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# Bond Graphs - A graphical language for the analysis of multiphysical systems

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# Bond Graphs

*A graphical language for the analysis of multiphysical systems*

→

Pierre Haessig, CentraleSupélec (campus of Rennes)

SG6: Dec. 2021 – Jan. 2022

Slide deck 1: the language

# Course outline

- **Bond graph objectives**
- **The bond graph language**
  - Bonds and power variables: the physical analogy
  - Elements
- Practice: reading & creating bond graphs
- Causality and derivation of mathematical models

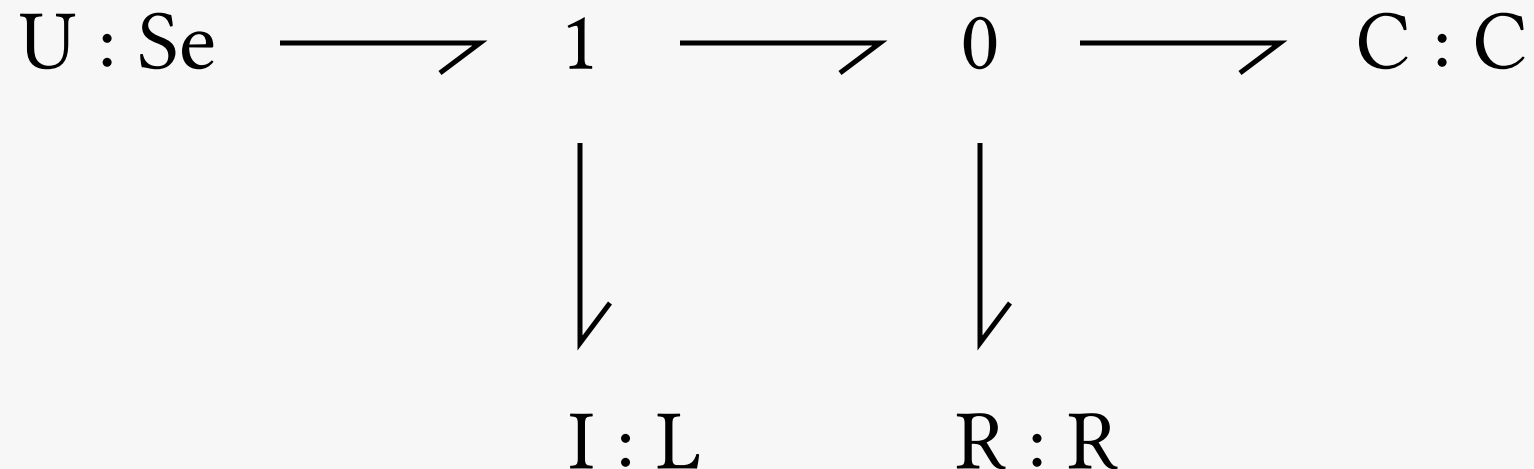
# Bond graph objectives

- a simple unified graphical language for many physical domains
- acausal models
  - to preserve the physical structure of the real physical system
  - which highlight energy exchange
- but with the (optional) superposition of a *computational causality* information

# Bond graph model structure is *hybrid*

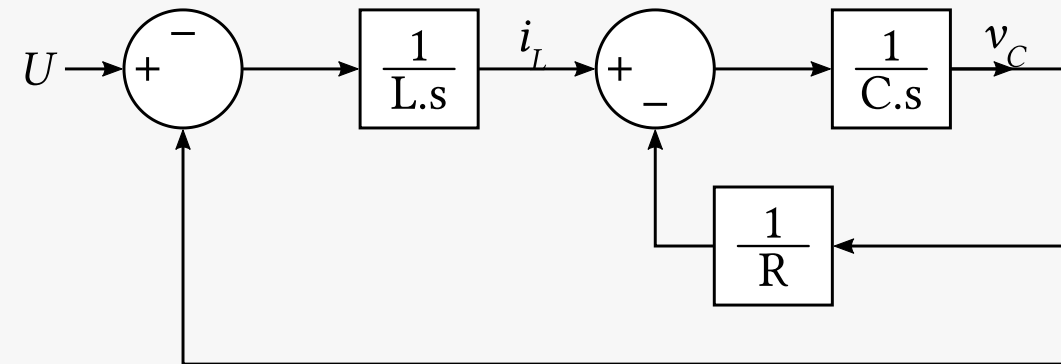
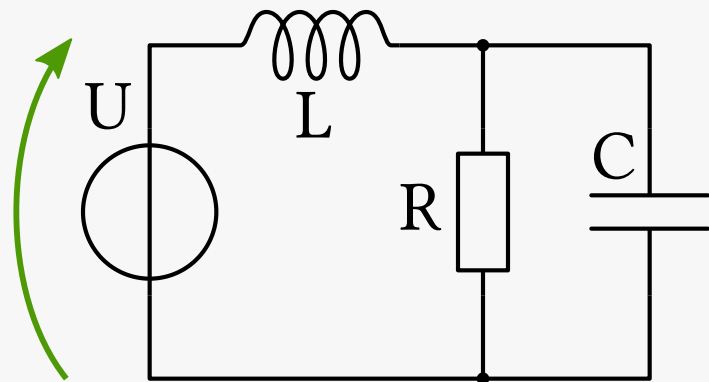
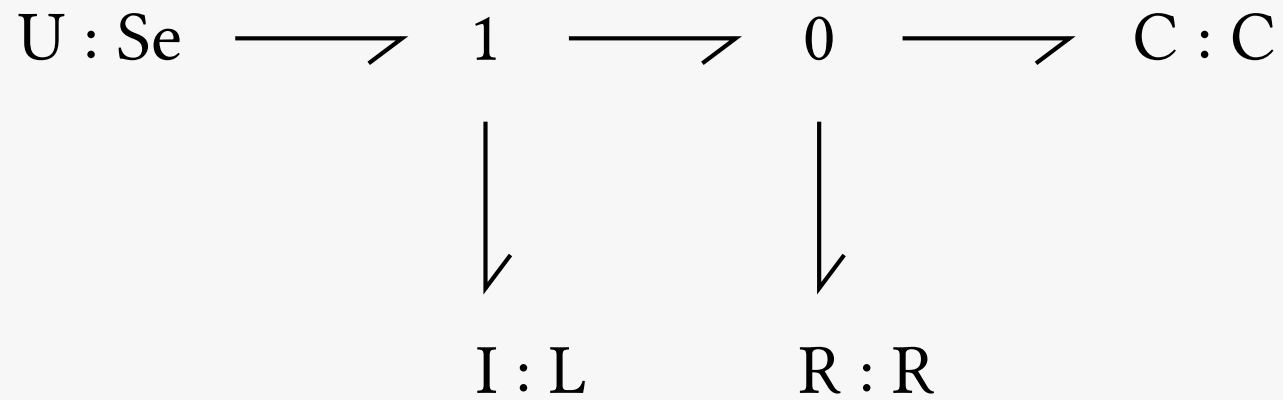
Two extremal structures of model:

1. Block diagrams, with very clear computational structure
  - but lost physical structure
2. Physical network-type diagrams (electrical, mechanical)
  - but no computational information



# Model comparisons

BG, Circuit (acausal), Block diagram (causal)

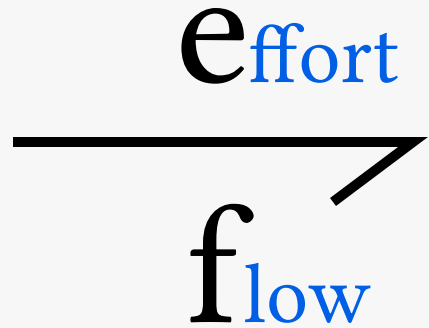


# The bond graph language

- Bonds & power variables
- Elements

# Bonds

Bonds model the physical interaction of **two** elements which **exchange energy**.



The interaction happens through two generalized physical variables: *effort* & *flow*, collectively named the “power variables”.



# Physical analogy in bond graphs

Each physical domain has a specific choice for the generalized *effort* and *flow* variables of each bond:

	<b>Effort <math>e</math></b>	<b>Flow <math>f</math></b>
Translational mechanics	Force $F$ (N)	Velocity $v$ (m/s)
Rotational mechanics	Torque $\Gamma$ (N.m)	Angular velocity $\omega$ (rad/s)
Electricity	Voltage $u$ (V)	Current $i$ (A)
Thermal transfers	Temperature $T$ (K)	Entropy flow rate $\dot{S}$ (J/K/s)
Hydraulic	Pressure $P$ (N/m <sup>2</sup> )	Volume flow rate $Q_v$ (m <sup>3</sup> /s)

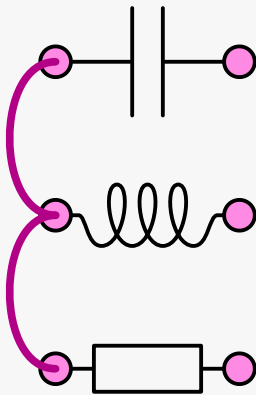
**Property:** the two variables of each bond are chosen such that:

$$Effort \times Flow = Power \text{ (Watt)}$$

# Reminder: Modelica's physical analogy

Modelica's analogy is based on the port's *connection* behavior

	Potential	Flow
Translat. mech.	<b>Position</b> $s$ (m)	Force $f$ (N)
Rotational mech.	Angular <b>position</b> $\varphi$ (rad)	Torque $\Gamma$ (N.m)
Electricity	Voltage $u$ (V)	Current $i$ (A)
Thermal transfers	Temperature $T$ (K)	<b>Heat</b> flow $Q$ (J/s=W)
Hydraulic	Pressure $P$ (N/m <sup>2</sup> )	<b>Mass</b> flow rate $Q_m$ (kg/s)



$$\begin{aligned} V_1 &= V_2 = V_3 \\ i_1 + i_2 + i_3 &= 0 \end{aligned}$$

⚠ Differences with bond graph:

- Force and Torque are **switched**: BG's Effort → Modelica's Flow
- Different vocabulary in **bold**: in particular position vs speed

# Comparison with Modelica's analogy

BG & Modelica:

- both introduce an analogy between variables across different physical domains
- but using a different **classification** & **vocabulary**, because each is built on different foundations

**BG's** analogy: group variables as “*effort*” or “*flow*”

- by *preserving common physical sense* (ex.: voltage  $\leftrightarrow$  force, current  $\leftrightarrow$  speed)
- with constraint  $e \times f = \text{Power}$

**Modelica's** analogy: group variables as “*potential*” or “*flow*”

- by *preserving the connection topology* of graphical diagrams (ex.: voltage  $\leftrightarrow$  position because both are equal at interconnection of ports)
- (with no constraint on the product  $e \times f$ )

# Relation between BG's and Modelica's analogies

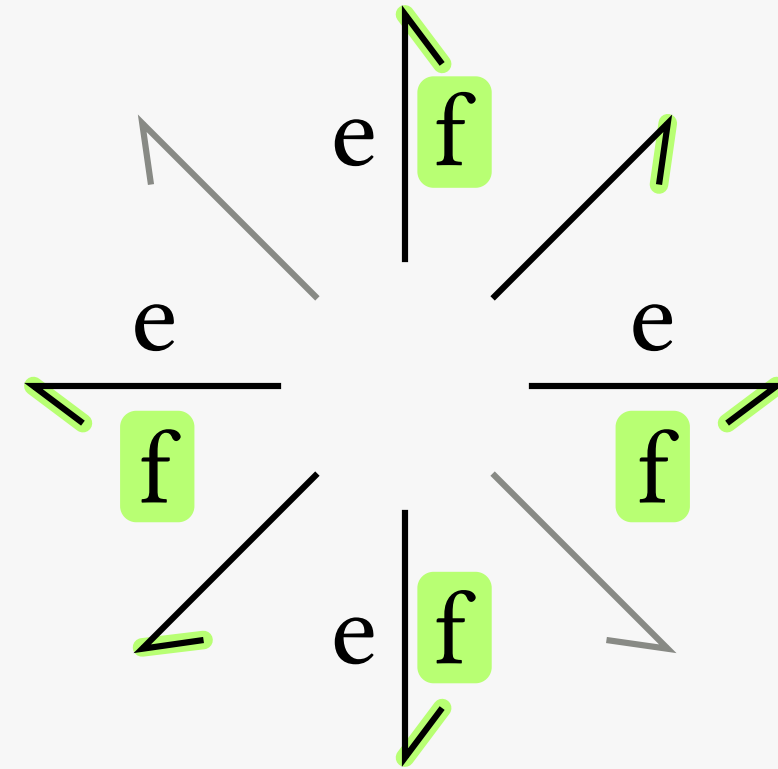
Domain	Mechanics	Others (e.g. electricity)
BG effort	Mod flow	Mod potential
BG flow	Mod der(potential)	Mod flow (*)

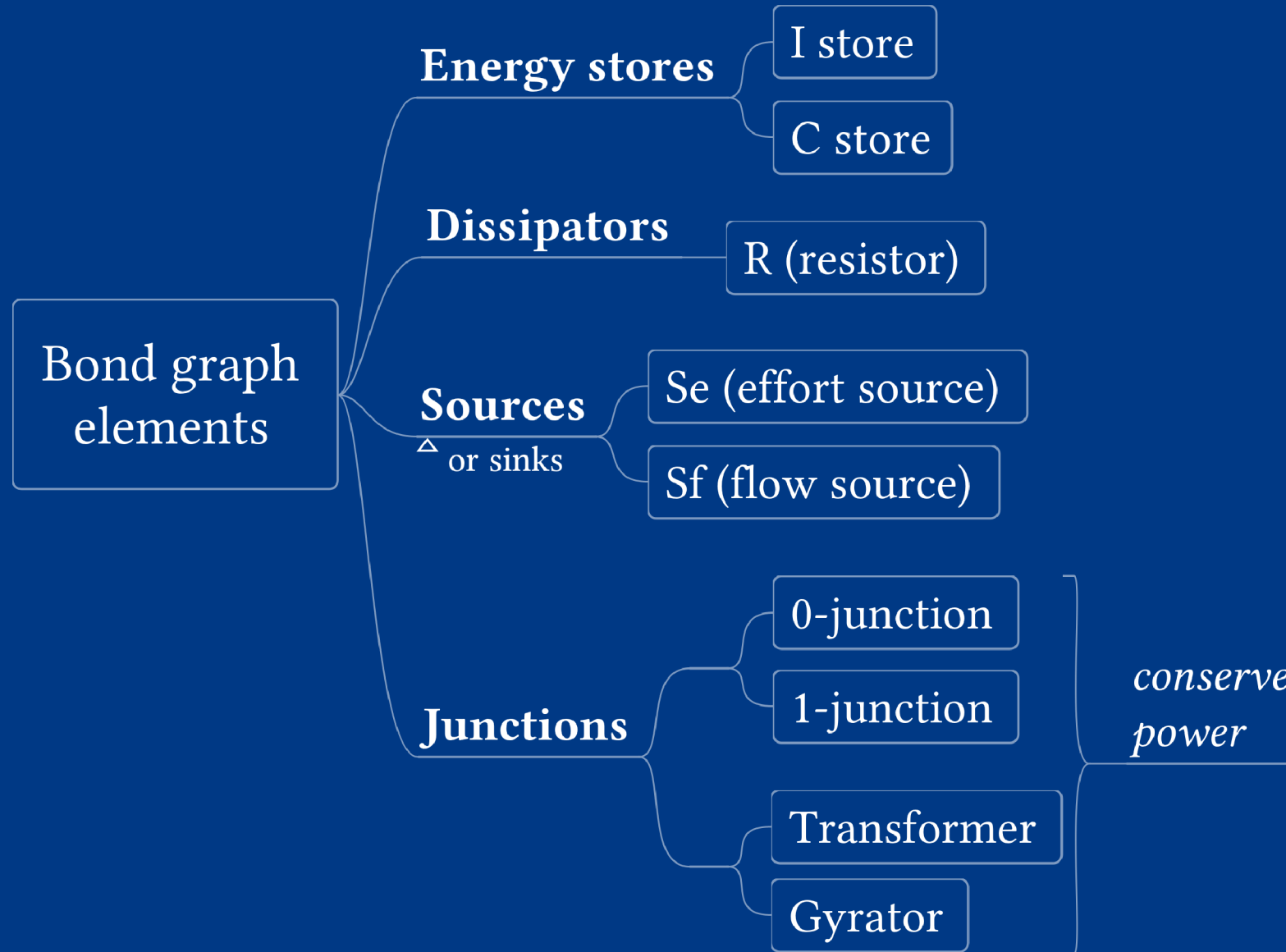
Observations:

- for all domains except mechanics, we have:
  - BG effort = Modelica potential
  - BG flow = Modelica flow (\*)
- but for mechanics:
  - it's reversed
  - and we have an extra derivative: speed = der(position)
- (\*) thermal domain is an exception : heat flow (J/s) vs entropy flow (J/K/s)

# Bonds

- Exchange of energy between elements
- Half arrow = direction of positive power flow
- Drawing conventions:
  - e above/left, f below/right
  - half arrow on the side of the flow (i.e. below/right)





# Junctions

Unlike in network-type diagrams, the connection of elements is not achieved with the topology of links (e.g. loops of wires), but using explicitly one of the two junction elements:

- “0” junction: common effort
- “1” junction: common flow

Also in the junction category: transformers and gyrators

Common property: **instantaneous power is conserved**

# 0 junction

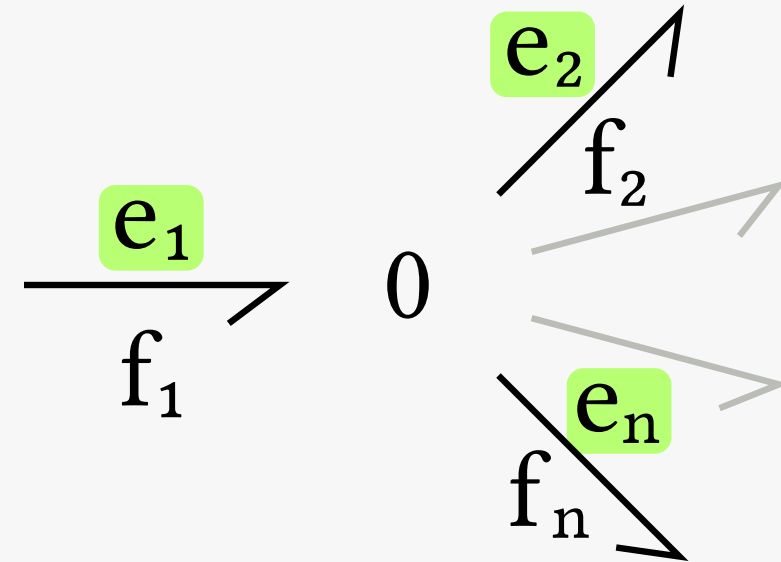
“Common effort” junction:

$$e_1 = e_2 = \dots = e_n$$

Flows are distributed (incoming sum = outgoing sum), according to the *orientation* of the bonds.

On the example:

$$f_1 = f_2 + \dots + f_n$$





# 1 junction

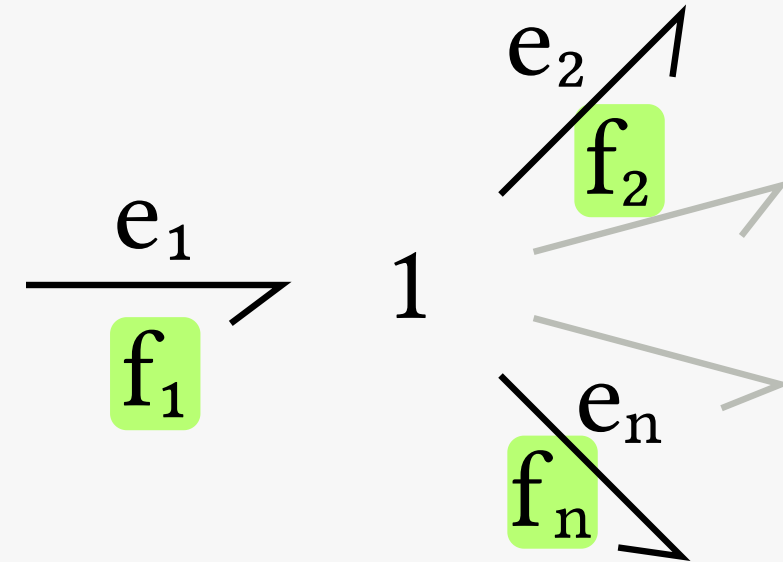
“Common flow” junction:

$$f_1 = f_2 = \dots = f_n$$

Efforts are distributed (incoming sum = outgoing sum), according to the *orientation* of the bonds.

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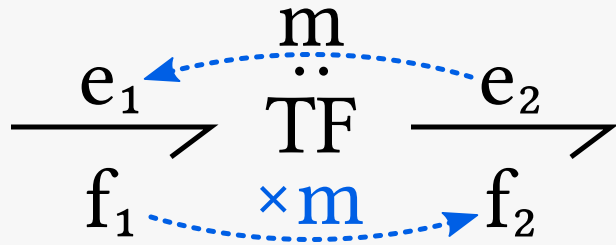


# Two-port junctions: transformers & gyrators

Transmit power with a scaling of efforts & flows:

- in the same domain (unitless scaling)
- between two domains (scaling with a physical unit)

# Transformer (TF)



$$f_2 = m \cdot f_1$$

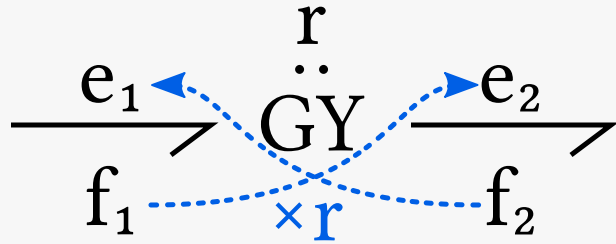
$$m \cdot e_2 = e_1$$

$m$  is the transformer ratio

Examples:

- Mechanical gear pair:  $\omega_2 = (r_2/r_1) \cdot \omega_1$
- Cable – Pulley:  $v = r \cdot \omega$
- Electrical **transformer**:  $v_1 = m \cdot v_2$  (⚠ inverted definition of the transformer ratio)

# Gyrator (GY)



$$e_2 = r \cdot f_1$$

$$e_1 = r \cdot f_2$$

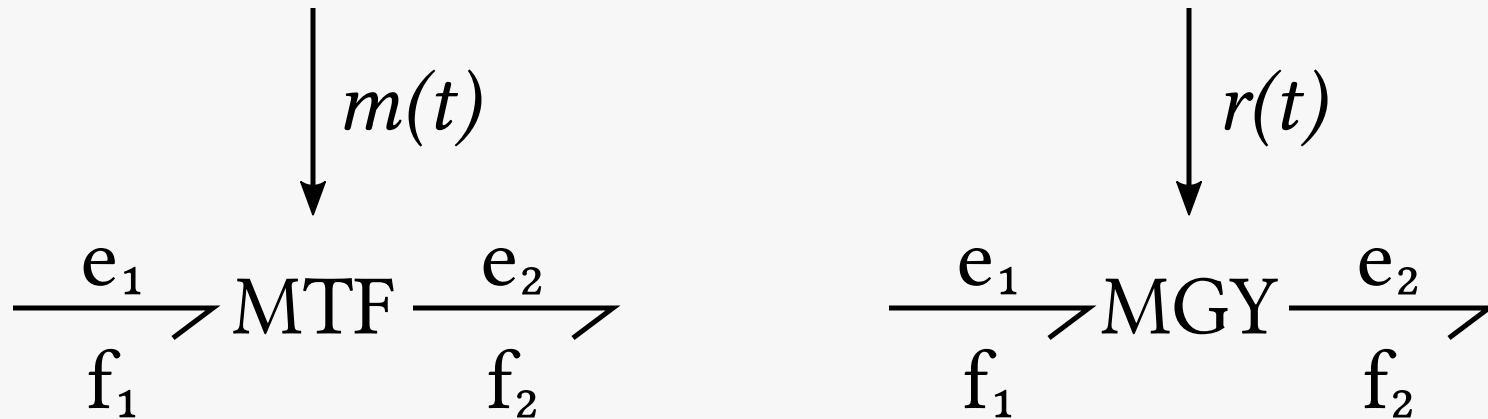
$r$  is the gyrator ratio

Example:

- EMF of a DC motor:  $e = K \cdot \Omega$  and  $C = K \cdot i$

# Modulation of Transformers (Gyrators)

The transformer (gyrator) can be *modulated by a signal* (using a *signal arrow*  $\rightarrow$ ):



Examples: crank-slider mechanism, averaged DC-DC converter.

# One-port elements

There are 3 basic energy consuming/storing devices:

- Dissipator: **R**esistor
- Energy stores:
  - **C** store, also called **C**ompliance or **C**apacitor
  - **I** store, also called **I**ntertia

In addition, there are two sources: **S<sub>e</sub>** (effort) and **S<sub>f</sub>** (flow source)

Remark: in electricity, *one-port* element = device with *two* electrical pins

# Resistor (R)

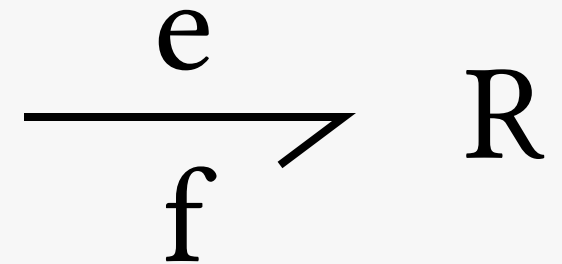
Relation (linear case):

$$e = R.f$$

Property: power is irreversibly dissipated (as heat)

Examples:

- Electrical resistor:  $u = R.i$
- Mechanical damper:  $f = d.v$



# C energy store (also called Compliance or Capacitor)

Using the “generalized displacement”  $q$  (an “energy” variable):

$$q = \int f \cdot dt$$

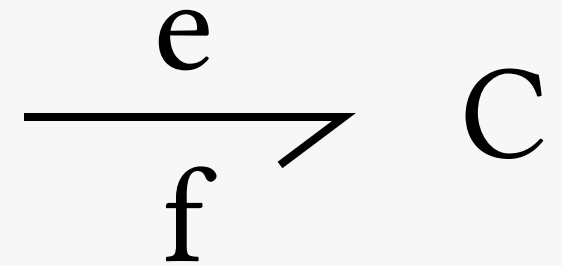
C store relation:

$$q = \Phi_C(e)$$

Linear C store:

$$q = C \cdot e$$

→ consequence:  $f = C \cdot de/dt$





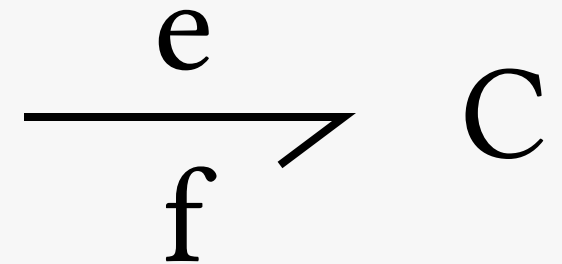
# C store examples (linear)

Mechanics:

- displacement = (kinematic) displacement  $x = \int v \cdot dt$
- relation:  $x = (1/k) \cdot f$
- C store = **spring**

Electricity:

- displacement = charge  $q = \int i \cdot dt$  (Coulomb)
- relation  $q = C \cdot u$
- C store = (electrical) **capacitor**



# I energy store (also called Inertia)

Using the “generalized momentum”  $p$  (an “energy” variable):

$$p = \int e \cdot dt$$

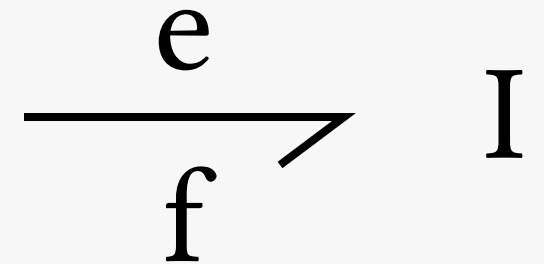
I store relation:

$$p = \Phi_I(f)$$

Linear I store:

$$p = I \cdot f$$

→ consequence:  $e = I \cdot df/dt$



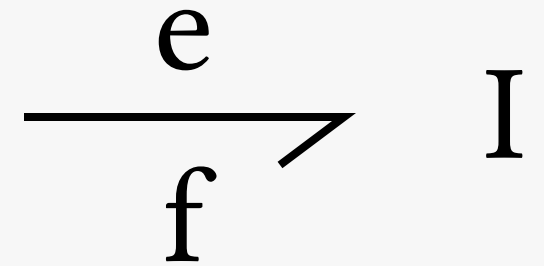
# I store examples (linear)

Mechanics:

- momentum = mechanical momentum  $p = \int f \cdot dt$
- relation:  $p = m \cdot v$ , that is  $f = m \cdot dv/dt$  (inertial force)
- I store = mechanical **inertia**

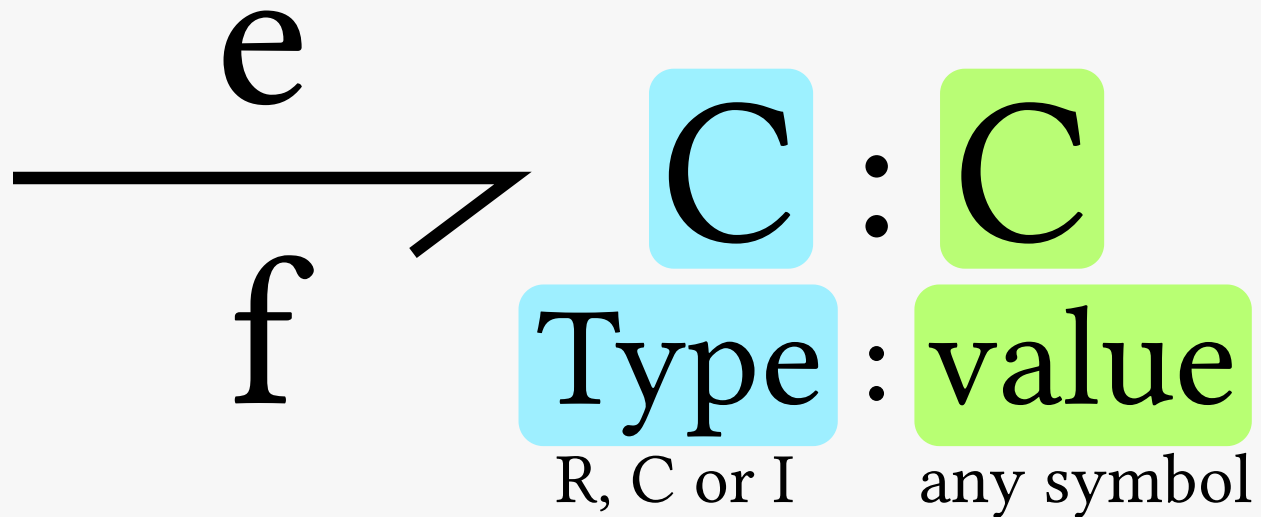
Electricity:

- momentum = magnetic flux linkage  $\lambda = \int u \cdot dt$  (V.s = Wb)
- relation:  $\lambda = L \cdot i$ , that is  $v = L \cdot di/dt$
- I store = **inductor**



# Parametrization of R, C, I elements

The value of a linear R/C/I element is appended with the notation “:  $x$ ”.



Same notation is used for the value of a source (next slide).

# Sources

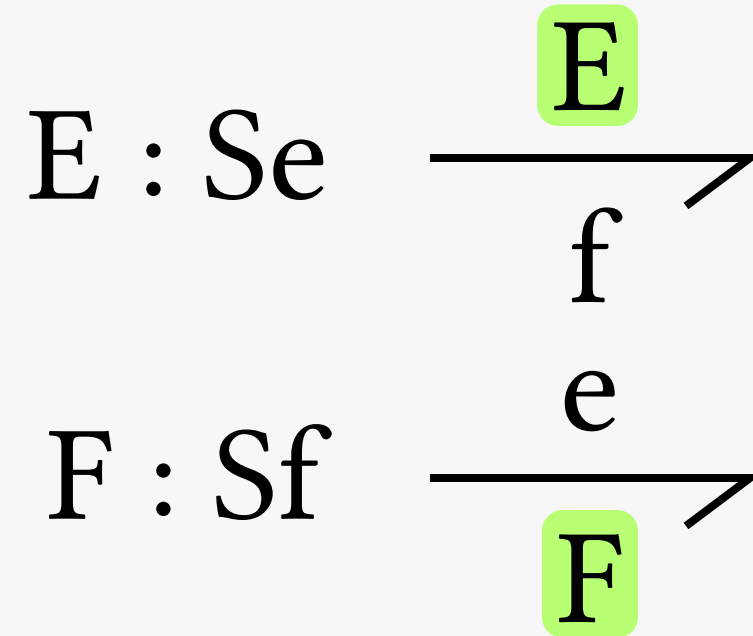
Sources either impose the effort (**Se**) or the flow (**Sf**).

Se examples:

- Electricity: voltage source
- Mechanics: imposed torque or force

Sf examples:

- Electricity: current source
- Mechanics: imposed speed



# Bond Graphs practice

*A graphical language for the analysis of multiphysical systems*



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Slide deck 2: practice

# Course outline

- Bond graph objectives
- The bond graph language
  - Bonds and power variables: the physical analogy
  - Elements
- **Practice: reading & creating bond graphs**
- Causality and derivation of mathematical models

# Practice

Objectives: being able to

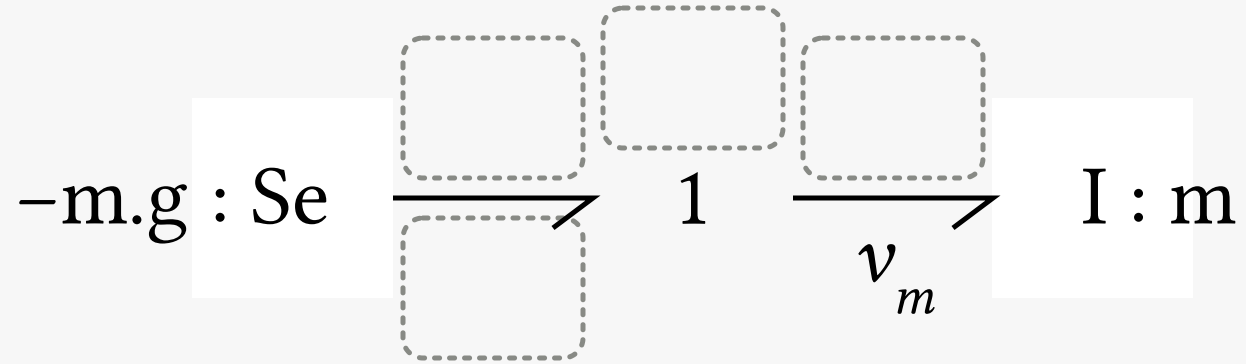
## 1. Read BGs

- recognize classical structure

## 2. Create BGs from network diagrams (electrical, mechanical)



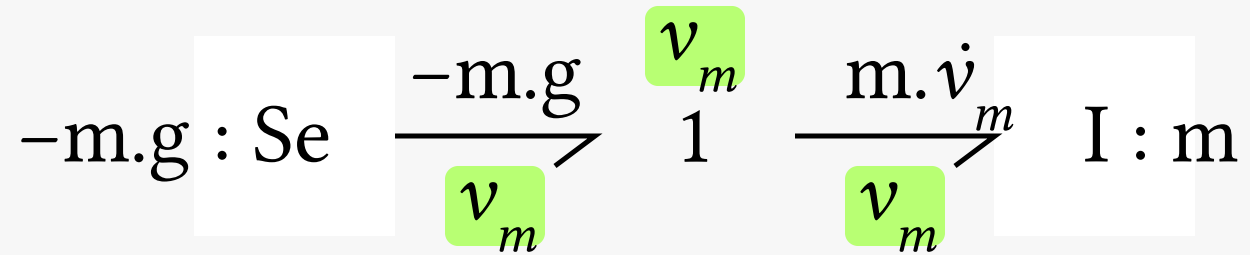
# M1: 1 junctions represent “velocity points”



Questions:

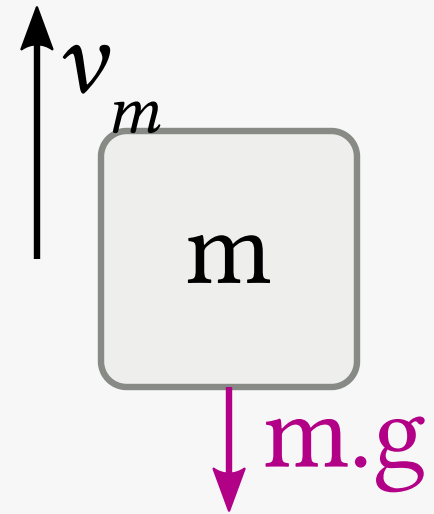
- Fill the blanks on the BG
- Represent the physical system
- Derive the overall mechanical equation of the system

# M1: 1 junctions represent “velocity points” (S)

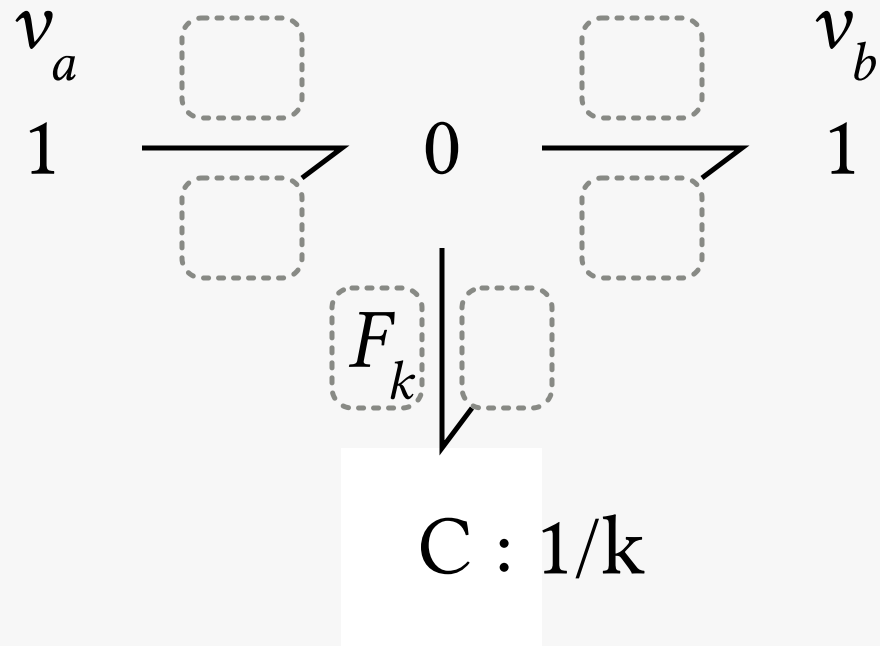


This BG represents the **free fall** of a mass  $m$  with equation:

$$m \cdot \frac{dv_m}{dt} = -m \cdot g$$



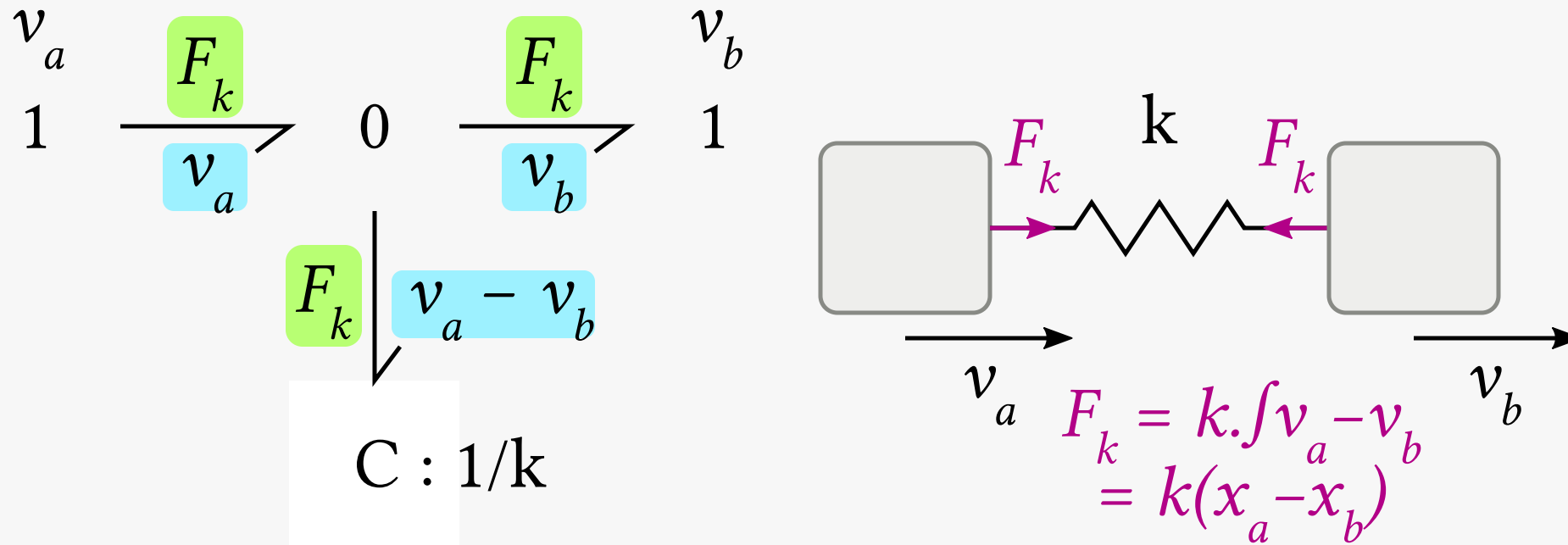
# M2: Springs and dampers typically on 0 junctions



Questions:

- Fill the blanks on the BG
- Represent the physical system
- Give the expression of  $F_k$

# M2: Springs and dampers typically on 0 junctions (Sol.)



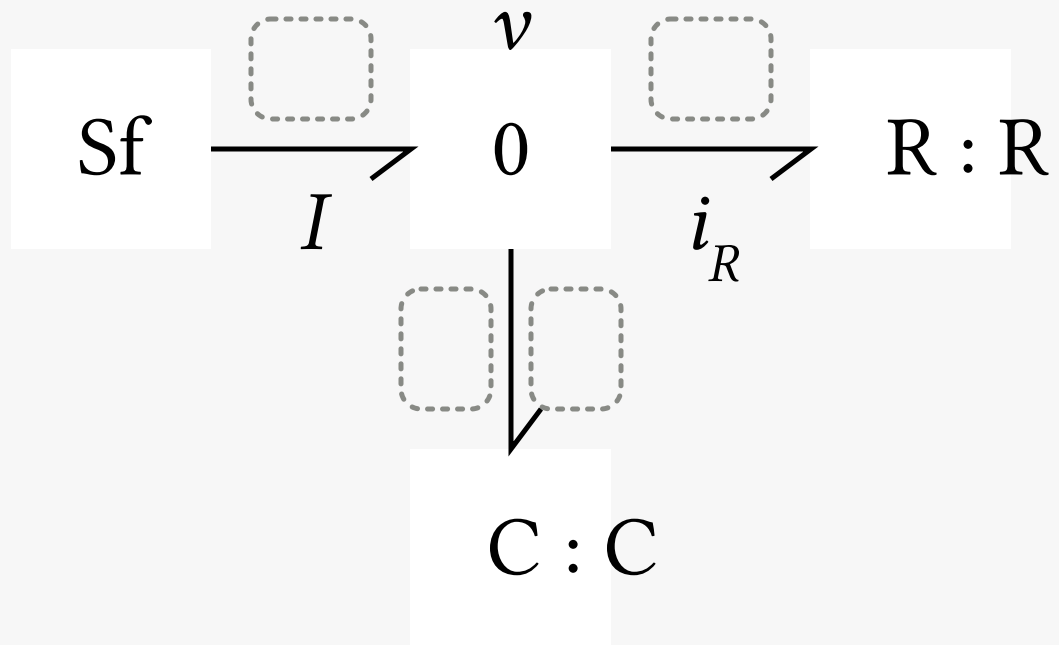
This BG represents a spring of stiffness  $k$  between two points A and B.

$$F_k = k \int (v_a - v_b) \cdot dt = k(x_a - x_b)$$

# Mechanical BGs: main structures

- velocity points represented by 1 junctions
  - an Inertia can attach directly to its corresponding 1 junction
- components which reacts on a velocity/position differences (spring and dampers) are typically connected to a *0 junction placed between two 1 junctions*

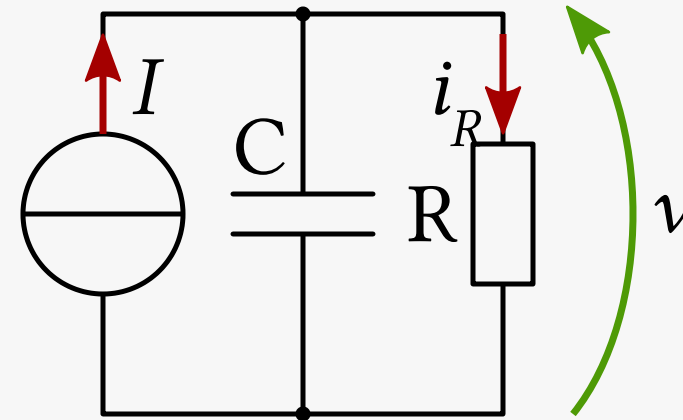
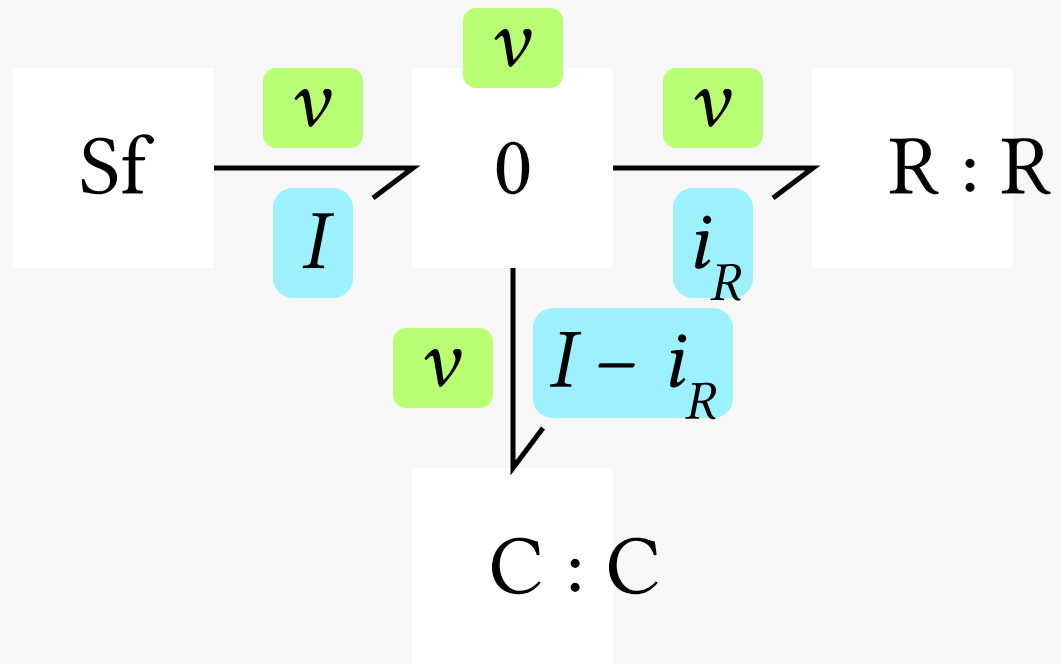
# E1: 0 junctions represent voltage nodes



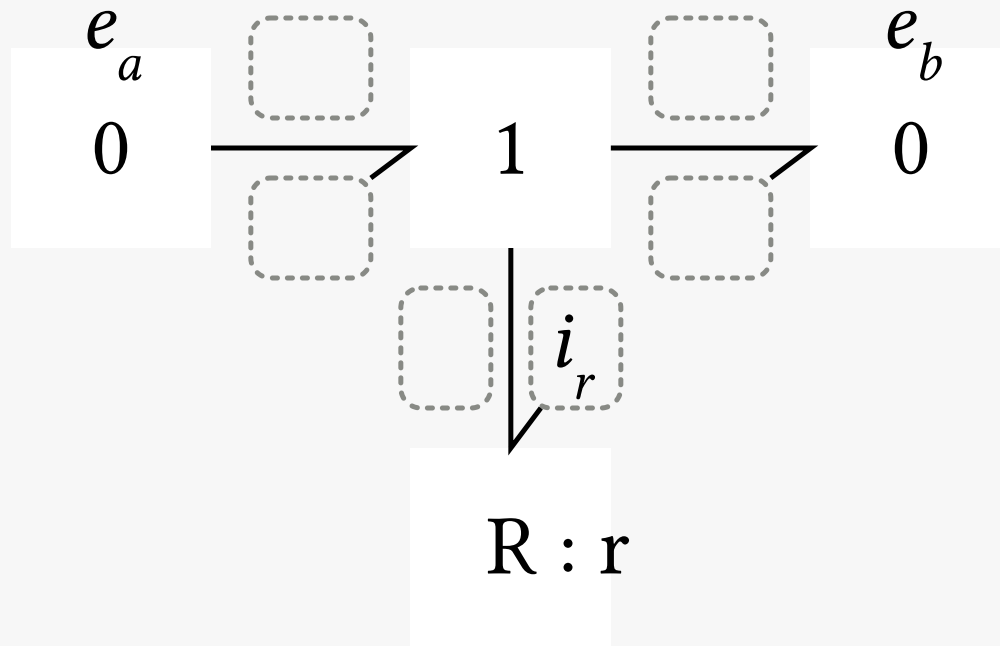
Questions:

- Fill the blanks on the BG
- Represent the physical system

# E1: 0 junctions represent voltage nodes (Sol.)



## E2: 1 junction for voltage drop on a RLC component

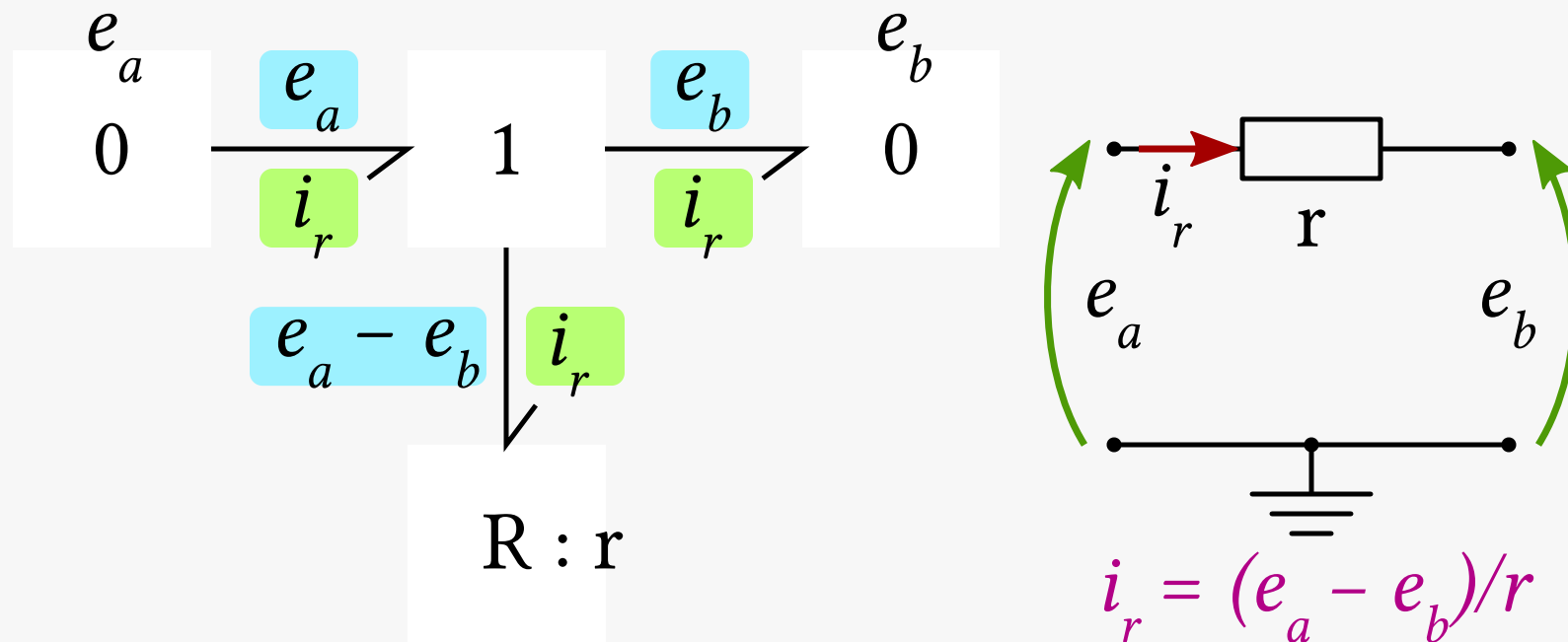


Questions:

- Fill the blanks on the BG
- Represent the physical system
- Compute  $i_r$



# E2: 1 junction for voltage drop on a RLC component (Sol.)



$$i_r = (e_a - e_b)/r$$

$$i_r = (e_a - e_b)/r$$

# E3: Voltage source

...

# Bond Graphs practice

*A graphical language for the analysis of multiphysical systems*

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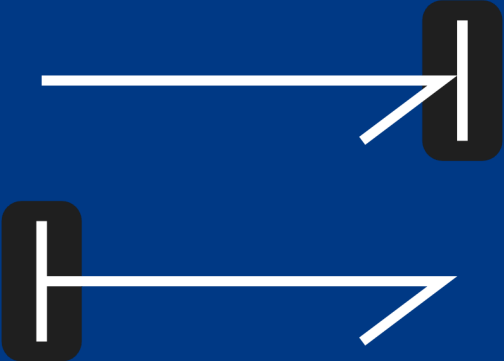
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Slide deck 3: causality

# Course outline

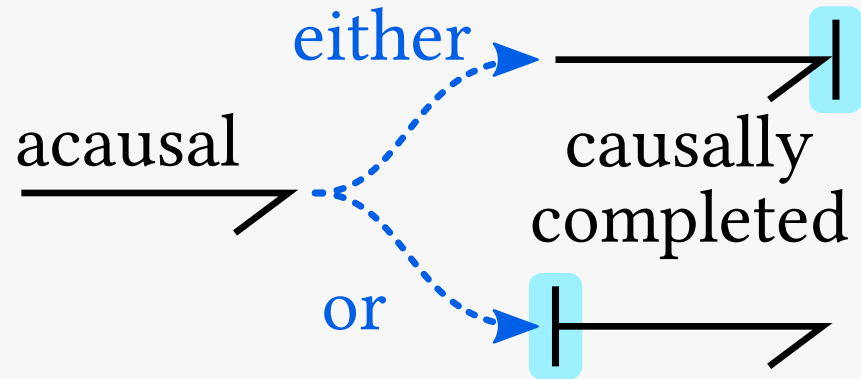
- Bond graph objectives
- The bond graph language
  - Bonds and power variables: the physical analogy
  - Elements
- Practice: reading & creating bond graphs
- Causality and derivation of mathematical models
  - **Principles**
  - Practice

# Computational causality & derivation of mathematical models



# Causally completed bonds

Reminder: bonds express acausal physical links between components

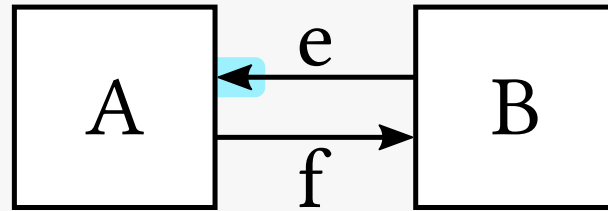
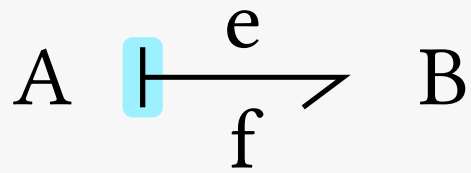
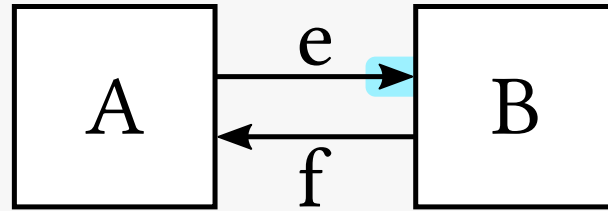
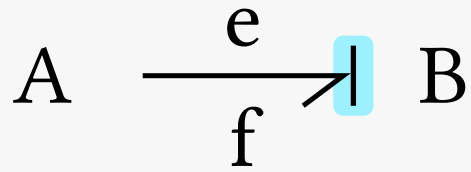


Optionnaly, bonds can be completed with “**causality stroke/arrow**” which indicate the direction of the computation flow.

Remark: this *computational* causality doesn't express a *physical* cause → consequence relationship

# Meaning and significance of causality in BGs

BG to block diagrams fragments



**Mnemonic:** the causality arrow is the direction of the *effort signal arrow*

To be continued...

# Rules for each class of BG components

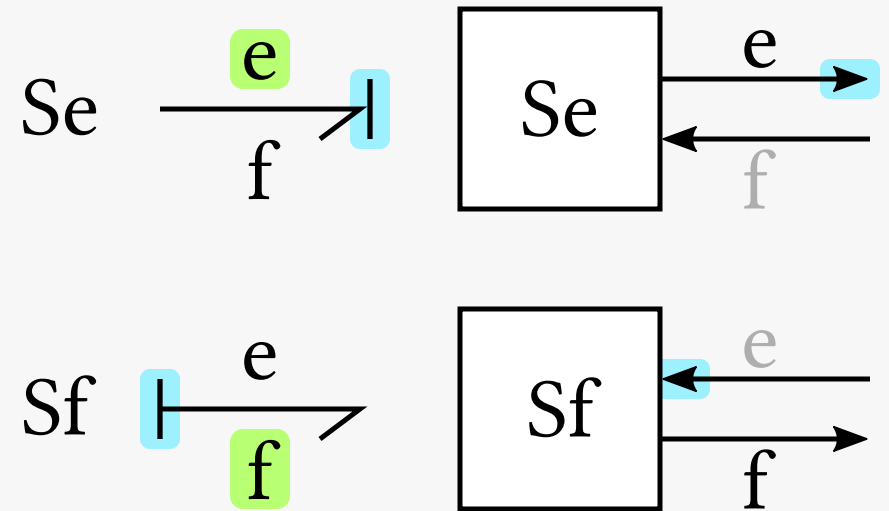
- sources
- junctions
- energy stores & dissipators



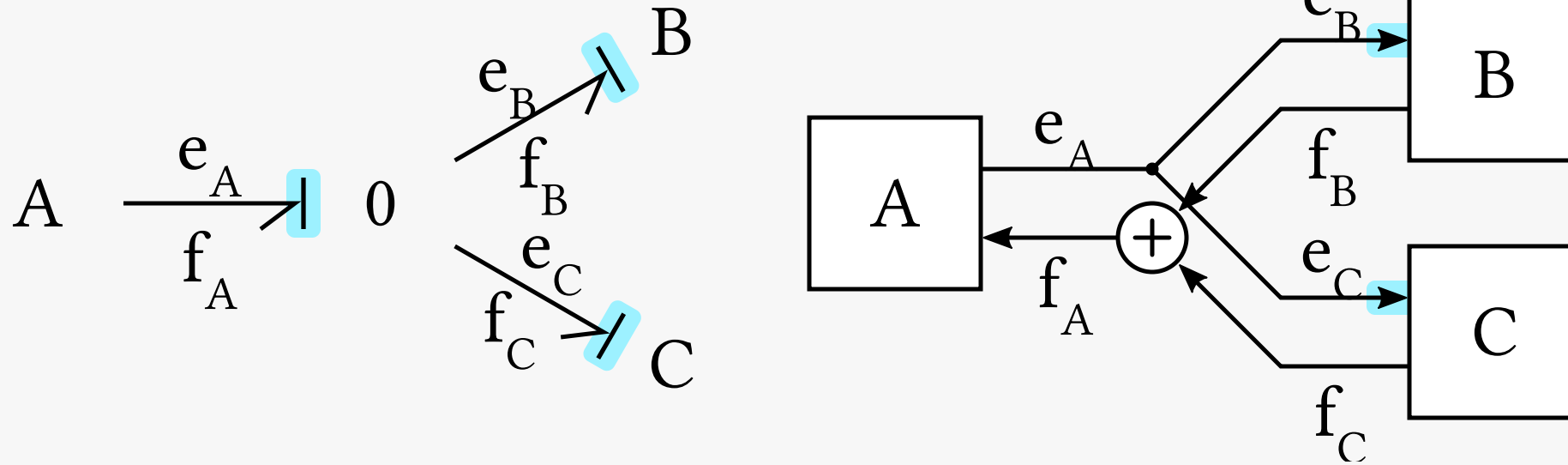
# Causality for Sources

For sources, causality orientation is **compulsory**:

- $S_e$  imposes the effort  
→ outgoing effort
- $S_f$  imposes the flow  
→ outgoing flow



# Causality for junctions: 0 junction

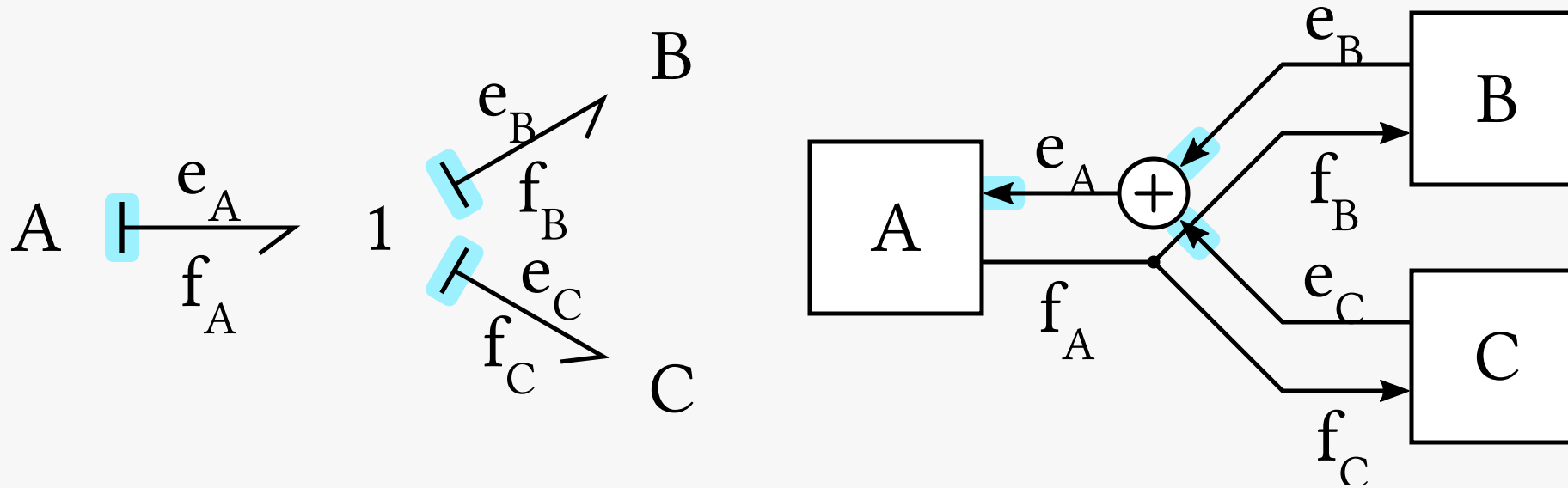


Compulsory rule

For 0 junction (share effort): *“One and only one incoming effort”*

Remark: causality arrows unrelated to bond orientations

# Causality for junctions: 1 junction

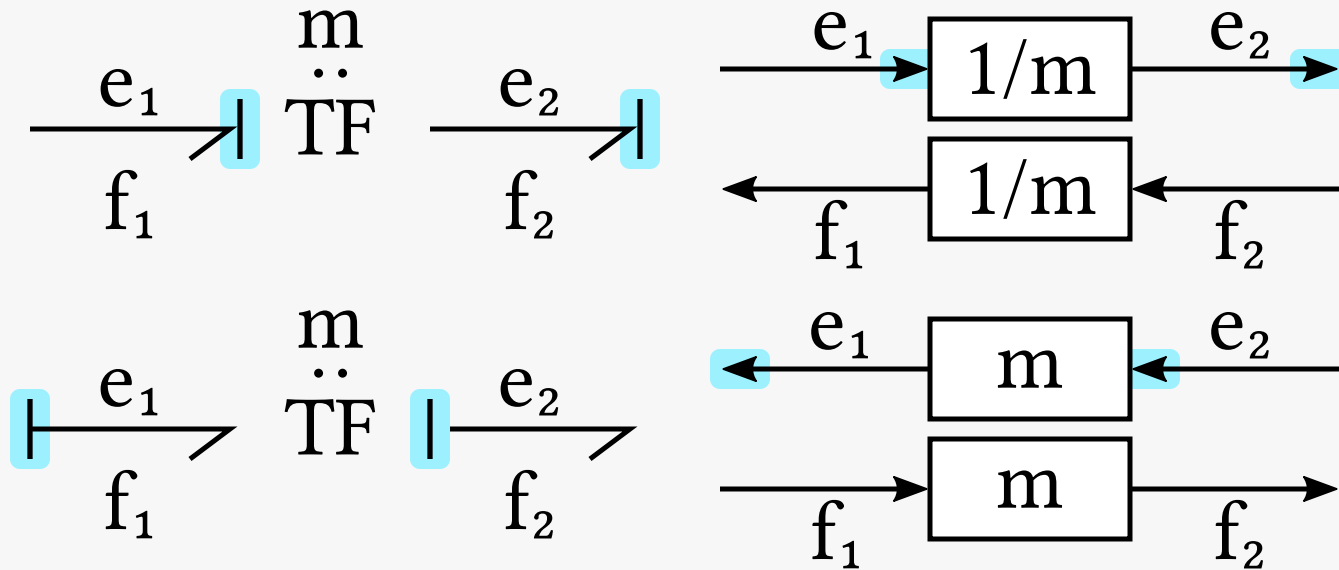


Compulsory rule

For 1 junction (share flow): *“One and only one incoming flow”*

Remark: causality arrows unrelated to bond orientations

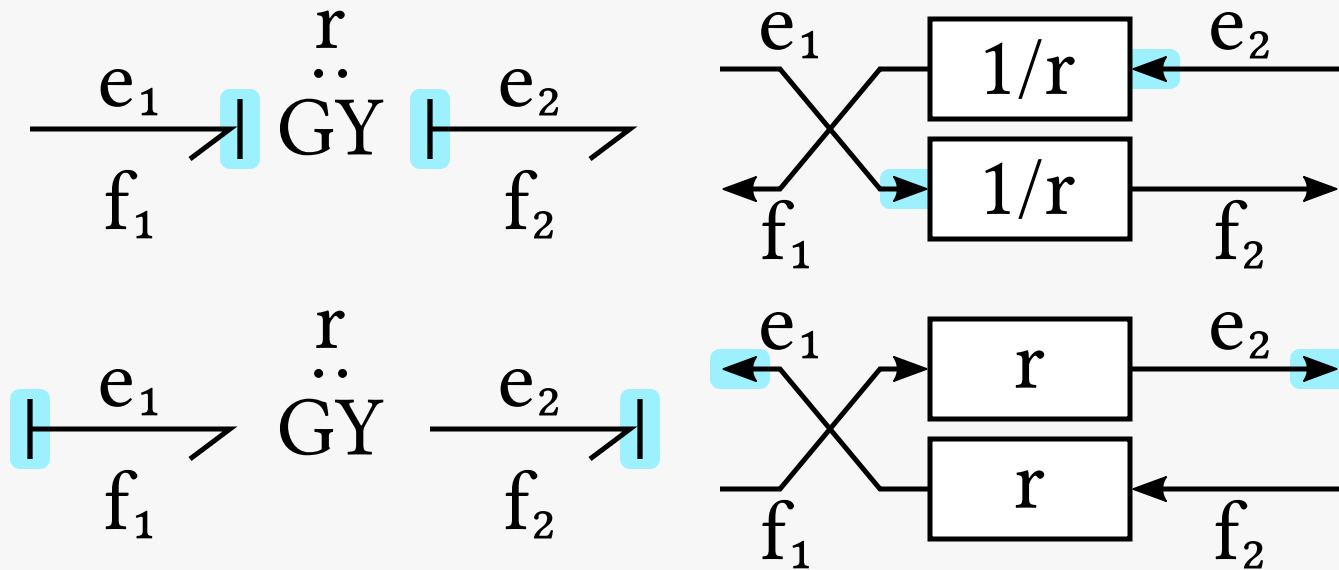
# Causality for junctions: TF



[Compulsory] Transformers **preserve** the direction of the e&f signals

- one incoming effort
- one incoming flow

# Causality for junctions: GY



[Compulsory] Gytrators **reverse** the direction of the e&f signals

- either two incoming efforts
- or two incoming flows

# Causality for Energy stores

C and I energy stores can accept the two possible causalities:

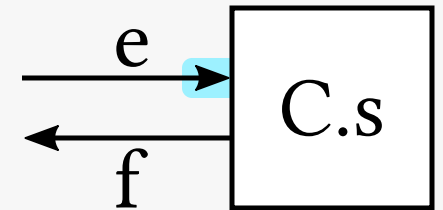
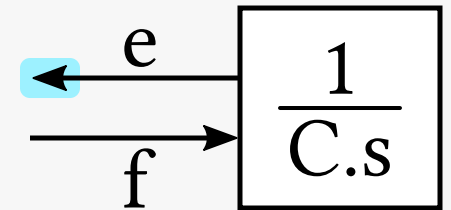
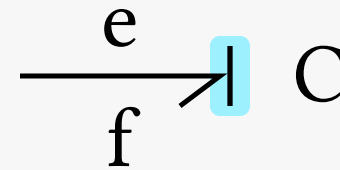
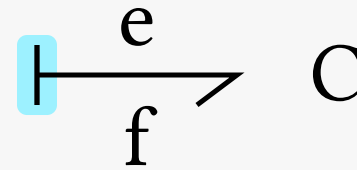
- **Integral causality** is the one yielding an integrator block
- **Derivative causality** is the one yielding an derivator block

For computational easiness, integrators are preferred, so  
Integral causality = **Preferred** causality

# Causality for Energy stores: C

Integral causality: receives a flow, imposes an effort

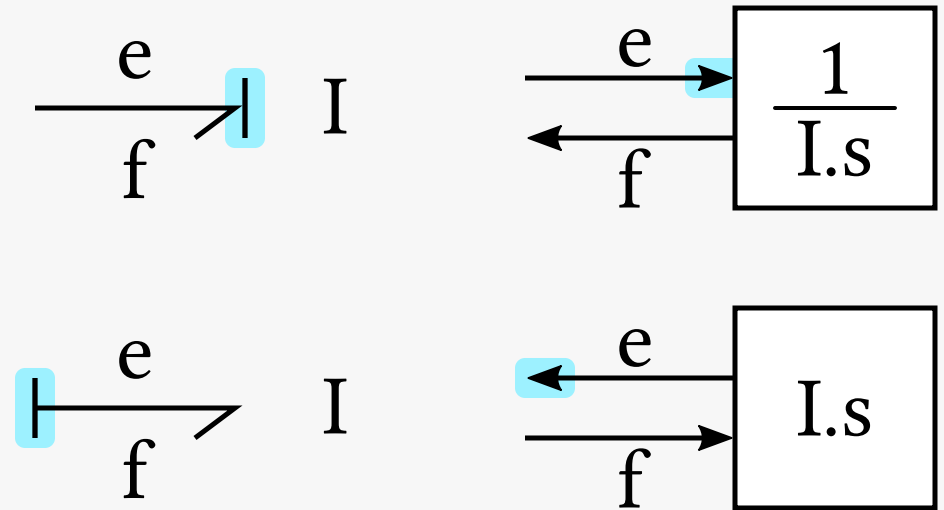
- Capacitor: current  $\rightarrow$  voltage
- Spring: speed/position  $\rightarrow$  force



# Causality for Energy stores: I

Integral causality: receives a flow, imposes an effort

- Inductor: voltage  $\rightarrow$  current
- Intertia: force  $\rightarrow$  speed

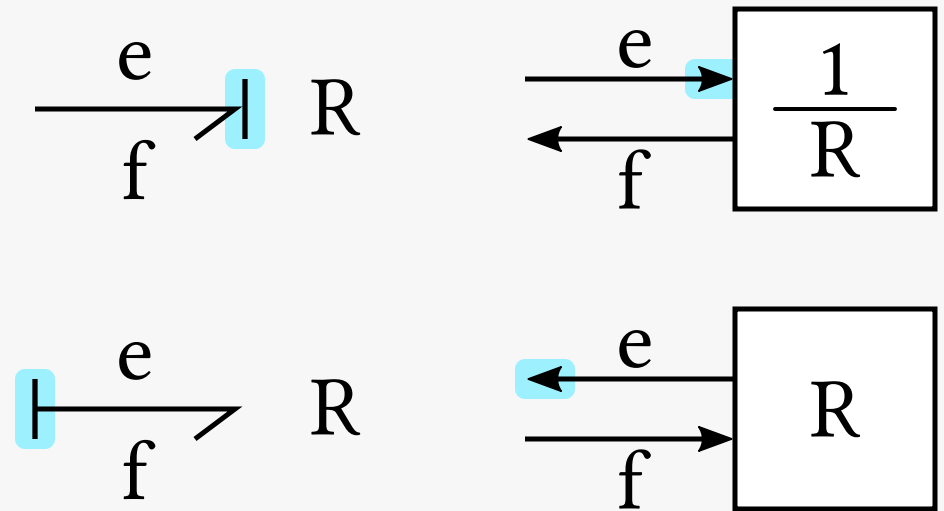




# Causality for R (dissipators)

For dissipators, all orientations are fine.

Sometimes called “resistance” vs  
“conductance” causality



# Sequential assignement rules

SCAP procedure:

Rule 1: assign sources

Rule 2: assign integral causality to Energy stores

# Causality assignment examples

- Ex1: Rule 1 (sources+propagation) is enough
- Ex2: Rule 1+2 (preferred causality for energy stores) are enough

# Bond Graphs practice

*A graphical language for the analysis of multiphysical systems*



Pierre Haessig, CentraleSupélec (campus of Rennes)

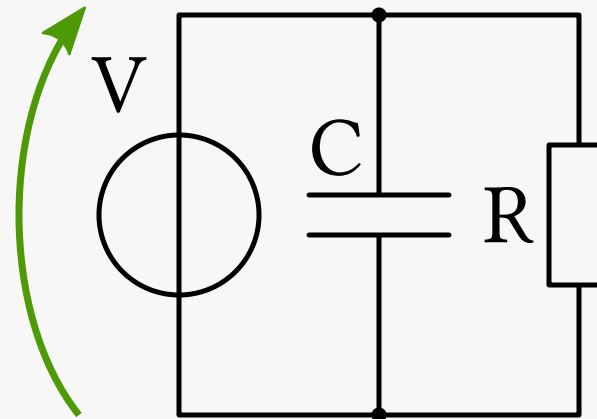
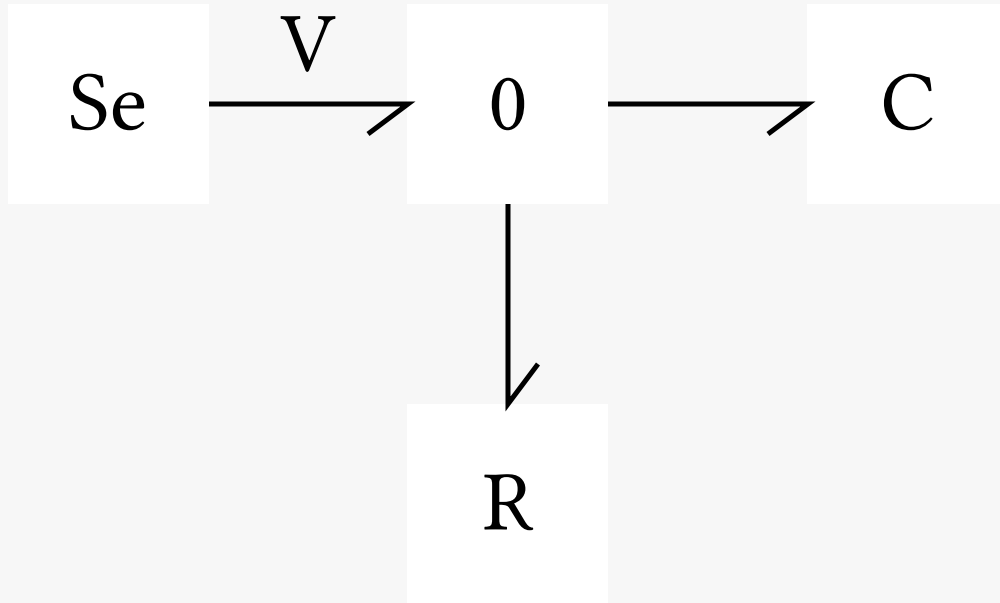
SG6: Dec. 2021 – Jan. 2022

Slide deck 4: causality practice

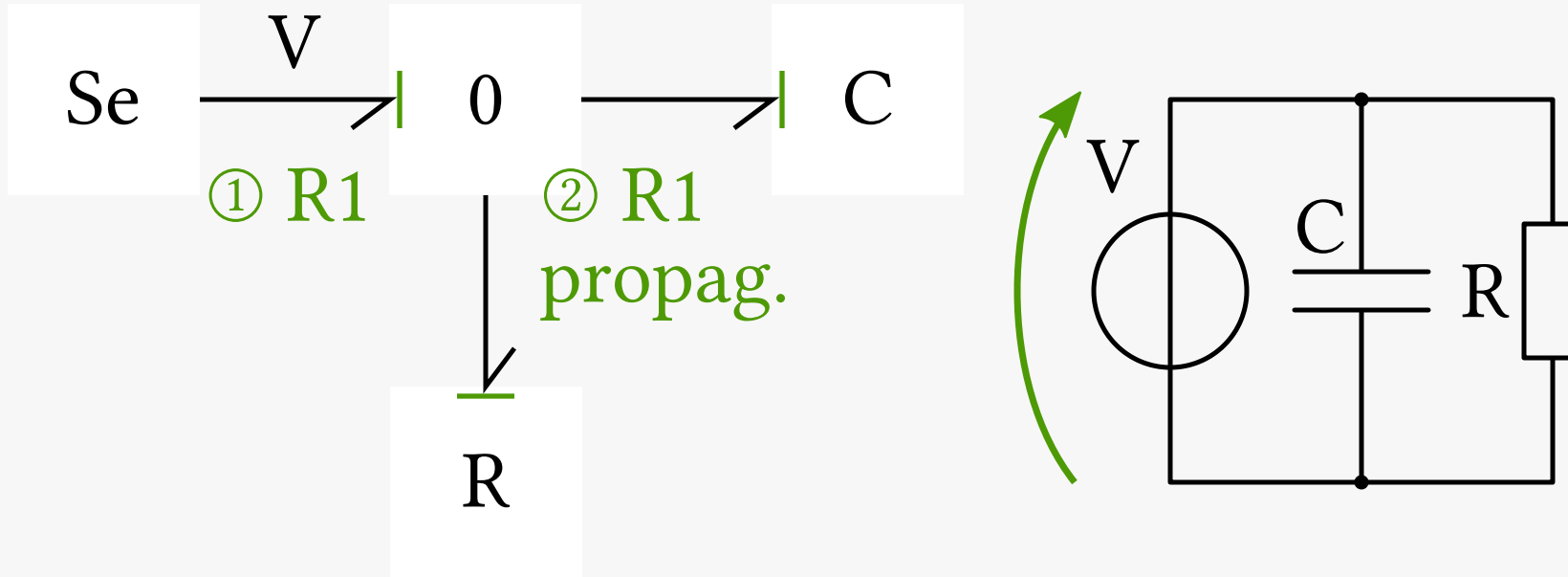
# Course outline

- Bond graph objectives
- The bond graph language
  - Bonds and power variables: the physical analogy
  - Elements
- Practice: reading & creating bond graphs
- Causality and derivation of mathematical models
  - Principles
  - **Practice**

# Causality assignment Ex1

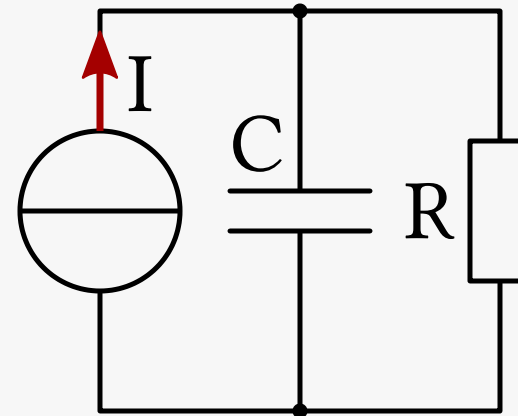
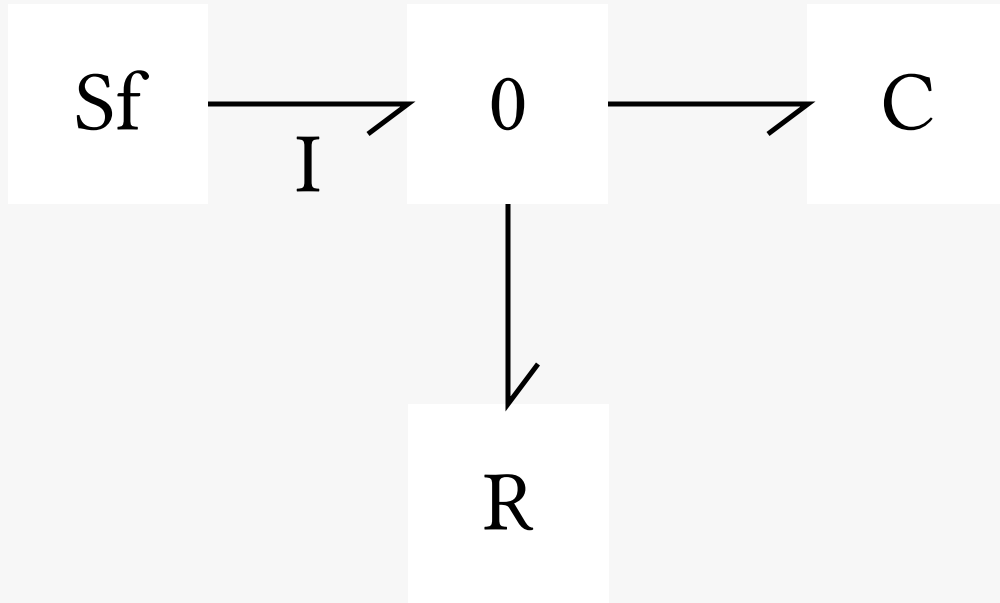


# Causality assignment Ex1 (Sol.)



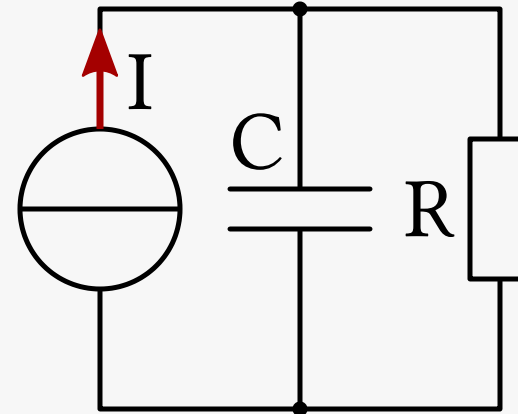
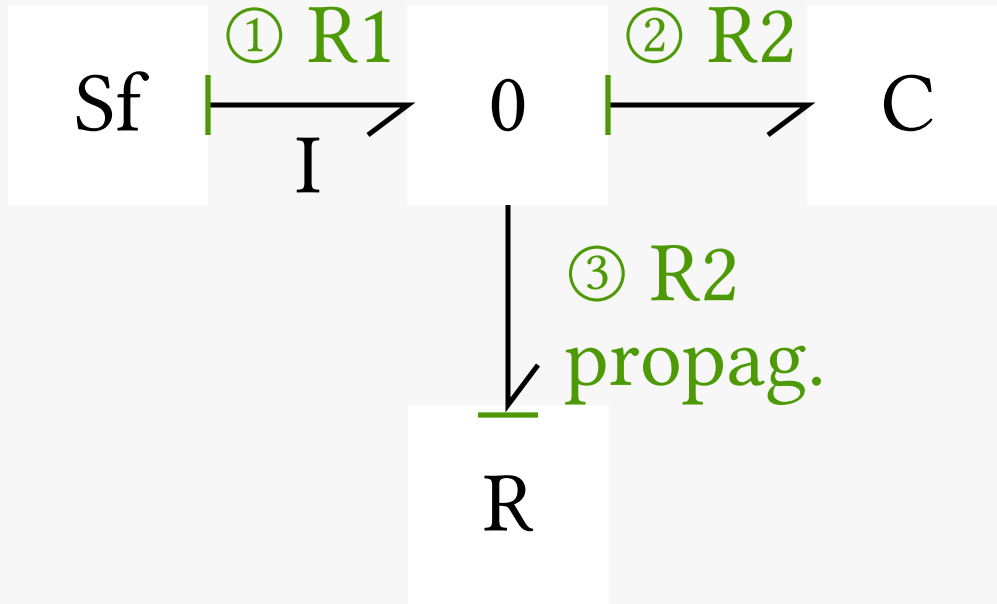
1.  $R1$ :  $Se$  sets the effort on the  $0$  junction
2. Propagation: effort propagates through the  $0$  junction

# Causality assignment Ex2





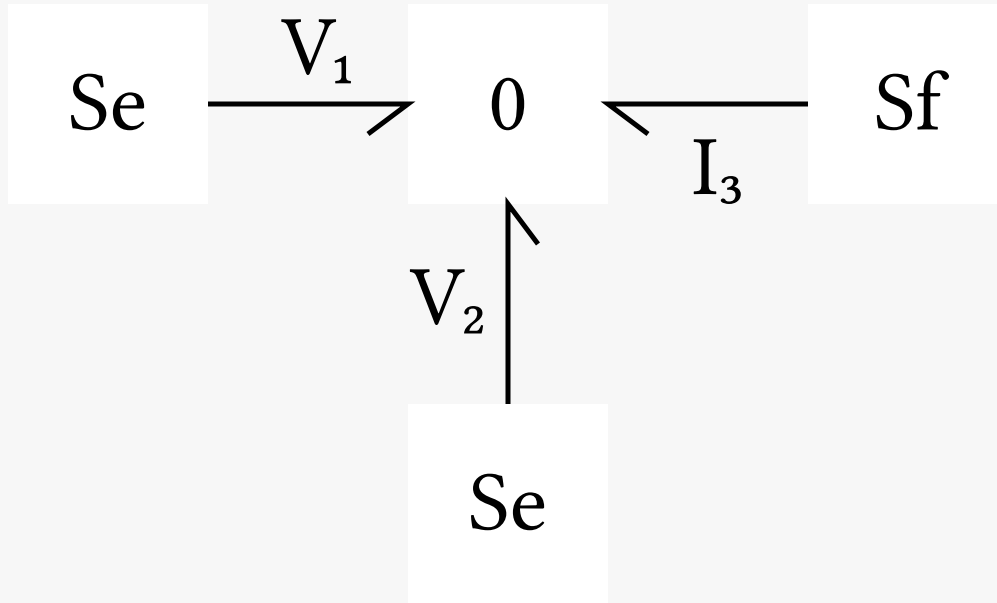
# Causality assignment Ex2 (Sol.)



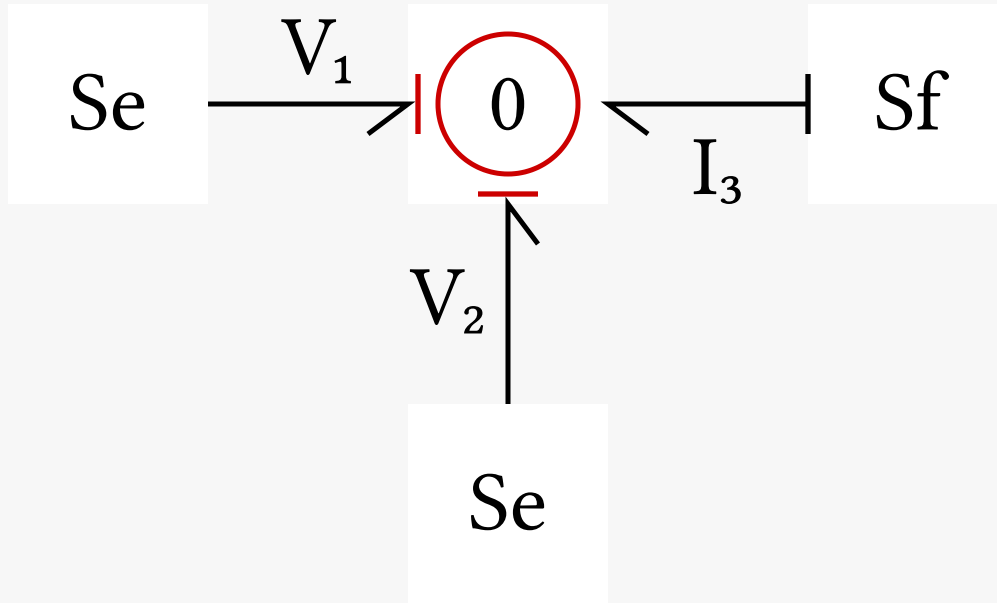
1. R1: Sf sets the flow on the 0 junction.
  - (no propagation)
2. R2: Set integral causality for C
  - $\rightarrow$  C sets the effort on the 0 junction

3. Propagation of effort to R through the 0 junction

# Impossible connections exercise



# Impossible connections exercise (Sol.)



The physical impossibility of parallel voltage sources emerges as a **causality conflict** on the 0 junction (“only one incoming effort allowed”).