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Measurement of surfaces topography

Noël Brunetière¹

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Outline

- 1 Introduction
- 2 Description of the surface roughness
- 3 Roughness parameters
 - Definition of the usual parameters
 - Influence of the machining process
- 4 Optical instruments for surface topography
 - Some basics of optics
 - Interferometry
 - Phase shifting interferometry
 - Vertical scanning interferometry
 - Focus variation
- 5 Other instruments
- 6 Comparison of instruments
- 7 Summary
- 8 Application

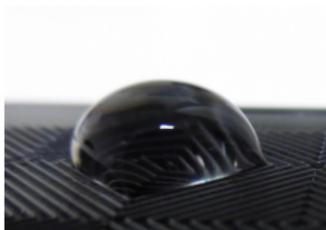
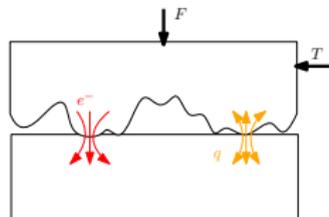
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Why measuring surface topography?

The topography of the surfaces will affect many physical phenomena and surface functions:

- light reflexion
- contact area between solids
 - adhesion
 - electricity conduction
 - heat conduction
 - friction
- tactile perception
- liquid spreading on the surface (wettability)



Why measuring surface topography?

- Control and verify the surface:
According to Leach[Lea11], it is essential to measure surface texture: “Surface texture plays a vital role in the functionality of a component. It is estimated that surface effects causes 10% of manufactured parts to fail and can contribute significantly to an advance nation’s GDP (Gross Domestic Product)”
- Check the surface morphology change with time

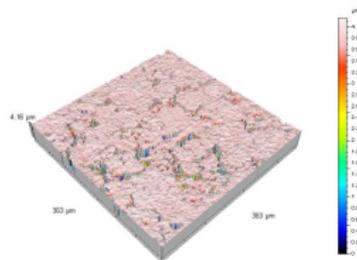


Figure: New surface

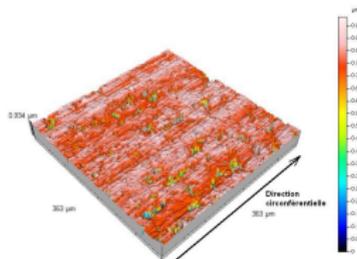


Figure: Surface after friction

Content of the course:

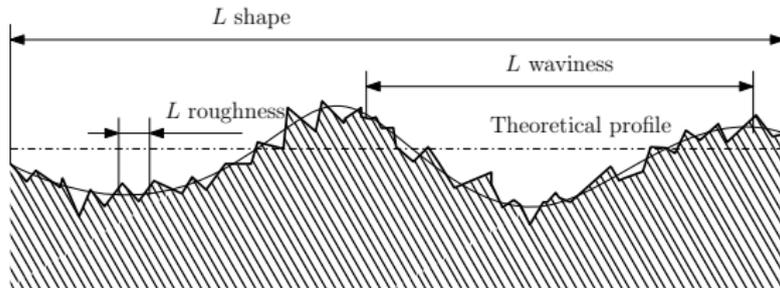
- Description of the surface roughness: the different types of surface deviation according to their scales and what will be measured.
- Roughness parameters: the parameters to describe roughness, identify different surface topography, link of the parameters with the production method
- Optical instruments for surface topography: different optical methods (mainly interferometry)
- Other instruments: other instruments available at Pprime
- Comparison of instruments: limits of the different instruments and influence on measurement results.

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Topography of surfaces: case of profiles

- Due to the production processes, the conditions of use, the wear, the surfaces exhibit defects which are characterized by deviations in altitude from their theoretical profile;
- Defects are generally identified according to their wavelength L :
 - $L \sim$ surface size: shape defect
 - $L \sim 0.01$ to $0.001m$: waviness
 - $L < 0.001m$: **roughness**.



Example of surfaces measured with different instruments

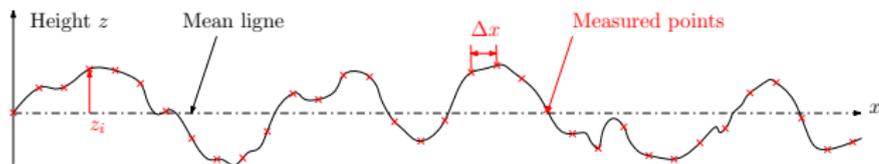


Figure: The surface is digitized by the instrument: a given number of points equally spaced and on given area (or length)

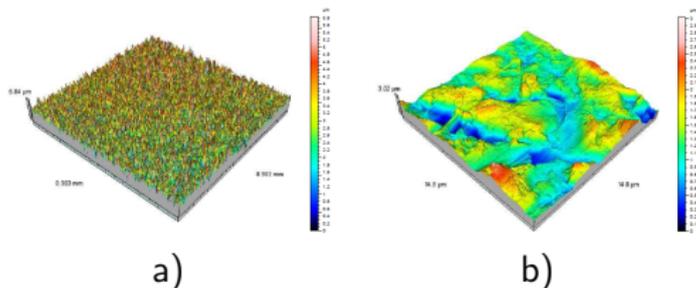


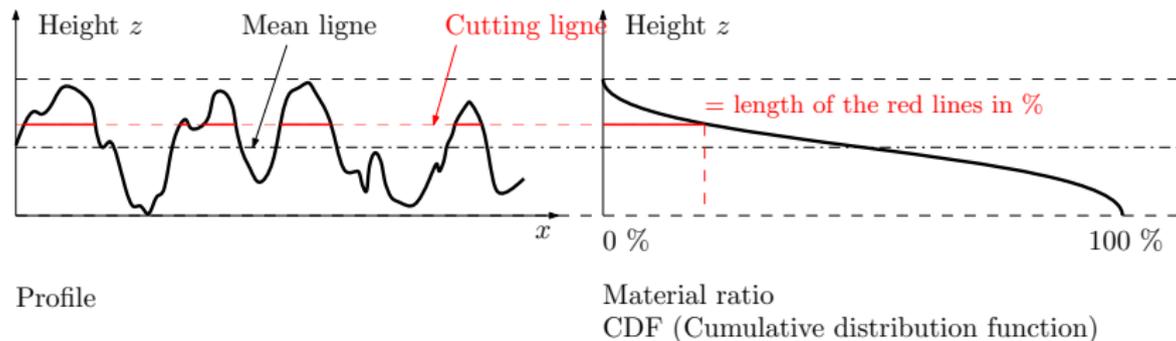
Figure: Same surface with: a) White Light Interferometer: z scale = $5.8 \mu\text{m}$, $L = 0.9\text{mm}$, $\Delta x = 0.9\mu\text{m}$, b) Atomic Force Microscope: z scale = $3 \mu\text{m}$, $L = 14.8\mu\text{m}$, $\Delta x = 0.029\mu\text{m}$

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Topography of surfaces: case of profiles

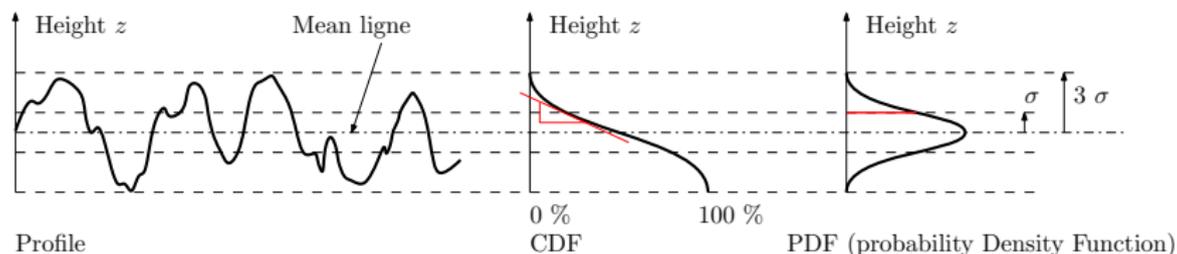
- Profile: height z as a function of the horizontal length x ;
- CDF = Cumulative Distribution Function: ratio of material above a cutting line of height z ;



Topography of surfaces: case of profiles

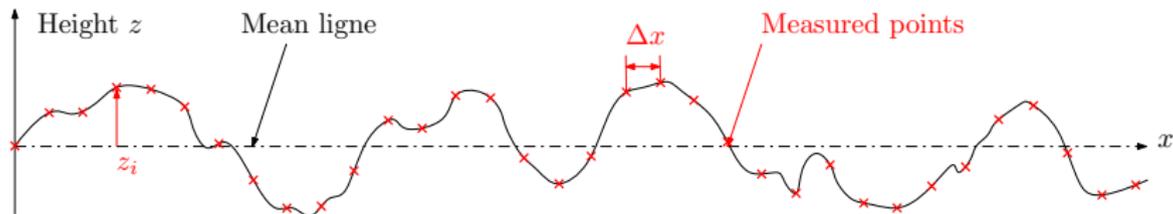
- PDF = Probability Density Function \propto derivative with respect to z of the CDF
- For most rough surfaces, the probability density follows a Gaussian law: :

$$f(z) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left[-\frac{(z-\mu)^2}{2\sigma^2}\right];$$
- μ is the mean and σ the standard deviation (Rq in ISO metrology);
- The profile points are in the range -3σ to $+3\sigma$.



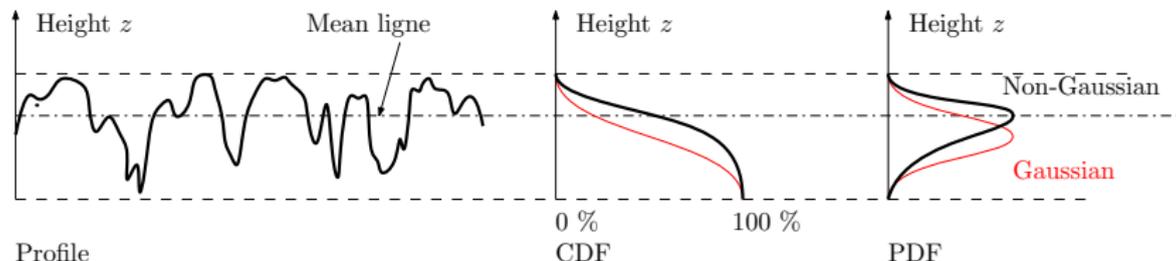
Height statistical parameters: case of profiles

- The profile is measured by a device which gives a series of n points z_i laterally spaced of Δx ;
- Height (m or μm): Arithmetic average $Ra = \frac{1}{n} \sum_1^n |z_i|$
- Height (m or μm): Quadratic average $Rq = \sqrt{\frac{1}{n} \sum_1^n z_i^2}$ is equivalent to σ ;
- Skewness parameter (non-dimensional): $RSk = \frac{1}{nRq^3} \sum_1^n z_i^3$;
- Kurtosis parameter (non-dimensional): $RKu = \frac{1}{nRq^4} \sum_1^n z_i^4$;



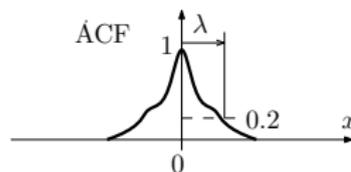
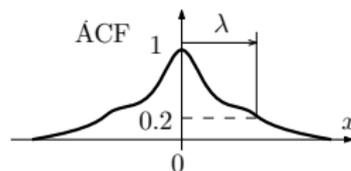
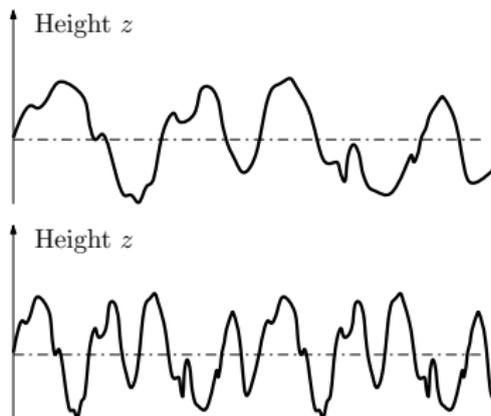
Height statistical parameters: case of profiles

- Due to the manufacturing process, surfaces can be Non-Gaussian.
- Gaussian surface : $Rq \approx \sqrt{\frac{\pi}{2}} Ra \approx 1.25Ra$ $RSk = 0$ $RKu = 3$
- Non Gaussian surface (process with pics removal): $RSk < 0$ et $RKu > 3$.



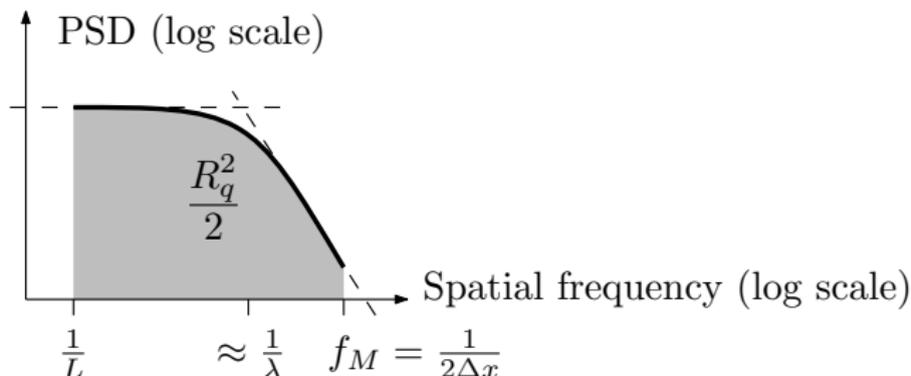
Lateral size of asperities: case of profiles

- ACF = Auto Correlation Function: degree of resemblance of a surface to itself shifted by x ;
- $FAC(x = j\Delta x) = \frac{1}{nRq^2} \sum_1^n z_i \times z_{i+j}$
- For many rough surfaces, it is a decreasing function tending to zero.
- The correlation length λ is the characteristic size of asperities.
- Is it the horizontal distance to reach ACF = 0.2.



Roughness characterization: PSD

- PSD = Power Spectrum Density is the Fourier transform of the ACF.
- Distribution of the space frequencies;
- Lowest frequency $\frac{1}{L}$ where L is the sample length;
- Highest frequency $f_M = \frac{1}{2\Delta x}$ where Δx is the sampling length.
- The slope changes occurs at $\frac{1}{\lambda}$
- The area below the curve corresponds to $\frac{R_q^2}{2}$

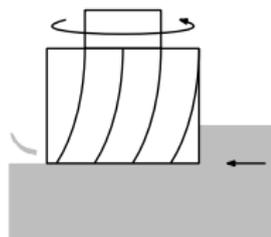


From profiles to surfaces

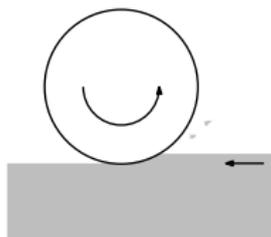
The roughness parameters are defined in the ISO standard. Many metrological tools measure the surface (and not the profile). Surface parameters have the same definition but different name:

Parameter	Profile	Surface
Arithmetic average	R_a	S_a
Quadratic average	R_q	S_q
Skewness	RSk	SSk
Kurtosis	RKu	SKu
Correlation length (lowest value)	λ	Sal
Correlation length (biggest value)	λ	Sal/Str

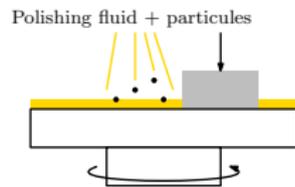
Usual machining process



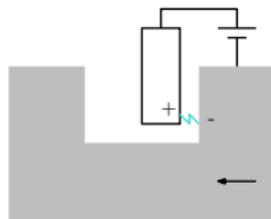
Milling



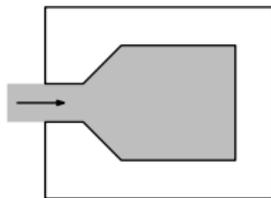
Grinding



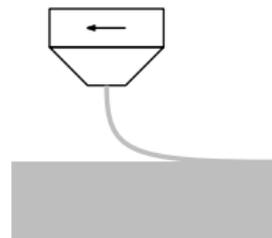
Honing and polishing



Electroerosion



Molding

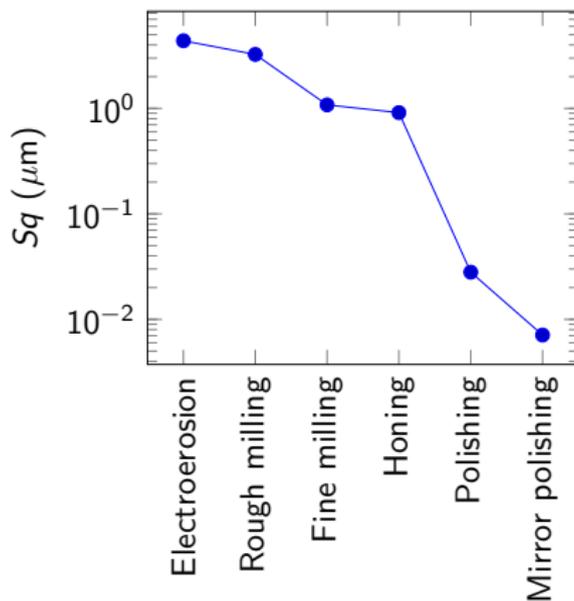


3D printing

Figure: Some usual surfaces production methods

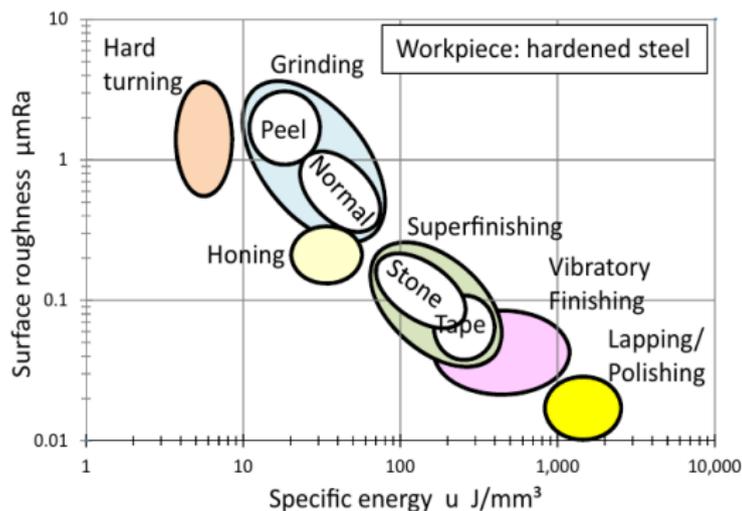
Impact of machining on topography

Impact of the machining process on the roughness height S_q (WLI, $L = 0.9\text{mm}$, $\Delta x = 0.9\mu\text{m}$):



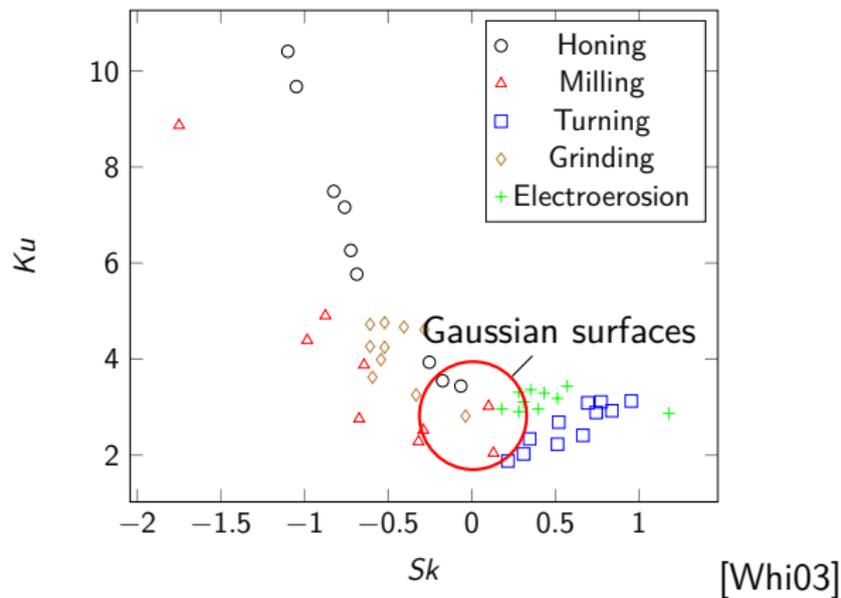
Impact of machining on topography

The high surface finish (or low roughness height level) corresponds to processes that require more energy [Has+16]. The amount of energy is inversely proportional to Ra :

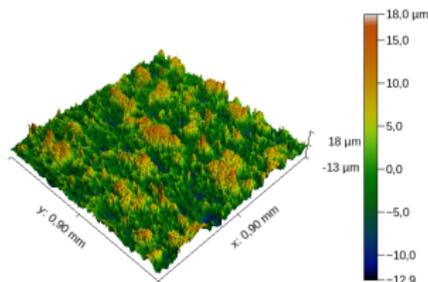


Impact of machining on topography

Impact of the machining process on the Gaussian nature of the surface:

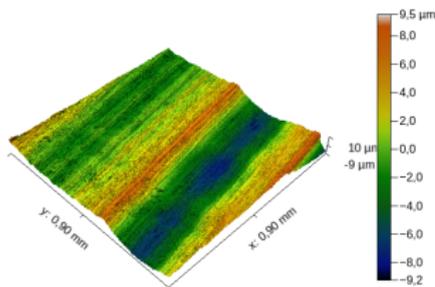


Examples of surfaces



Paramètre	Valeur
Sq (μm)	4.38
Sa (μm)	3.55
SSk	0.60
SKu	2.793
Sal (μm)	37
Str	0.56

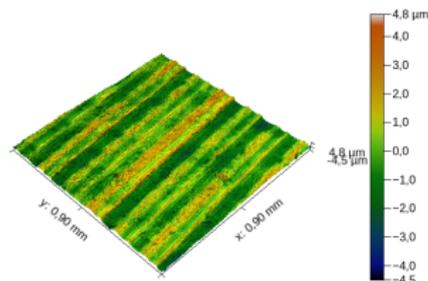
Figure: Surface obtained by electroerosion ($z \times 5$), WLI, $L = 0.9\text{mm}$, $\Delta x = 0.9\mu m$.



Paramètre	Valeur
Sq (μm)	3.25
Sa (μm)	2.63
SSk	-0.04
SKu	2.51
Sal (μm)	93.8
Str	< 0.2

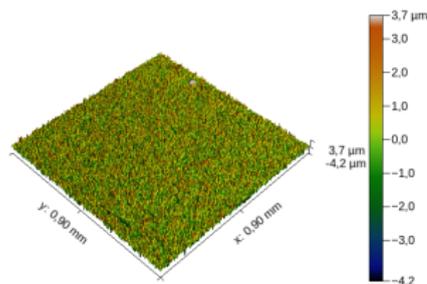
Figure: Surface obtained by rough milling ($z \times 5$), WLI, $L = 0.9\text{mm}$, $\Delta x = 0.9\mu m$.

Examples of surfaces



Paramètre	Valeur
Sq (μm)	1.08
Sa (μm)	0.89
SSk	0.034
SKu	2.49
Sal (μm)	24.3
Str	< 0.2

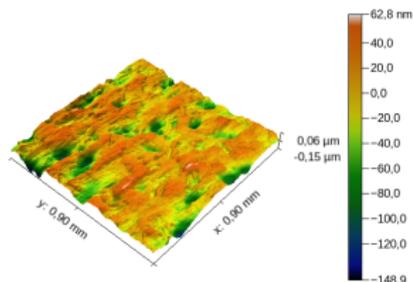
Figure: Surface obtained by fine milling ($z \times 5$), WLI, $L = 0.9\text{mm}$, $\Delta x = 0.9\mu\text{m}$.



Paramètre	Valeur
Sq (μm)	0.912
Sa (μm)	0.709
SSk	-0.24
SKu	3.93
Sal (μm)	4.54
Str	0.7

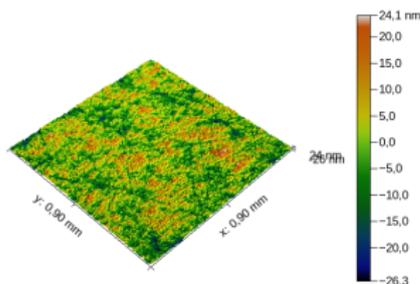
Figure: Surface obtained by honing ($z \times 5$), WLI, $L = 0.9\text{mm}$, $\Delta x = 0.9\mu\text{m}$.

Examples of surfaces



Paramètre	Valeur
Sq (μm)	0.028
Sa (μm)	0.022
SSk	-0.92
SKu	4.09
Sal (μm)	44.8
Str	0.35

Figure: Surface obtained by polishing ($z \times 500$), WLI, $L = 0.9mm$, $\Delta x = 0.9\mu m$.

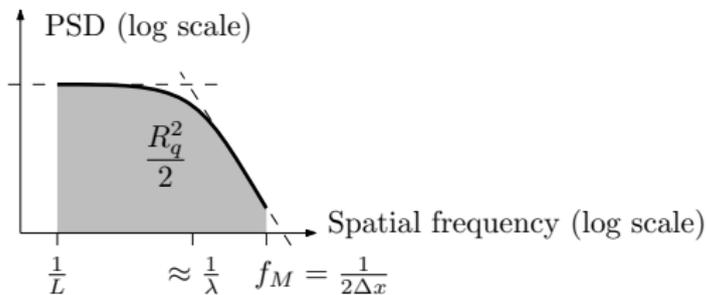


Paramètre	Valeur
Sq (μm)	0.0071
Sa (μm)	0.0057
SSk	-0.09
SKu	3.14
Sal (μm)	23.22
Str	0.6

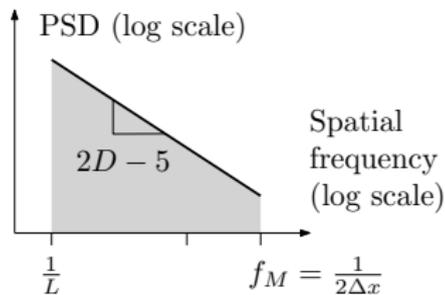
Figure: Surface obtained by mirror polishing ($z \times 500$), WLI, $L = 0.9mm$, $\Delta x = 0.9\mu m$.

Can surfaces be fractal ?

- A fractal surface: $\text{PSD} \propto f^{2D-5}$ where D is the fractal dimension (between 1 and 2 for a profile)
- According to Whitehouse[Whi01], λ or Sa is linked to the process (density of grain during polishing, etc)
- A surface appears to be fractal if $L < \lambda$



PSD of a machined surface



PSD of a fractal surface

Can surfaces be fractal ?

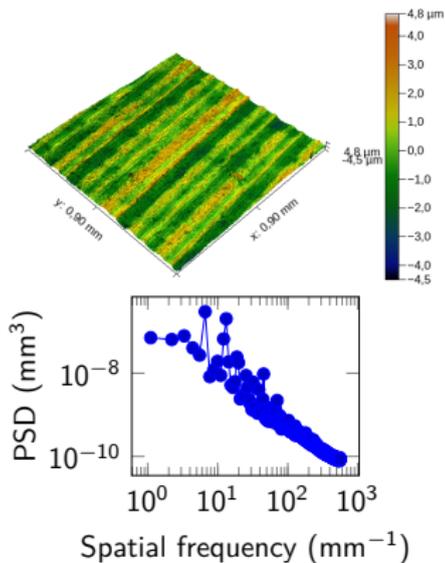


Figure: Engineered surface: slope change at $1/\lambda$

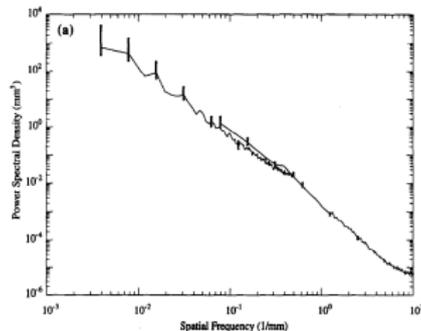
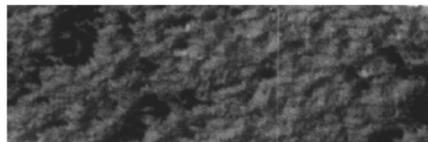


Figure: Natural material (fractured granite): fractal[PD97]

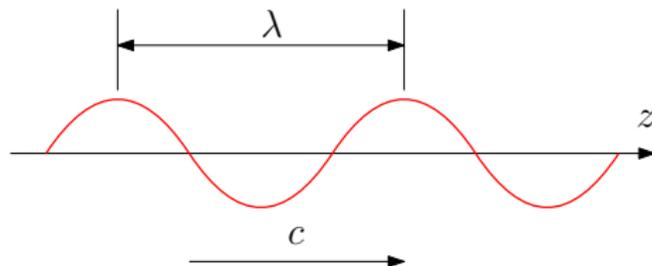
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Some basics of optics

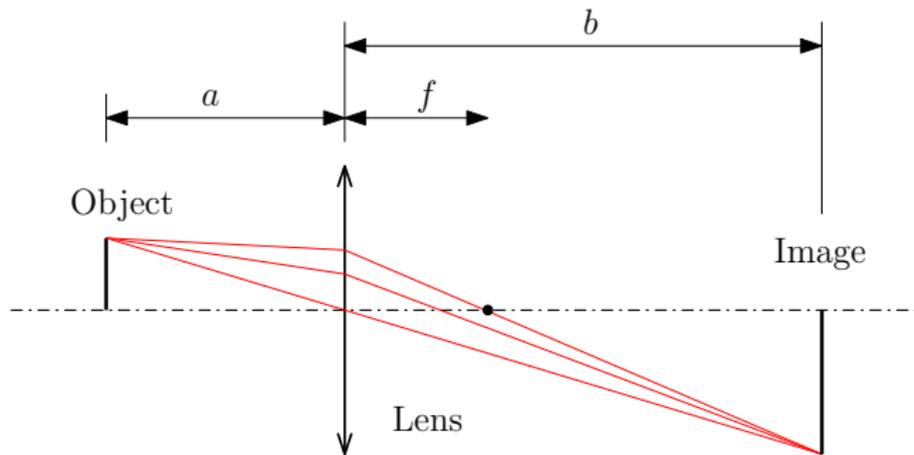
Monichromatic light wave (in air): $s = a \cos \left[\frac{2\pi}{\lambda} (ct - z) + \phi \right]$

- c speed of light
- λ wave length
- t time, z position
- ϕ phase



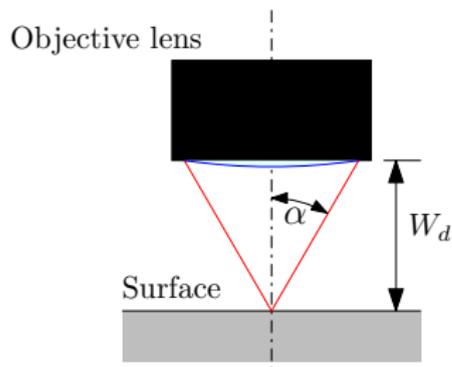
Some basics of optics

- f focal length
- Magnification $M = \frac{b}{a}$



Some basics of optics

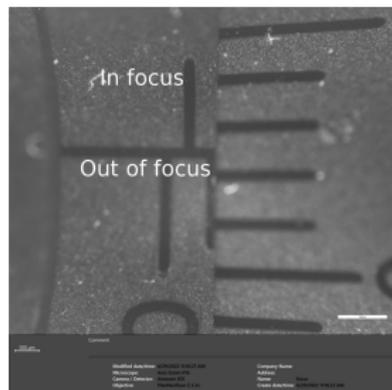
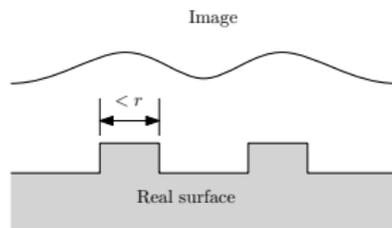
- A_N Numerical aperture of a microscope objective
- n refractive index of the media ($n = 1$ in air)
- $A_N = n \sin \alpha$
- This parameter will affect the performance of the objective (working distance W_d and others)



Some basics of optics

- Spatial resolution r = minimum distance between two features [Lea11]
- $r = 0.61 \frac{\lambda}{A_N}$ (Rayleigh)
- $r = 0.82 \frac{\lambda}{A_N}$ (Sparrow)
- Depth of field Z = depth along with the object is in focus [Lea11]
- $Z = \frac{n\lambda}{A_N^2}$

The numerical aperture has a strong impact on the optical performance



Michelson Interferometer

- $L_2 = L_1 + d$
- Beam 1:

$$s_1 = a \cos \left[\frac{2\pi}{\lambda} (ct - z) + \phi \right]$$
- Beam 2:

$$s_2 = a \cos \left[\frac{2\pi}{\lambda} (ct - z - 2d) + \phi \right]$$
- Resulting beam: $s = s_1 + s_2 =$

$$2a \cos \left[\frac{2\pi}{\lambda} (ct - z - d) \right] \cos \left(\frac{2\pi}{\lambda} d \right)$$
- Light intensity:
- $I = \frac{1}{T} \int_0^T s^2 dt = 2a^2 \cos^2 \left(\frac{2\pi}{\lambda} d \right)$
- $I = a^2 \cos \left(\frac{4\pi}{\lambda} d \right) + a^2$

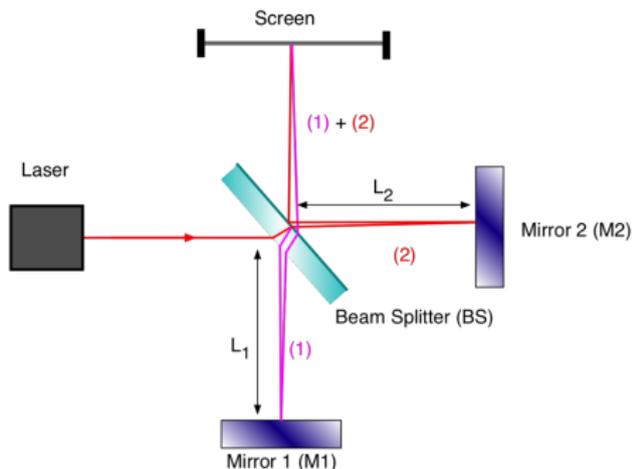


Figure:

<https://commons.wikimedia.org/wiki/File:Michelson Interferometer scheme.png>, Polytec GmbH, CC BY-SA 3.0 via Wikimedia Commons

Michelson Interferometer

- $L_2 = L_1 + d$
- Beam 1:

$$s_1 = a_1 \cos \left[\frac{2\pi}{\lambda} (ct - z) + \phi \right]$$
- Beam 2:

$$s_2 = a_2 \cos \left[\frac{2\pi}{\lambda} (ct - z - 2d) + \phi \right]$$
- Light intensity $I = \frac{1}{T} \int_0^T s^2 dt$ with
 $s = s_1 + s_2$
- $I = a_1 a_2 \cos \left(\frac{4\pi}{\lambda} d \right) + 0.5 (a_1^2 + a_2^2)$
- It $a_1 \ll a_2$, there is a problem of signal detection and accuracy

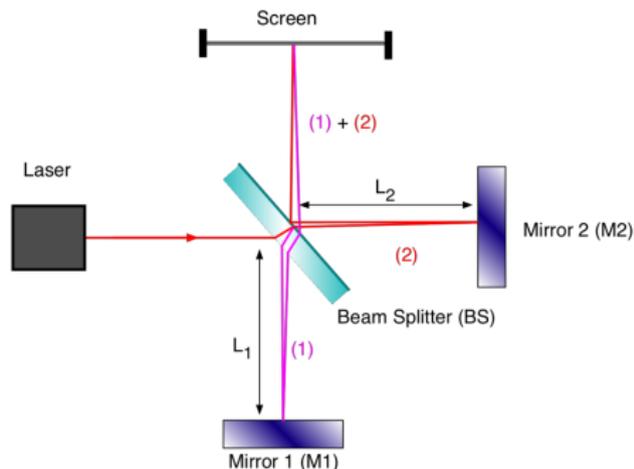


Figure:

<https://commons.wikimedia.org/wiki/File:Michelson Interferometer scheme.png>, Polytec GmbH, CC BY-SA 3.0 via Wikimedia Commons

Michelson Interferometer

- If the mirror is moved, there will be alternate of dark and light signal on the screen
- The wave length is $\lambda/2$

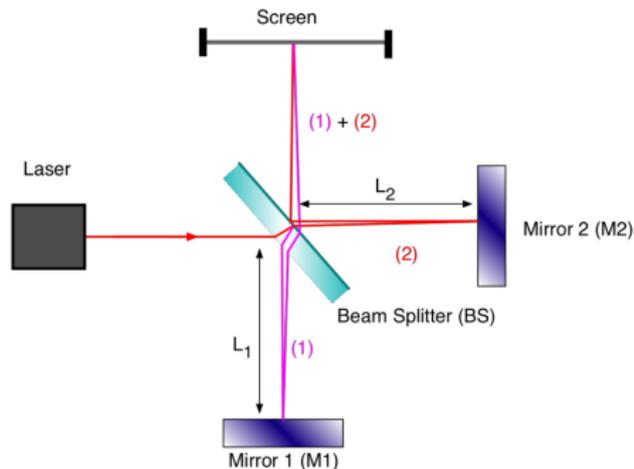
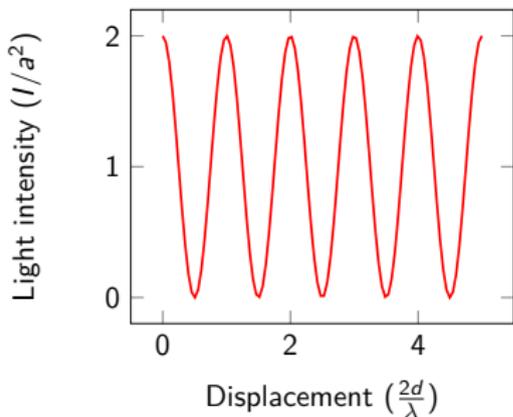


Figure:

<https://commons.wikimedia.org/wiki/File:Michelson Interferometer scheme.png>, Polytec GmbH, CC BY-SA 3.0 via Wikimedia Commons

Michelson Interferometer

- If one mirror is spherical, Newton's ring are obtained
- The lines are level lines with a height difference of $\lambda/2$ (with λ from 380 to 780 nm for visible light.)
- Is it convex or concave shape ?

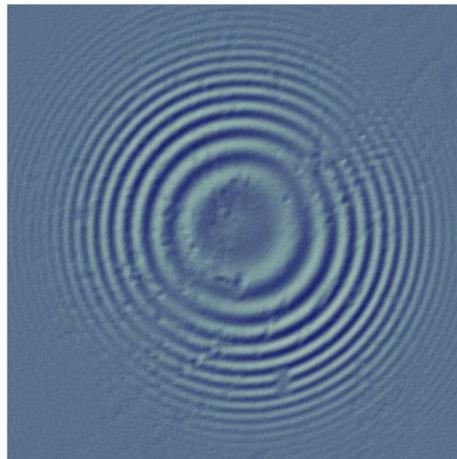


Figure: Interferences on ceramic ball (from a ball bearing)

Michelson Interferometer with white light

- White light has a spectrum of wave lengths (380–780 nm).
- If the mirror is moved, there will be alternate of dark and light signal on the screen but with decreasing amplitude (on spatial range of a few microns)

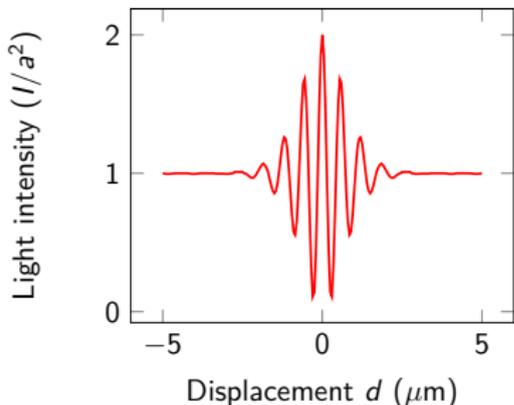


Figure: White light through a prism (https://upload.wikimedia.org/wikipedia/commons/0/0d/Dispersion_of_a_mercury_vapor_lamp_with_a_flint_glass_prism_IPNr_0125.jpg, D-Kuru, CC BY-SA 3.0 via Wikimedia Commons)

Phase shifting interferometry (monochromatic light)

- The real position is found by imposing controlled displacement u_i (using a piezo actuator or an accurate device)
- $I = a^2 \cos(\theta + \phi_i) + a^2$
- $\theta = \frac{4\pi}{\lambda} d$ and $\phi_i = \frac{4\pi}{\lambda} u_i$
- It is possible to calculate θ (with an unknown constant $k\pi$) and then d .
Unwrapping is necessary

Position is found from relative position to neighbor points

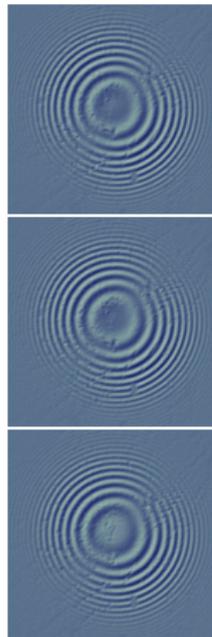


Figure: Interferences on ceramic ball at different ϕ values

Phase shifting interferometry (monochromatic light)

Advantage

- Very fast measurement (limited number of images)
- Vertical accuracy of less than 0.1 nm, independent of magnification

Disadvantages:

- Limited to flat surface (must be in focus everywhere)
- Limited to smooth surface ($Sq < 30$ nm), otherwise interference fringes are not visible.
- Does not work with steps $> \frac{\lambda}{2}$

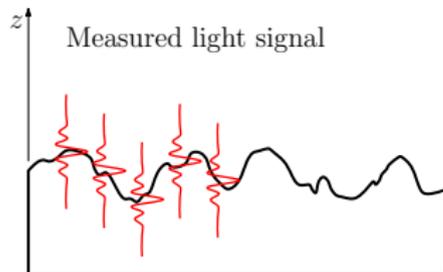
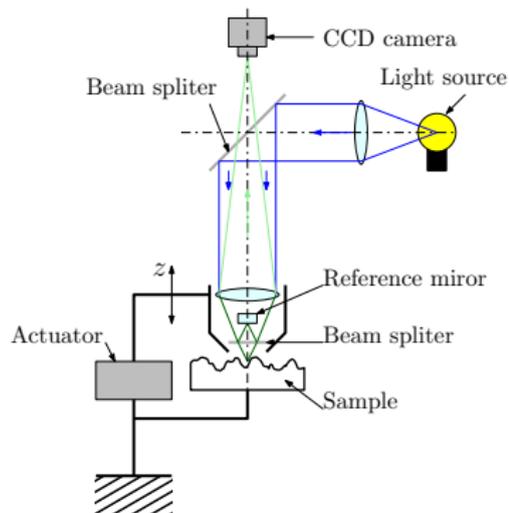


Figure: Example of surface adapted to phase shifting interferometry: hard drive disk surface

Vertical scanning interferometer (white light)

White light interferometer:

- Large vertical displacement (compared to phase shifting)
- Full interferogram for each pixel
- Height calculation independent of the neighbors



Vertical scanning interferometer (white light)

Advantage

- Vertical range not limited by depth of field
- Works with rough and stepped surfaces

Disadvantages:

- More images to record and analyse
- Slower than phase shifting

Position is independent of relative position to neighbor points



Figure: Interferences on ceramic ball during vertical scanning measurement

Interferometry lens

Lens	A_N	W_d (mm)	L (mm)	Δx (μm)	r (μm)	Z (μm)	Max slope
5x	0.13	9.3	3.6	3.6	2.12	20	3.5°
10x	0.30	7.4	1.8	1.8	0.92	3.73	8°
20x	0.40	4.7	0.9	0.9	0.69	2.10	14.5°
50x	0.55	3.6	0.36	0.36	0.50	1.11	22°

Table: Lens of the Talysurf CCI with $\delta z < 0.1$ nm

Focus variation

- A focus variation microscope uses confocal light and actuator for the lens
- The light reflected will vary depending on the focus level

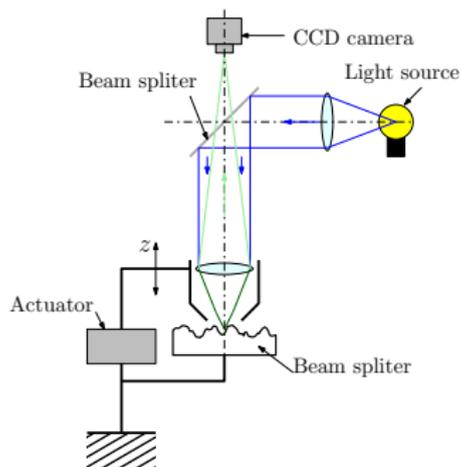


Figure: Focus variation microscope

Focus variation

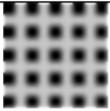
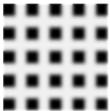
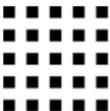
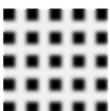
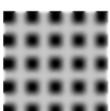
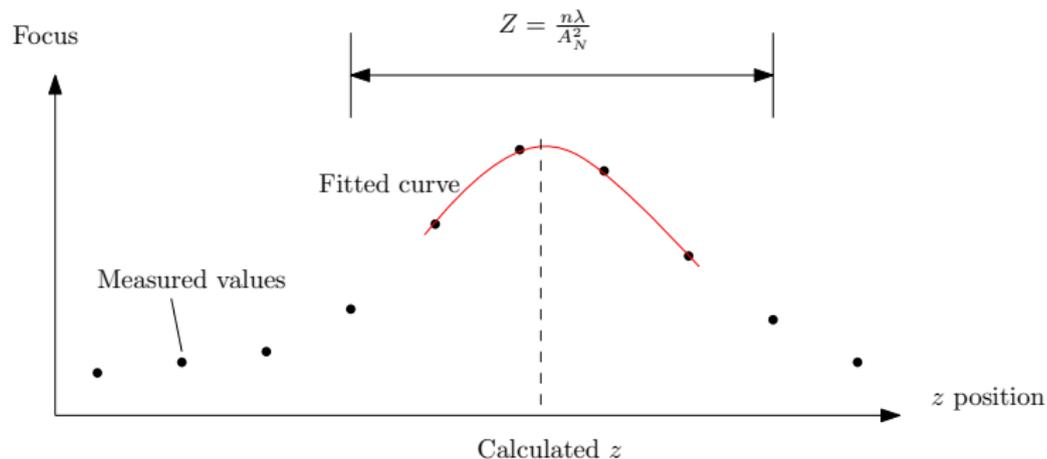
Focus	Image	Light RMS (neighbors of central pixel)
Out		10
Almost		20
Focus		50
Almost		20
Out		10

Figure: Effect of displacement of the objective on the image

Focus variation

- Identification of the maximum by curve fitting
- Vertical resolution dependent on the depth of field Z
- Necessity to use neighbor points information



Focus variation lens

Lens	A_N	Δx (μm)	r (μm)	Z (μm)	Δz (nm)	Max slope
10x	0.30	1	1.09	8.2	100	87°
20x	0.40	0.5	0.82	4.6	50	87°
50x	0.6	0.2	0.54	2.05	20	87°

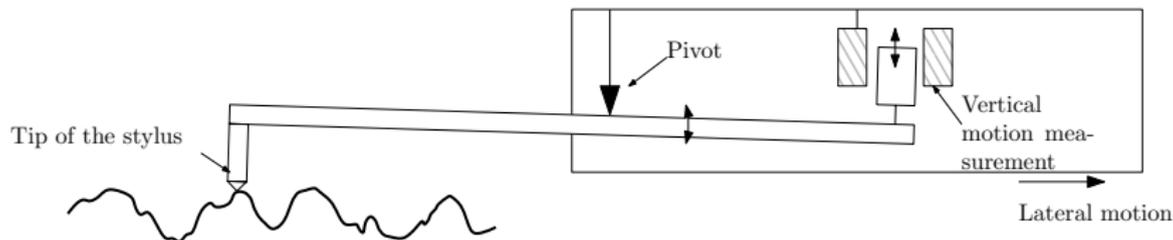
Table: Lens of the Alicona Infinte focus (Low vertical resolution but high surface slope limit)

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Stylus profiler:

- Vertical resolution $\Delta z = 1nm$;
- Lateral resolution $\Delta x = 2\mu m =$ curvature radius of the tip;
- Sample length $L_x = 10cm$;
- Requires profile or surface scan (can create scratches in soft materials)



Atomic force microscope

- Vertical resolution $\Delta z < 0.1nm$;
- Lateral resolution $\Delta x \simeq 1nm =$ curvature radius of the tip;
- Sample length $L_x < 0.1mm$;
- Requires profile or surface scan.

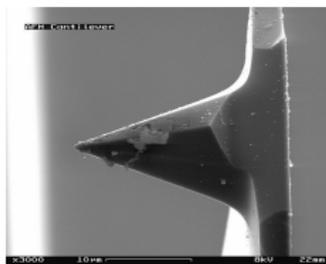
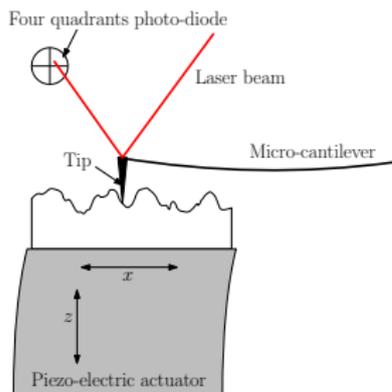


Figure: Left: Principle of an AFM - Right: AFM tip³

³: [https://commons.wikimedia.org/wiki/File:AFM_\(used\)_cantilever_in_Scanning_Electron_Microscope,_magnification_3000x.JPG](https://commons.wikimedia.org/wiki/File:AFM_(used)_cantilever_in_Scanning_Electron_Microscope,_magnification_3000x.JPG): SecretDisc, derivative work: MaterialsScientist, CC BY-SA 3.0, via Wikimedia Commons

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Comparison of instrument

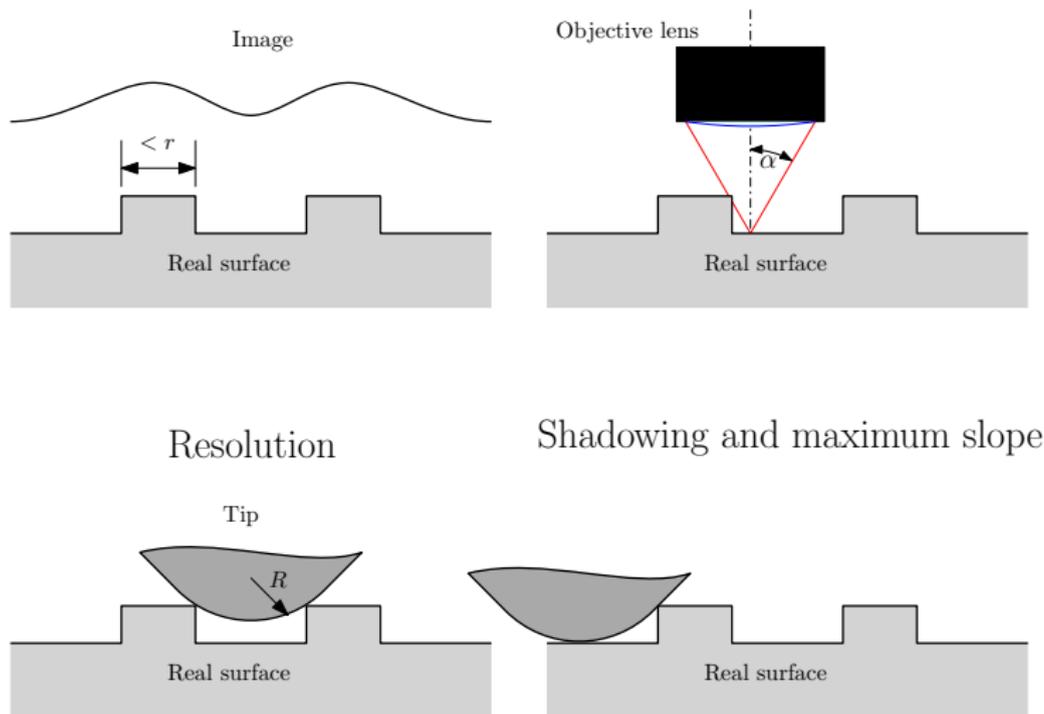


Figure: Usual limitations of optical and contacting measurement systems

Comparison of instrument

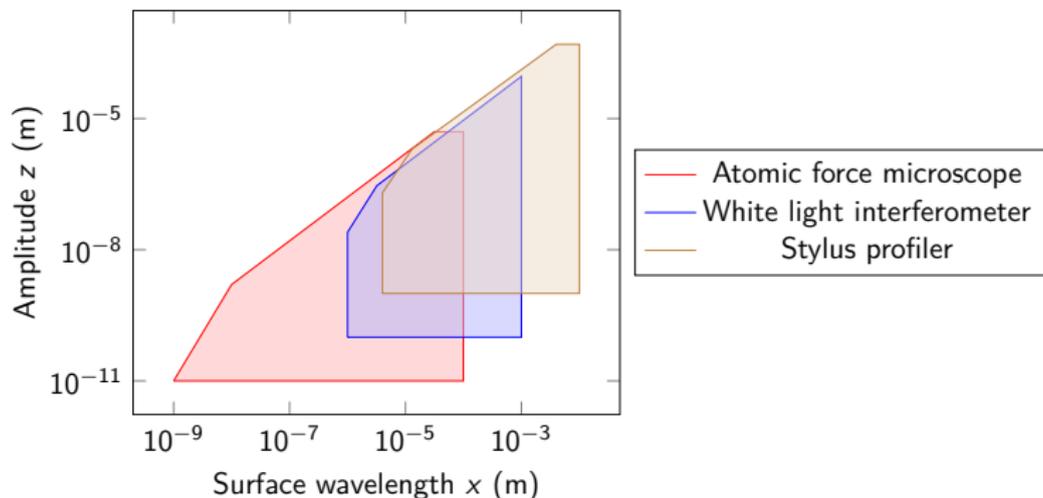
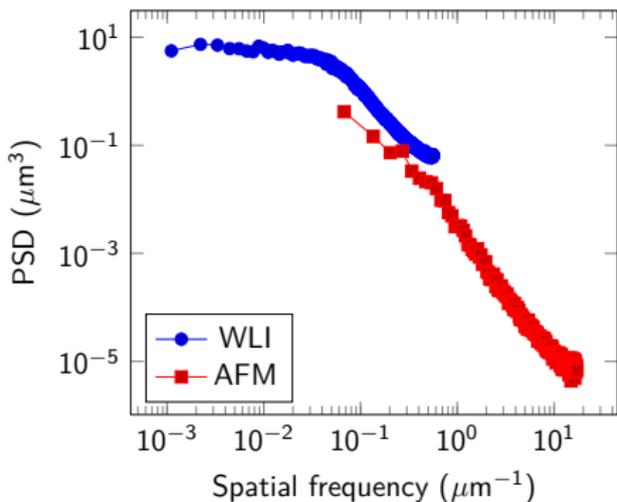


Figure: Comparison of the ranges of measurement of instruments (including slope and radius limits)[Ste87]

Comparison of instruments



Necessity to specify the instrument
(sample length and lateral sampling)
when giving S_q values

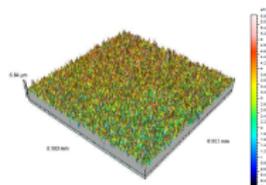


Figure: WLI (White Light Interferometer),
 $S_q = 0.93\mu\text{m}$, $L = 0.9\text{mm}$, $\Delta x = 0.9\mu\text{m}$

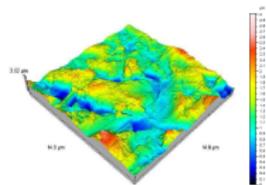


Figure: AFM (Atomic Force Microscope),
 $S_q = 0.37\mu\text{m}$, $L = 14.8\mu\text{m}$, $\Delta x = 0.029\mu\text{m}$

Comparison of instrument

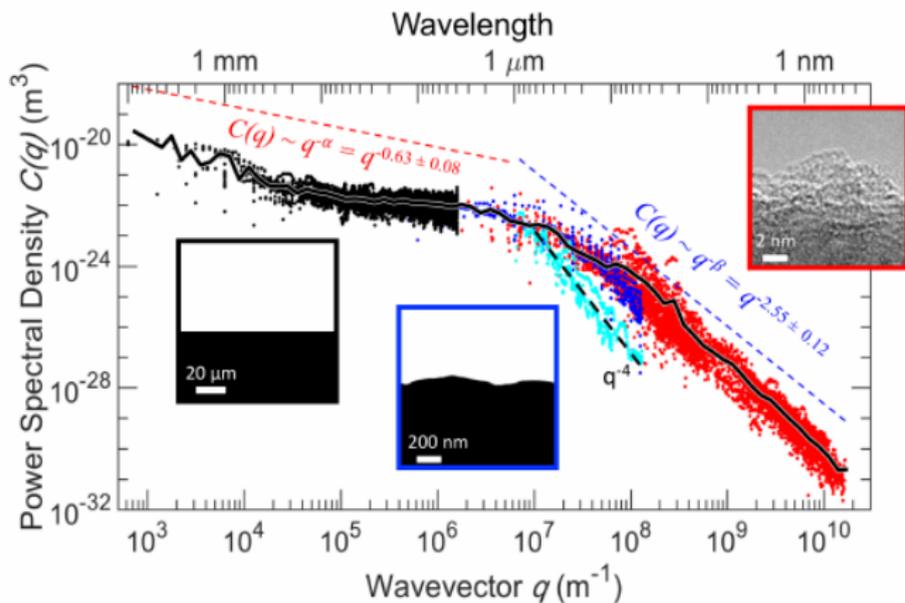


Figure: Comparison of different instruments on a surface: black = stylus profiler, blue = AFM, red = Transmission Electron Microscope, figure from [Guj+18] (open access on <https://d-scholarship.pitt.edu/37274/>)

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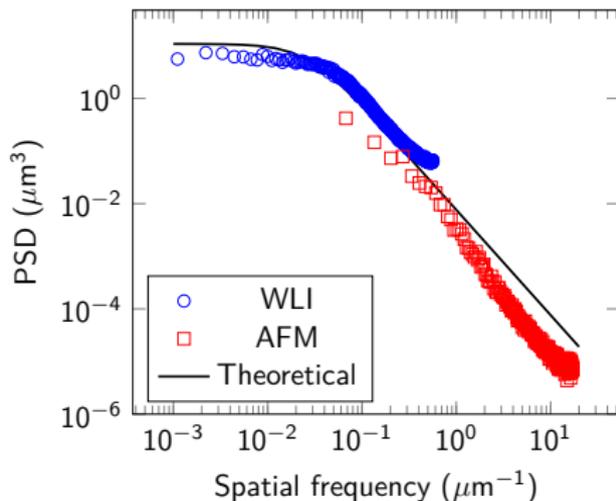
Summary

- Solid surface = Interface between a solid and another media;
- The topography is affected by the machining process, wear, etc;
- The topography influences surface functions and physical phenomena ;
- The surface topography is characterized by several parameters including Rq or Sq (roughness height) and Sa (lateral scale of asperities);
- These parameters depend on the measuring instrument;
- The instruments are defined by their bandwidth of spatial frequencies;
- This information must be given with the surface roughness parameters;
- Optical instruments offer areal measurements but some artifacts due to light reflection are possible
- Contact instruments requires a sampling of the surface (long process)

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Application



If the ACF is exponential, the PSD can be expressed: $PSD = \frac{2Rq_0^2\lambda}{1+4\pi^2\lambda^2f^2}$. Here $Rq_0 = 0.95\mu\text{m}$ and $\lambda = 6\mu\text{m}$

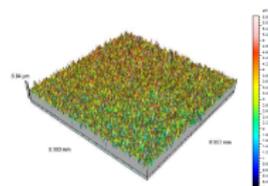


Figure: WLI (White Light Interferometer),
 $Sq = 0.93\mu\text{m}$, $L = 0.9\text{mm}$, $\Delta x = 0.9\mu\text{m}$

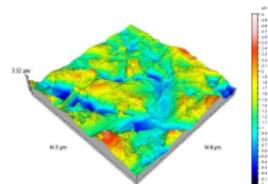


Figure: AFM (Atomic Force Microscope),
 $Sq = 0.37\mu\text{m}$, $L = 14.8\mu\text{m}$, $\Delta x = 0.029\mu\text{m}$

Application

- 1 We assume that $PSD = \frac{2Rq_0^2\lambda}{1+4\pi^2\lambda^2f^2}$
- 2 Express the Rq value for frequencies in the range f_1 and f_2 . We use:

$$\frac{Rq^2}{2} = \int_{f_1}^{f_2} PSDdf$$
- 3 Calculate the theoretical values of Rq for the AFM and WLI and compare to the measured values.

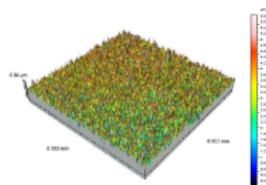


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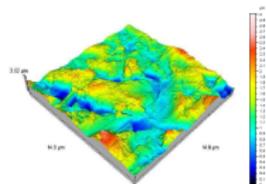


Figure: AFM (Atomic Force Microscope),
 $Sq = 0.37\mu\text{m}$, $L = 14.8\mu\text{m}$, $\Delta x = 0.029\mu\text{m}$

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