Causality for $R$ (dissipators)

For dissipators, all orientations are fine.

Sometimes called “resistance” vs “conductance” causality.
Sequential assignment rules

SCAP procedure:

Rule 1: assign sources

Rule 2: assign integral causality to Energy stores
Causality assignment examples

- Ex1: Rule 1 (sources+propagation) is enough
- Ex2: Rule 1+2 (prefered causality for energy stores) are enough
Bond Graphs practice

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Slide deck 4: causality practice
Course outline

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- Practice: reading & creating bond graphs
- Causality and derivation of mathematical models
  - Principles
  - Practice
Causality assignment Ex1

Se → V → 0 → C

R

V

C

R

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Causality assignment Ex1 (Sol.)

1. R1: Se sets the effort on the 0 junction
2. Propagation: effort propagates through the 0 junction
Causality assignment Ex2

\[ \text{Sf} \xrightarrow{I} 0 \xrightarrow{} C \]

\[ \text{R} \]

\[ \text{I} \]

\[ \text{C} \quad \text{R} \]
1. R1: Sf sets the flow on the 0 junction.
   ○ (no propagation)

2. R2: Set integral causality for C
   ○ → C sets the effort on the 0 junction

3. Propagation of effort to R through the 0 junction
Impossible connections exercise

Se $\rightarrow$ $V_1$ $0$ $\rightarrow$ $I_3$ $Sf$

$V_2$

Se

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Impossible connections exercise (Sol.)

The physical impossibility of parallel voltage sources emerges as a causality conflict on the 0 junction ("only one incoming effort allowed").