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Sound radiation of planar structure and nearfield acoustical holography : k-space analysis

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RECHERCHE



INSTITUT NATIONAL DES SCIENCES APPLIQUÉES DE LYON

Sound radiation of planar structure and nearfield acoustical holography: k-space analysis

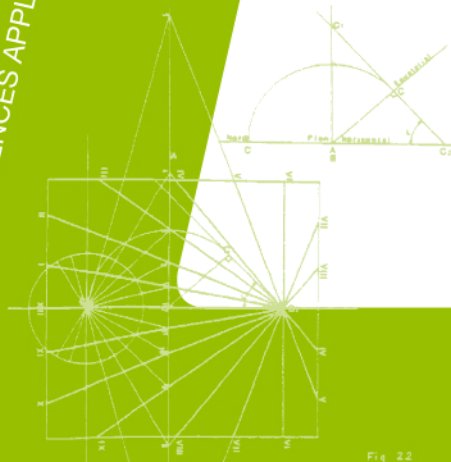


Fig. 22

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Centre Lyonnais
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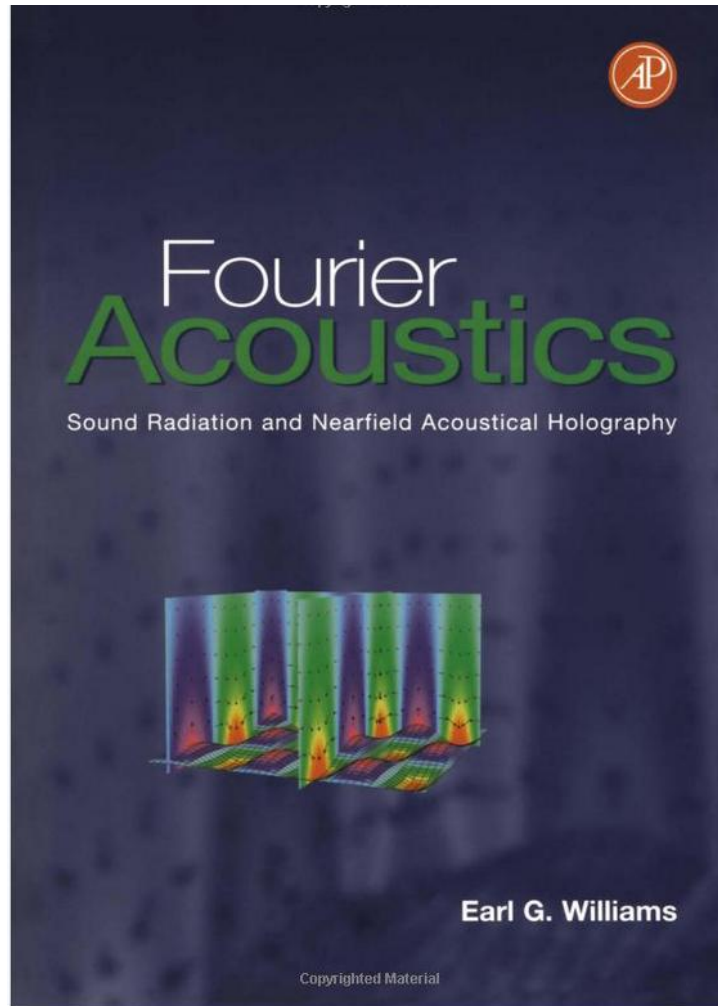
UNIVERSITE DE LYON



laboratoire
vibrations
acoustique



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DES SCIENCES
APPLIQUÉES
LYON



“Fourier Acoustics: Sound Radiation and Nearfield Acoustical Holography” by Earl G. Williams, 1999.

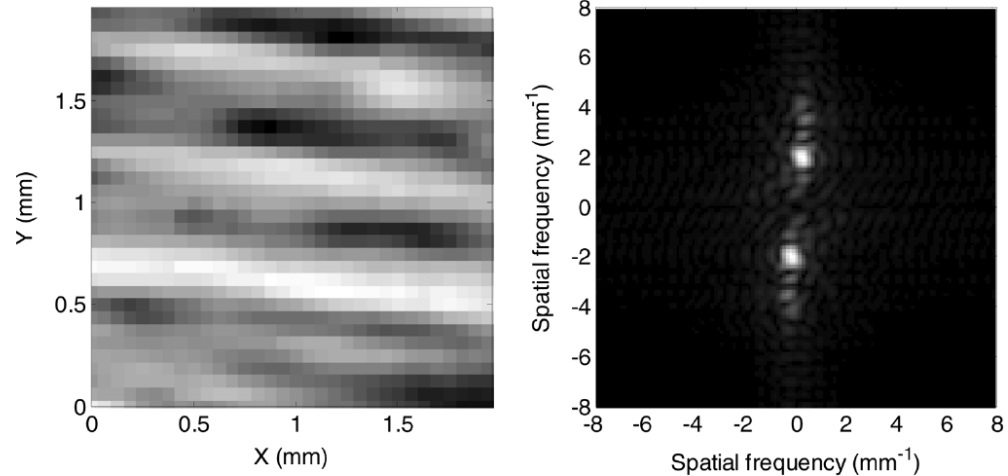
k-space analysis \approx spatial frequency analysis \approx wavenumber analysis

Illustration of k-space analysis for different applications

- Ultrasonics

Tendinopathy Discrimination by Use of Spatial Frequency Parameters in Ultrasound B-Mode Images

Gregory R. Bashford^{*}, Senior Member, IEEE, Nicholas Tomsen, Shruti Arya, Judith M. Burnfield, and Kornelia Kulig

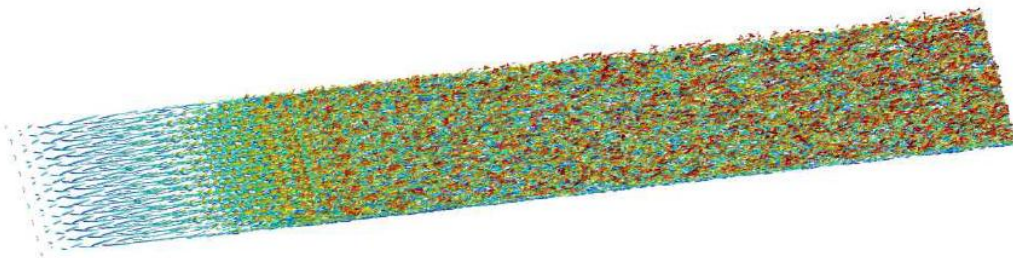


Example image kernel and spatial frequency spectrum.

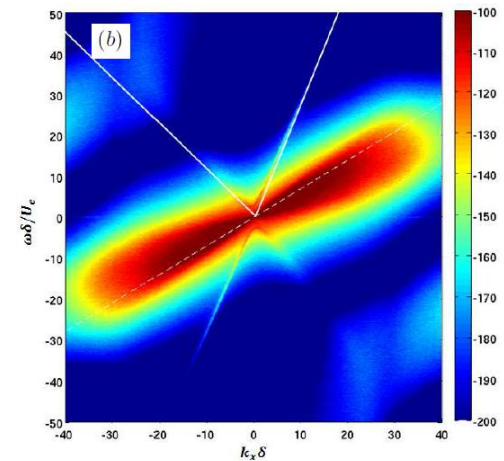
Left, typical 32×32 pixel kernel within the Achilles tendon of a control subject.

Right, spatial frequency spectrum obtained by taking the 2-D FFT of the left image.

- Aeroacoustics



LES of spatially developing TBL at Mach 0.5
Gloerfelt and Margnat (2014).



Wavenumber-frequency spectrum $10 \log_{10} \Phi_{pp}(\mathbf{k}, \omega)$ of wall pressure beneath a ZPG Mach 0.5 turbulent boundary layer

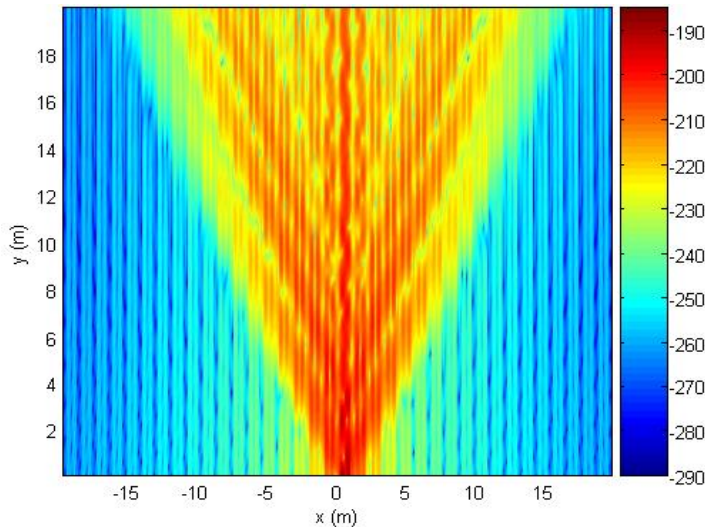
- Structural acoustics (vibro-acoustics)

Examples of industrial structures for which the K-space analysis is of interest for understanding how these structures radiate

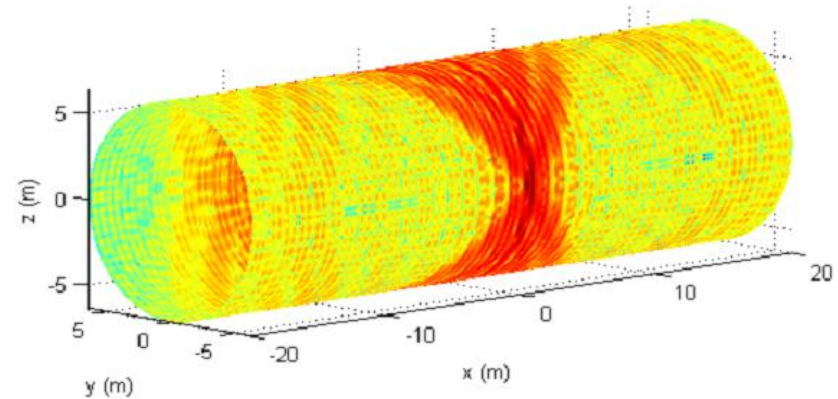


- Structural acoustics (vibro-acoustics)

Modelling the acoustic radiation from infinite structures



Transversal velocity field of an infinite periodically stiffened panel excited by an harmonic point force

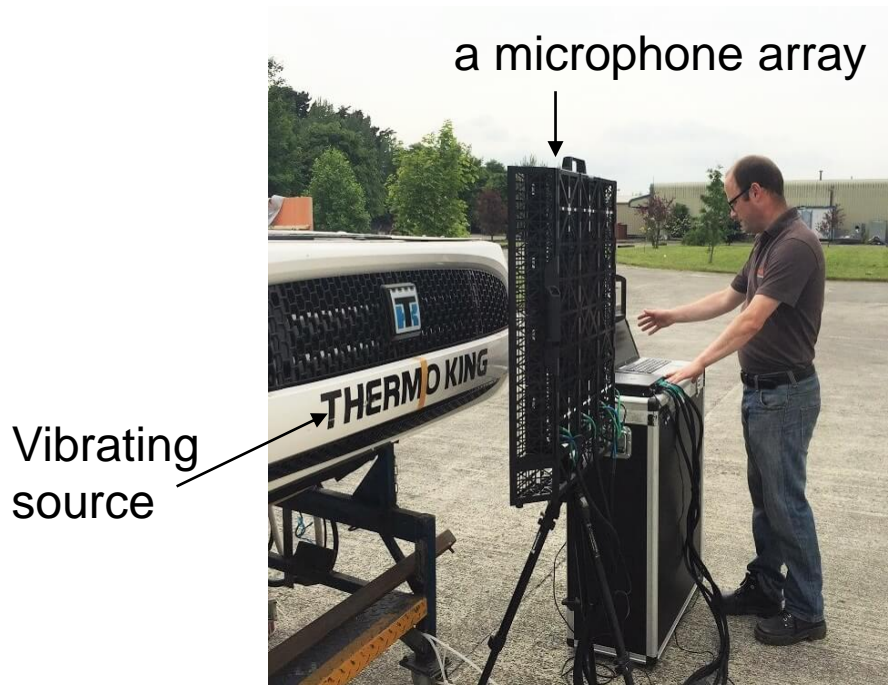


Radial velocity field of an infinite periodically stiffened shell excited by an harmonic point force

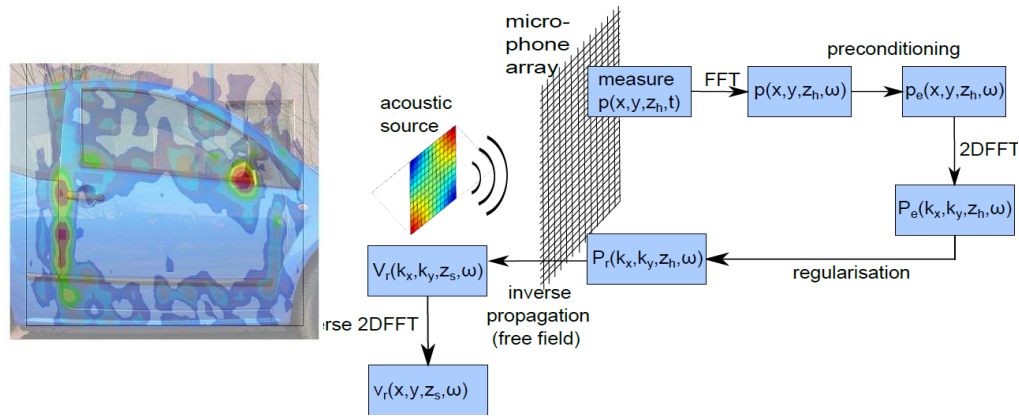
K-space analysis :

- **A tool for understanding how structures radiate in the far field**
- **Resolution of vibroacoustic problem**

Nearfield Acoustical Holography (NAH)



Measurement of the radiated pressure with a microphone array



Inverse problem: knowing the measured pressures and having a radiation model of the structure, what is the velocity field of the vibrating structure?

OUTLINE

I – Helmholtz equation

II – Vibro-acoustic behavior of infinite thin panels

III – Vibro-acoustic behavior of finite thin panels

IV – Nearfield Acoustical Holography (NAH)

I – Helmholtz equation

Assumptions on the acoustic domain:

- stagnant uniform fluid or gas
- perfect fluid (i.e. no viscosity, or heat conduction)
- No mass source or external volume force terms

Fluid equations with the previous assumptions:

- Mass conservation

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{u}) = 0$$

- The momentum conservation law (second Newton's law)

$$\rho \left(\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} \right) = -\nabla p$$

- The energy conservation law (adiabatic-reversible = isentropic processus)

$$\frac{\partial s}{\partial t} + \mathbf{u} \cdot \nabla s = 0$$

- An equation of state (thermodynamics)

$$p = p(\rho, s)$$

$$\text{Isentropic flow} \quad \left(\frac{\partial p}{\partial s} \right)_{\rho} = 0$$

$$c^2 = \left(\frac{\partial p}{\partial \rho} \right)_s \quad \text{the square of the isentropic speed of sound}$$

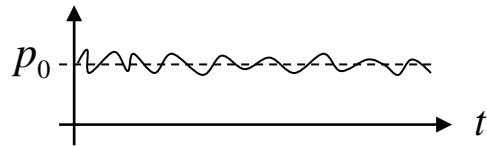
*S.W. Rienstra & A. Hirschberg, An Introduction to Acoustics, Eindhoven University of Technology, 2014

I – Helmholtz equation

Linearization and wave equation:

Small perturbations around the equilibrium position:

$p = p_0 + p'$, $\rho = \rho_0 + \rho'$, $s = s_0 + s'$, $\mathbf{u} = \mathbf{u}_0 + \mathbf{u}'$ with $\mathbf{u}_0 = 0$ for stagnant fluid



Linearized equations (taking the 1st order terms only):

$$\frac{\partial \rho'}{\partial t} + \rho_0 \nabla \cdot \mathbf{u}' = 0 \quad (1) \quad \rho_0 \frac{\partial \mathbf{u}'}{\partial t} + \nabla p' = 0 \quad (2) \quad p' = c_0^2 \rho' \quad (3) \quad \frac{\partial s'}{\partial t} = 0 \quad (4)$$

Wave equation (introducing (3) in (1) and writing $\nabla \cdot (2) - \frac{\partial}{\partial t} (1)$)

$$\nabla^2 p' - \frac{1}{c_0^2} \frac{\partial^2 p'}{\partial t^2} = 0$$

Helmholtz equation (steady state harmonic variation $p' = \bar{p} e^{i\omega t}$)

$$\nabla^2 \bar{p} + \frac{\omega^2}{c_0^2} \bar{p} = 0$$

II – Vibro-acoustic behavior of infinite thin panels

II.1 - Natural propagative waves

II.1.a - For an acoustic medium

Helmholtz equation $\nabla^2 p + \frac{\omega^2}{c_0^2} p = 0$

Dispersion equation $k = f(\omega)$

$$k_x^2 + k_y^2 + k_z^2 = k_0^2$$

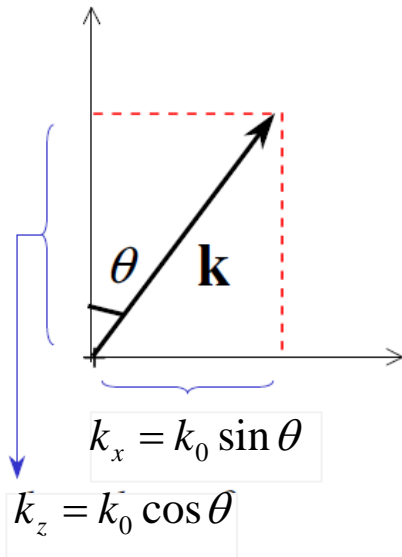
with the acoustic (natural) wavenumber $k_0 = \frac{\omega}{c_0}$

Propagative plane wave $p = P e^{-ik_x x - ik_y y - ik_z z}$

Wavevector \mathbf{k} : $\mathbf{k} = \begin{pmatrix} k_x \\ k_y \\ k_z \end{pmatrix}$

Dispersion equation: $\|\mathbf{k}\|_2 = k_0$

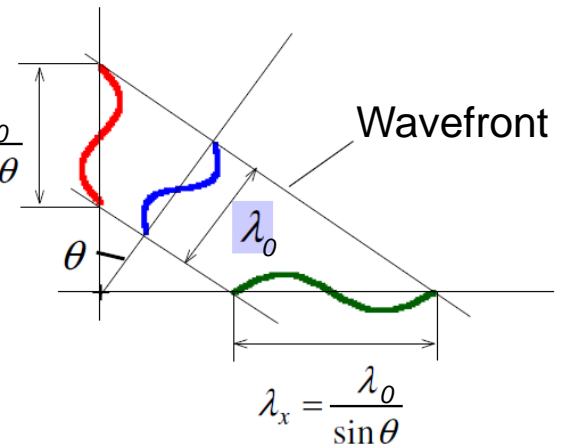
an example with $k_y = 0$



$$\lambda_\zeta = \frac{2\pi}{k_\zeta}$$

$$\lambda_z = \frac{\lambda_0}{\cos \theta}$$

with ζ either stands for $x, z, 0$



II.1 - Natural propagative waves

II.1.b - For a thin flexible plate



Infinite thin plate made of an isotropic elastic material

Plate equation of motions with Love-Kirchhoff assumptions and time-harmonic dependence (pure bending, rotational inertia and shear stress not take into account, only adapted for “low” frequencies, use Mindlin assumptions for “high” frequencies):

$$D \left(\frac{\partial^4 w}{\partial x^4} + 2 \frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4} \right) - \omega^2 \rho h w = 0 \quad \text{with the flexural rigidity } D = Eh^3 / 12(1 - \nu^2)$$

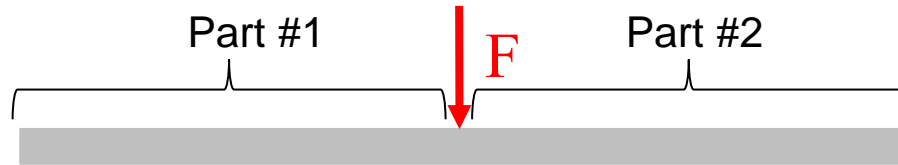
Propagative plane wave $w = We^{-ik_x x - ik_y y}$

$$\text{Dispersion equation } (k_x^2 + k_y^2)^2 = \frac{\omega^2 \rho h}{D}$$

$$\rightarrow \text{For } k_y = 0, k_x^4 = \frac{\omega^2 \rho h}{D} \rightarrow 4 \text{ solutions: } k_x = \begin{cases} k_f \\ -k_f \\ ik_f \\ -ik_f \end{cases} \begin{cases} \text{Propagative waves} \\ \text{Evanescent waves} \end{cases}$$

$$\text{Flexural (natural) wavenumber } k_f = \sqrt{\omega \sqrt{\frac{\rho h}{D}}}$$

Forced response using the wave decomposition:



Infinite thin plate excited by a normal line force

Wave decomp. for each part (with non reflective conditions at infinite):

$$\begin{cases} w_1(x) = A_1 e^{ik_f x} + B_1 e^{k_f x}, x \in]-\infty, 0[\\ w_2(x) = A_2 e^{-ik_f x} + B_2 e^{-k_f x}, x \in [0, \infty[\end{cases}$$

Continuity relations at the junction:

$$\begin{cases} w_1(0) = w_2(0) \\ \frac{\partial w_1}{\partial x}(0) = \frac{\partial w_2}{\partial x}(0) \\ D \frac{\partial^2 w_1}{\partial x^2}(0) = D \frac{\partial^2 w_2}{\partial x^2}(0) \\ D \frac{\partial^3 w_1}{\partial x^3}(0) - F = D \frac{\partial^3 w_2}{\partial x^3}(0) \end{cases}$$

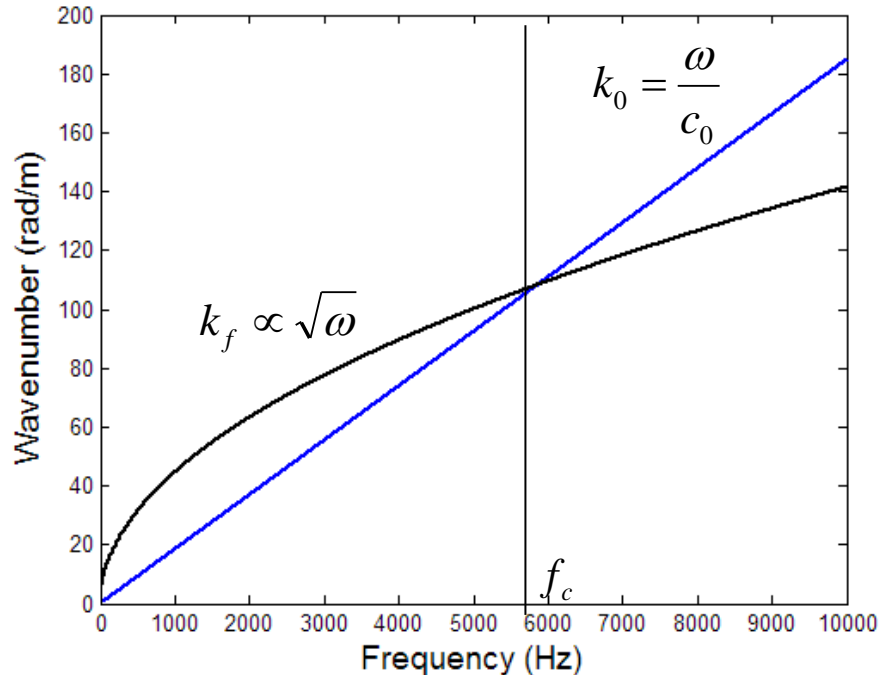
$$\begin{aligned} &\Rightarrow \begin{cases} A_1 + B_1 = A_2 + B_2 \\ iA_1 + B_1 = -iA_2 - B_2 \\ -A_1 + B_1 = -A_2 + B_2 \\ -ik_f^3 A_1 + k_f^3 B_1 + \frac{F}{D} = ik_f^3 A_2 - k_f^3 B_2 \end{cases} \\ &\Rightarrow \begin{cases} A_1 = A_2 \\ B_2 = -iA_2 \\ B_1 = -iA_2 \\ A_2 = \frac{F}{4iDk_f^3} \end{cases} \end{aligned}$$

Analytical expression of the plate displacement:

$$w_2(x) = \frac{F}{4iDk_f^3} e^{-ik_f x} - \frac{F}{4Dk_f^3} e^{-k_f x}, x \in [0, \infty[$$

II.1 - Natural propagative waves

II.1.c - The critical frequency (or acoustic coincidence frequency)



Critical frequency (flexural celerity = acoustic celerity): $k_0 = k_f$

$$f_c = \frac{c_0^2}{2\pi} \sqrt{\frac{\rho h}{D}} = \frac{c_0^2}{2\pi} \sqrt{\frac{12\rho(1-\nu^2)}{Eh^2}} \rightarrow 5.8 \text{ kHz for a 2mm-thick steel plate}$$

II.2 – Mathematical formulation and resolution in the k-space

Time dependence: $e^{i\omega t}$

Semi-infinite acoustic medium

$$\Delta p + k_0^2 p = 0$$

Infinite plate

$$\frac{\partial p}{\partial z} = -\rho_0 (i\omega) v_z$$

$$v_z = i\omega w$$

$$D(1+i\eta) \left(\frac{\partial^4 w}{\partial x^4} + 2 \frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4} \right) - \omega^2 \rho h w = F \delta - p \Big|_{z=0}$$

Resolution using the spatial Fourier transform:

$$f(x, y) \rightarrow \tilde{f}(k_x, k_y) = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} f(x, y) e^{-ik_x x} e^{-ik_y y} dx dy$$

$$\tilde{f}(k_x, k_y) \rightarrow f(x, y) = \frac{1}{4\pi^2} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \tilde{f}(k_x, k_y) e^{ik_x x} e^{ik_y y} dk_x dk_y$$

Property of the Fourier transform on the space derivative:

$$\frac{\partial^p \partial^q f}{\partial x^p \partial y^q}(x, y) \rightarrow (ik_x)^p (ik_y)^q \tilde{f}(k_x, k_y)$$

...

Vibroacoustic problem in the wavenumber space (i.e. k-space):

$$\left\{ \begin{array}{l} \tilde{v}_z(k_x, k_y) = i\omega \tilde{w}(k_x, k_y) \\ Z_p \tilde{v}(k_x, k_y) = \tilde{F}(k_x, k_y) - \tilde{p}(k_x, k_y, 0) \text{ with } Z_p = \left[D(1+i\eta)(k_x^2 + k_y^2)^2 - \omega^2 \rho h \right] / i\omega \\ \frac{\partial^2 \tilde{p}}{\partial z^2}(k_x, k_y, z) + (k_0^2 - k_x^2 - k_y^2) \tilde{p}(k_x, k_y, z) = 0 \\ \frac{\partial \tilde{p}}{\partial z}(k_x, k_y, 0) = -\rho_0(i\omega) \tilde{v}(k_x, k_y) + \text{Sommerfeld conditions} \end{array} \right.$$

Resolution of the fluid equations:

Homogeneous 2nd order differential equation about z:

$$\frac{\partial^2 \tilde{p}}{\partial z^2}(k_x, k_y, z) + (k_0^2 - k_x^2 - k_y^2) \tilde{p}(k_x, k_y, z) = 0$$

Solution on the following form: $\tilde{p}(k_x, k_y, z) = \tilde{p}(k_x, k_y, 0) e^{ik_z z}$ with k_z the vertical wavenumber is unknown

→ Characteristic equation of the differential equation: $k_z^2 = k_0^2 - k_x^2 - k_y^2$

Solutions satisfying the Sommerfeld conditions:

$$k_z = \begin{cases} -\sqrt{k_0^2 - k_x^2 - k_y^2} & \text{if } \sqrt{k_x^2 + k_y^2} < k_0 & \rightarrow \text{Propagative waves} \\ i\sqrt{k_x^2 + k_y^2 - k_0^2} & \text{if } \sqrt{k_x^2 + k_y^2} > k_0 & \rightarrow \text{Evanescent waves} \end{cases}$$

Euler equation:

$$\tilde{p}(k_x, k_y, 0) = Z_a \tilde{v}(k_x, k_y) \quad \text{with} \quad Z_a = -\frac{\rho_0 \omega}{k_z}$$

$$\text{then} \quad \tilde{p}(k_x, k_y, z) = Z_a \tilde{v}(k_x, k_y) e^{ik_z z}$$

Vibroacoustic problem in the wavenumber space (i.e. k-space):

$$\left\{ \begin{array}{l} Z_p \tilde{v}(k_x, k_y) = \tilde{F} - \tilde{p}(k_x, k_y, 0) \quad \text{with} \quad Z_p = \left[D(1+i\eta)(k_x^2 + k_y^2)^2 - \omega^2 \rho h \right] / i\omega \\ \tilde{p}(k_x, k_y, z) = Z_a \tilde{v}(k_x, k_y) e^{ik_z z} \quad \text{with} \quad Z_a = -\frac{\rho_0 \omega}{k_z}, \quad k_z = \begin{cases} -\sqrt{k_0^2 - k_x^2 - k_y^2} & \text{if } \sqrt{k_x^2 + k_y^2} < k_0 \\ i\sqrt{k_x^2 + k_y^2 - k_0^2} & \text{if } \sqrt{k_x^2 + k_y^2} > k_0 \end{cases} \end{array} \right.$$

Spectral velocity (i.e. velocity in the wavenumber space):

$$\tilde{v}(k_x, k_y) = \frac{\tilde{F}(k_x, k_y)}{Z_p(k_x, k_y) + Z_a(k_x, k_y)}$$

Spectral pressure at z (i.e. pressure in the wavenumber space for z given):

$$\tilde{p}(k_x, k_y, z) = \Gamma(k_x, k_y, z) \tilde{v}(k_x, k_y)$$

with the spectral propagator $\Gamma(k_x, k_y, z) = Z_a(k_x, k_y) e^{ik_z z}$

RK: Spectral force for a normal point force at (x_0, y_0)

$$\tilde{F}(k_x, k_y) = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \delta(x - x_0) \delta(y - y_0) e^{-ik_x x} e^{-ik_y y} dx dy = e^{-ik_x x_0} e^{-ik_y y_0}$$

II.3 – Estimation of the plate velocity in the physical space

Inverse space - Fourier transform:

$$\tilde{f}(k_x, k_y) \xrightarrow{I.F.T.} f(x, y) = \frac{1}{4\pi^2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \tilde{f}(k_x, k_y) e^{j(k_x x + k_y y)} dk_x dk_y$$

→ No analytical expressions for the Fourier transform of the spectral displacement/pressure in general

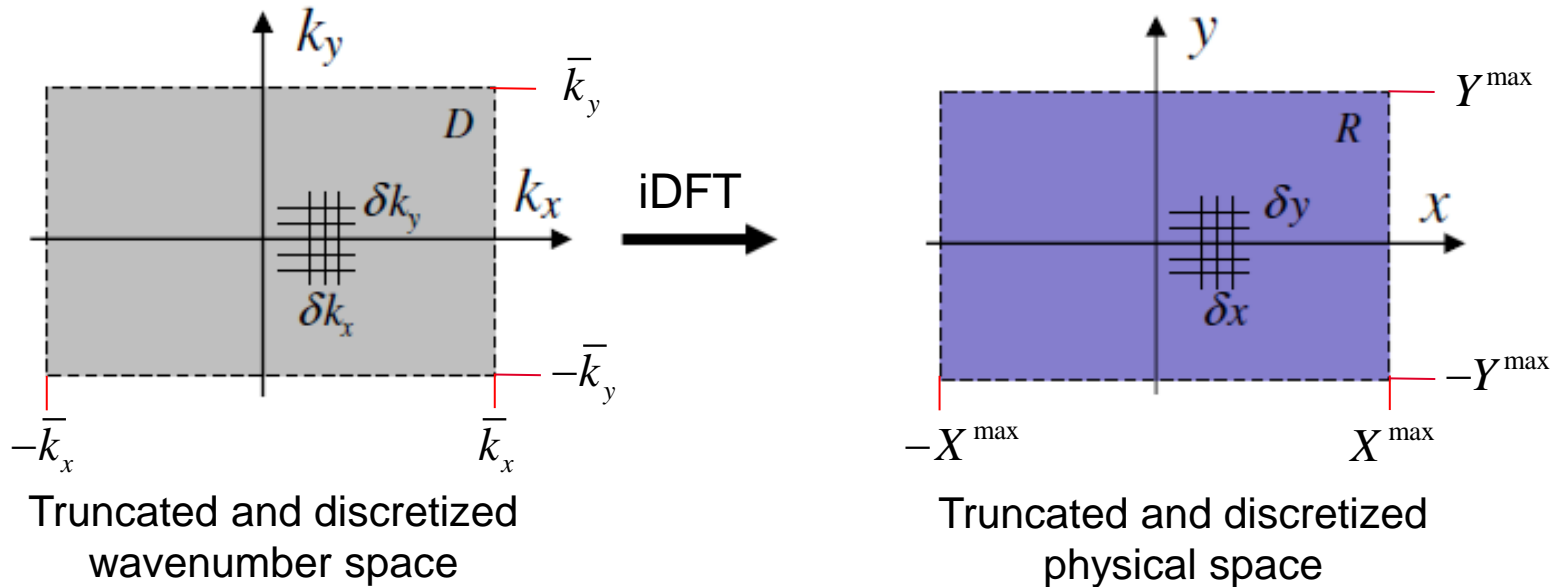
→ Alternative: use an inverse Discrete Fourier Transform (iDFT) for approximating the solution

Truncation and sampling of the wavenumber domain D :

$$\tilde{F}_{n_{k_x}, n_{k_y}} = \tilde{f}(n_{k_x} \delta k_x, n_{k_y} \delta k_y), \quad \forall (n_x, n_y) \in \left(\left[-\frac{\bar{k}_x}{\delta k_x}, \frac{\bar{k}_x}{\delta k_x} \right] \times \left[-\frac{\bar{k}_y}{\delta k_y}, \frac{\bar{k}_y}{\delta k_y} \right] \right)$$

where $\delta k_x, \delta k_y$ are the wavenumber resolution, and,

\bar{k}_x, \bar{k}_y the cut-off wavenumbers of D .



Properties of the iDFT:

$$\delta x = \frac{2\pi}{\Delta k_x} = \frac{2\pi}{k_x^{\max} - k_x^{\min}} \longrightarrow \delta x = \frac{\pi}{k_x}, \quad \delta y = \frac{\pi}{k_y}$$

$$\Delta X = X^{\max} - X^{\min} = \frac{2\pi}{\delta k_x} \longrightarrow X^{\max} = \frac{\pi}{\delta k_x}, \quad Y^{\max} = \frac{\pi}{\delta k_y}$$

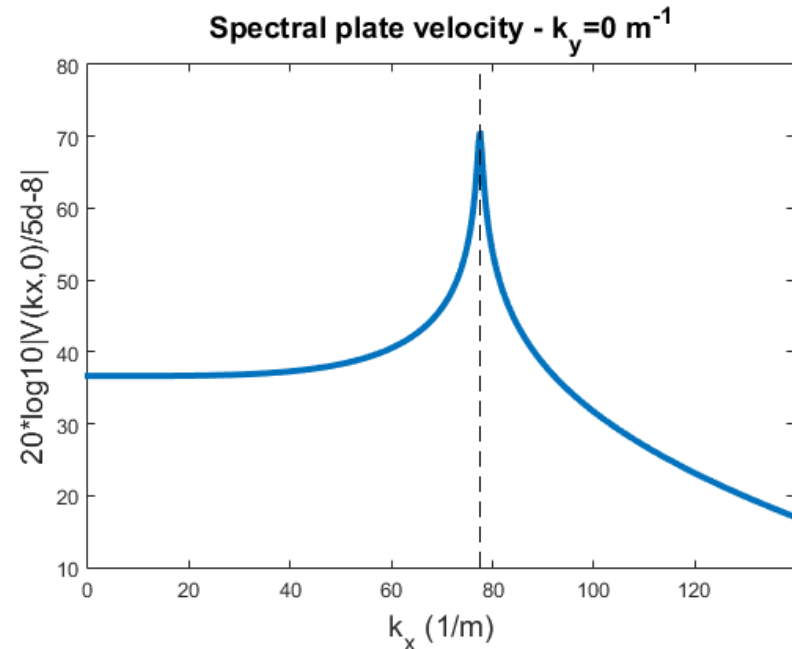
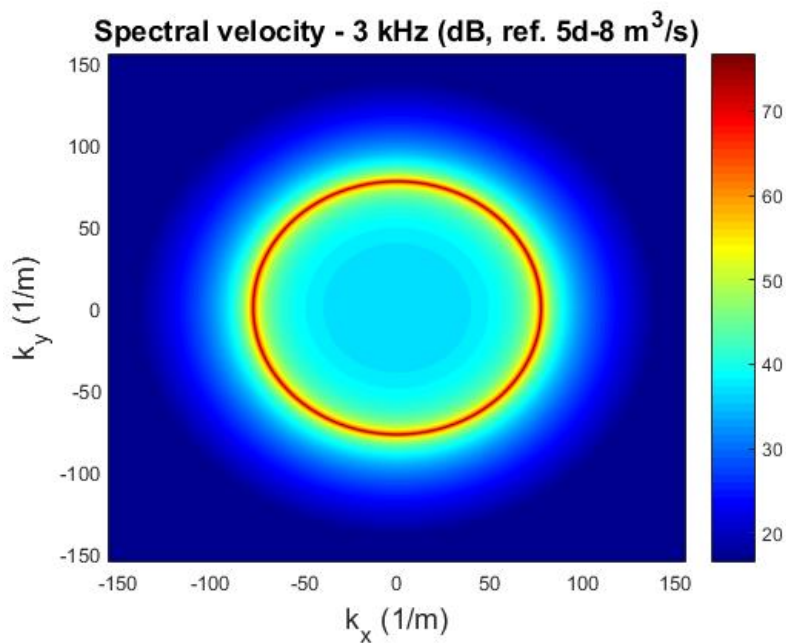
➔ The wavenumber resolutions and the cut-off wavenumbers should be chosen carefully to avoid the loss of information and the aliasing effect.

II.4 – Analysis of the plate velocity in the k-space WITHOUT fluid

Spectral velocity response of an *in-vacuo* plate excited by a point force at (0,0)

$$\tilde{v}_{in-vacuo}(k_x, k_y) = \frac{F_0}{Z_p(k_x, k_y)} \quad Z_p = \left[D(1+i\eta)(k_x^2 + k_y^2)^2 - \omega^2 \rho h \right] / i\omega$$

Steel plate - Thickness= 2 mm



Dash line, flexural wavenumber (kf)

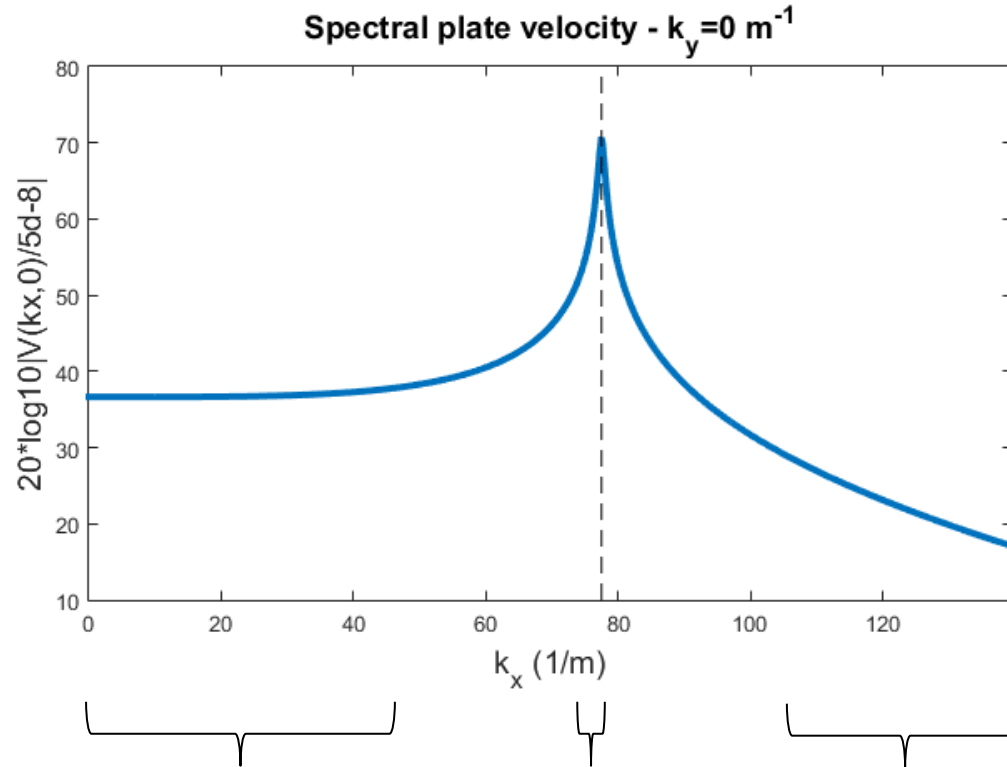
➔ $k_y=0$: line force along y at x=0

Spectral velocity response of an *in-vacuo* plate excited by a point force at (0,0)

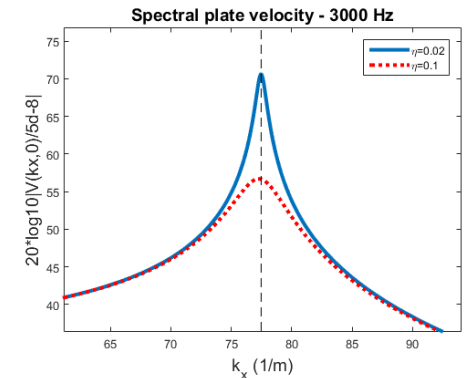
$$\tilde{v}_{in-vacuo}(k_x, 0) = \frac{F_0}{Z_p(k_x, 0)}$$

$$Z_p(k_x, 0) = [D(1+i\eta)k_x^4 - \omega^2 \rho h] / i\omega$$

$$Z_p(k_x, 0) = D[(1+i\eta)k_x^4 - k_f^4] / i\omega$$



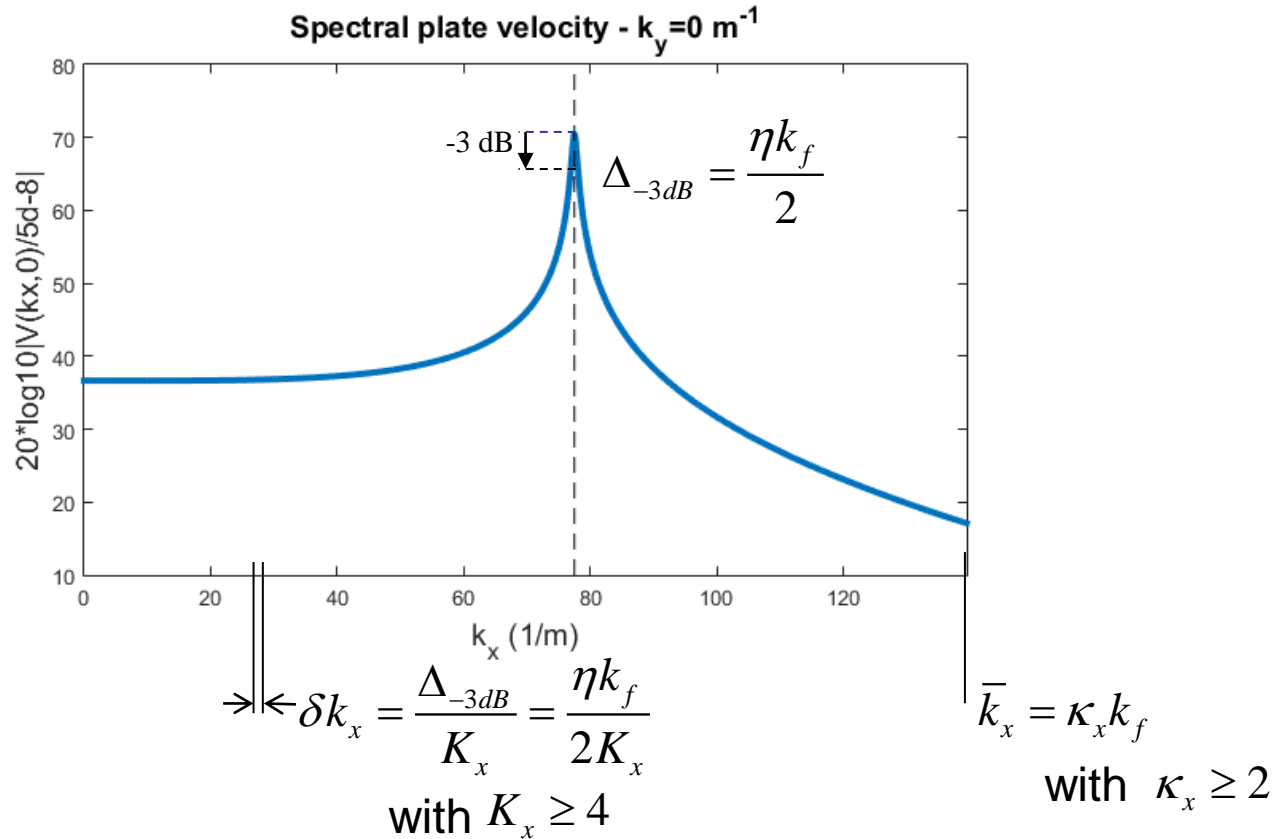
$$k_f = \sqrt{\omega} \sqrt{\frac{\rho h}{D}}$$



$$Z_p(k_x, 0) \approx [-\omega^2 \rho h] / i\omega \quad Z_p(k_f, 0) \approx iD\eta k_f^4 / i\omega \quad Z_p(k_x, 0) \approx Dk_x^4 / i\omega$$

Mass controlled Damping controlled Stiffness controlled

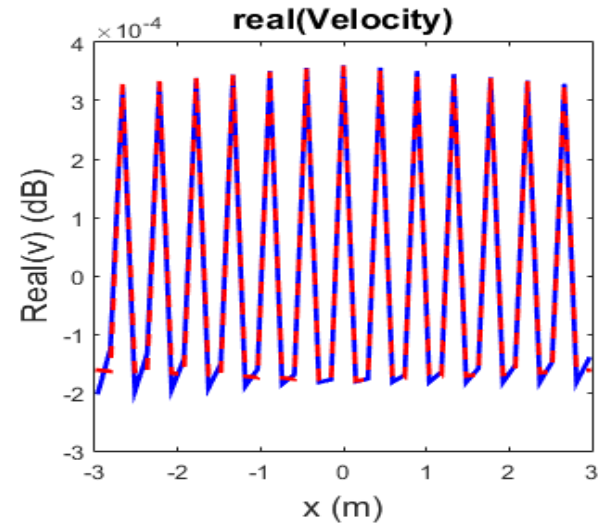
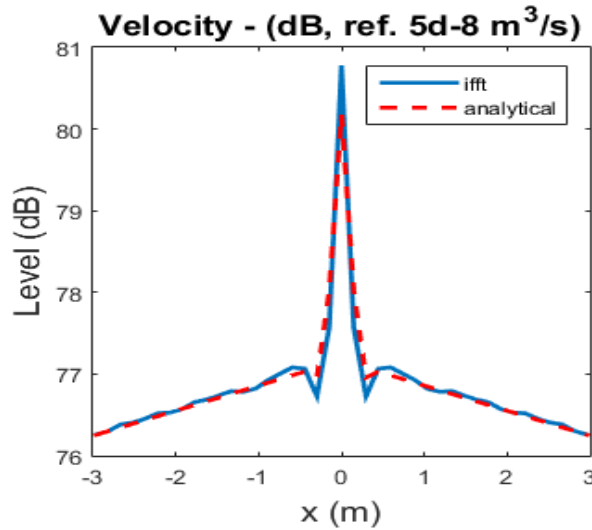
Criteria for choosing the wavenumber resolutions and the cut-off wavenumbers:



Comparison with the analytical result (Matlab code: *plate_velocity.m*)

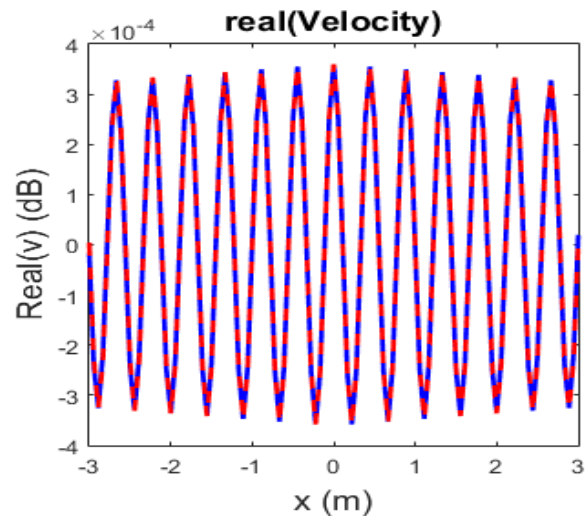
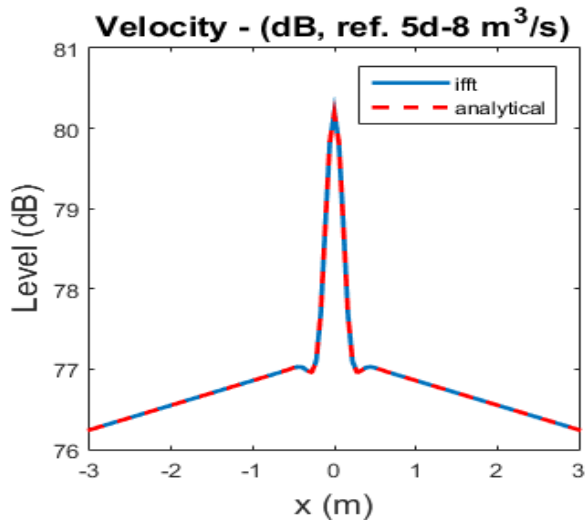
$$K_x = 2$$

$$K_x = 1.5$$



$$K_x = 4$$

$$K_x = 4$$



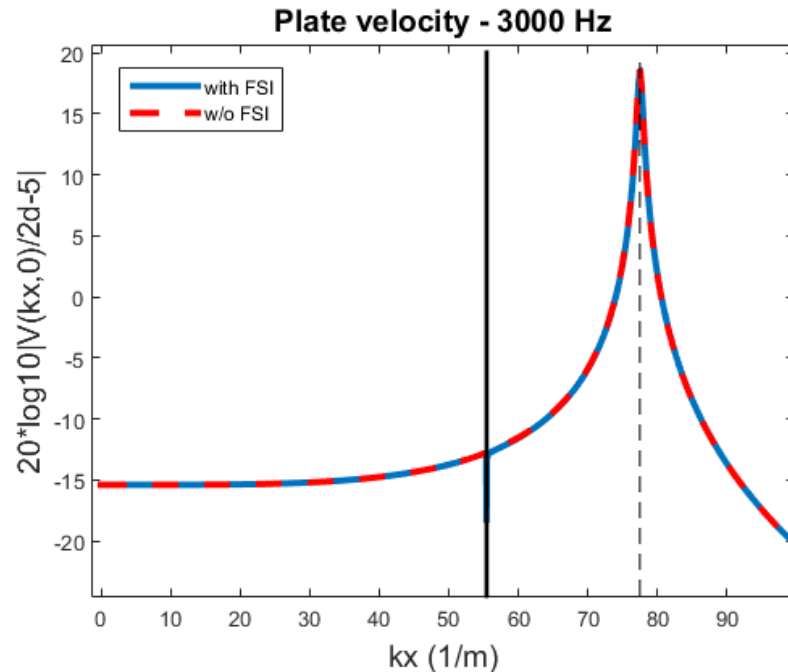
II.5 – Analysis of the plate velocity in the k-space WITH fluid

Spectral velocity response of an **air** loaded plate excited by a point force at (0,0)

Steel plate in air - Thickness= 2 mm - Critical frequency= 5.8kHz

$$\tilde{v}(k_x, 0) = \frac{F_0}{Z_p(k_x, 0) + Z_a(k_x, 0)}$$

Matlab code: *spectral_fluid_loaded_plate_velocity.m*



**! Below the
critical
frequency !**

Dash line, flexural wavenumber (kf)
Solid line, acoustic wavenumber (k₀)

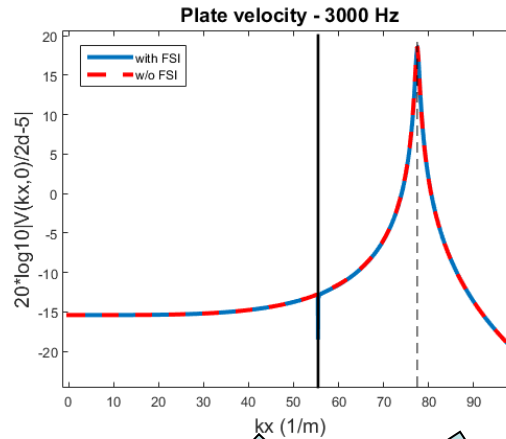
Spectral velocity response of an **air** loaded plate excited by a point force at (0,0)

Steel plate in air - Thickness= 2 mm - Critical frequency= 5.8kHz

Below the critical frequency

$$\tilde{v}(k_x, 0) = \frac{F_0}{Z_p(k_x, 0) + Z_a(k_x, 0)}$$

$$\lim_{k_x \rightarrow k_0} Z_a(k_x, 0) = \infty$$



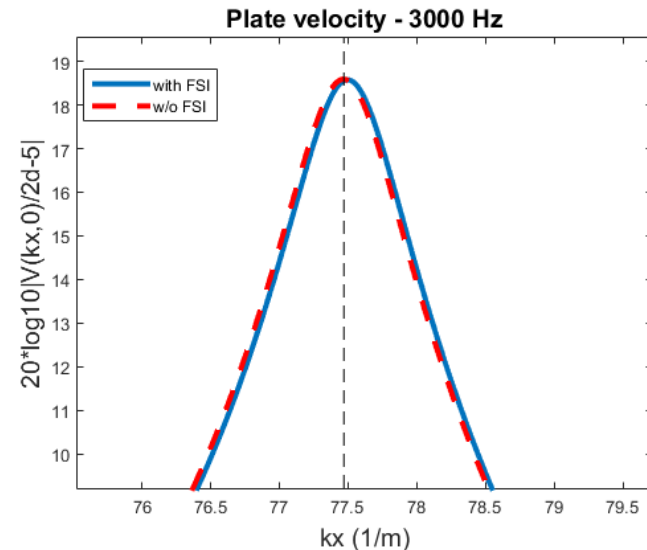
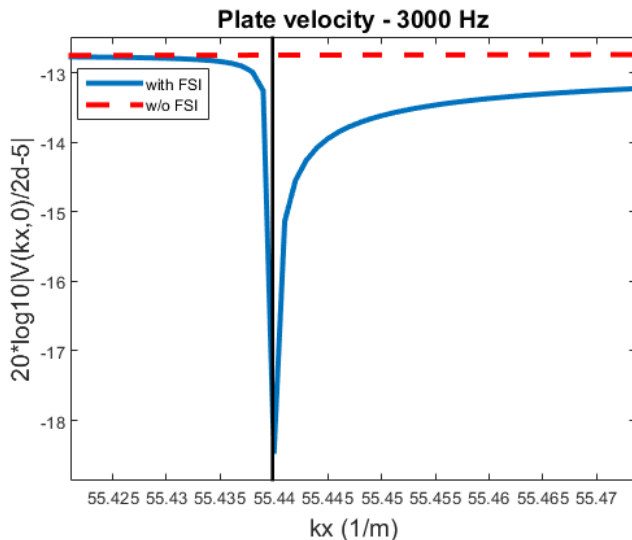
! Below the critical frequency !

for $f < f_c$

$$Z_a(k_f, 0) \in \mathbb{R}^-$$

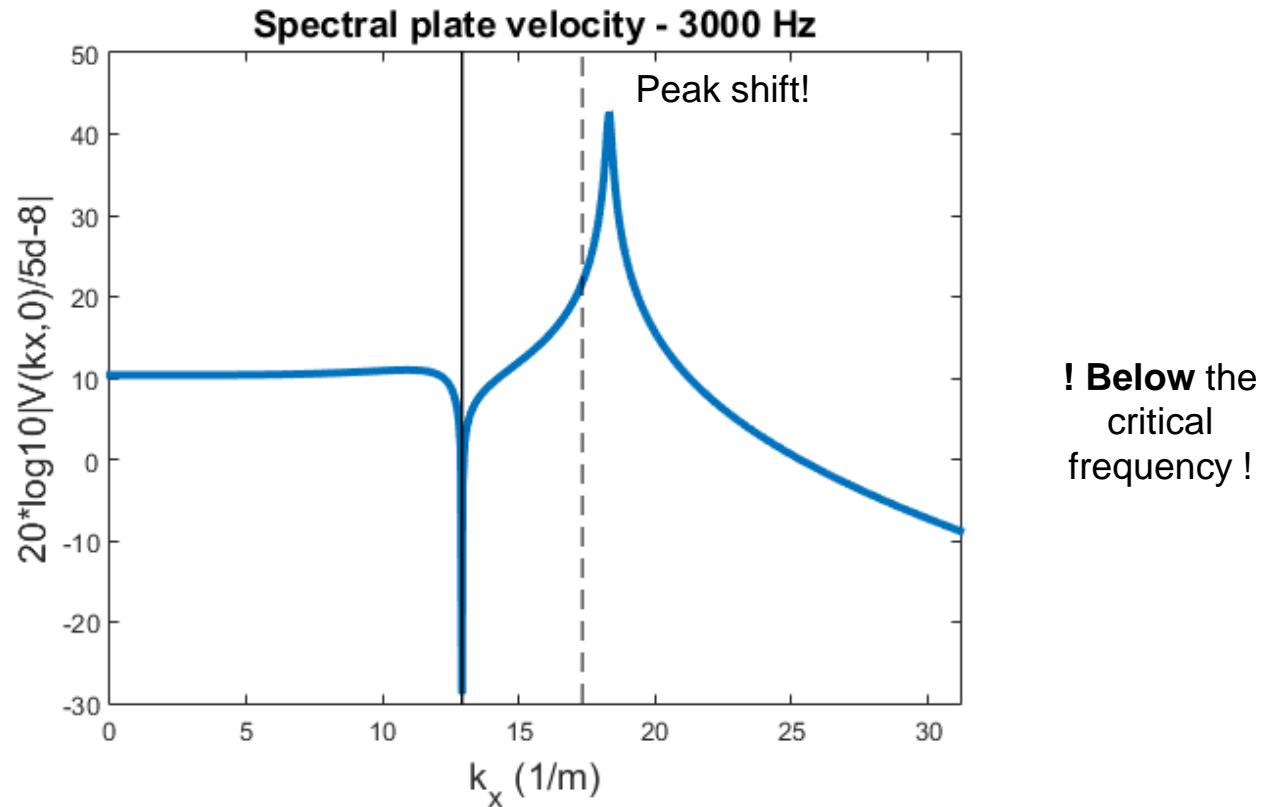
$$|Z_p(k_f, 0)| \gg Z_a(k_f, 0)$$

for light fluid (like air)



Spectral velocity response of a **water** loaded plate excited by a point force at (0,0)

Steel plate in water - Thickness= 40 mm - Critical frequency= 5.7kHz



Dash line, flexural wavenumber (k_f)
Solid line, acoustic wavenumber (k_0)

Why the peak has shift?

As $f < f_c \rightarrow k_f > k_0$

$$\text{For } k_x \approx k_f \rightarrow \left\{ \begin{array}{l} k_z = i\sqrt{k_x^2 - k_0^2} \\ \hat{Z}_p(k_x, 0) + \hat{Z}_a(k_x, 0) = \left[Dk_x^4 - \omega^2 \left(\underbrace{\rho h + \frac{\rho_0}{\sqrt{k_x^2 - k_0^2}}}_{\text{Fluid added mass}} \right) \right] / i\omega \end{array} \right.$$

Fluid added mass

(inertial effect of the fluid on the shell)

For an air loaded plate, $\frac{\rho_0}{\sqrt{k_x^2 - k_0^2}}$ is generally negligible compared to ρh .

Estimation of an equivalent mass density for the panel:

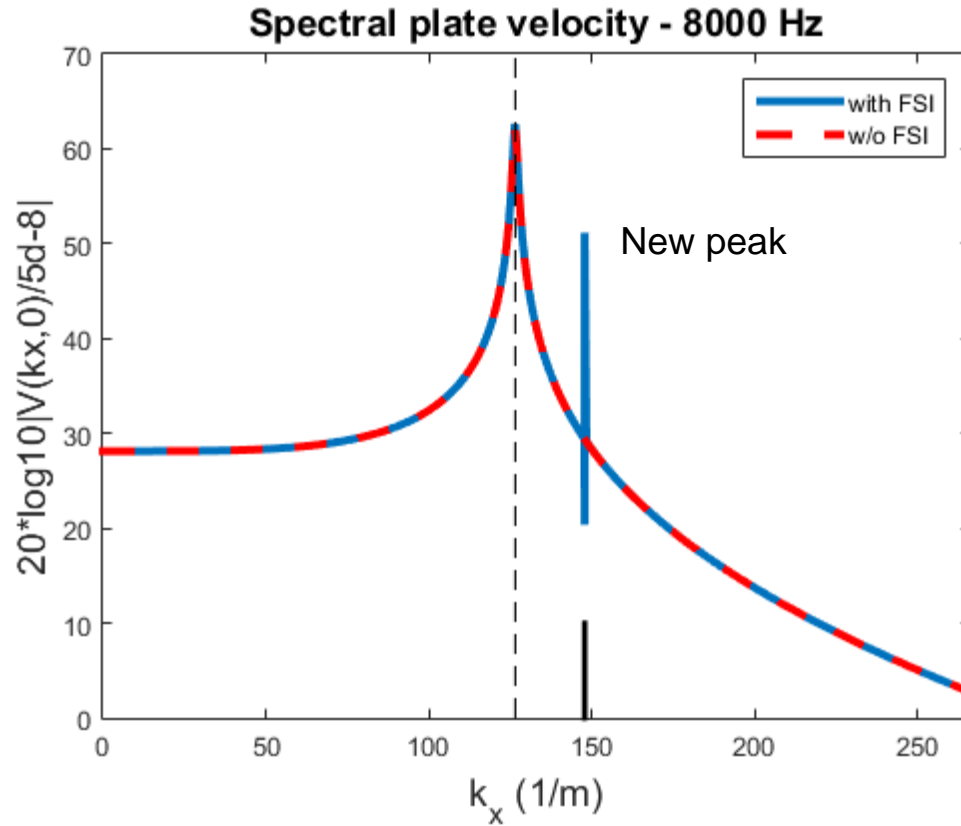
Considering the fluid added mass term with, $k_x = k_f$, one deduces $\rho_{eq} \approx \rho + \frac{\rho_0}{h\sqrt{k_f^2 - k_0^2}}$

Estimation of the flexural wavenumber taking the fluid added mass effect:

$$k_f^{FL} = \sqrt{\omega \sqrt{\frac{\rho_{eq} h}{D}}} \quad \text{At 3 kHz, } k_f^{FL} = 18.4 \text{ m}^{-1} \quad k_f = 17.3 \text{ m}^{-1}$$

Spectral velocity response of an **air** loaded plate excited by a point force at (0,0)

Steel plate in air - Thickness= 2 mm - Critical frequency= 5.7kHz



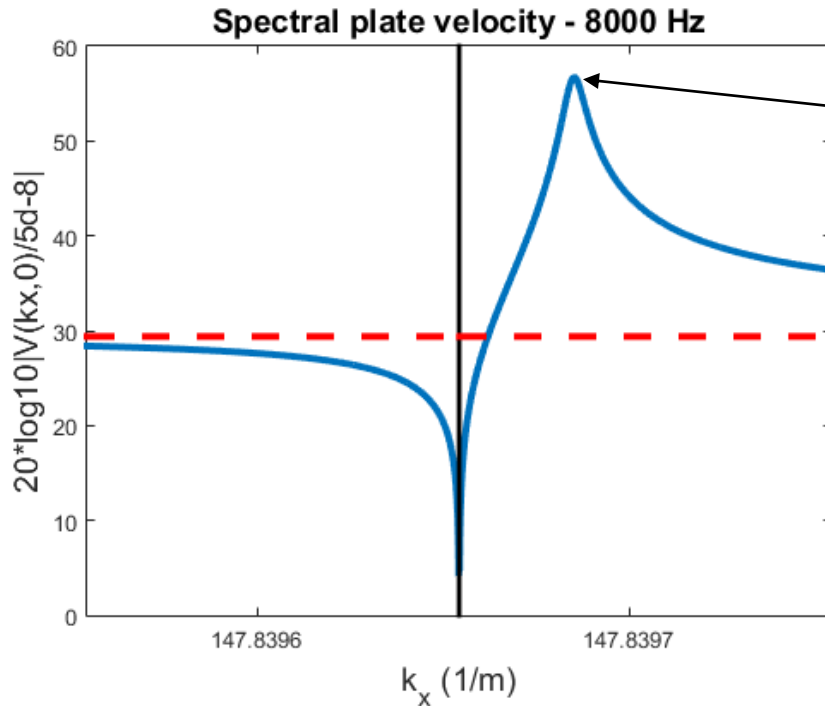
! Above the critical frequency !

Dash line, flexural wavenumber (k_f)
Solid line, acoustic wavenumber (k_0)

Spectral velocity response of a **air** loaded plate excited by a point force at (0,0)

Steel plate in air - Thickness= 2 mm - Critical frequency= 5.7kHz

Focus on wavenumbers close to the acoustic wavenumber (i.e. zoom)



Scholte-Stoneley wave:

- Surface wave (appears at the fluid-structure interface)
- Fluid-borne wave
- Evanescent in the normal direction of the panel
- Fewly damped (as the fluid is no damped) → Very narrow peak

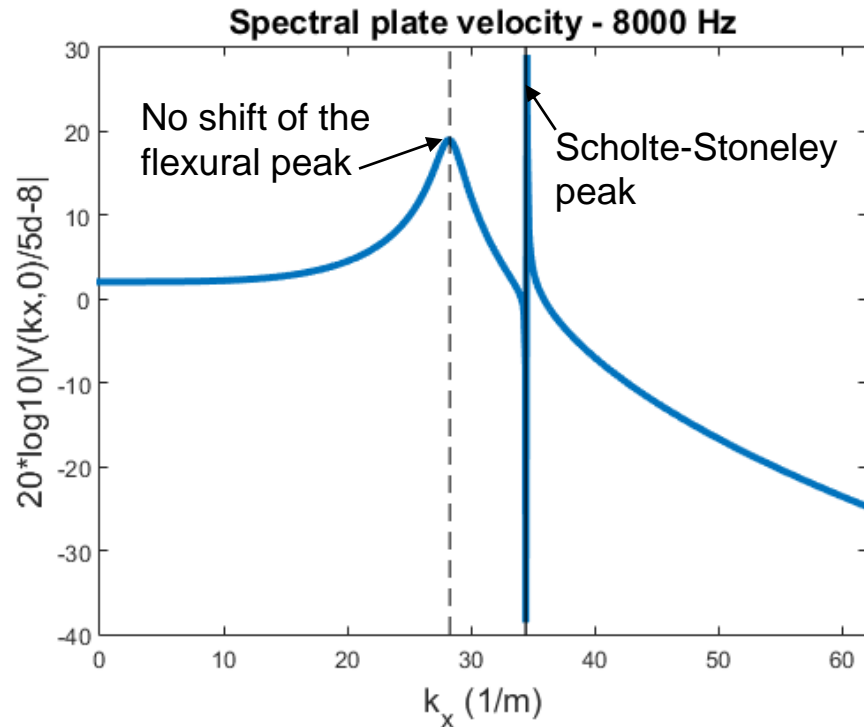
$$k_{Scholte} = 147.839685 \text{ m}^{-1}$$

$$k_0 = 147.839654 \text{ m}^{-1}$$

$$\lim_{k_x \rightarrow k_0} Z_a(k_x, 0) = \infty$$

Spectral velocity response of a **water** loaded plate excited by a point force at (0,0)

Steel plate in water - Thickness= 40 mm - Critical frequency= 5.7kHz



Dash line, flexural wavenumber (kf)

→ Contrary to the frequencies below the critical frequency, the flexural peak is not shifted, but it is less narrow.

Why the peak does not shift and is larger?

As $f > f_c \rightarrow k_f < k_0$

$$\text{For } k_x \approx k_f \rightarrow \left\{ \begin{array}{l} k_z = -\sqrt{k_0^2 - k_x^2} \\ Z_p(k_x, 0) + Z_a(k_x, 0) = \left[D(1+i\eta)k_x^4 + i \underbrace{\frac{\rho_0 \omega^2}{\sqrt{k_0^2 - k_x^2}} - \omega^2 \rho h}_{\text{Fluid added damping}} \right] / i\omega \end{array} \right.$$

Fluid added damping
(panel radiation into the fluid)

Estimation of an equivalent damping for the panel:

Considering the fluid added damping term with, $k_x = k_f$, one deduces

$$\eta_{eq} \approx \eta + \frac{\rho_0 \omega^2}{D k_f^4 \sqrt{k_0^2 - k_f^2}}$$

At 8 kHz, with $\eta = 0.02 \rightarrow \eta_{eq} \approx 0.18 \rightarrow$ The internal panel damping is negligible compared to the radiation damping.

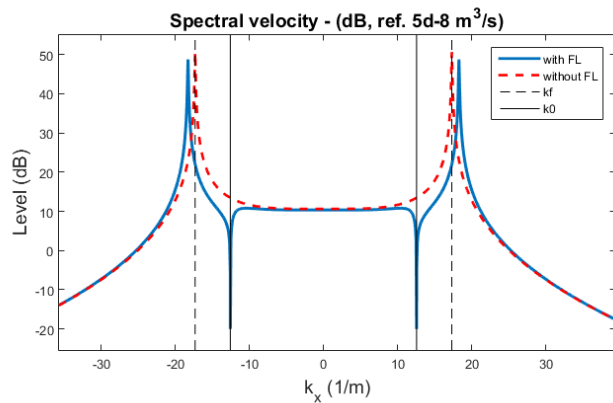
RK: for an air loaded plate, $\eta_{eq} \approx 0.0211 \rightarrow$ The radiation damping is negligible compared to the internal panel damping.

II.6 – Analysis of the plate velocity in the physical space

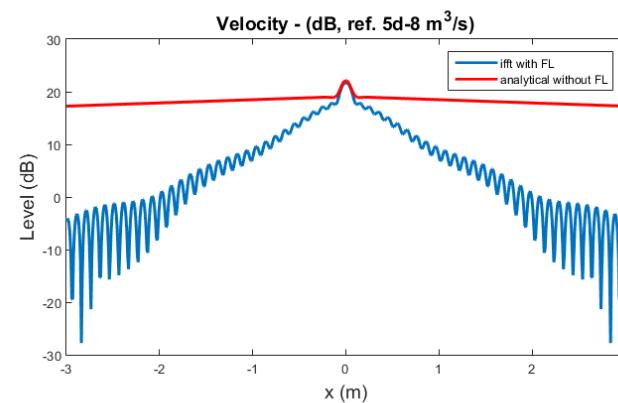
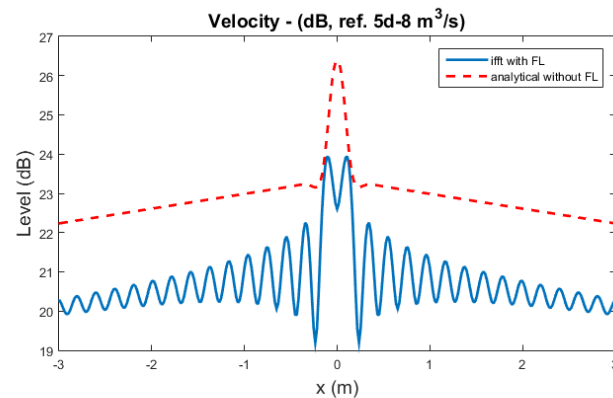
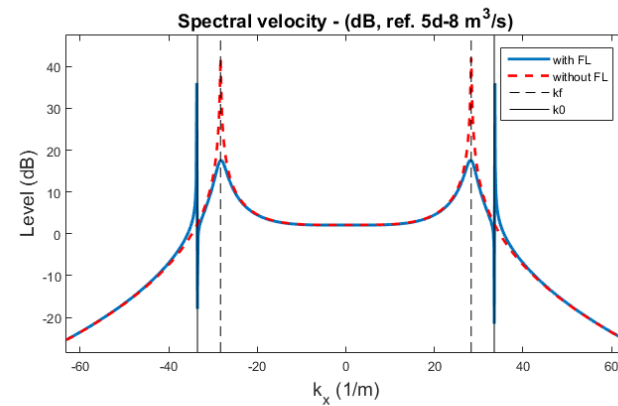
Matlab code: *fluid_loaded_plate_velocity.m*

Steel plate in water - Thickness= 40 mm - Critical frequency= 5.7kHz

3 kHz



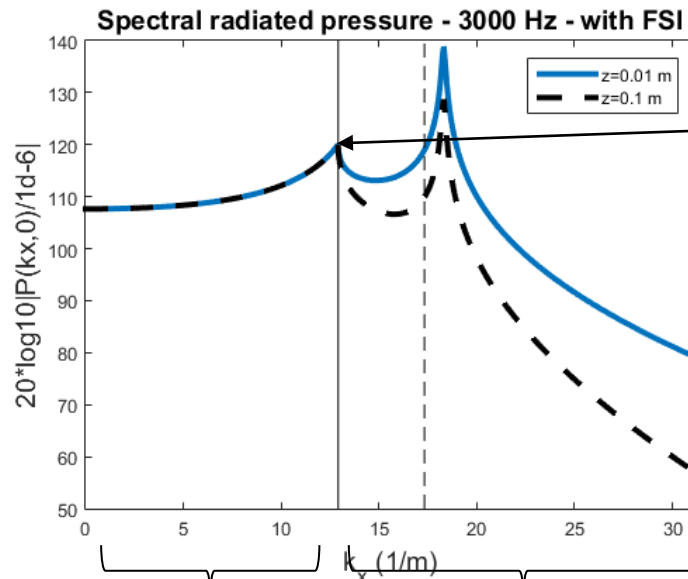
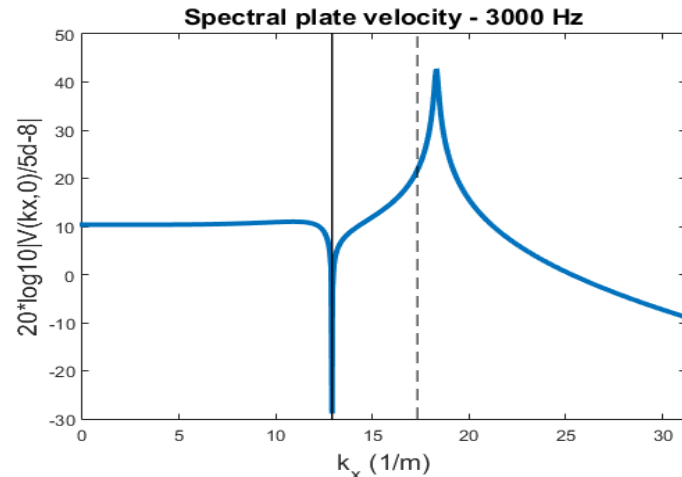
8 kHz



II.7 – Analysis of the radiated pressure in the k-space

Spectral **pressure** response of a **water** loaded plate excited by a point force at (0,0)

Steel plate in water - Thickness= 40 mm - Critical frequency = 5.7kHz



Matlab code:

fluid_loaded_plate_pressure.m

**! Below the
critical frequency !**

$$p(k_x, 0, z) = F_0 \frac{Z_a(k_x, 0)}{Z_p(k_x, 0) + Z_a(k_x, 0)} e^{ik_z z}$$

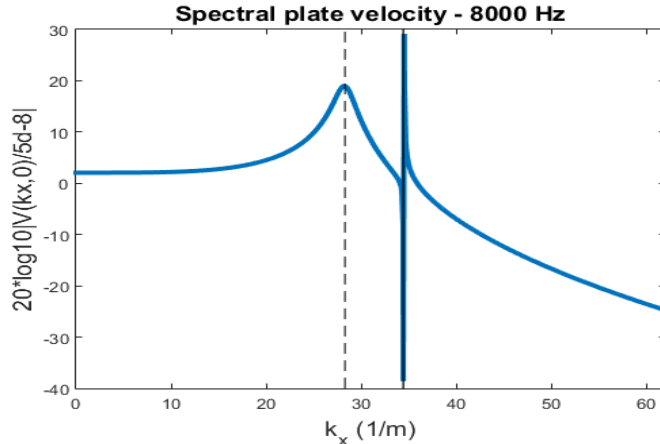
$$\lim_{k_x \rightarrow k_0} Z_a(k_x, 0) = \infty \quad \text{as} \quad \lim_{k_x \rightarrow k_0} k_z = 0$$

$$\rightarrow \lim_{k_x \rightarrow k_0} p(k_x, 0, z) = F_0 \rightarrow 120 \text{ dB for } F_0 = 1 \text{ N}$$

→ Below f_c , propagative flexural waves induced evanescent waves into the fluid

Spectral **pressure** response of a **water** loaded plate excited by a point force at (0,0)

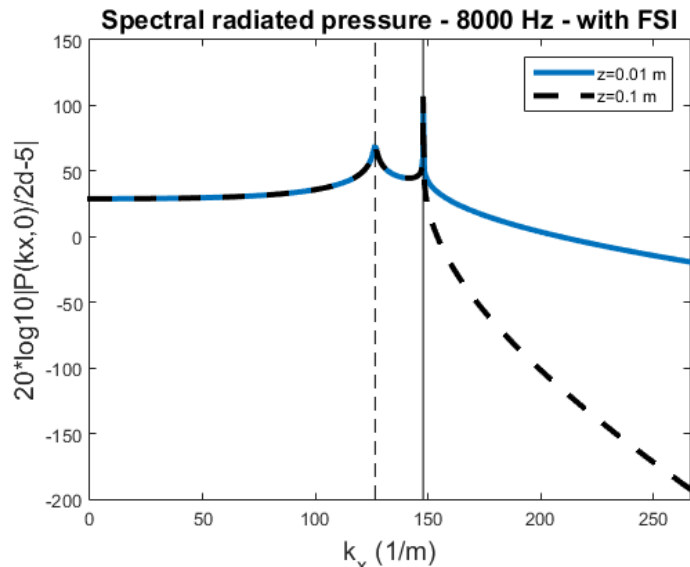
Steel plate in water - Thickness= 40 mm - Critical frequency = 5.7kHz



! Above the critical frequency !

$$k_{Scholte} = 147.839685 \text{ m}^{-1}$$

$$k_0 = 147.839654 \text{ m}^{-1}$$



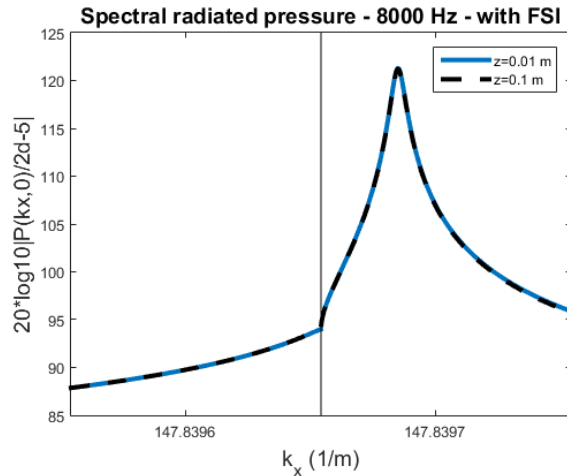
Propagative radiation k_x Evanescent radiation

➔ Above f_c , propagative flexural waves induced propagative waves into the fluid

Spectral **pressure** response of a **water** loaded plate excited by a point force at (0,0)

Steel plate in water - Thickness= 40 mm - Critical frequency = 5.7kHz

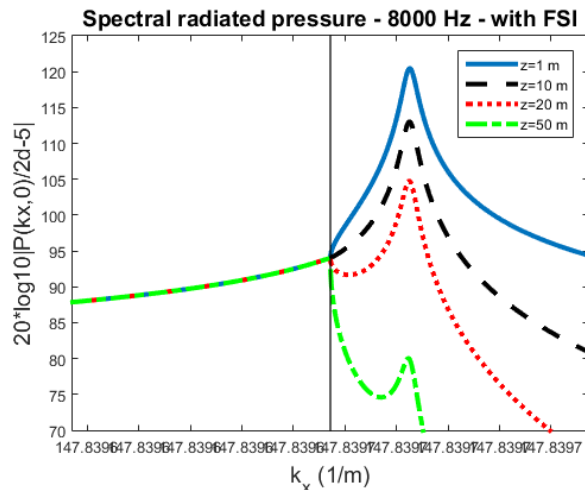
Focus on wavenumbers close to the acoustic wavenumber (i.e. zoom)



! Above the critical frequency !

$$k_{Scholte} = 147.839685 \text{ m}^{-1}$$

$$k_0 = 147.839654 \text{ m}^{-1}$$



→ Scholte-Stoneley waves induced evanescent waves into the fluid, but only very slightly decreasing

→ Effect of the damping/viscosity of the fluid? Open question?...

Propagative radiation Evanescent radiation

Case of a monochromatic space excitation:

Pressure induced by a wall plane wave of wavevector (\bar{k}_x, \bar{k}_y) : $F(x, y) = e^{i\bar{k}_x x + i\bar{k}_y y}$

Spectral force:

$$\tilde{F}(k_x, k_y) = 4\pi^2 \delta(k_x - \bar{k}_x) \delta(k_y - \bar{k}_y) \text{ because } F(x, y) = \frac{1}{4\pi^2} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \tilde{F}(k_x, k_y) e^{ik_x x} e^{ik_y y} dk_x dk_y$$

Spectral pressure:

$$\tilde{p}(k_x, k_y, z) = \frac{4\pi^2 Z_a(k_x, k_y)}{Z_p(k_x, k_y) + Z_a(k_x, k_y)} e^{ik_z z} \delta(k_x - \bar{k}_x) \delta(k_y - \bar{k}_y)$$

Pressure in the physical space:

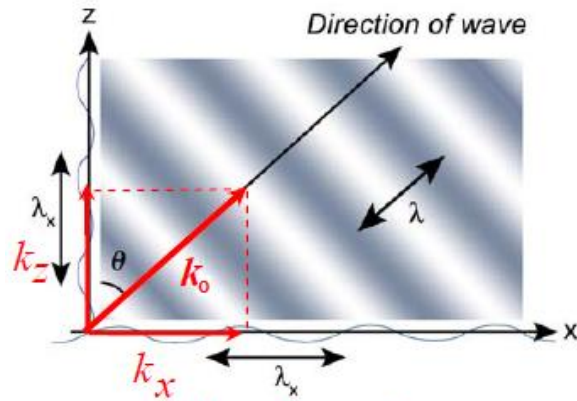
$$p(x, y, z) = \frac{Z_a(\bar{k}_x, \bar{k}_y)}{Z_p(\bar{k}_x, \bar{k}_y) + Z_a(\bar{k}_x, \bar{k}_y)} e^{i\bar{k}_z z} e^{i\bar{k}_x x + i\bar{k}_y y}$$

Examples of the different modes of radiation ($\bar{k}_y = 0$)

$$0 < \bar{k}_x < k_0$$

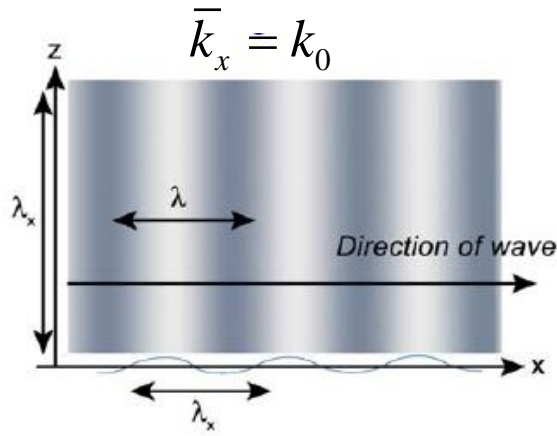
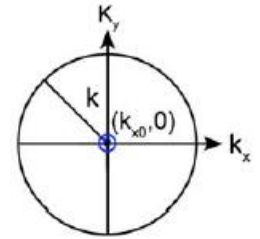
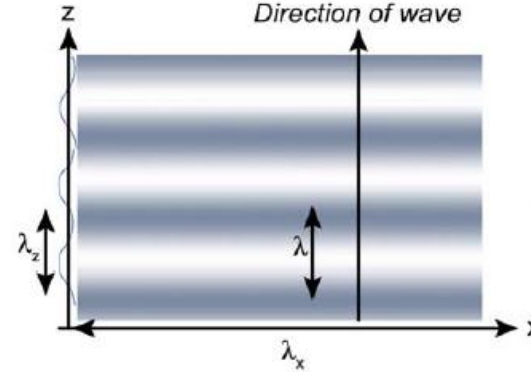
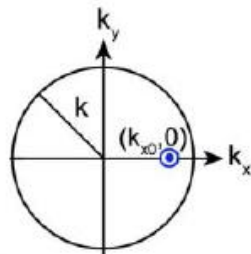
$$p(x, 0, z)$$

$$\bar{k}_x = 0$$

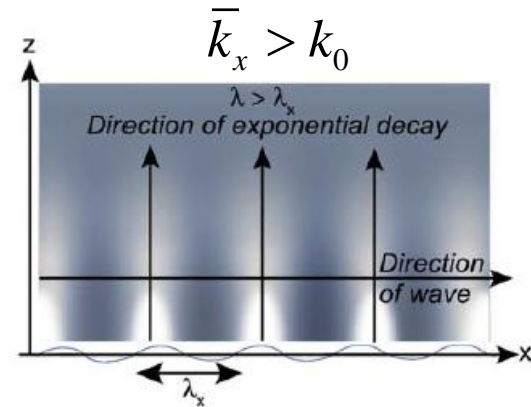
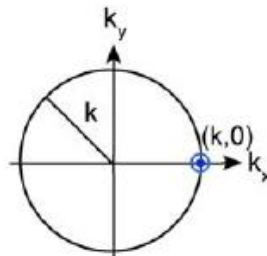


$$k_x = k_0 \sin \theta$$

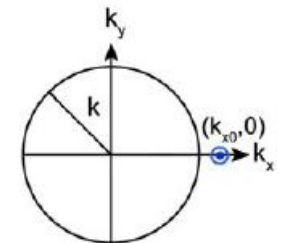
$$k_z = k_0 \cos \theta$$



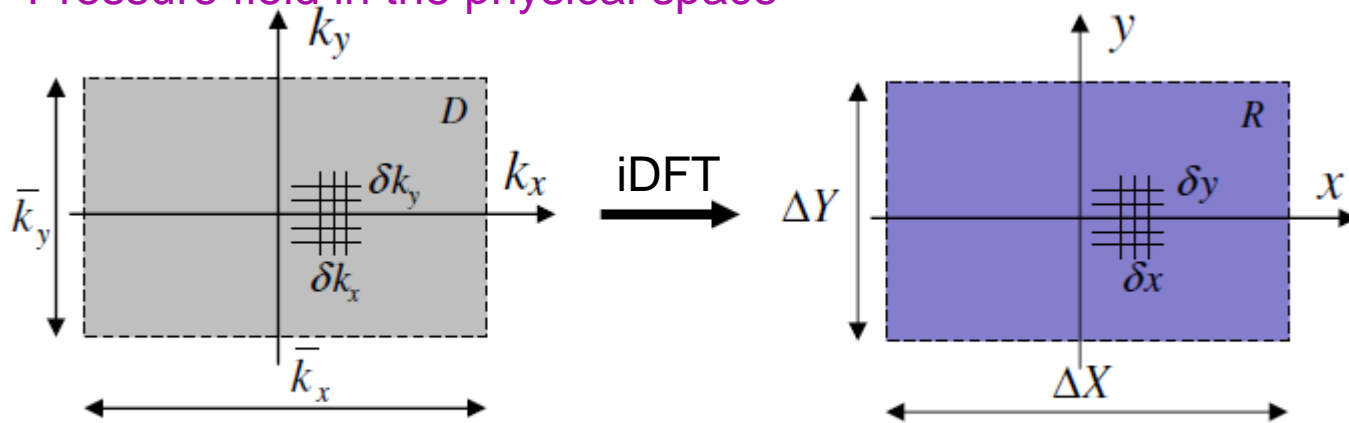
Grazing plane wave



Evanescent wave



II.8 – Pressure field in the physical space



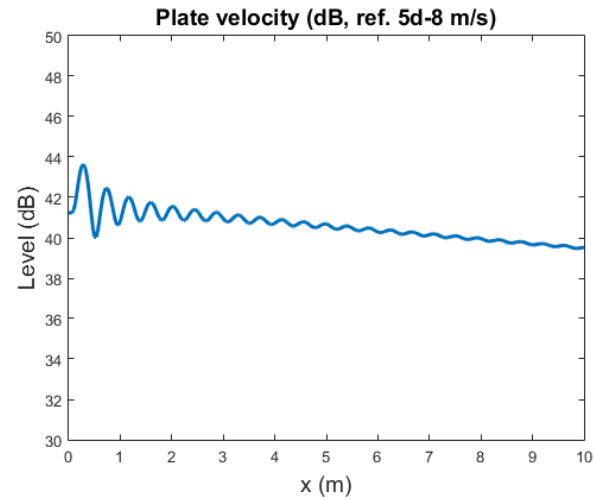
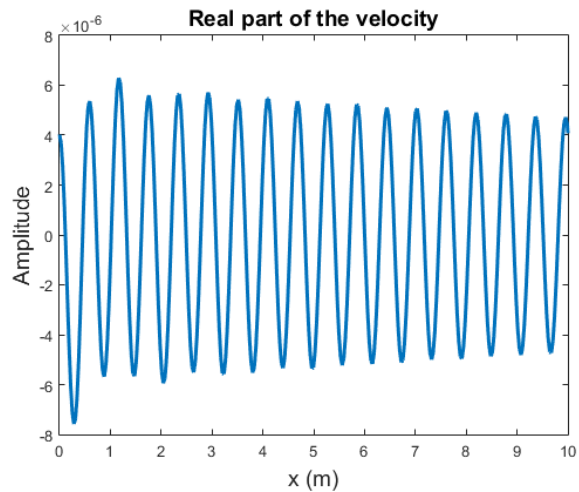
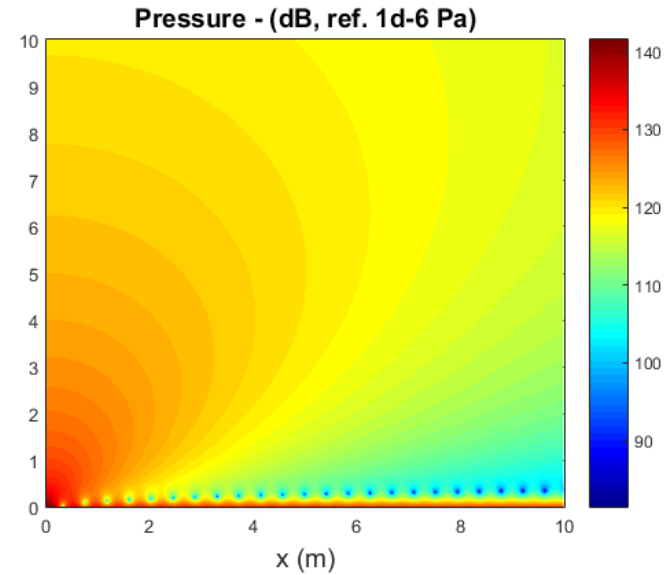
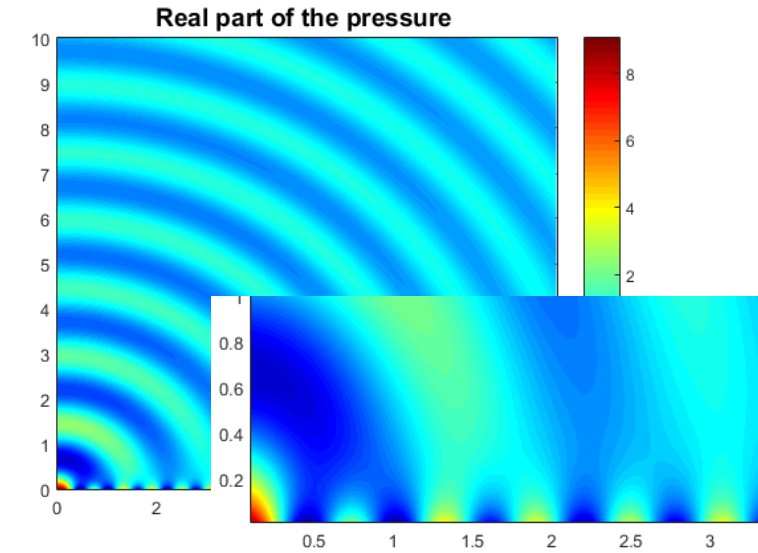
Discrete domain definitions: D, wavenumber space; R, physical space.

$$\delta x = \frac{2\pi}{\bar{k}_x}, \quad \delta y = \frac{2\pi}{\bar{k}_y}, \quad \Delta X = \frac{2\pi}{\delta k_x}, \quad \Delta Y = \frac{2\pi}{\delta k_y}$$

MATLAB codes: - *fluid_loaded_plate_pressure.m* (pressure for a given Z in function of X) → Study the influence of the wavenumber resolution

- *fluid_loaded_plate_pressure_map.m*
(pressure map in function of X and Z)

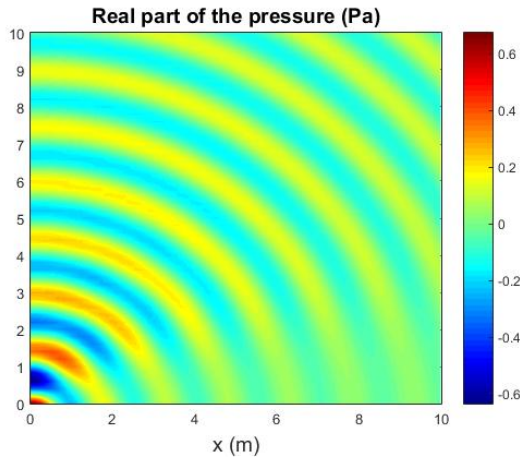
Line excitation ($k_y=0$) 1000 Hz



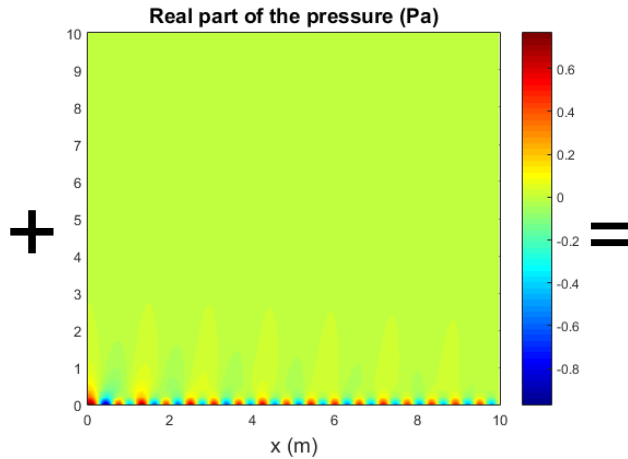
Line excitation ($k_y=0$)

1000 Hz

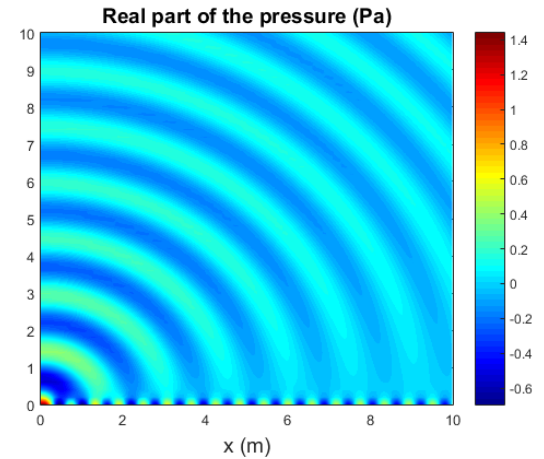
Supersonic components



Subsonic components



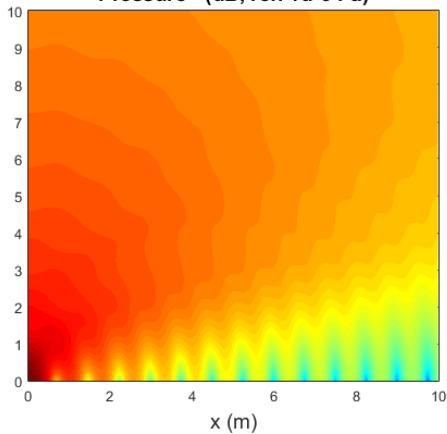
Total



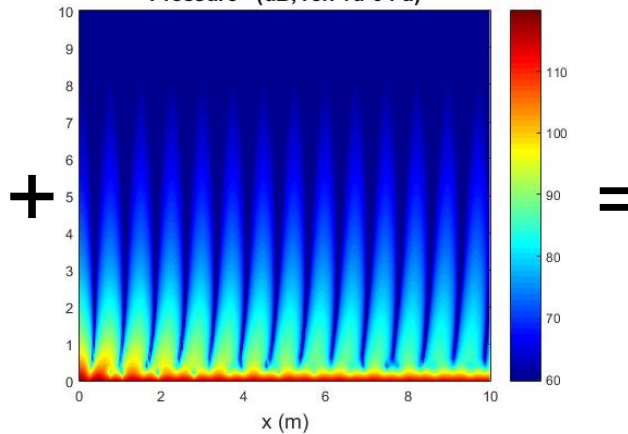
+

=

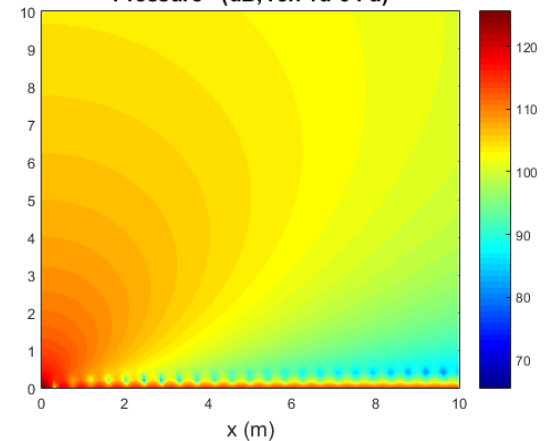
Pressure - (dB, ref. 1d-6 Pa)



Pressure - (dB, ref. 1d-6 Pa)



Pressure - (dB, ref. 1d-6 Pa)

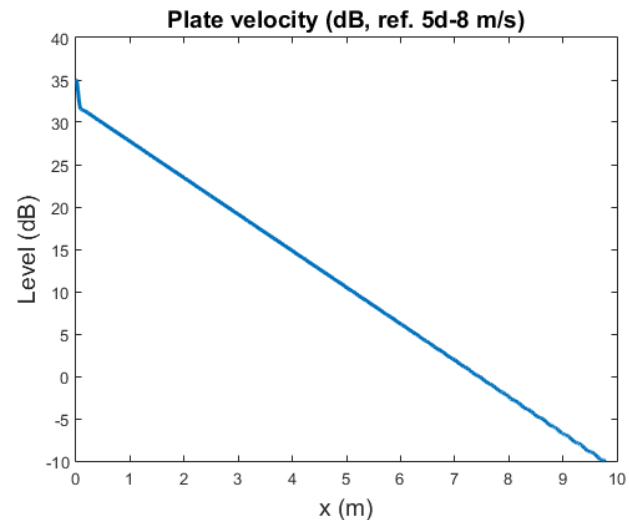
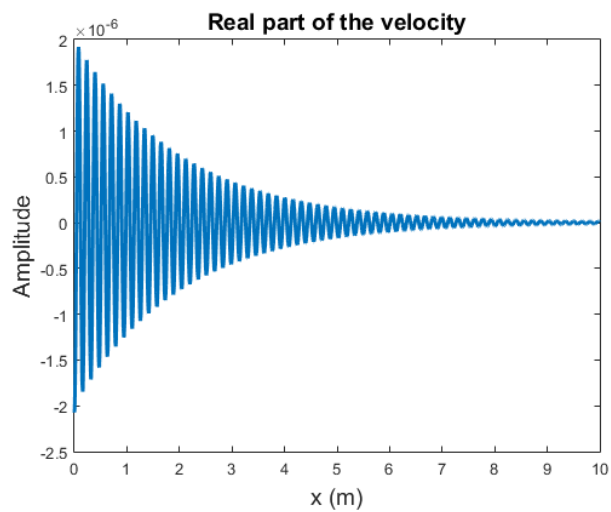
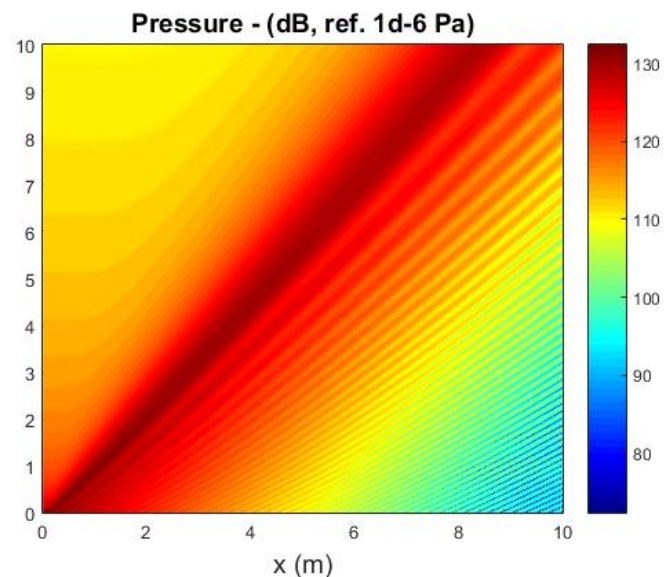
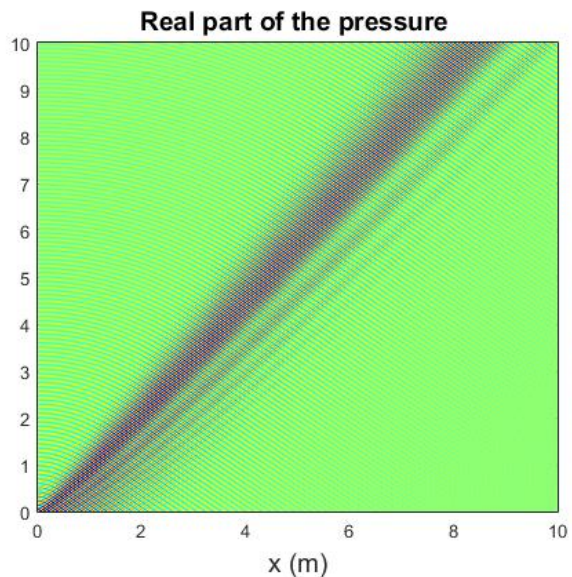


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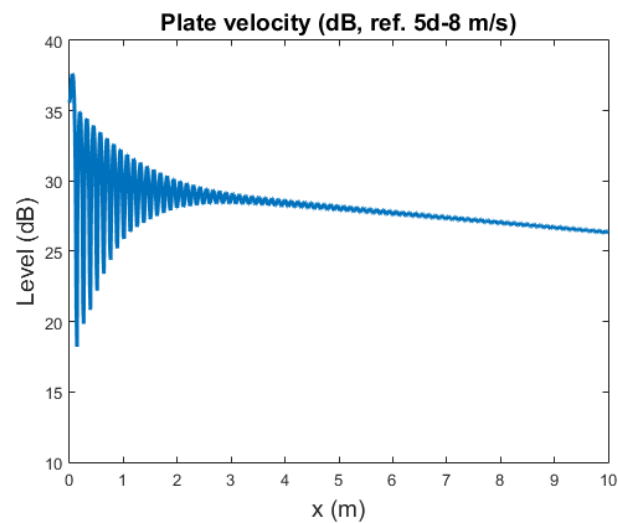
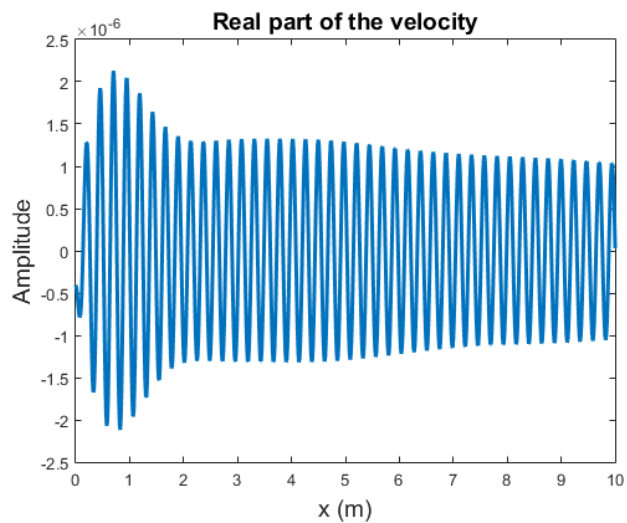
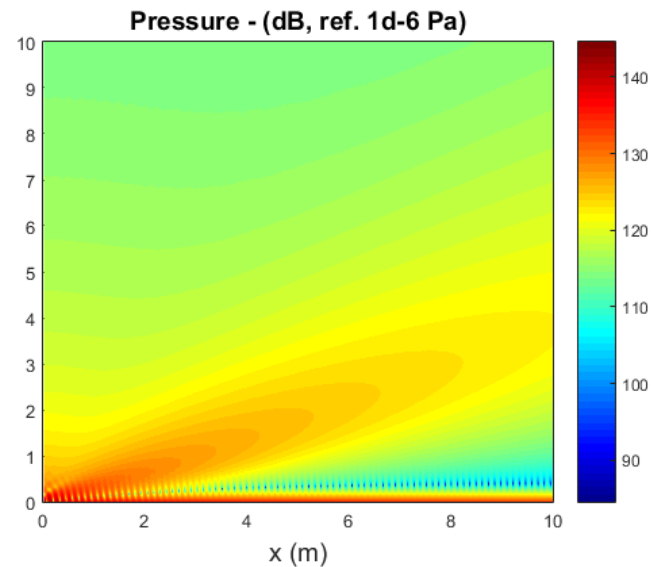
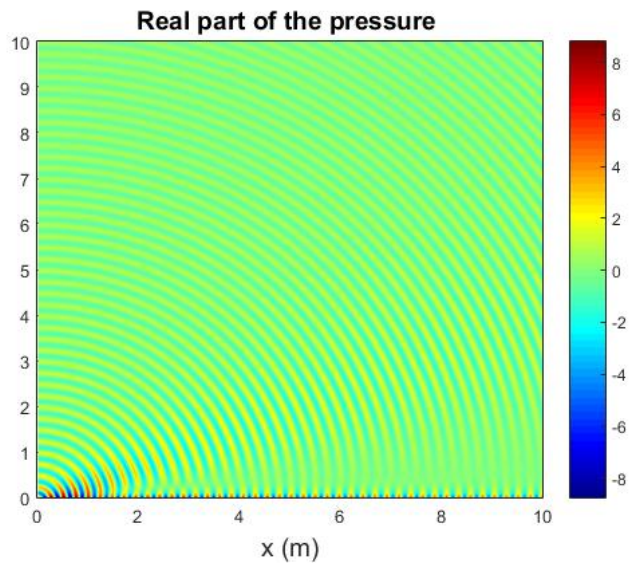
Line excitation ($k_y=0$)

16000 Hz



Line excitation ($k_y=0$)

6000 Hz



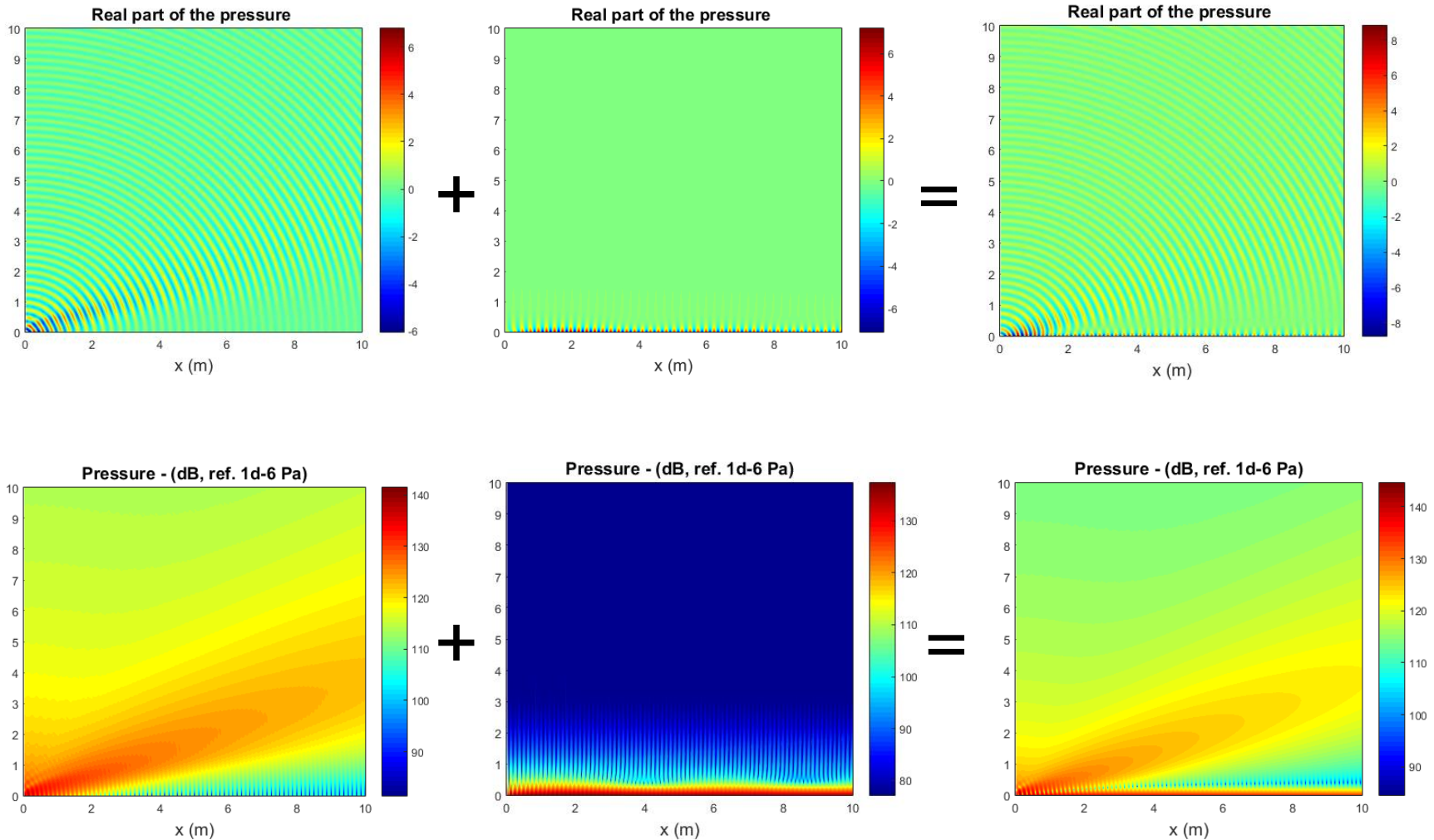
Line excitation ($k_y=0$)

6000 Hz

Supersonic components

Subsonic components

Total



II.9 – Radiation power and radiation efficiency

Reminder: Parseval-Plancherel formula

$$\int_{-\infty}^{\infty} f(x) g^*(x) dx = \frac{1}{2\pi} \int_{-\infty}^{\infty} \tilde{f}(k_x) \tilde{g}^*(k_x) dk_x$$

$$\int_{-\infty}^{\infty} |f(x)|^2 dx = \frac{1}{2\pi} \int_{-\infty}^{\infty} |\tilde{f}(k_x)|^2 dk_x$$

Demo:

$$\begin{aligned} \int_{-\infty}^{\infty} f(x) g^*(x) dx &= \int_{-\infty}^{\infty} f(x) \frac{1}{2\pi} \int_{-\infty}^{\infty} \tilde{g}(k_x) dk_x dx \\ &= \frac{1}{2\pi} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(x) \tilde{g}(k_x) dk_x dx \\ &= \frac{1}{2\pi} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(x) \tilde{g}(k_x) dx dk_x \\ &= \frac{1}{2\pi} \int_{-\infty}^{\infty} \tilde{g}(k_x) \int_{-\infty}^{\infty} f(x) dx dk_x \\ &= \frac{1}{2\pi} \int_{-\infty}^{\infty} \tilde{g}(k_x) \tilde{f}(k_x) dk_x \end{aligned}$$

Radiation efficiency:

$$\sigma = \frac{\Pi_{rad}}{\rho_0 c_0 S V^2} \quad \text{with}$$

- Sum of the quadratic velocity:
$$SV^2 = \lim_{T \rightarrow \infty} \frac{1}{T} \int_{-T/2}^{T/2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \bar{v}^2(x, y, t) dx dy dt$$

- Radiated power:
$$\Pi_{rad} = \lim_{T \rightarrow \infty} \frac{1}{T} \int_{-T/2}^{T/2} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \bar{p}(x, y, 0, t) \bar{v}(x, y, t) dx dy dt$$

Harmonic excitation, stationary response
and complex notation

$$\bar{v}(x, y, t) = \text{Re} \{ v(x, y) e^{j\omega t} \}$$

$$\bar{p}(x, y, z, t) = \text{Re} \{ p(x, y, z) e^{j\omega t} \}$$

$$SV^2 = \frac{1}{2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} |v(x, y)|^2 dx dy$$

$$\Pi_{rad} = \frac{1}{2} \text{Re} \left\{ \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} p(x, y, 0) v^*(x, y) dx dy \right\}$$

Using the Parseval-Plancherel formula

$$SV^2 = \frac{1}{2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} |v(x, y)|^2 dx dy = \frac{1}{8\pi^2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} |\tilde{v}(k_x, k_y)|^2 dk_x dk_y$$

$$\Pi_{rad} = \frac{1}{8\pi^2} \operatorname{Re} \left\{ \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \tilde{p}(k_x, k_y, 0) \tilde{v}^*(k_x, k_y) dk_x dk_y \right\}$$

$$\tilde{p}(k_x, k_y, 0) = Z_a(k_x, k_y) \tilde{v}(k_x, k_y)$$

$$\Pi_{rad} = \frac{1}{8\pi^2} \operatorname{Re} \left\{ \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} Z_a(k_x, k_y) |\tilde{v}(k_x, k_y)|^2 dk_x dk_y \right\}$$

$$\begin{cases} Z_a \in \square & \text{if } \sqrt{k_x^2 + k_y^2} \leq k_0 \\ Z_a \in i\square & \text{if } \sqrt{k_x^2 + k_y^2} > k_0 \end{cases}$$

$$\Pi_{rad} = \frac{1}{8\pi^2} \operatorname{Re} \left\{ \iint_{\Omega_a} Z_a(k_x, k_y) |\tilde{v}(k_x, k_y)|^2 dk_x dk_y \right\}$$

with $\Omega_a = \left\{ (k_x, k_y) \in \square^2, \sqrt{k_x^2 + k_y^2} \leq k_0 \right\}$

Numerical evaluation of the previous quantities:

- Truncation of the wavenumber space with a cut-off wavenumber (only \bar{k}_y for velocity)

$$k_x \in [-\bar{k}, \bar{k}], k_y \in [-\bar{k}, \bar{k}] \quad \bar{k} = \kappa \max[k_0, k_f] \quad \kappa > 2$$

- Discretisation of the wavenumber space with a wavenumber resolution δk

Remark: Other interpretation of the radiated power

No energy dissipation into the fluid

Integrating the radial component of the intensity vector over a large sphere of radius R concentric with the sound source

Intensity estimated from plane wave approximation:

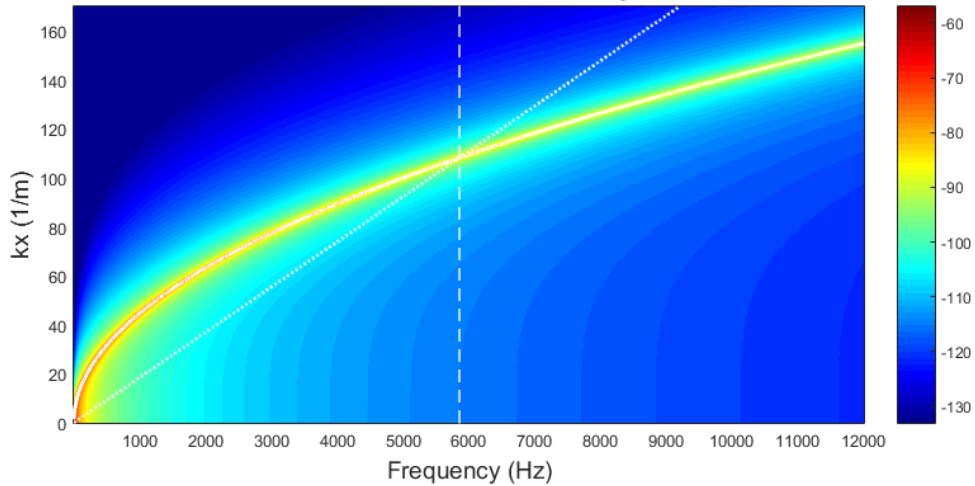
$$Z = \frac{p}{v} = \rho_0 c_0 \quad I = \frac{1}{2} p v^* = \frac{1}{2 \rho_0 c_0} |p|^2$$

$$\Pi_{rad} = \frac{1}{2 \rho_0 c_0} \iint_{\Sigma^\infty} |p|^2 dS \quad \Sigma^\infty \text{ Hemi sphere of large radius}$$

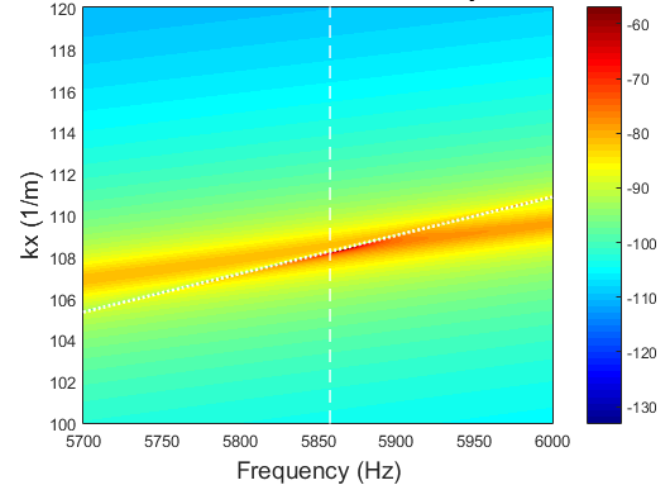
Radiation power and radiation efficiency – Air loaded panel

Steel plate in air - Thickness= 2 mm - Critical frequency= 5.8kHz

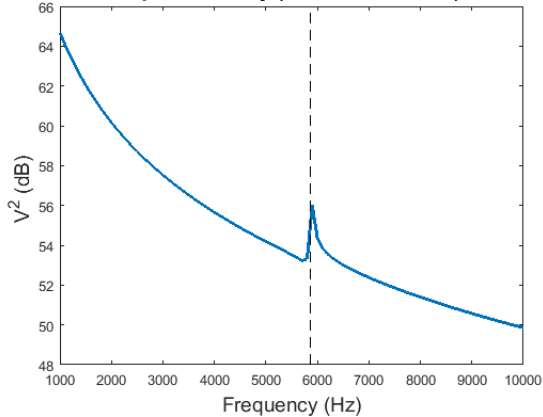
Spectral velocity (dB, ref. $1 \text{ m}^3/\text{s}$, $k_y=0 \text{ m}^{-1}$)



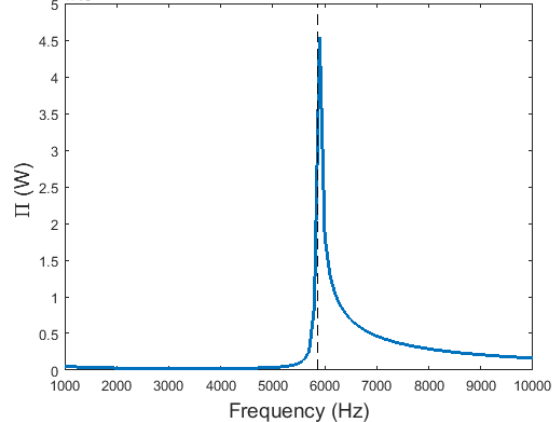
Spectral velocity (dB, ref. $1 \text{ m}^3/\text{s}$, $k_y=0 \text{ m}^{-1}$)



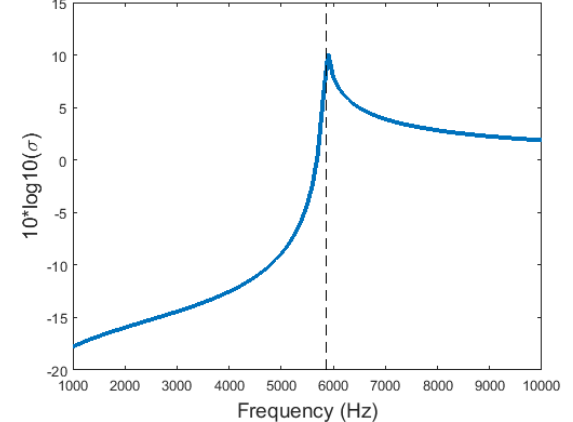
Square velocity (dB, ref. 5.10^{-8} m/s)



Radiation power for a mech. line force

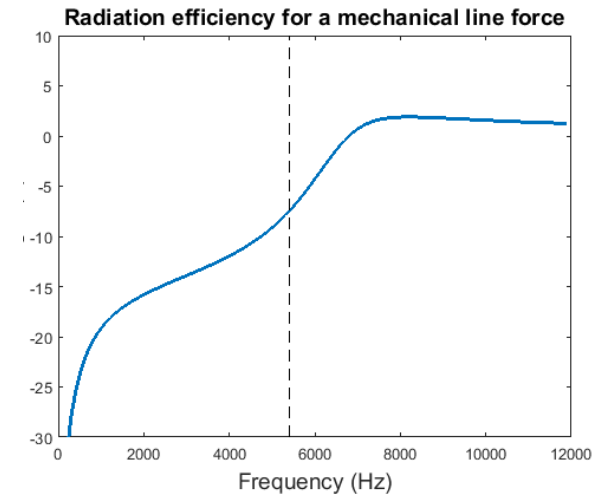
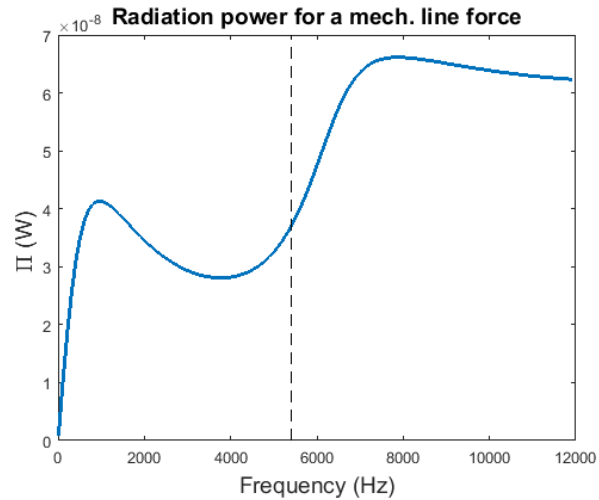
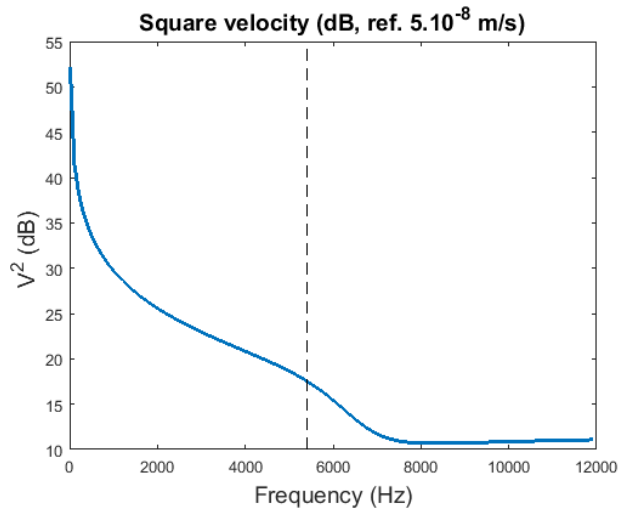
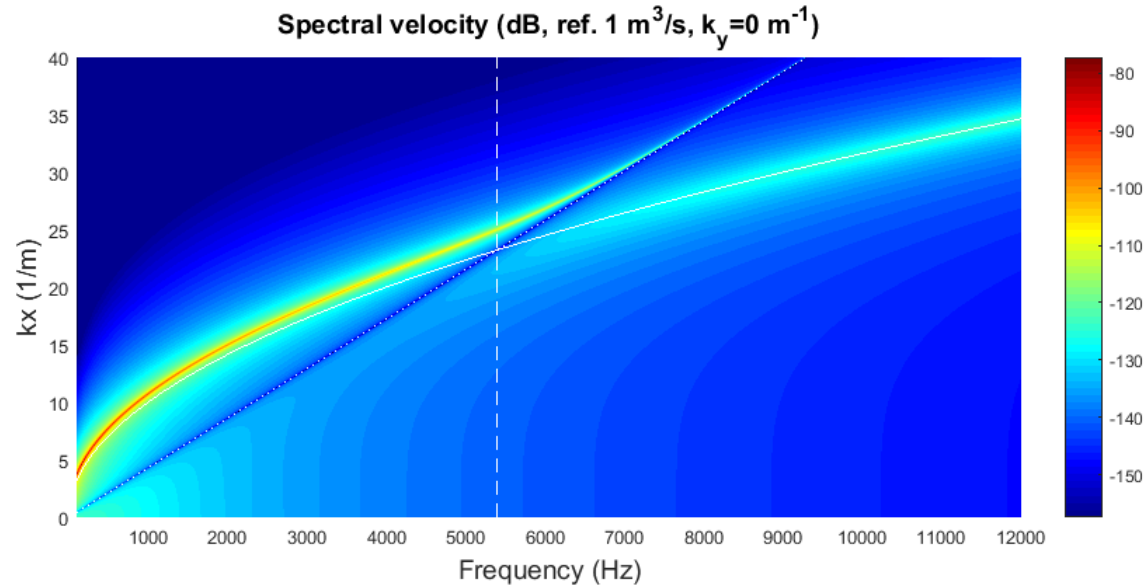


Radiation efficiency for a mechanical line force



Radiation power and radiation efficiency – Water loaded panel

Steel plate in water - Thickness= 40 mm - Critical frequency = 5.4kHz



ON THE RADIATION EFFICIENCY OF INFINITE PLATES SUBJECT TO A POINT LOAD IN WATER

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9500 MacArthur Boulevard, West Bethesda, Maryland 20817-5700, U.S.A.*

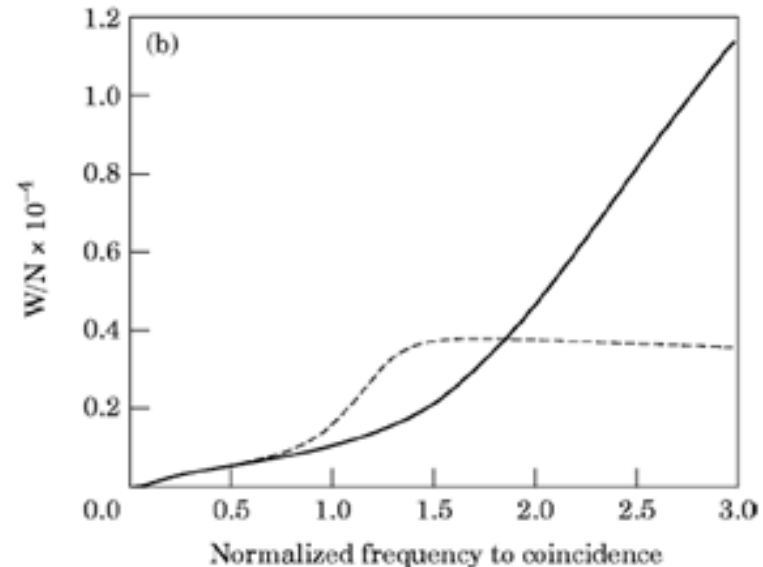
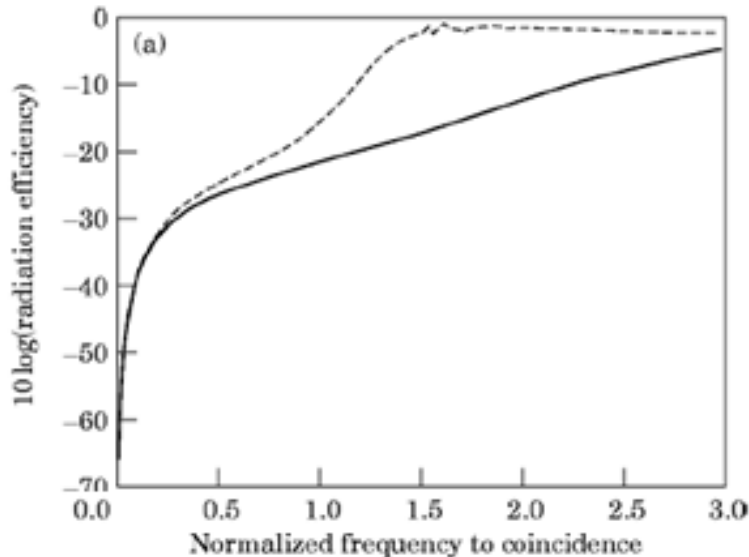
(Received 17 July 1995, and in final form 7 July 1997)

Journal of Sound and Vibration (1997) 208, 441-445.

In this paper, an approach to calculate the radiation efficiency for infinite plates has been developed and validated. The comparison between the results from the thin plate theory and those from the thick plate theory indicates the thin plate theory is adequate for radiated power calculation up to 0.7 times the coincidence frequency but starts to underestimate significantly for frequencies larger than twice the coincidence frequency. As far as the radiation efficiency is concerned, the thin plate theory is adequate up to 0.2 times the coincidence frequency, and for higher frequencies, it overestimates the efficiency (see Figure 2). Consequently, it predicts a lower frequency at which the radiation efficiency approaches unity when compared to the thick plate theory. It is concluded that the heavier fluid loading causes the radiation efficiency to approach unity at a higher frequency.

The results obtained are valid for the radiation efficiency of finite plates of real material excited by a point load in water for frequencies near and above the coincidence frequency.

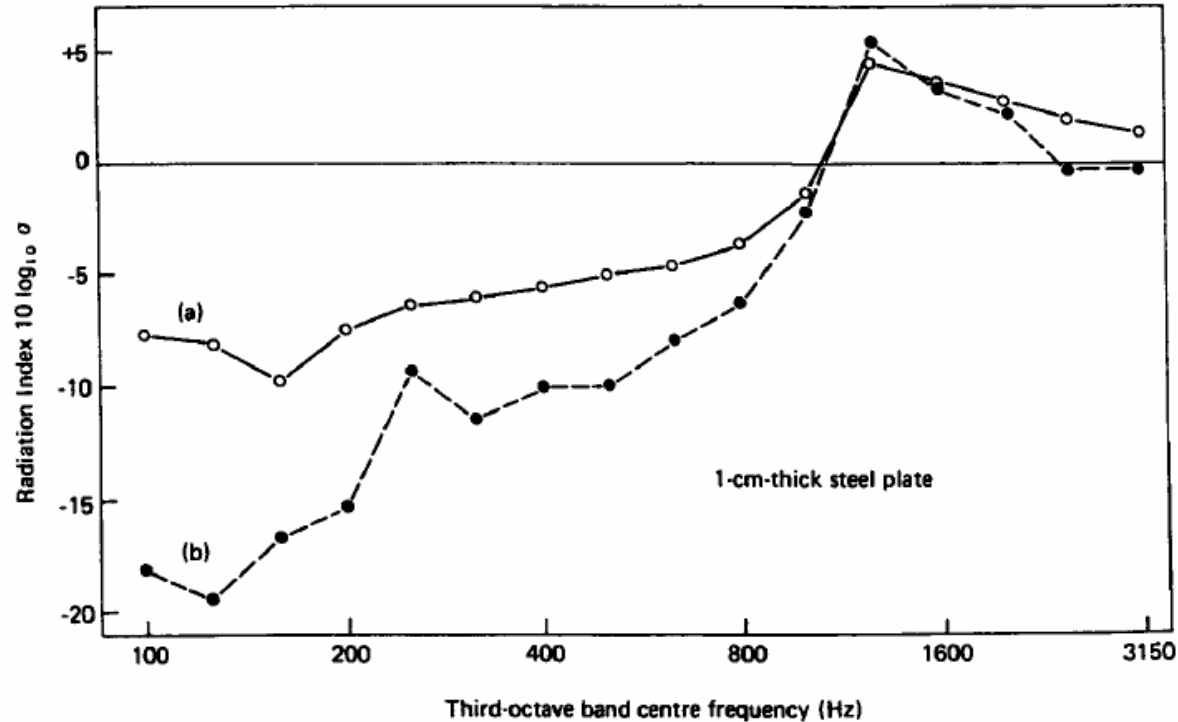
Results for a brass 12.7mm thick plate excited by a point excitation
_Critical frequency in water: 26 420 Hz



The calculated (a) radiation efficiency and (b) radiated power of an infinite plate loaded with water on one side and subjected to a point load. —, By the "thick plate" theory; ---, by the "thin plate" theory.

- As shown on this example, the thin plate (love-kirchhoff) model may not be adapted around and above the critical frequency
- Calculation in the k-space with the thick plate (mindlin) model can be performed

Influence of the type of excitation on the radiation efficiency



Radiation efficiency: (a), acoustic excitation; (b), mechanical point force.
Macadam 1977

→ Radiation efficiency depend on the type of excitation below the critical frequency

II.10 – Evaluation of the radiated pressure in the far field

Stationary Phase theorem:

Let $\varphi(k_x)$ a continuous function and $\psi(k_x)$ a continuous with a single extremum \bar{k}_x such that $\frac{\partial^2 \psi}{\partial k_x^2}(\bar{k}_x) \neq 0$.

The integral $I = \int_{-\infty}^{+\infty} \phi(k_x) e^{j\psi(k_x)} dk_x$ can be approximated by:

$$I \approx \begin{cases} \frac{\sqrt{2\pi} \varphi(\bar{k}_x) e^{j\frac{\pi}{4} + j\psi(\bar{k}_x)}}{\sqrt{\left| \frac{\partial^2 \psi}{\partial k_x^2}(\bar{k}_x) \right|}}, & \text{si } \frac{\partial^2 \psi}{\partial k_x^2}(\bar{k}_x) > 0, \\ \frac{\sqrt{2\pi} \varphi(\bar{k}_x) e^{-j\frac{\pi}{4} + j\psi(\bar{k}_x)}}{\sqrt{\left| \frac{\partial^2 \psi}{\partial k_x^2}(\bar{k}_x) \right|}}, & \text{si } \frac{\partial^2 \psi}{\partial k_x^2}(\bar{k}_x) < 0. \end{cases}$$

Principle: - φ varies slowly than $e^{j\psi(k_x)}$;

- when the phase of ψ varies of 2π , the integral is null;

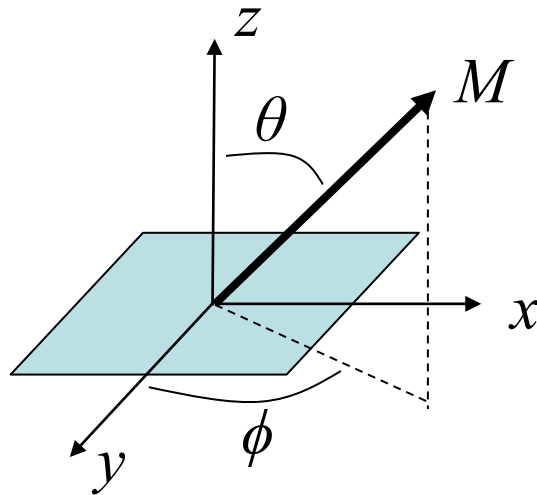
- only the « points » when the phase is stationary (null derivative)

contribute to the integral.

$$p(x, y, z) = \frac{1}{8\pi^2} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} Z_a(k_x, k_y) \tilde{v}(k_x, k_y) e^{ik_x x + ik_y y + ik_z z} dk_x dk_y$$

$$p(x, y, z) = \frac{1}{8\pi^2} \int_{-k_0}^{+k_0} \int_{-k_0}^{+k_0} Z_a(k_x, k_y) \tilde{v}(k_x, k_y) e^{ik_x x + ik_y y - i\sqrt{k_0^2 - k_x^2 - k_y^2} z} dk_x dk_y$$

$$\psi(k_x, k_y) = k_x x + k_y y - \sqrt{k_0^2 - k_x^2 - k_y^2} z$$



$$\begin{cases} x = R \sin \theta \cos \phi \\ y = R \sin \theta \sin \phi \\ z = R \cos \theta \end{cases}$$

Radiated pressure in the far field (i.e. $k_0 R \gg 1$):

$$p_\infty(R, \theta, \phi) = \frac{i\omega\rho_0}{2\pi R} \tilde{v}(\bar{k}_x, \bar{k}_y) e^{-ik_0 R} \quad \text{with} \quad \begin{cases} \bar{k}_x = k_0 \sin \theta \cos \phi \\ \bar{k}_y = k_0 \sin \theta \sin \phi \end{cases}$$

Directivity of the far-field radiated noise– Water loaded panel

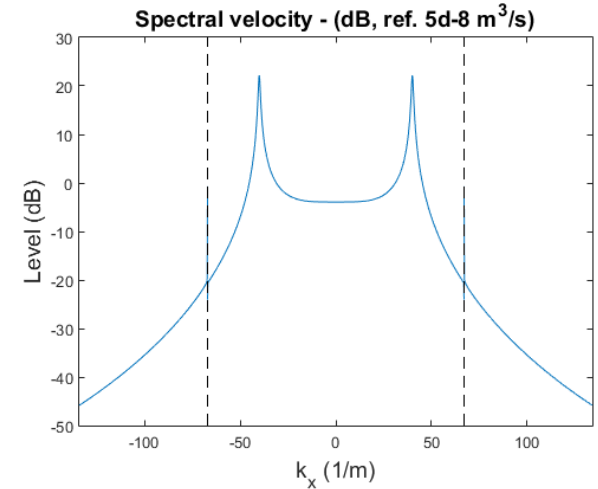
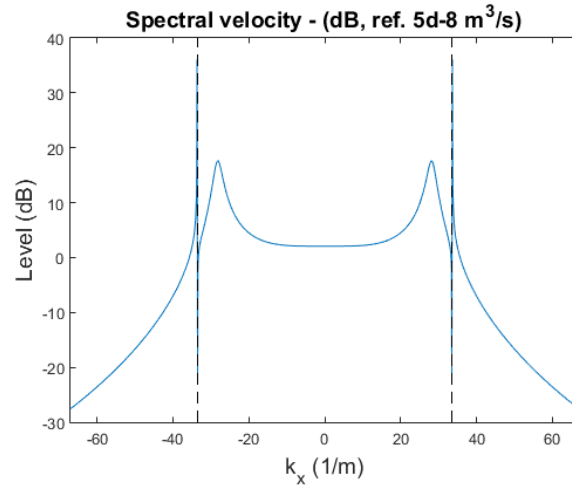
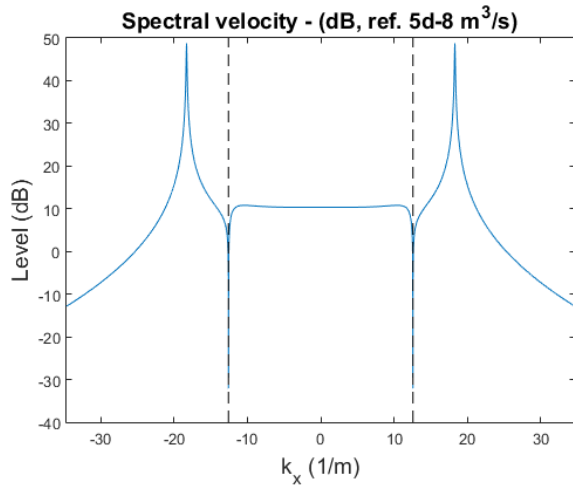
Steel plate in water - Thickness= 40 mm - Critical frequency = 5.4kHz – R=10 m –

Line excitation ($k_y=0$)

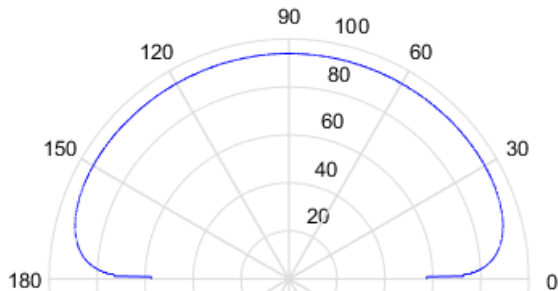
3000 Hz

8000 Hz

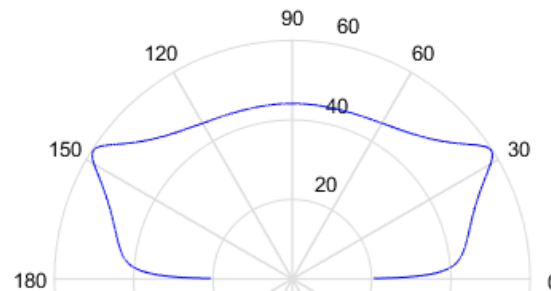
16000 Hz



Radiated pressure at 10 m - (dB, ref. $1d-6 Pa$)

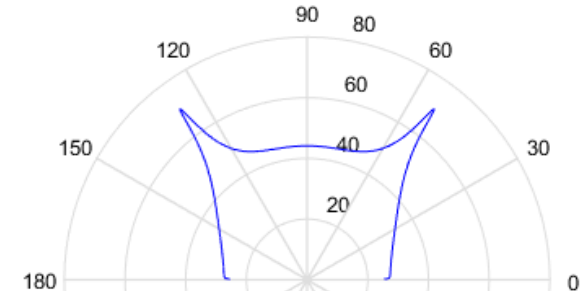


Radiated pressure at 10 m - (dB, ref. $1d-6 Pa$)



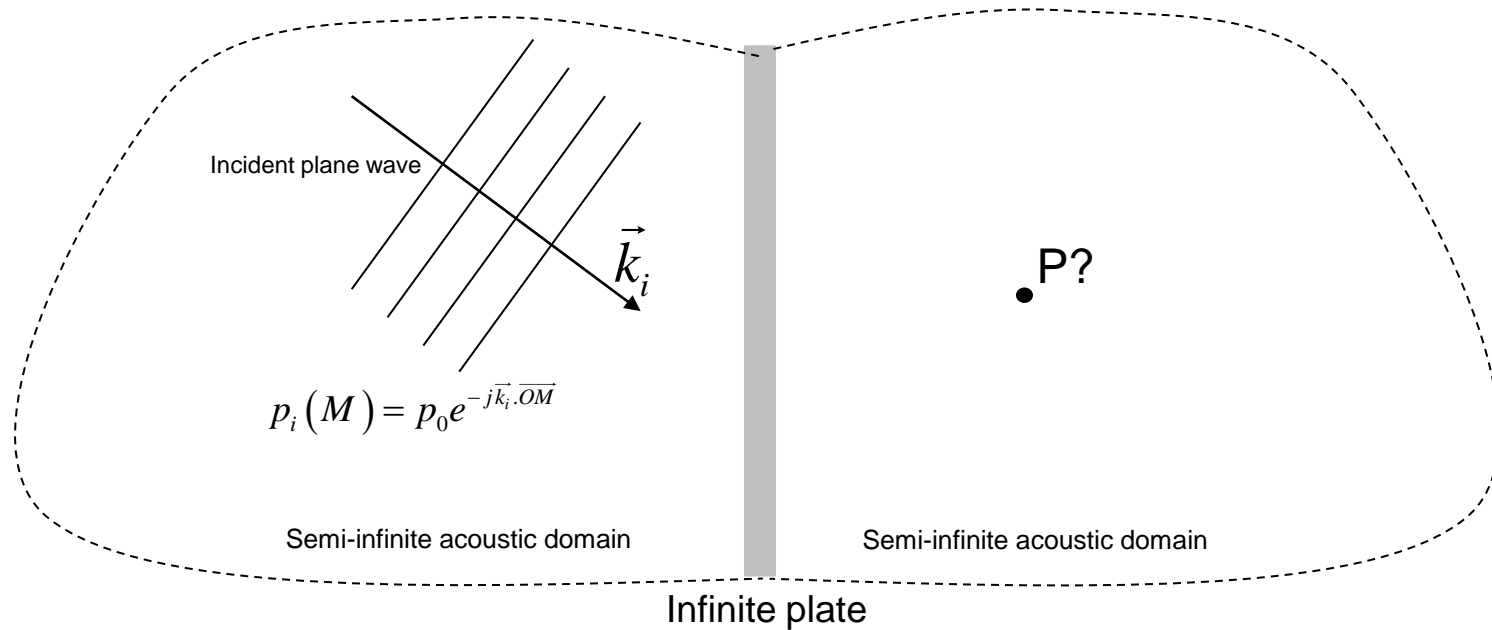
+ 50 dB

Radiated pressure at 10 m - (dB, ref. $1d-6 Pa$)



+ 50 dB

II.11 - Transmission loss of an infinite plate



How can we introduce the plane wave excitation in the VA problem?

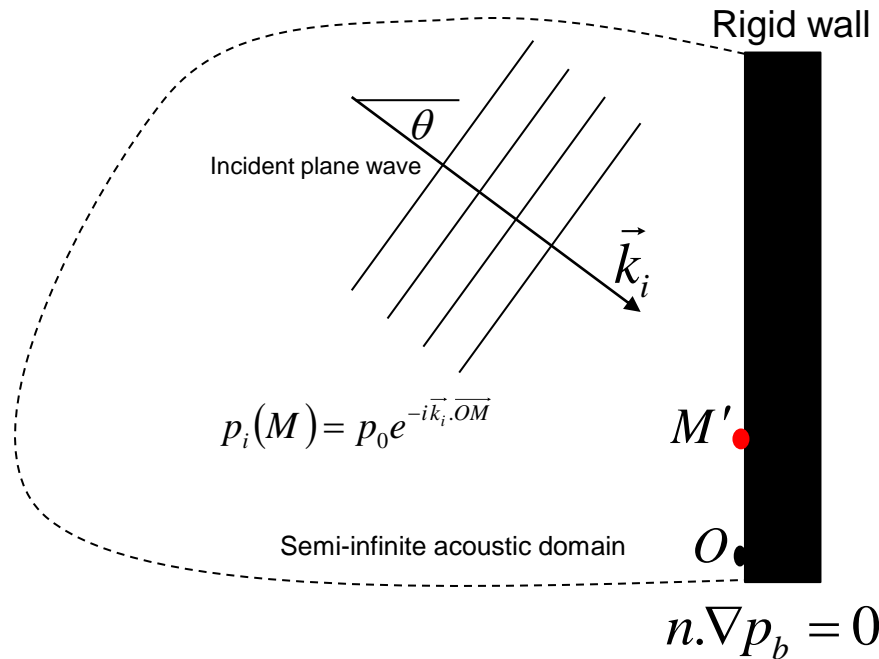
- With a source term q in the right member of the Helmholtz equation:

$$\nabla^2 p - \frac{1}{c_0^2} \frac{\partial^2 p}{\partial t^2} = q \quad \rightarrow \text{Definition of } q?$$

$$n \cdot \nabla p = \rho_0 \omega^2 w$$

- Using the « blocked » pressure induced by the external source

Description of an external source by its « blocked » pressure



Problem with rigid wall:

$$\nabla^2 p_b - \frac{1}{c_0^2} \frac{\partial^2 p_b}{\partial t^2} = q$$

$$n \cdot \nabla p_b = 0$$

Blocked pressure on the wall:
(source-image theory)

$$p_b(M') = 2p_0 e^{-i\vec{k}_i \cdot \overline{OM}'} = 2p_0 e^{-ik_x x}$$

with $k_x = k_0 \sin \theta$

Pressure decomposition: $p = p_b + p_{rad}$



Linear operator

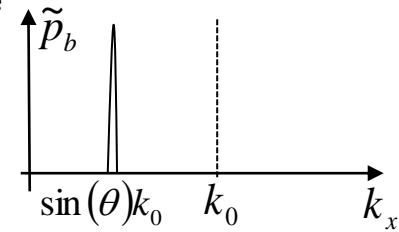
$$\nabla^2 p_{rad} - \frac{1}{c_0^2} \frac{\partial^2 p_{rad}}{\partial t^2} = 0 \quad n \cdot \nabla p_{rad} = \rho_0 \omega^2 w$$

p_{rad} , the radiated pressure respects the homogeneous Helmholtz eq. and the Euler eq.

➔ If the blocked pressure is known, it is not necessary to define the source term q

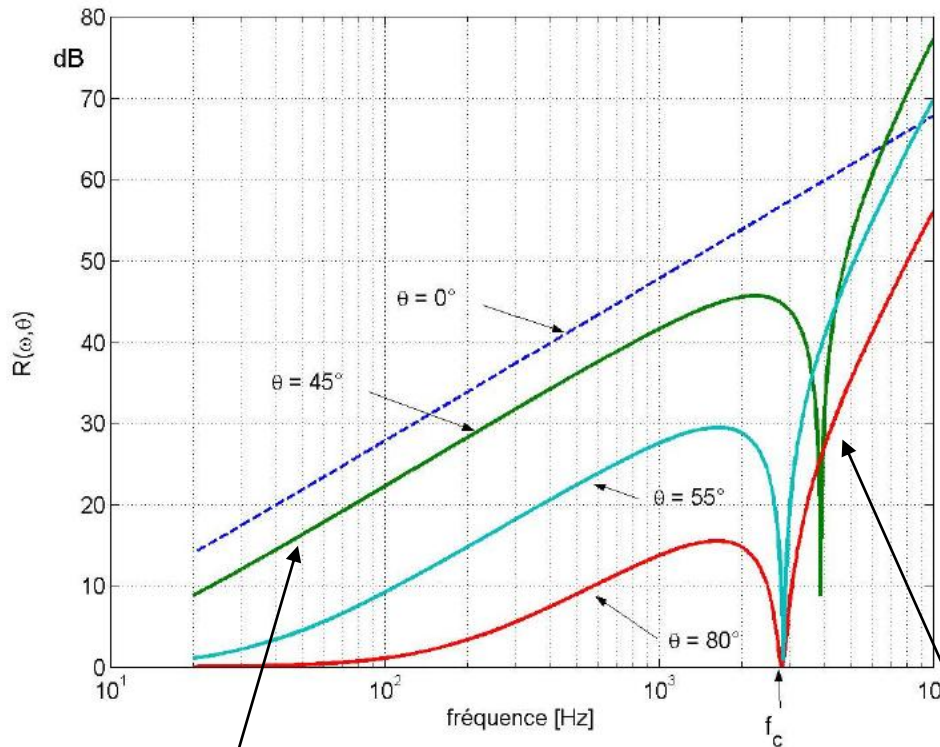
Transmission loss of an infinite plate for an incident acoustic wave

Resolution in the wavenumber space with $\tilde{p}_b(k_x, k_y) = 2p_0\delta(k_x - \sin(\theta)k_0)$



Incident angle from the normal

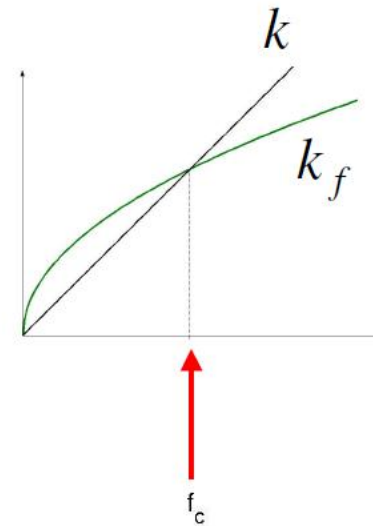
$$\text{Transmission loss: } R = 10\log_{10}\left(\frac{W_t}{W_i}\right) = 10\log_{10}\left(\frac{P_t^2}{P_o^2}\right)$$



Mass law (6dB/oct)

Damping control

Stiffness law (18dB/oct)

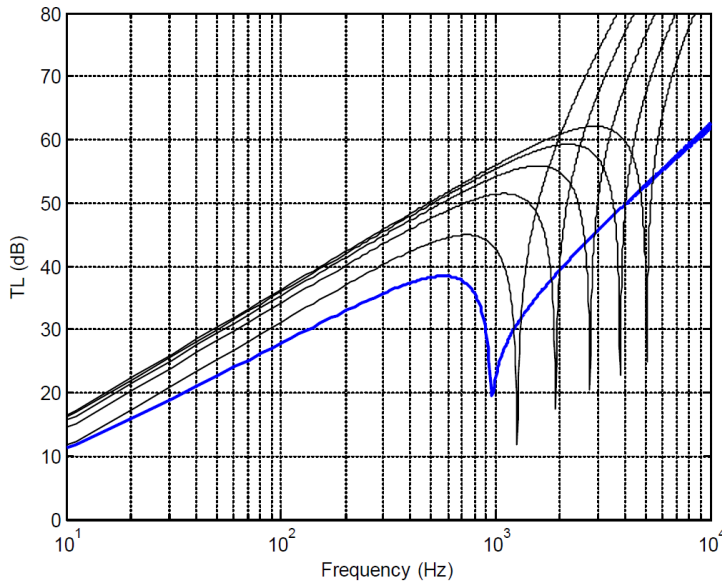


Transmission loss of an infinite plate under a diffuse field

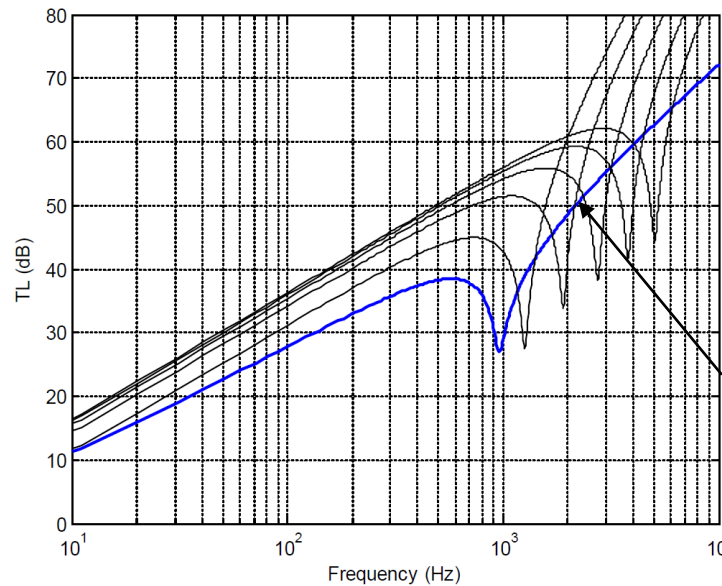
Diffuse field = an infinite set of uncorrelated acoustic plane waves with uniformly distributed incidence angles



Random diffuse field incidence TL for a 25 mm thick steel panel in air
blue line, TL for a random diffuse field; grey line, TL for various angles of incidence.



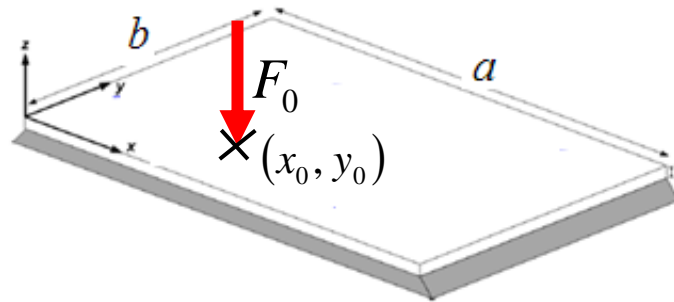
Panel damping loss factor: 0.005



Panel damping loss factor: 0.05

III – Vibroacoustic response of a finite panel excited by a mechanical force

III.1 – Plate velocity



Simply supported plate excited by a normal mechanical point force

Finite structure → Modal behavior

Modal information for the S-S plate:

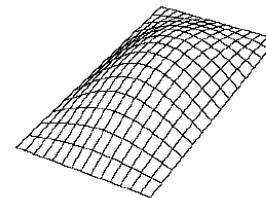
- Modal angular frequency:

$$\omega_{pq} = \sqrt{\frac{D}{\rho h} \left[\left(\frac{p\pi}{a} \right)^2 + \left(\frac{q\pi}{b} \right)^2 \right]}$$

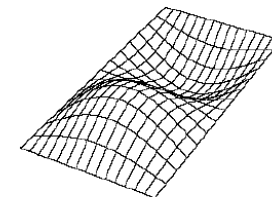
- Mode shape:

$$\phi_{pq}(x, y) = \sin(k_p x) \sin(k_q y) \text{ with } k_p = \frac{p\pi}{a}, k_q = \frac{q\pi}{b}$$

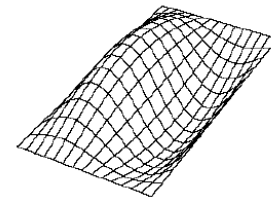
- Generalized mass: $M_{pq} = \rho h a b / 4$



$\phi_{11}(x, y)$



$\phi_{21}(x, y)$



$\phi_{12}(x, y)$

Plate displacement response (by modal expansion method):

$$v(x, y, t) = \sum_{p=1}^{\infty} \sum_{q=1}^{\infty} \frac{i\omega F_{pq} \phi_{pq}(x, y)}{M_{pq} (-\omega^2 + i\omega \omega_{pq} \eta_{pq} + \omega_{pq}^2)} e^{i\omega t}$$

with the generalized forces $F_{pq} = \int_0^a \int_0^b p(x, y) \phi_{pq}(x, y) dx dy$

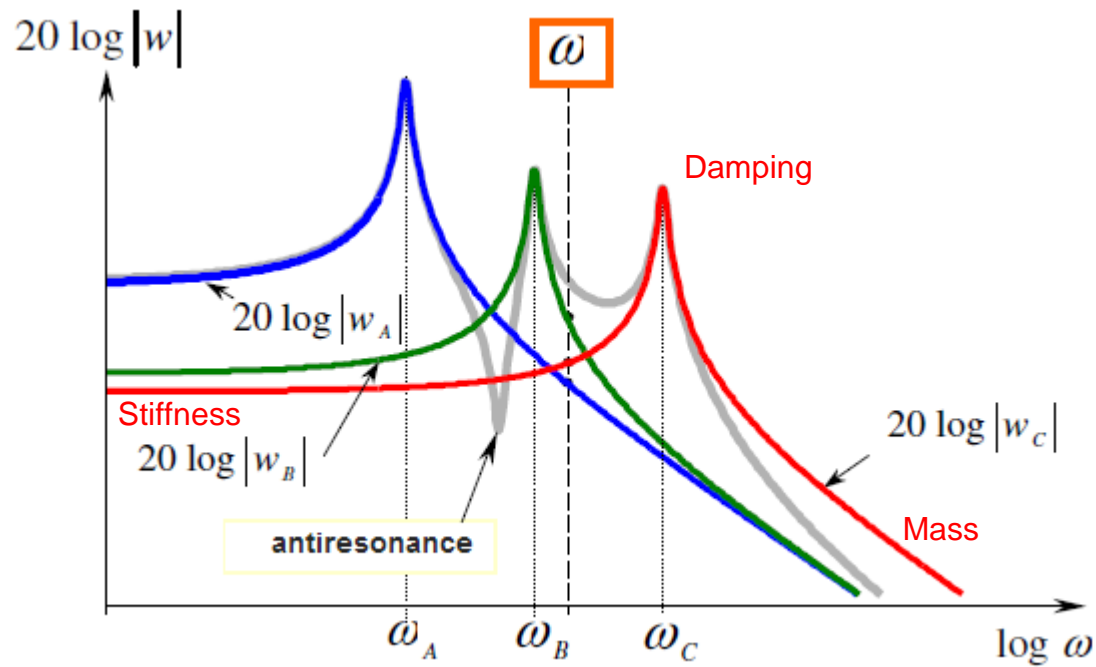
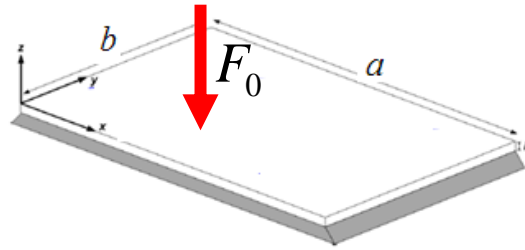


Illustration of the modal response

Example for a harmonic mechanical point force at (x_0, y_0)



$$v(x, y, \omega) = \sum_{p=1}^{\infty} \sum_{q=1}^{\infty} \frac{i\omega F_0 \varphi_{pq}(x_0, y_0) \varphi_{pq}(x, y)}{M_{pq} (-\omega^2 + i\omega \omega_{pq} \eta_{pq} + \omega_{pq}^2)}$$

with F_0 the magnitude of the force.

Displacement in the wavenumber space:

$$\tilde{v}(k_x, k_y, \omega) = \sum_{p=1}^{\infty} \sum_{q=1}^{\infty} \frac{i\omega F_0 \varphi_{pq}(x_0, y_0) \tilde{\varphi}_{pq}(k_x, k_y)}{M_{pq} (-\omega^2 + i\omega \omega_{pq} \eta_{pq} + \omega_{pq}^2)}$$

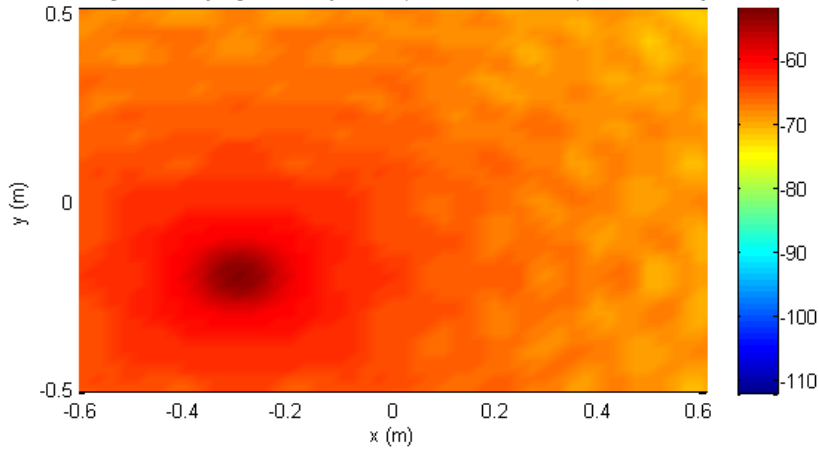
$$\int_0^a \sin(k_p x) e^{-ik_x x} dx = \int_0^a \frac{e^{ik_p x} - e^{-ik_p x}}{2i} e^{-ik_x x} dx = \dots = \frac{p\pi}{a} \frac{(-1)^p e^{-ik_x a} - 1}{k_x^2 - (p\pi/a)^2}$$

$$\tilde{\varphi}_{pq}(k_x, k_y) = k_p k_q \left[\frac{(-1)^p e^{-ik_x a} - 1}{k_x^2 - k_p^2} \right] \left[\frac{(-1)^q e^{-ik_y b} - 1}{k_y^2 - k_q^2} \right] \quad \text{with } k_q = \frac{q\pi}{b} \quad k_p = \frac{p\pi}{a}$$

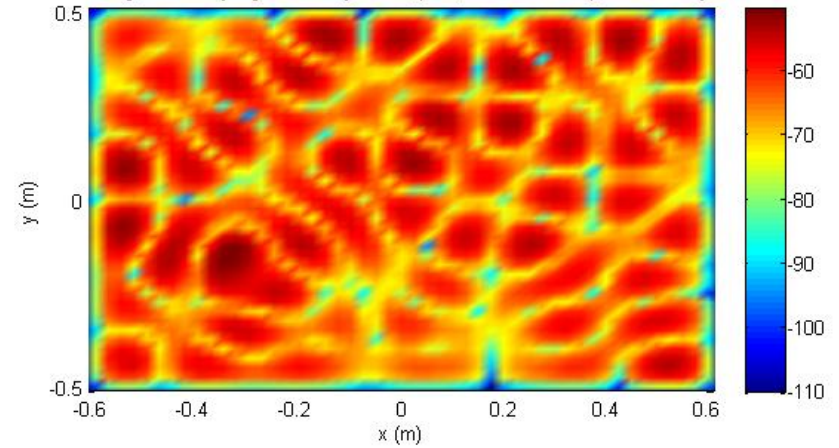
the modal wavenumbers

Plate response for a mechanical point force at (-0.3m, -0.2m) – 500 Hz

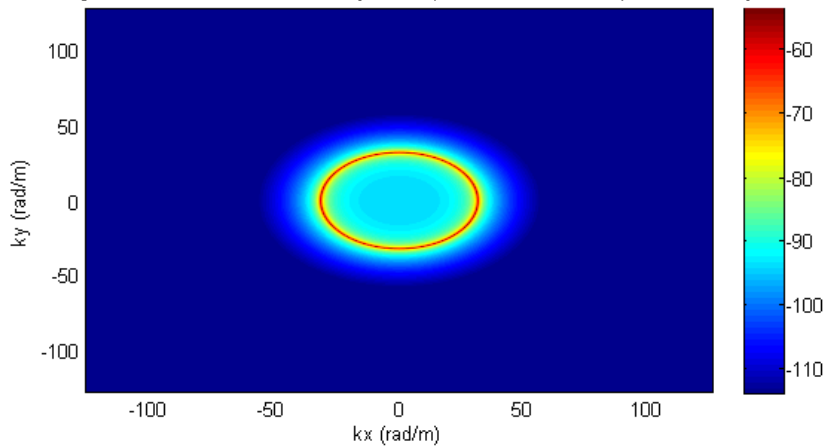
Velocity in the physical space (dB, ref. 1 m/s) - Infinite plate



Velocity in the physical space (dB, ref. 1 m/s) - Finite plate

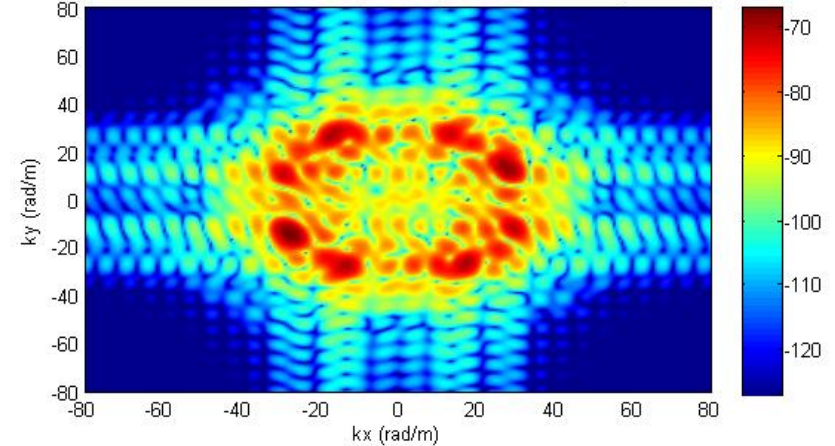


Velocity in the wavenumber space (dB, ref. 1 m³/s) - Infinite plate



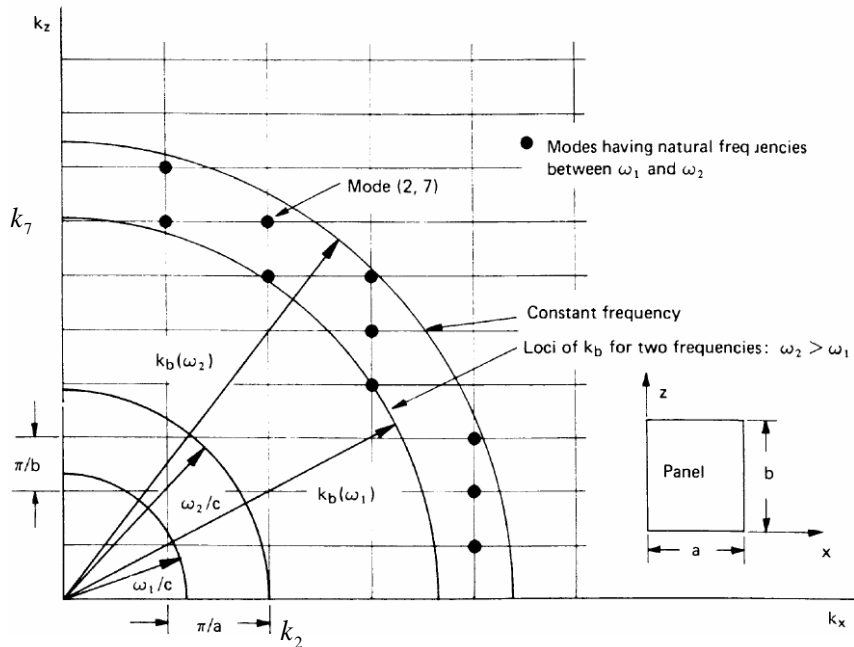
Infinite plate

Velocity in the wavenumber space (dB, ref. 1 m³/s) - Finite plate



Simply supported steel plate 1.2m x 1m x 2mm

$$\tilde{\varphi}_{pq}(k_x, k_y) = k_p k_q \left[\frac{(-1)^p e^{-ik_x a} - 1}{k_x^2 - k_p^2} \right] \left[\frac{(-1)^q e^{-ik_y b} - 1}{k_y^2 - k_q^2} \right]$$

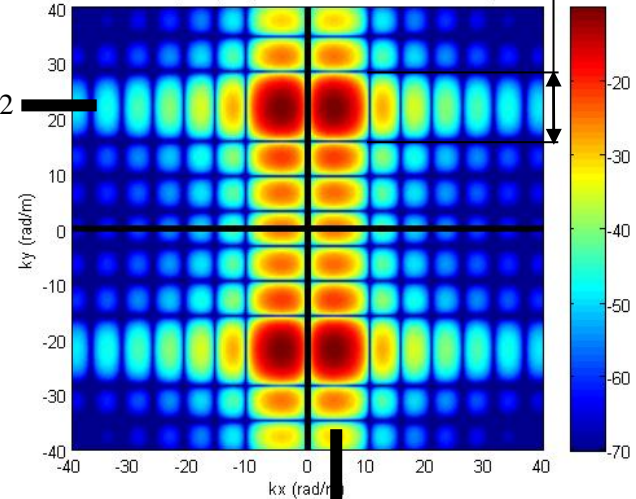


Modal wavenumbers of resonant modes, flexural wavenumber and acoustic wavenumber versus frequency
 Fahy, F. – Sound and structural vibration, 1985.

$$k_7 = \frac{7\pi}{b} \approx 22$$

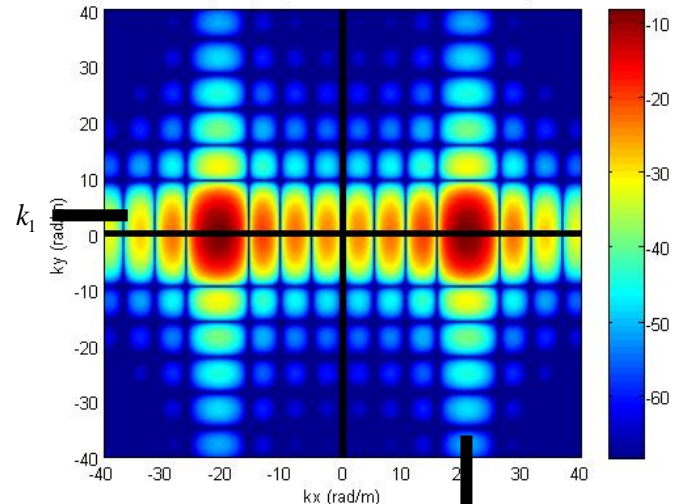
$$\Delta k = \frac{4\pi}{b} \approx 12.5$$

Mode shape (2,7) in the wavenumber space



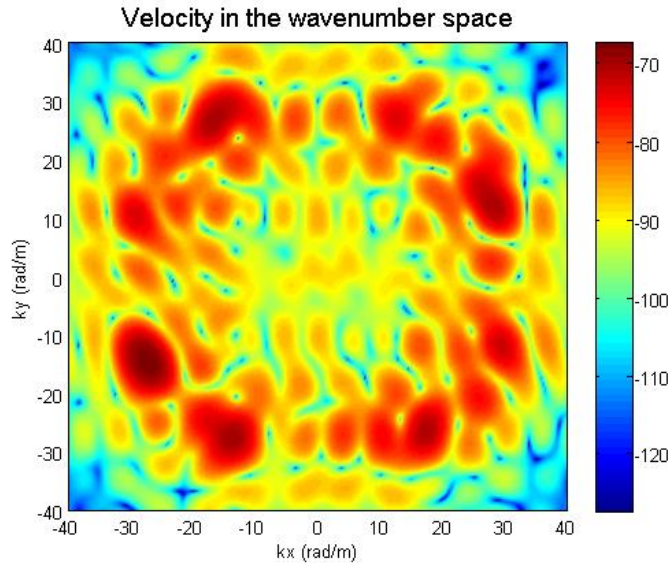
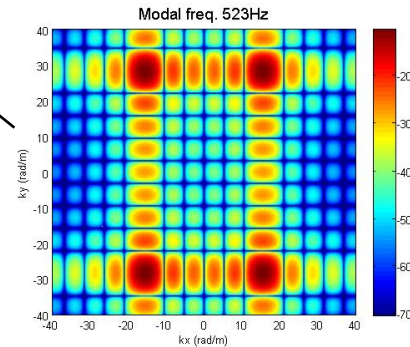
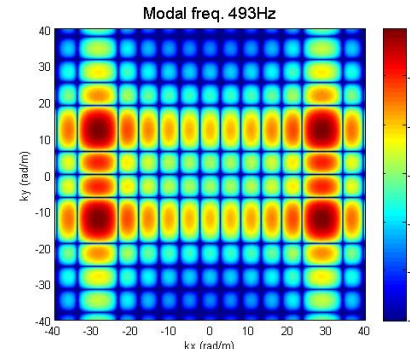
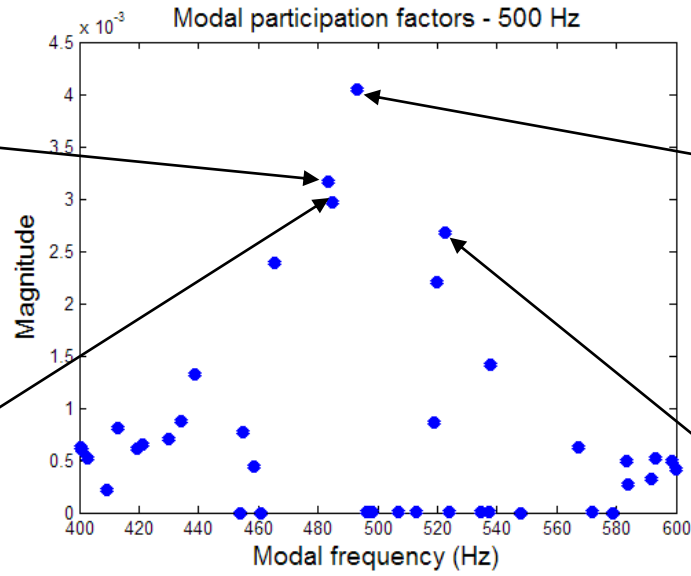
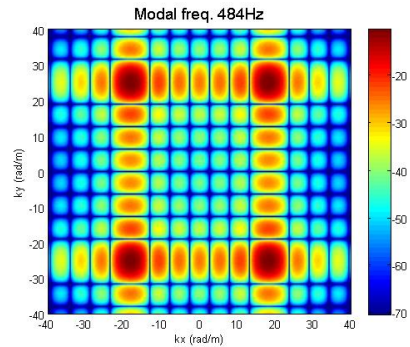
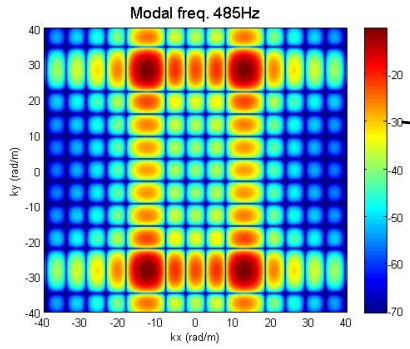
$$k_2 = \frac{2\pi}{a} \approx 5.2$$

Mode shape (8,1) in the wavenumber space



$$k_8$$

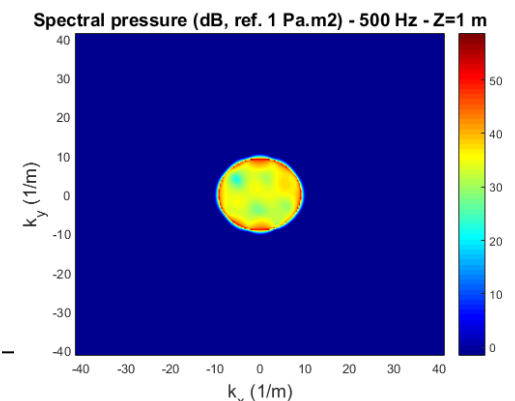
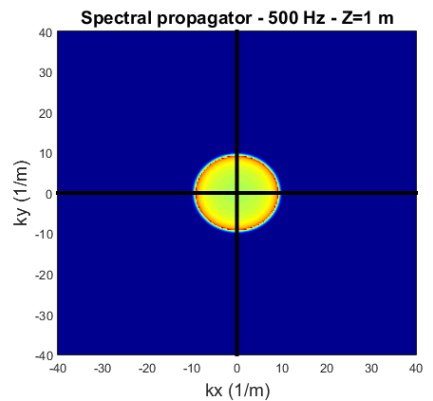
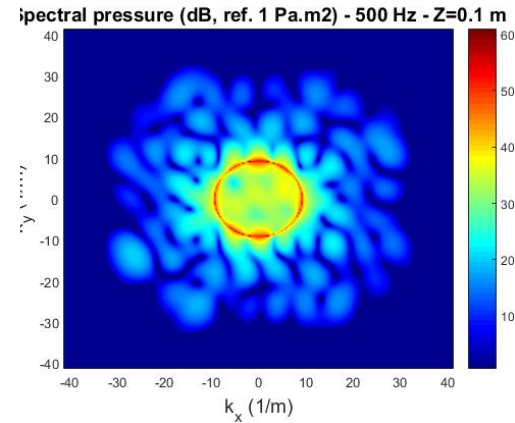
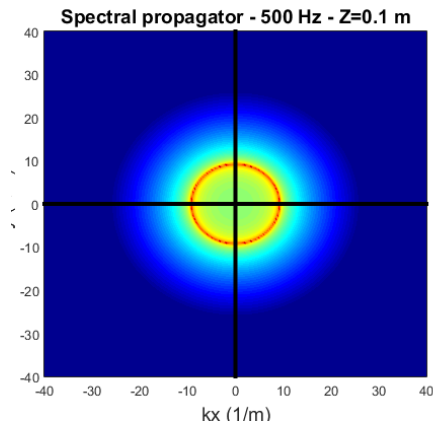
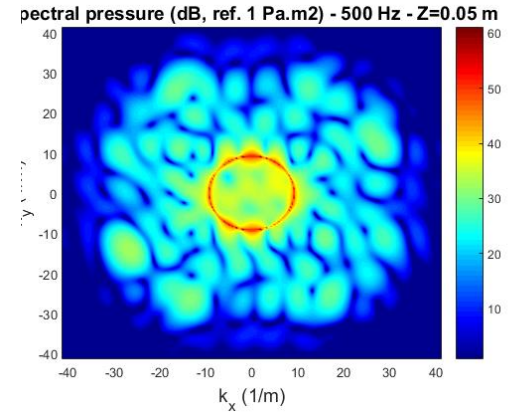
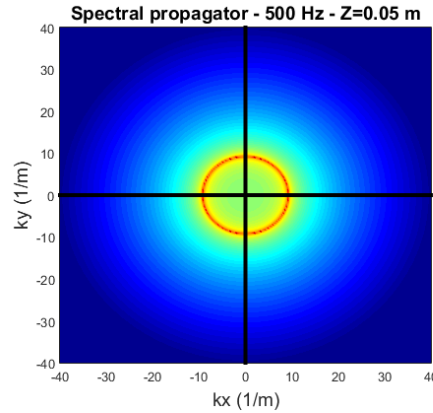
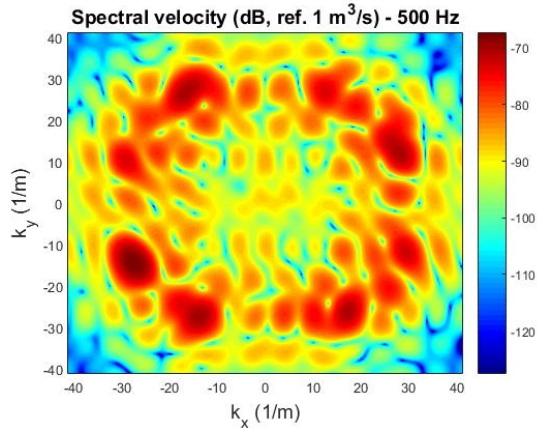
$$A_{pq} = \frac{i\omega F_{pq}}{M_{pq}(-\omega^2 + i\omega\omega_{pq}\eta_{pq} + \omega_{pq}^2)}$$



$$\tilde{v}(k_x, k_y, \omega) = \sum_{p=1}^{\infty} \sum_{q=1}^{\infty} \frac{i\omega F_{pq} \tilde{\varphi}_{pq}(k_x, k_y)}{M_{pq}(-\omega^2 + i\omega\omega_{pq}\eta_{pq} + \omega_{pq}^2)}$$

Spectral radiated pressure at different distances from the panel

III.2 – Radiated pressure



$$p(k_x, k_y, z) \approx \Gamma(k_x, k_y) \tilde{v}(k_x, k_y)$$

with the spectral propagator:

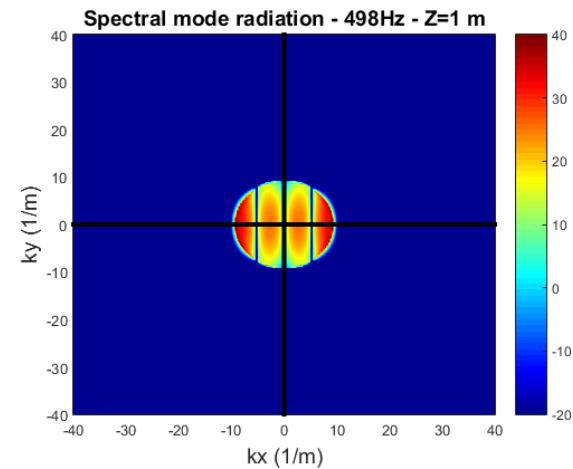
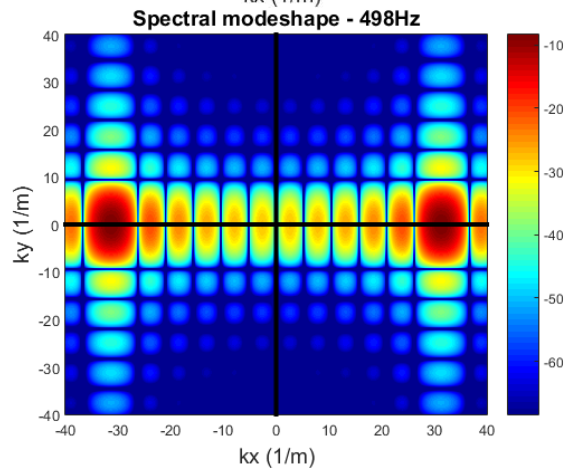
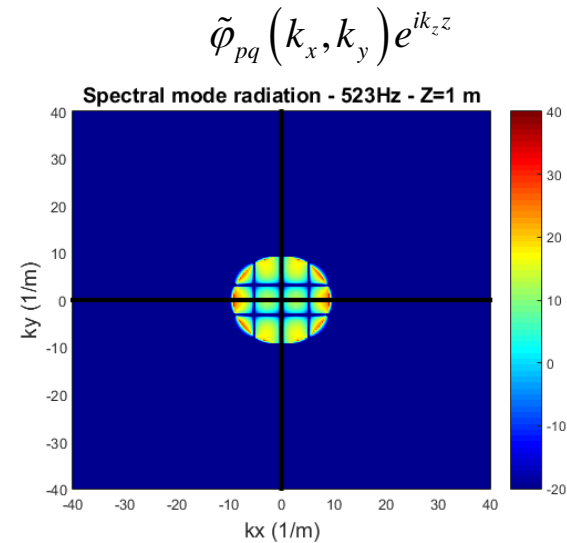
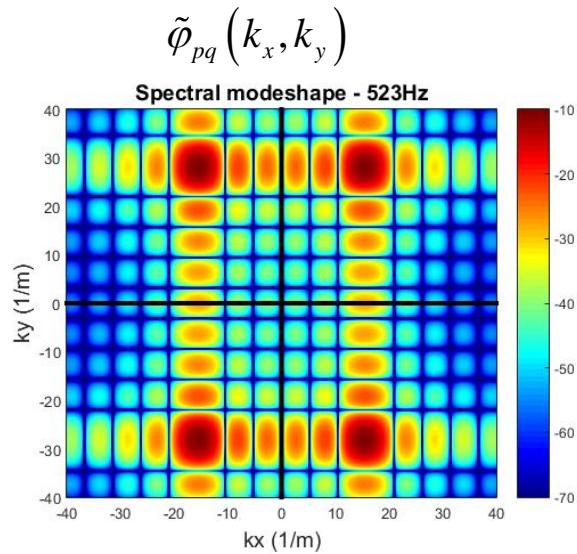
$$\Gamma(k_x, k_y) \approx -\frac{\rho_0 \omega}{k_z} e^{ik_z z}$$

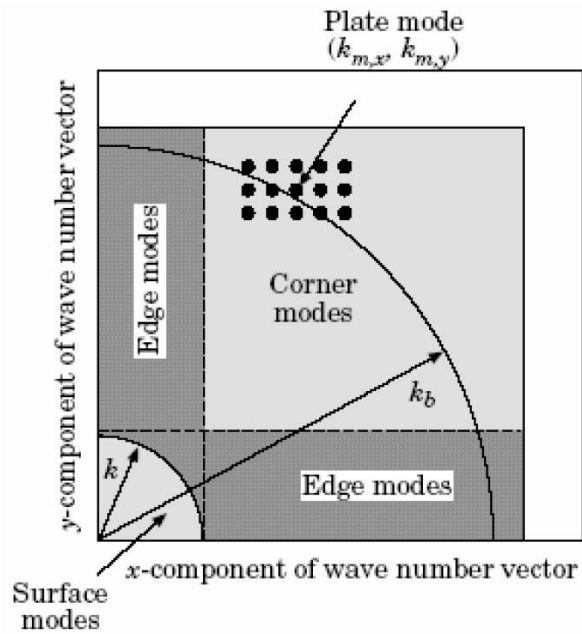
$$k_z = \begin{cases} -\sqrt{k_0^2 - k_x^2 - k_y^2} & \text{if } \sqrt{k_x^2 + k_y^2} < k_0 \\ i\sqrt{k_x^2 + k_y^2 - k_0^2} & \text{if } \sqrt{k_x^2 + k_y^2} > k_0 \end{cases}$$

→!! Light fluid assumption !!

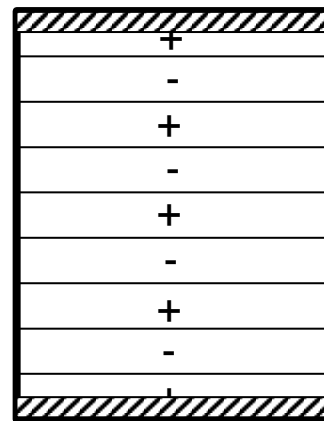
$$p(k_x, k_y, z) = Z_a \sum_{p=1}^{\infty} \sum_{q=1}^{\infty} A_{pq} \tilde{\varphi}_{pq}(k_x, k_y) e^{ik_z z}$$

$$A_{pq} = \frac{i\omega F_0 \varphi_{pq}(x_0, y_0)}{M_{pq} (-\omega^2 + i\omega\omega_{pq}\eta_{pq} + \omega_{pq}^2)}$$

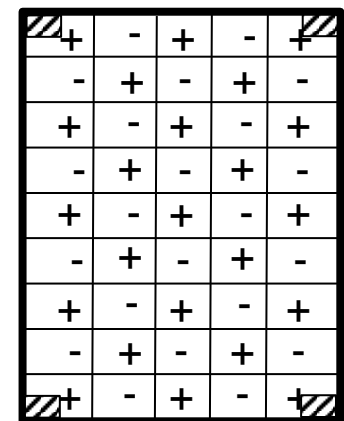




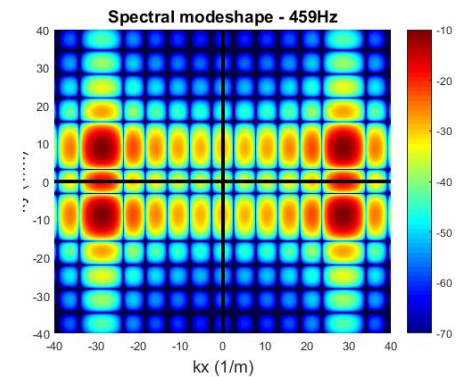
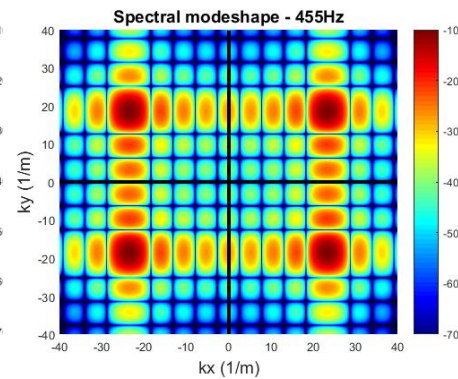
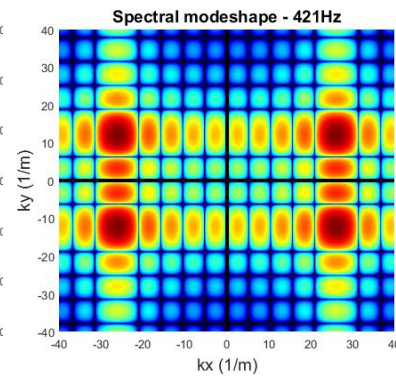
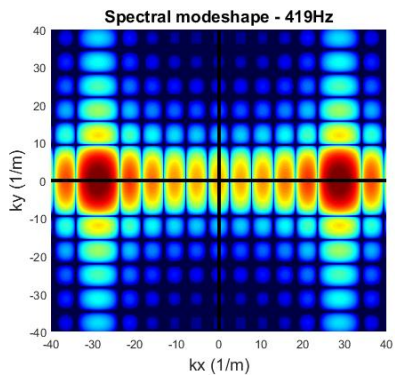
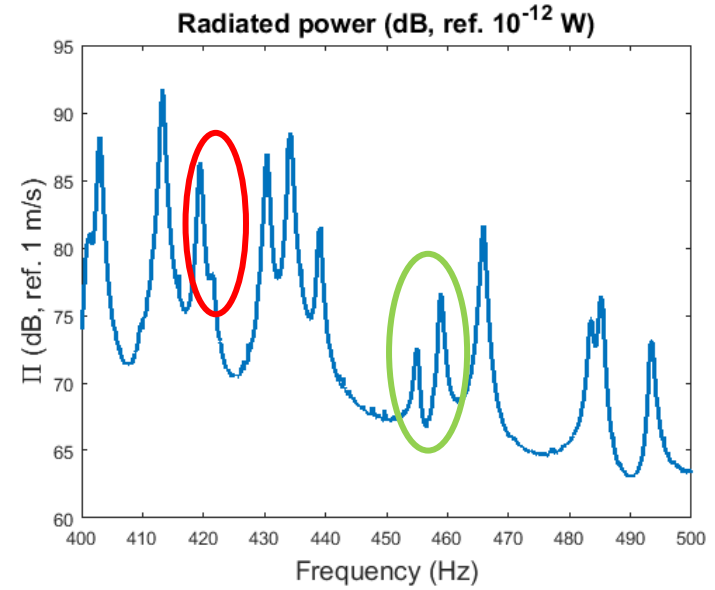
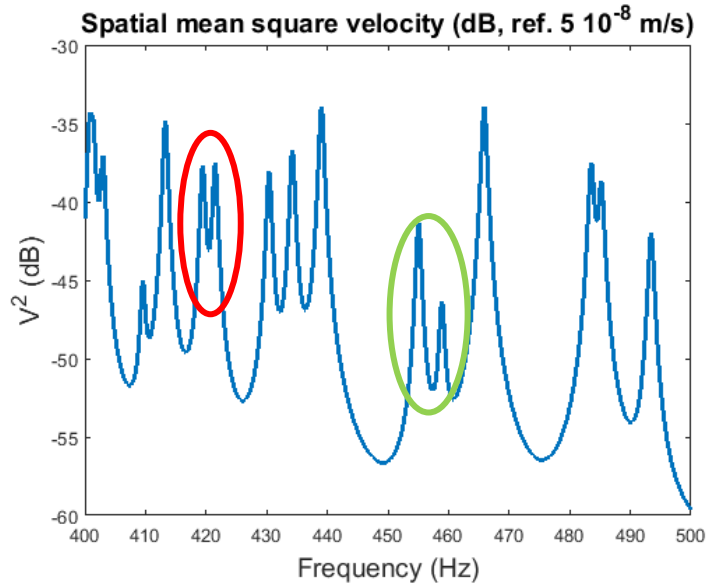
Edge modes

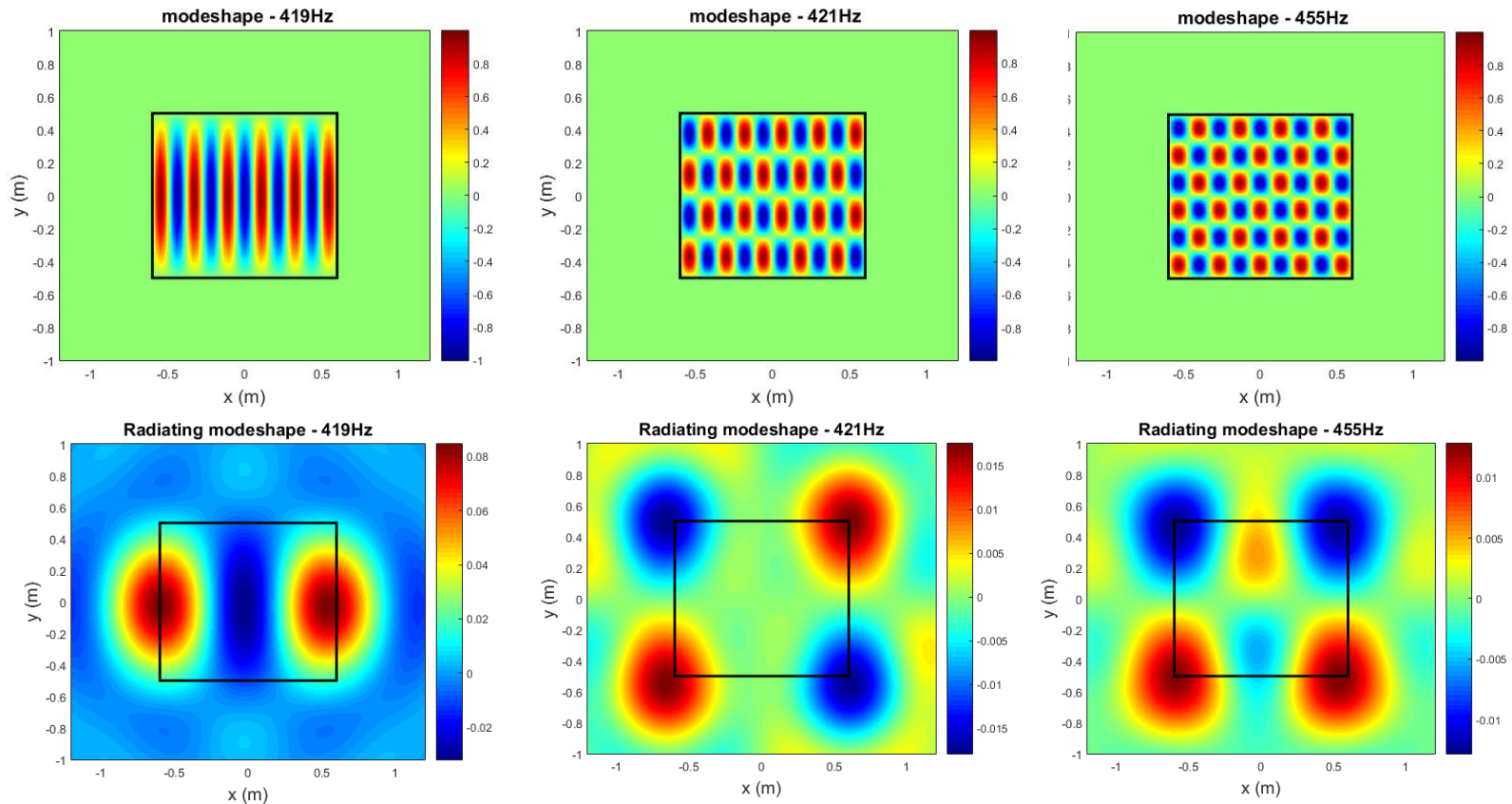


Corner modes



 Area radiating effectively in the far field



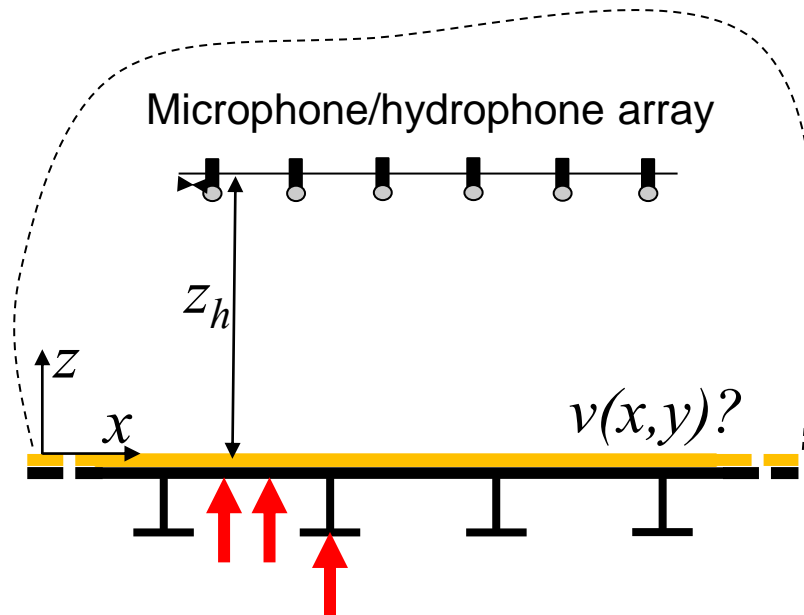


➔ Compare the amplitude between the standard modeshape and the radiating modeshape

(see in the literature the comparison between active intensity and non-negative intensity/supersonic intensity (Williams))

IV – Nearfield Acoustical Holography

IV.1 – Principle



- Williams and Maynard, 1980, Holographic Imaging without the wavelength resolution limit, Phys. Rev. Lett., 45, pp.554-557.
- Inverse problem: knowing the measured pressures in a plane at a given distance of the radiated structure, what is the velocity field of the structure?

Spectral pressure from the spectral velocity at $z=0$:

$$\tilde{p}(k_x, k_y, z) = \Gamma(k_x, k_y, z) \tilde{v}(k_x, k_y), \forall z \geq 0$$

with the spectral propagator: $\Gamma(k_x, k_y, z) = Z_a(k_x, k_z) e^{ik_z z}$

$$\text{with: } k_z = \begin{cases} -\sqrt{k_0^2 - k_x^2 - k_y^2} & \text{if } \sqrt{k_x^2 + k_y^2} < k_0 \\ i\sqrt{k_x^2 + k_y^2 - k_0^2} & \text{if } \sqrt{k_x^2 + k_y^2} > k_0 \end{cases}$$

Spectral pressure at z_h from the spectral velocity at $z=0$

$$\tilde{p}(k_x, k_y, z_h) = \Gamma(k_x, k_y, z_h) \tilde{v}(k_x, k_y)$$

Spectral velocity at $z=0$ from the spectral pressure at z_h

$$\tilde{v}_z(k_x, k_y) = G(k_x, k_y, z_h) \tilde{p}(k_x, k_y, z_h)$$

with the inverse spectral propagator:

$$G(k_x, k_y, z_h) = \frac{1}{\Gamma(k_x, k_y, z_h)} = \frac{e^{-ik_z z_h}}{Z_a(k_x, k_z)}$$

The process can be decomposed in 4 steps:

- (1) Measure the pressure field in a plane (called hologram) at z_h distance from the panel $p(k_x, k_y, z_h)$
- (2) Compute the spectral pressure $\tilde{p}(k_x, k_y, z_h)$
- (3) Multiply by the inverse propagator $G(k_x, k_y, z_h) \tilde{p}(k_x, k_y, z_h) = \tilde{v}_z(k_x, k_y)$
- (4) Compute the inverse Fourier transforms $\tilde{v}_z(k_x, k_y) \rightarrow v_z(x, y)$

Remark: possibility to reconstruct also the wall pressure and the normal acoustic intensity

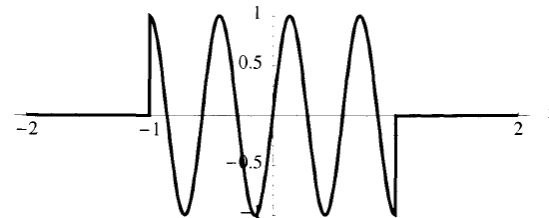
Because NAH solves an inverse problem, the mathematical solution must be approached with some caution to assure that the solution is unique and stable.⁴ That is, straightforward implementation of Eq. (3.4) will lead to disaster! How this equation must be modified to avoid disaster is the springboard for the NAH technique.

E.G. Williams, *FOURIER Acoustics: Sound Radiation and Nearfield Acoustical Holography*, Academic press, 1999.

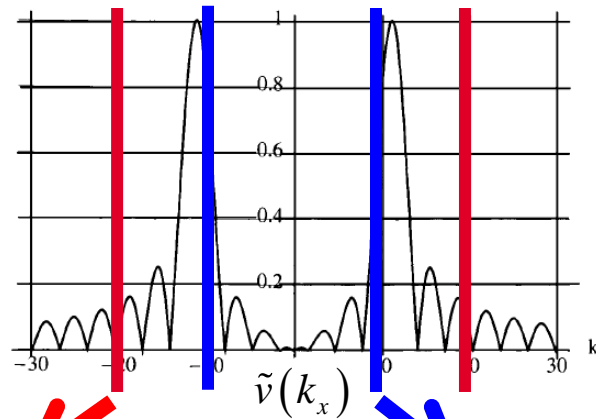
IV.2 – k-space filter

Arrays of microphone → Measurement at discrete points δx → Use of Discrete Fourier Transform for estimating the spectral pressure → introduction of a sampling wavenumber (i.e. Cutoff wavenumber) $\bar{k}_x = \frac{\pi}{\delta x}$

1D example: $v(x) = \sin(3.5\pi x) \Pi_{[-1,1]}(x)$

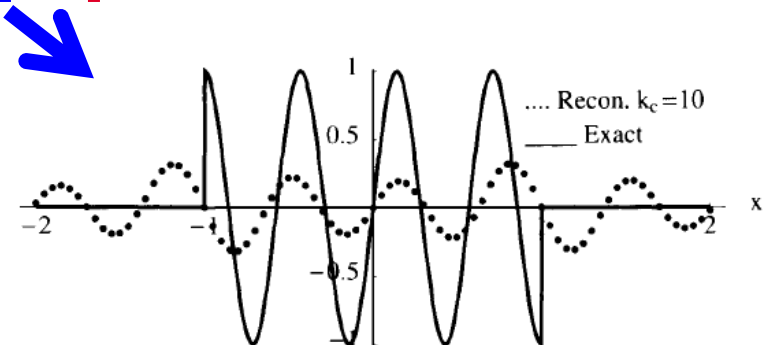
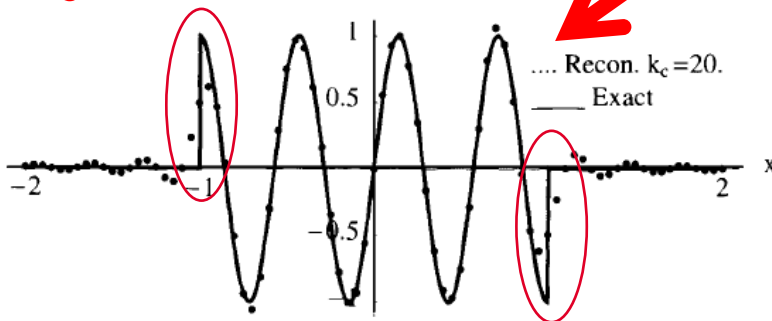


Velocity to reconstruct (target)



Spectral velocity: sinc function

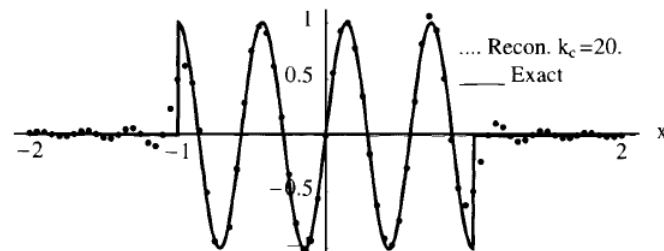
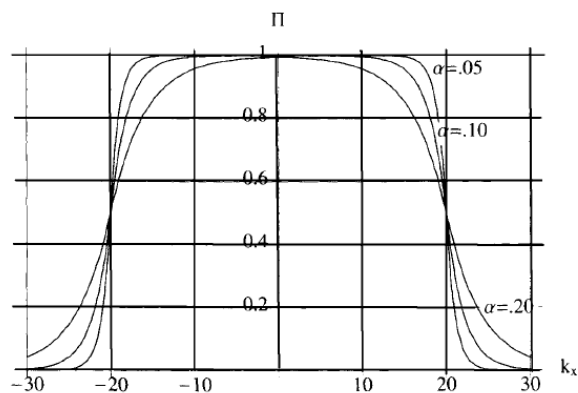
Edge reconstruction errors



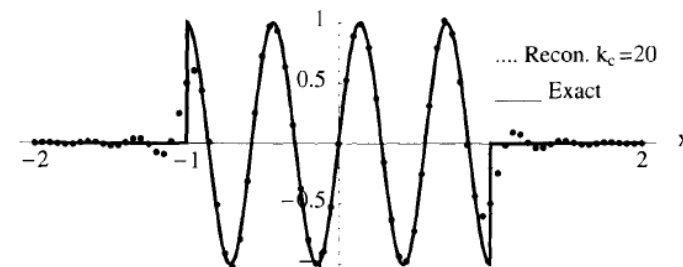
Large errors

Modification of the filter shape (for 'reducing' the edge reconstruction errors)

$$\bar{\Pi}(k_x/(2k_c)) = \begin{cases} 1 - \frac{1}{2}e^{-(1-|k_x|/k_c)/\alpha} & |k_x| < k_c \\ \frac{1}{2}e^{(1-|k_x|/k_c)/\alpha} & |k_x| > k_c \end{cases}$$



With rectangular filter

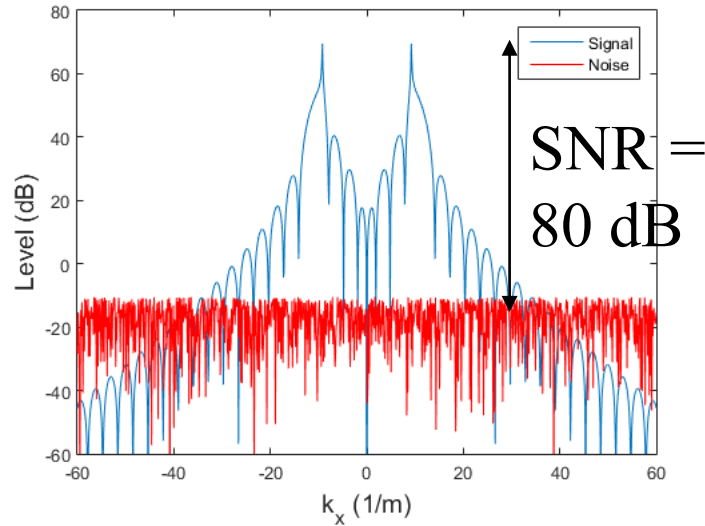


With tapered filter

- ➔ Very slight improvement with the tapered filter
- ➔ Use greater k_c for better reconstruction at the edges

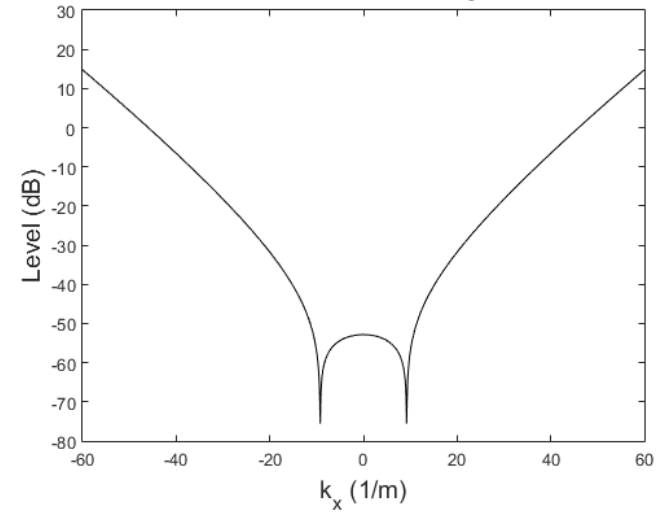
IV.3 – Influence of measurement noise

Spectral pressure hologram

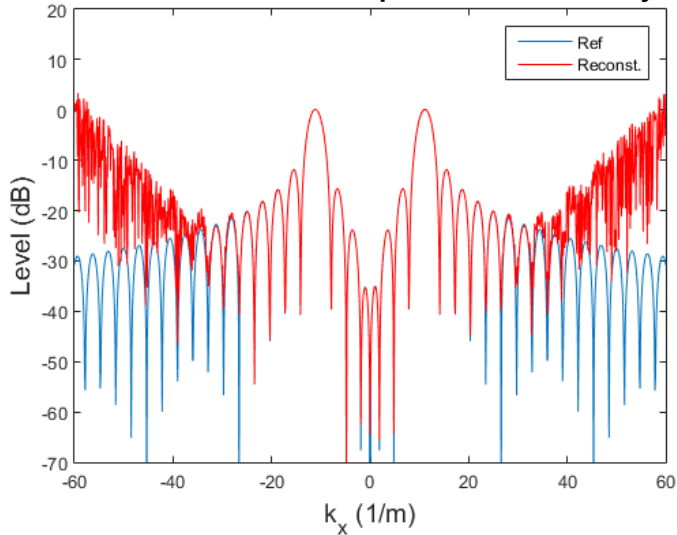


$z_h=0.1$ m, 500 Hz, Air

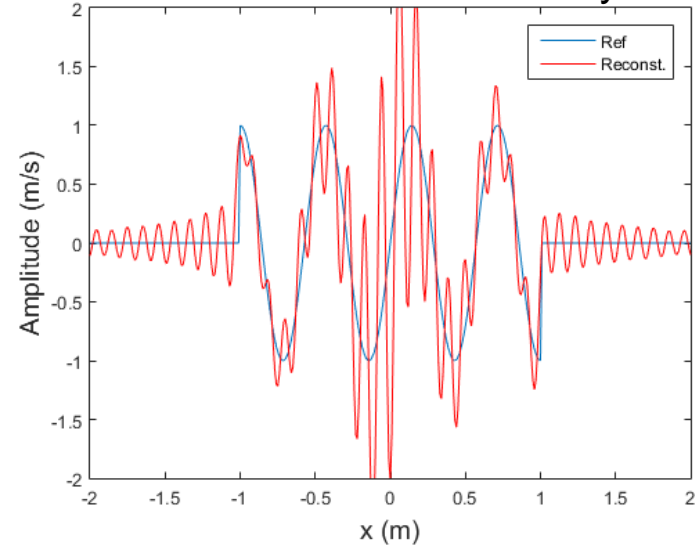
Inverse propagator



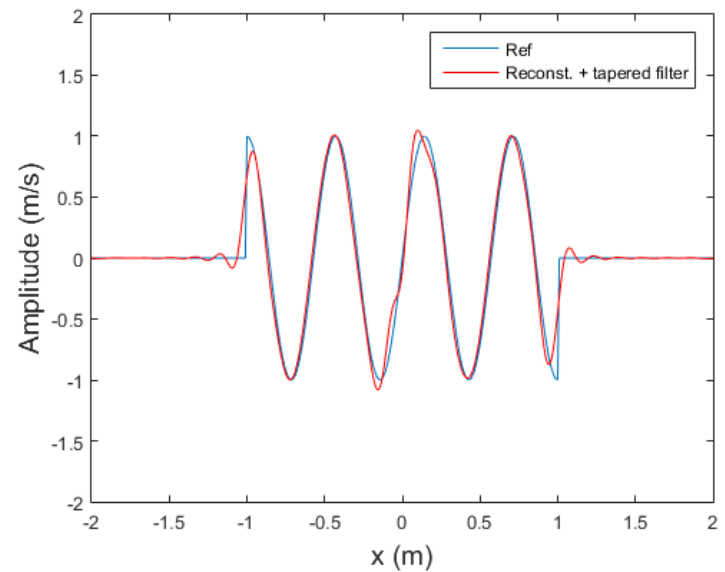
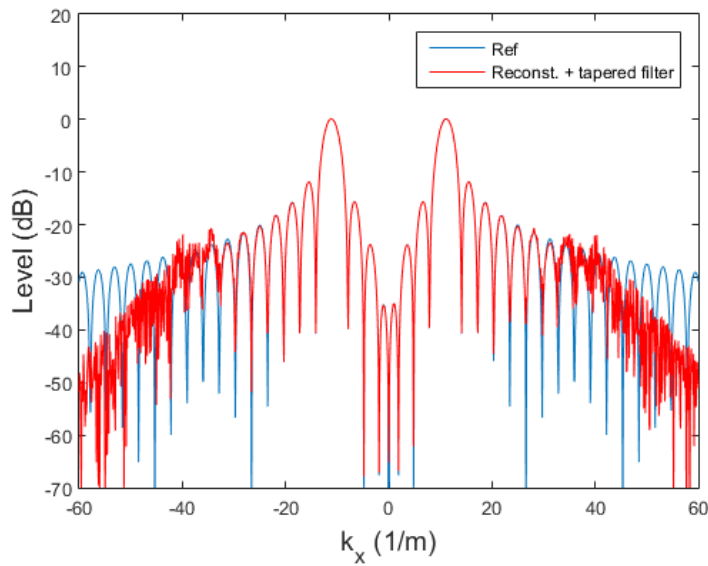
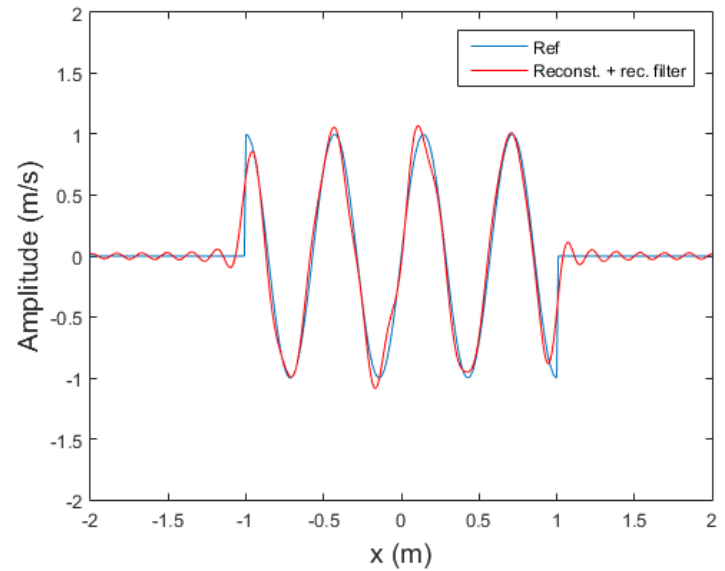
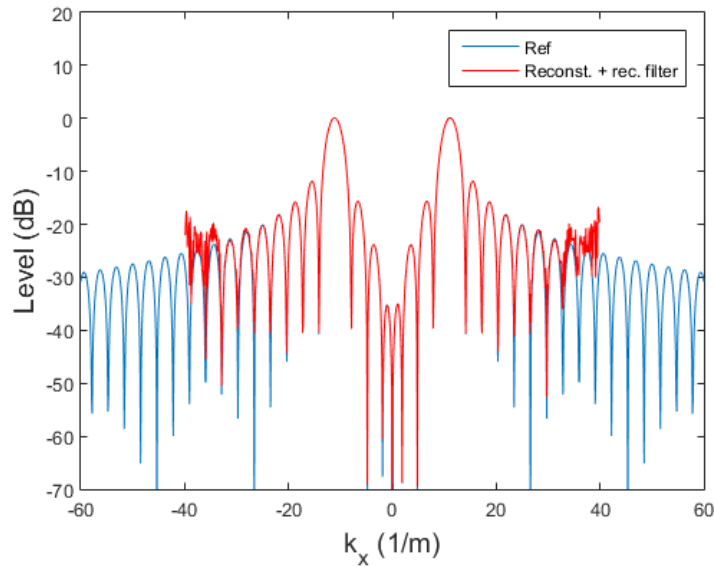
Reconstructed spectral velocity



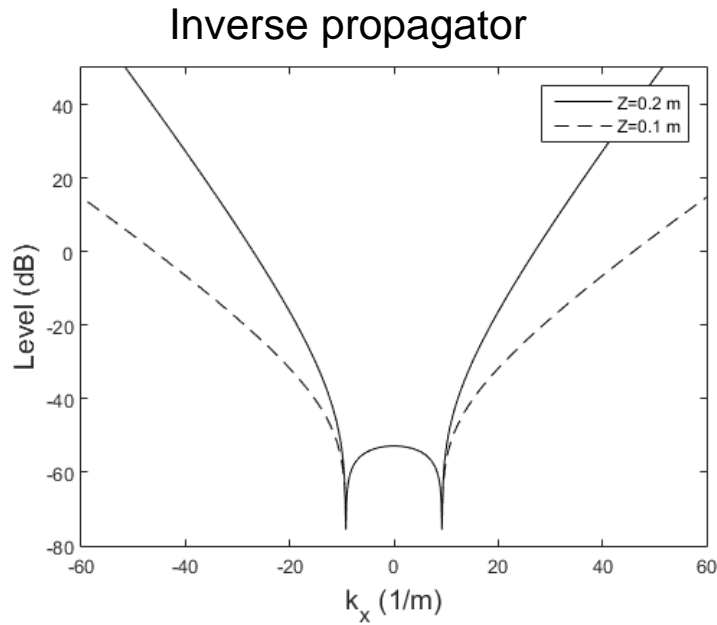
Reconstructed velocity



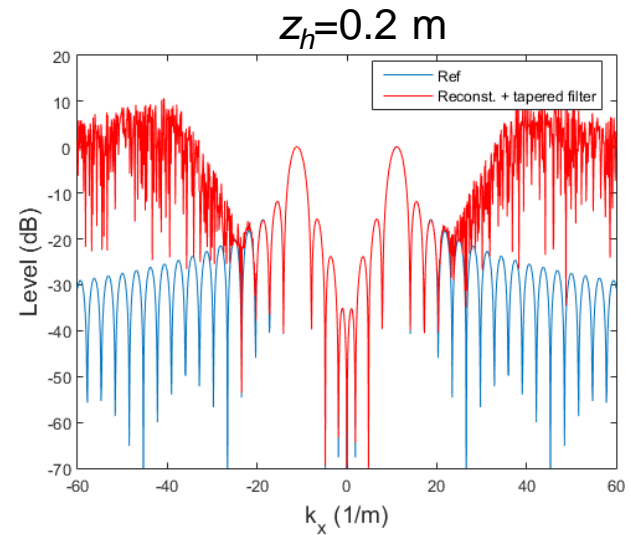
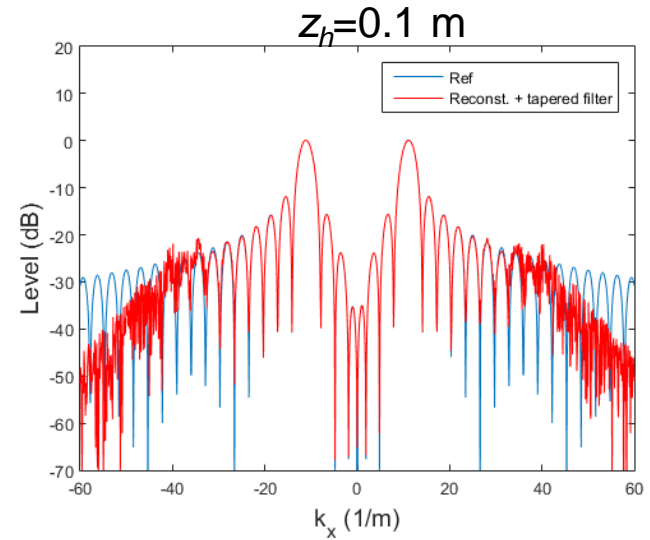
Influence of measurement noise: use of k-filter with cutoff wavenumber $k_c=40$ 1/m.



IV.4 – Influence of the standoff distance z_h



$$G(k_x, k_y, z_h) = \frac{e^{-ik_z z_h}}{Z_a(k_x, k_z)}$$



IV.5 – Finite Measurement Aperture Effect

Measurement on an infinite plane is not possible in practice

➔ Measurement on a rectangular area (called measurement aperture)

$$p_{mes}(x, y) = p(x, y) \Pi(x/L_x) \Pi(y/L_y)$$
$$\downarrow$$
$$\tilde{p}_{mes}(k_x, k_y) = \tilde{p}(k_x, k_y) * \tilde{\Pi}_{Aperture}(k_x, k_y)$$
$$\tilde{\Pi}_{Aperture}(k_x, k_y) = \text{sinc}(k_x L_x / 2) \text{sinc}(k_y L_y / 2)$$

The convolution with the sinc functions spread the spectrum of \tilde{p} .

Non-physical contribution may appear and amplified by the inverse propagator.

➔ In practice, the measurement aperture is always larger than the actual source. The size of the aperture should be larger than twice the actual source. This may guarantee that the measured pressure field drops off significantly towards the edges of the aperture.

IV.6 – Discretization and Aliasing

Microphone spacing should be chosen such that the aliasing effect may be negligible.

Conclusion

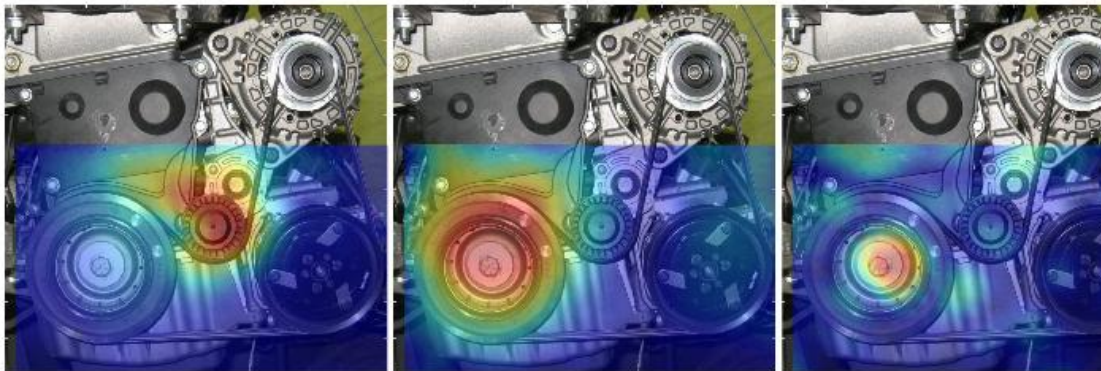
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For more details, see E.G. Williams, *FOURIER Acoustics: Sound Radiation and Nearfield Acoustical Holography*, Academic press, 1999.

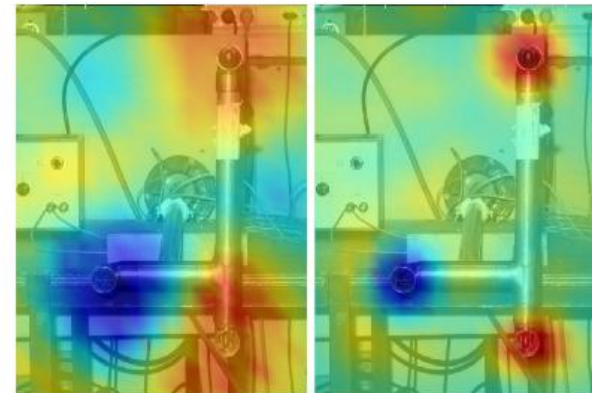
Quentin Leclere, quentin.leclere@insa-lyon.fr

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Laboratoire Vibrations Acoustique, INSA-Lyon



RMS velocity maps in the near-field region of a gasoline engine: left, 600 Hz; center, 1100 Hz; right, 4200 Hz.



Left: velocity of the membrane at 4 cm distance from the acoustic source at 2700 Hz. Right: back-propagated velocity field in the source plane using NAH. The chromatic scale represents negative, zero and positive acoustic values.

Appendix

SOUND RADIATION OF A PERIODICALLY STIFFENED PLATE EXCITED BY A POINT FORCE

I - INTRODUCTION

Stiffened structures are very common for practical applications

Example:

- Fuselage/wing of an airplane
- Railway car structure
- Submarine pressure hull

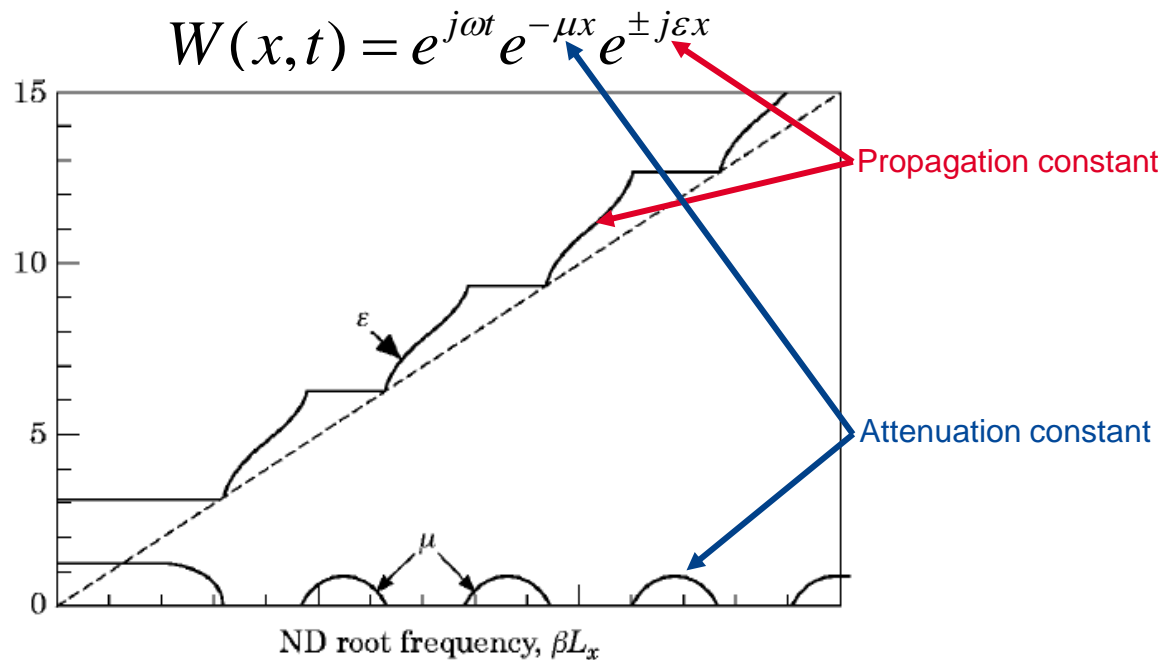
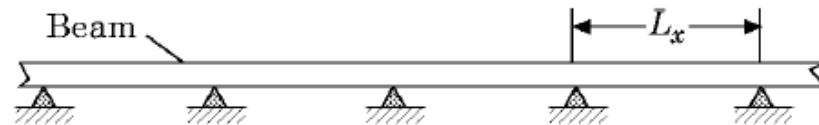


Need of « basic models » for understanding how stiffened structures radiated in the near field/far field

- effects of stiffener dynamics, of periodicity
- Influence of stop band/pass band (Block-Floquet wave)

II – FLEXURAL BLOCH-FLOQUET WAVES

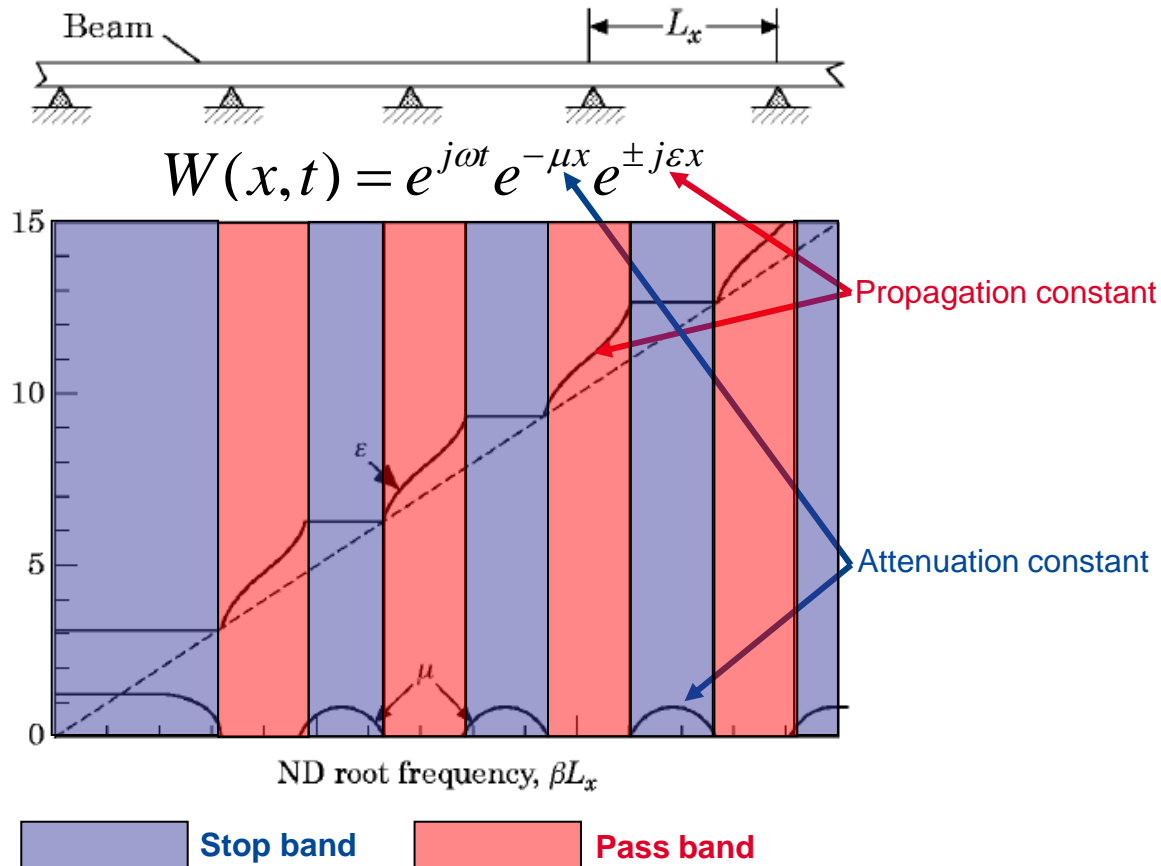
One-dimensional periodic structure: example of the simply supported beam



D.J. Mead, Wave propagation in continuous periodic structures: research contributions from Southampton 1964-1995, JSV (1996) 190, 495-524.

II – FLEXURAL BLOCH-FLOQUET WAVES

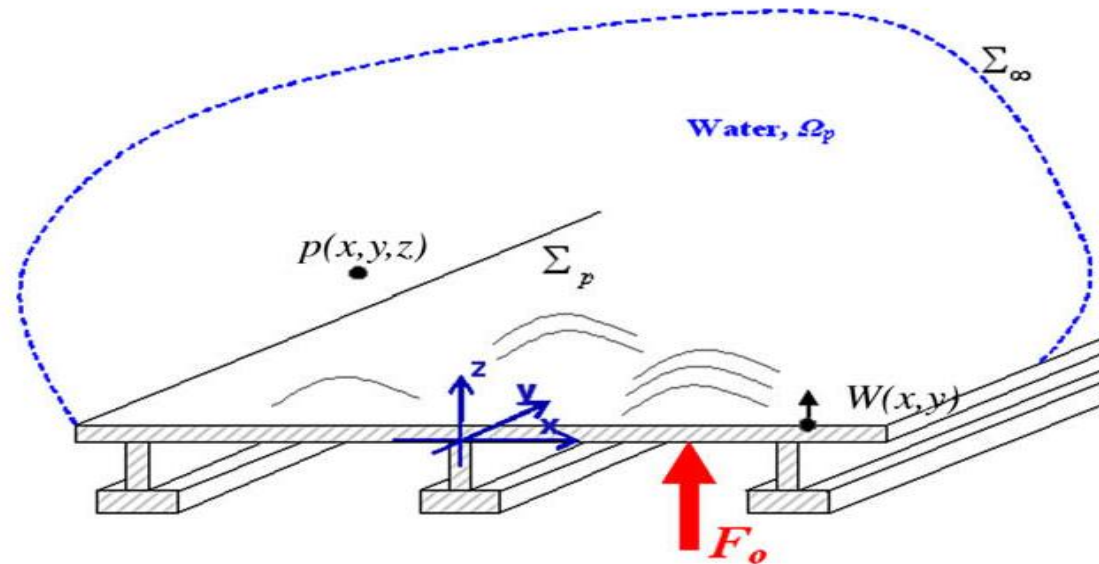
One-dimensional periodic structure: example of the simply supported beam



D.J. Mead, Wave propagation in continuous periodic structures: research contributions from Southampton 1964-1995, JSV (1996) 190, 495-524.

Despite the numerous studies of wave motion in continuous periodic systems over the past 40 years, a simple physical explanation has yet to be presented for the very existence of frequency-propagation zones and attenuation zones. However, even if there is no simple answer to the question "Why does wave motion of one frequency propagate freely while motion of another frequency is attenuated?", reliable prediction methods do exist for the properties of free motion and the magnitudes of forced motion.

II – PERIODICALLY STIFFENED PLATE



Fluid load panel periodically stiffened by line beams

Stiffened panel: 2-D system, periodic in one direction (i.e. x axis)

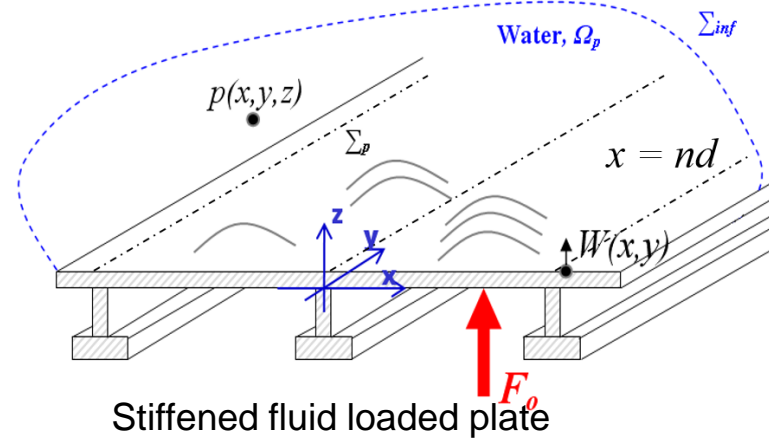


Space-Fourier transform about y-axis

For each wavenumber k_y , one-dimensional periodic system

(see L. MAXIT - Wavenumber space and physical space responses of a periodically ribbed plate to a point drive: A discrete approach. *Applied Acoustics*, 70 (2009) 563-578.) [10.1016/j.apacoust.2008.06.012](https://doi.org/10.1016/j.apacoust.2008.06.012)

Some elements for the spectral displacement calculation



Formulation of vibro-acoustic interaction (1):

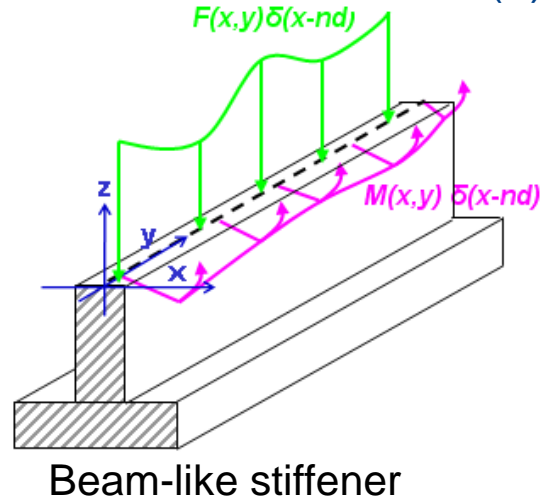
Love – Kirchhoff assumptions

$$D^* \left(\frac{\partial^4 W}{\partial x^4} (x, y) + 2 \frac{\partial^4 W}{\partial x^2 \partial y^2} (x, y) + \frac{\partial^4 W}{\partial y^4} (x, y) \right) - \omega^2 \rho h W(x, y) =$$

$$\underbrace{F_0 \delta(x - x_0) \delta(y)}_{\text{Point force } (x_0, 0)} - \underbrace{p(x, y, 0) - \sum_{n \in \mathbb{Z}} F(x, y) \delta(x - nd) + \sum_{n \in \mathbb{Z}} M(x, y) \delta'(x - nd)}_{\text{Forces and moments exerted by the stiffeners on the plate}}$$

↑
Wall pressure

Formulation of vibro-acoustic interaction (2):



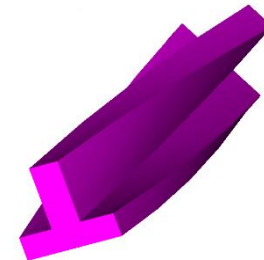
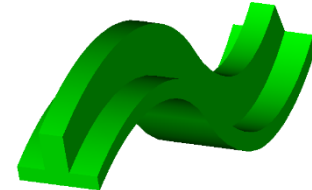
Beam assumptions (without warping)

- Bending motions:

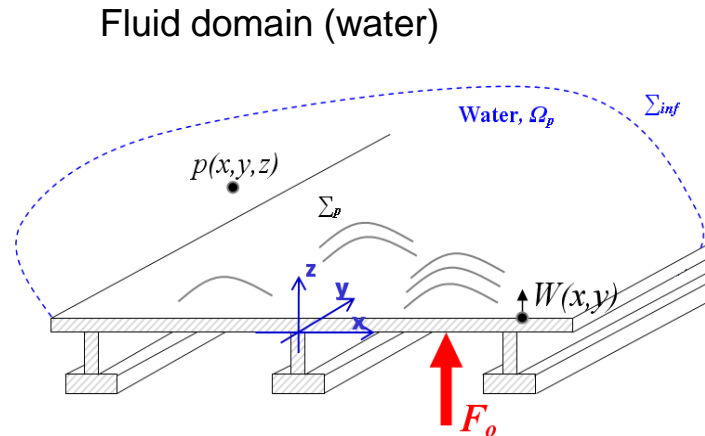
$$E_p^* I_x \frac{\partial^4 W}{\partial y^4}(x, y) - \omega^2 \rho_p S_p W(x, y) = F(x, y)$$

- Torsion motions:

$$-G_p^* J_p \frac{\partial^3 W}{\partial x \partial y^2}(x, y) - \omega^2 \rho_p I_0 \frac{\partial W}{\partial x}(x, y) = M(x, y)$$



Formulation of vibro-acoustic interaction (3):



- Helmholtz equation in the fluid domain:

$$\Delta p + \frac{\omega^2}{c_0^2} p = 0$$

- Euler equation and normal velocity continuity at $z=0$:

$$-\rho_0 \omega^2 W(x, y) = -\frac{\partial p}{\partial z}(x, y, 0)$$

- Sommerfeld radiation conditions on a fictive infinite surface, Σ_{inf} (no reflection)

Resolution using the Space-Fourier transform:

$$f(x, y) \xrightarrow{T.F.} \tilde{f}(k_x, k_y) = \int_{\square^2} f(x, y) e^{-j(k_x x + k_y y)} dx dy$$

Properties: $\frac{\partial^p \partial^q W}{\partial x^p \partial y^q}(x, y) \rightarrow (ik_x)^p (ik_y)^q \tilde{W}(k_x, k_y)$

$$\sum_{n \in \mathbb{Z}} \delta(x - nd) \xrightarrow{F.T.} \frac{1}{d} \sum_{n \in \mathbb{Z}} \delta\left(k_x + \frac{2\pi n}{d}\right) \quad (\text{Poisson formula})$$

Results: *Plate displacements in the wave number domain* (k_x, k_y)

$$\tilde{W}(k_x, k_y) = \tilde{W}_0(k_x, k_y) \Gamma(k_x, k_y)$$

Displacement of the bare plate $\tilde{W}_0(k_x, k_y) = \frac{F_0 e^{-jk_x x_0}}{Z(k_x, k_y)}$

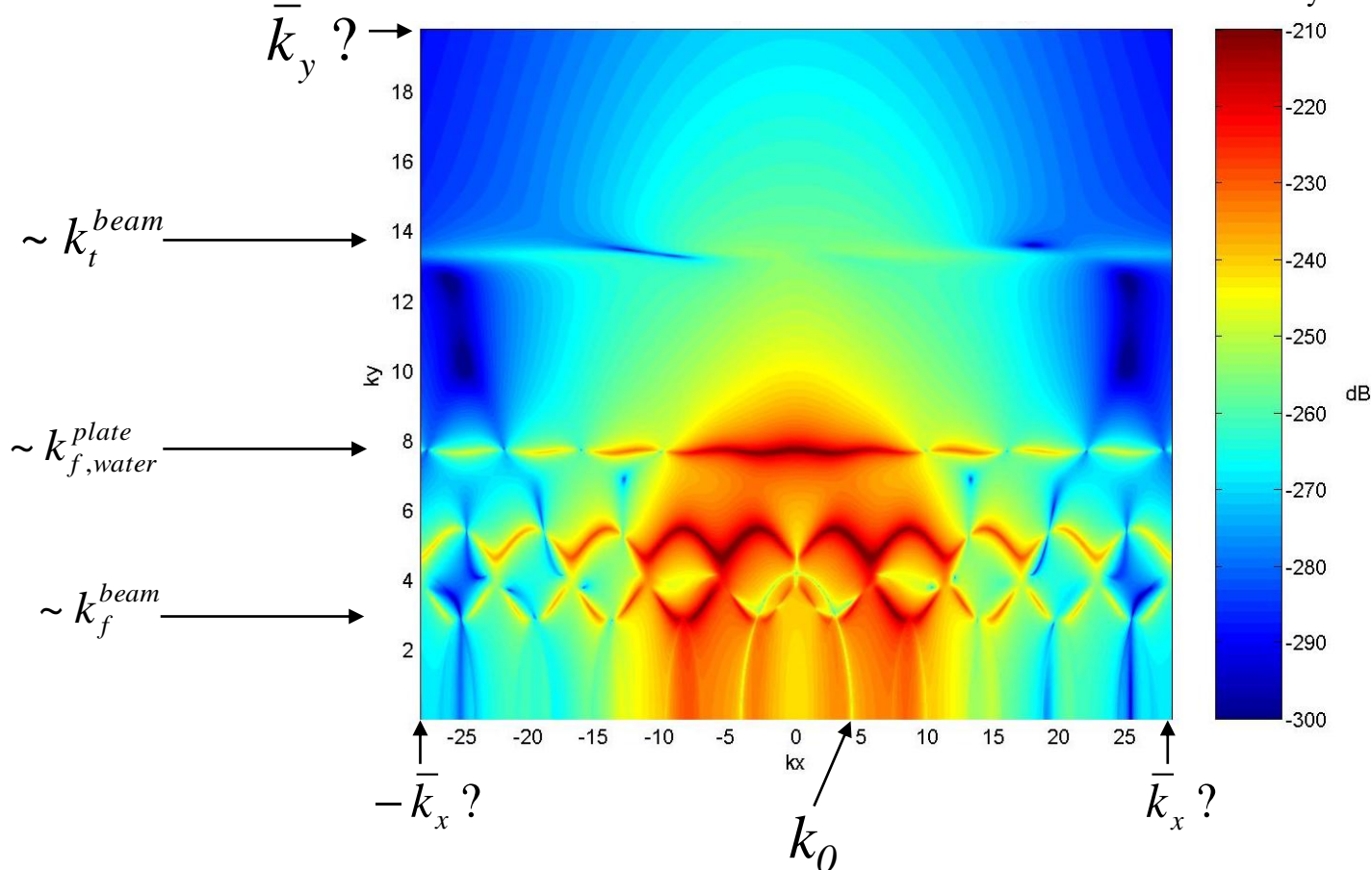
Rib effect:

$$\Gamma = \frac{A_p Z_p S_1 (S_1 - \hat{S}_1) - (1 + A_p S_2) [1 + Z_p (S_0 - \hat{S}_0)] - k_x A_p [Z_p (\hat{S}_0 S_1 - S_0 \hat{S}_1) - \hat{S}_1]}{A_p Z_p S_1^2 - (1 + A_p S_2)(1 + Z_p S_0)}$$

DEFINITION OF CRITERIONS

1 - CRITERION ON THE MAXIMUM WAVENUMBERS

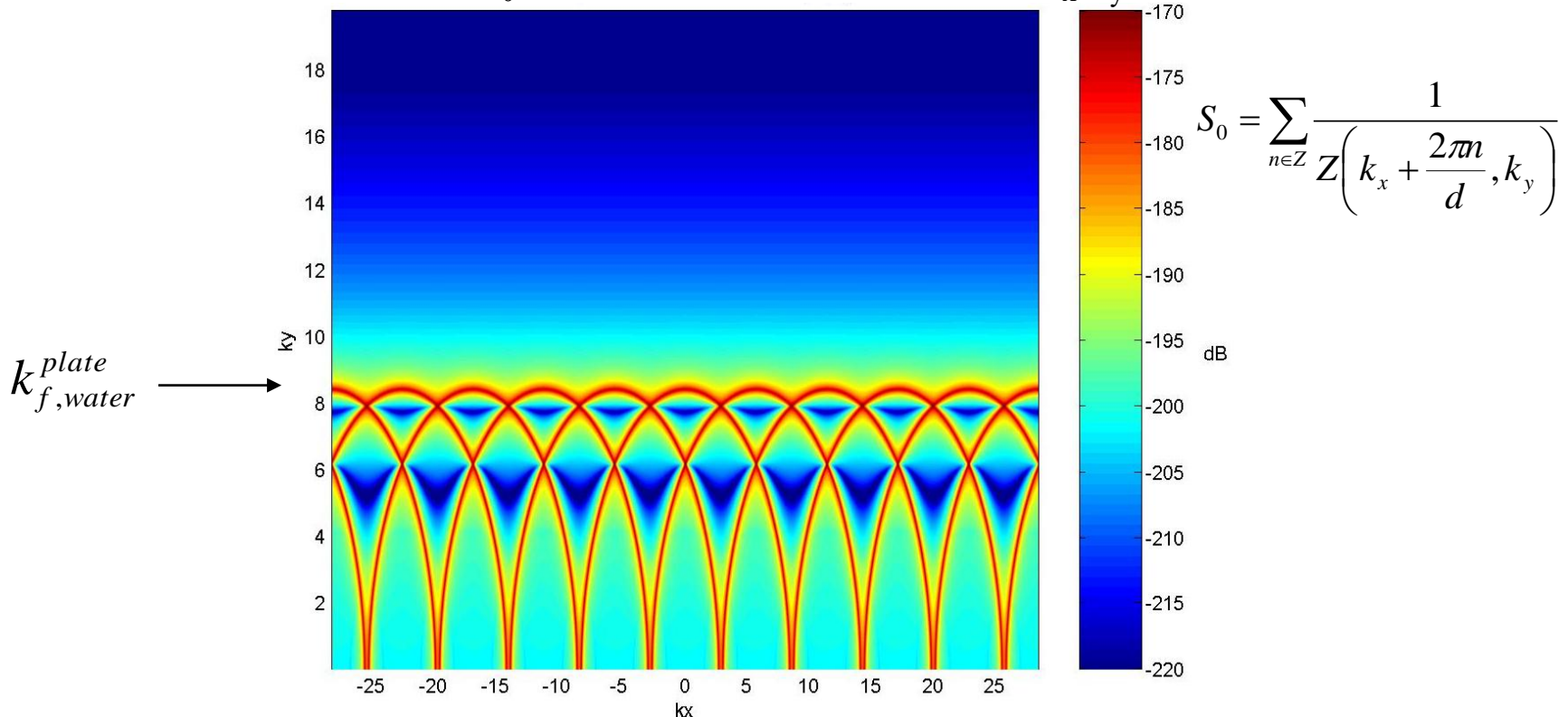
Level of the displacement in the wavenumber domain (k_x, k_y)



DEFINITION OF CRITERIONS

1 - CRITERION ON THE MAXIMUM WAVENUMBERS

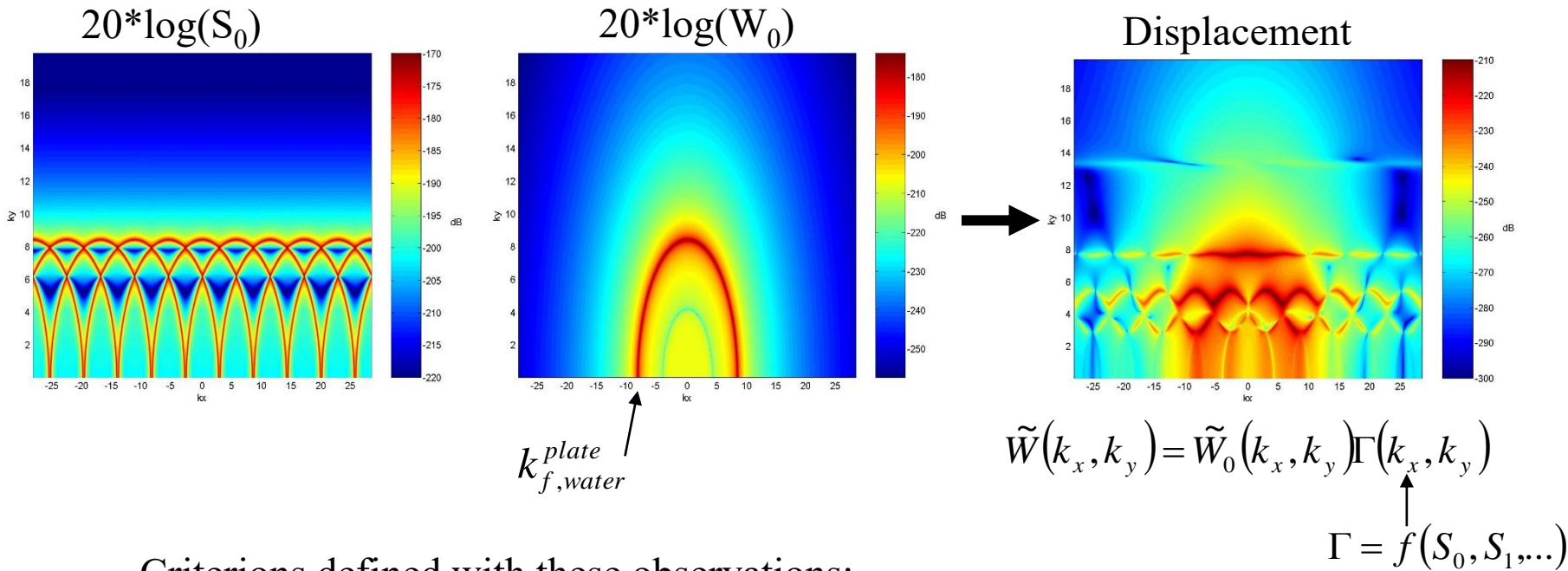
Term $20 \cdot \log(S_0)$ in the wavenumber domain (k_x, k_y)



Maximums for:
$$\sqrt{\left(k_x + \frac{2\pi n}{d}\right)^2 + k_y^2} \approx k_{f,water}^{plate}$$

DEFINITION OF CRITERIONS

1 - CRITERION ON THE MAXIMUM WAVENUMBERS



Criteria defined with these observations:

$$\bar{k}_x = \kappa_x \max \left[k_0, k_{f,water}^{plate} + \frac{2\pi\chi}{d} \right] \quad \bar{k}_y = \kappa_y \max \left[k_0, k_{f,water}^{plate}, k_f^{beam}, k_t^{beam} \right]$$

with $\kappa_x = 1,5 \quad \kappa_y = 1,5 \quad \chi = 2$

DEFINITION OF CRITERIONS

2 - DISCRETE SPACE CRITERION $\delta k_x, \delta k_y$

Half power bandwidths for a 4th order admittance
(plate/beam flexural motion)

Damping \swarrow \searrow Natural wavenumber

$$\Delta_{-3dB}^{(4)} = \frac{\eta k}{2}$$

Half power bandwidths for a 2nd order admittance
(beam torsion motion)

$$\Delta_{-3dB}^{(2)} = \eta \bar{k}$$

Discretisation space criterion:

$$\delta k_x = \frac{1}{K} \Delta_{-3dB}^{(4), plate} \quad \delta k_y = \frac{1}{K} \min \left(\left[\Delta_{-3dB}^{(2), beam}, \Delta_{-3dB}^{(4), plate}, \Delta_{-3dB}^{(4), beam} \right] \right)$$

with K greater than 3.

Only valid for the displacements and the pressure in the near field ($k_0 z \ll 1$)

$$\tilde{p}_0(k_x, k_y, z) = -\frac{\rho_0 \omega^2 F_0 e^{-j k_x x_0} e^{-\sqrt{k_x^2 + k_y^2 - k_0^2} z}}{Z(k_x, k_y) \sqrt{k_x^2 + k_y^2 - k_0^2}}$$

DEFINITION OF CRITERIONS

3 - ALIASING

Discrete Fourier transform → Aliasing phenomena

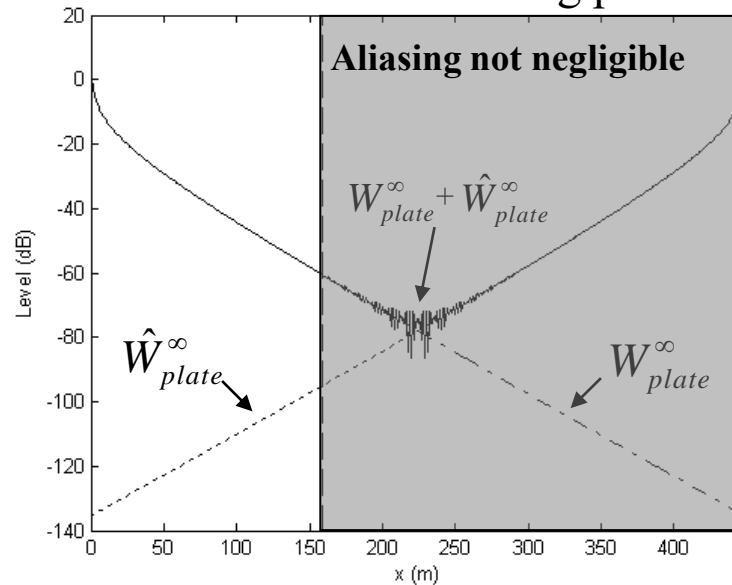


Illustration of the superposition of an original field W_{plate}^{∞}
with its aliasing \hat{W}_{plate}^{∞}

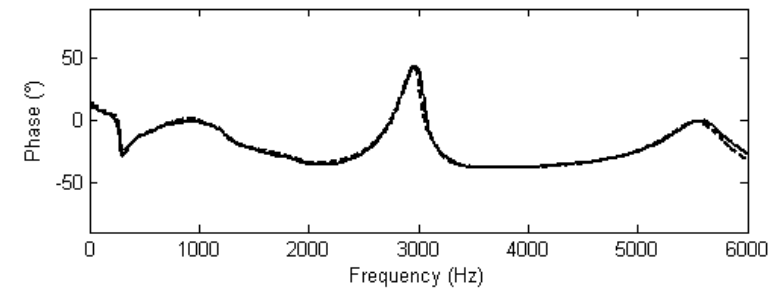
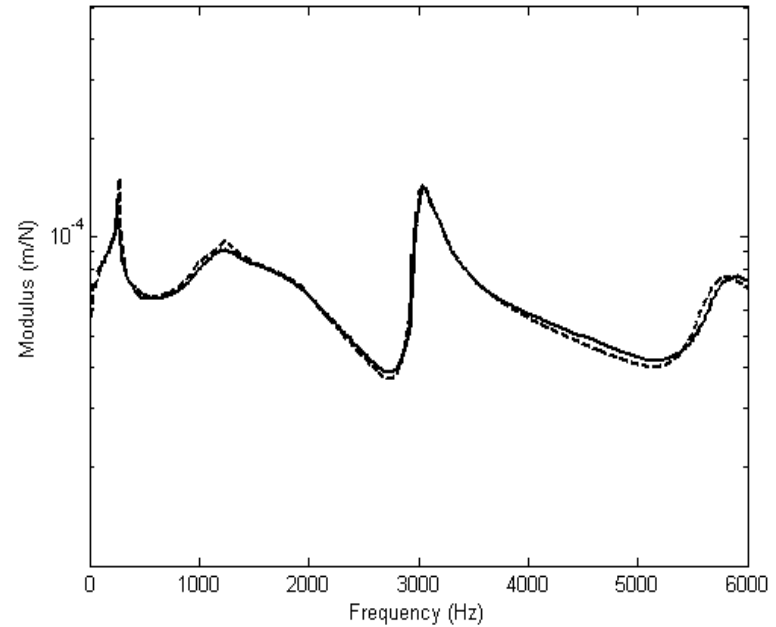
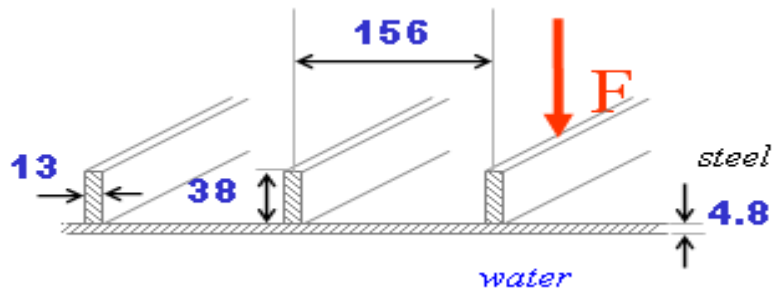
Asymptotic formulations of the spatial decrease of the displacement field for an infinite plate and an infinite beam (flexural, torsion)

→ Definition of criterions describing where the aliasing may be neglected

NUMERICAL VALIDATION

Test case in the literature:

Mace B.R., JSV n°73 (4), p.
487-504, 1980

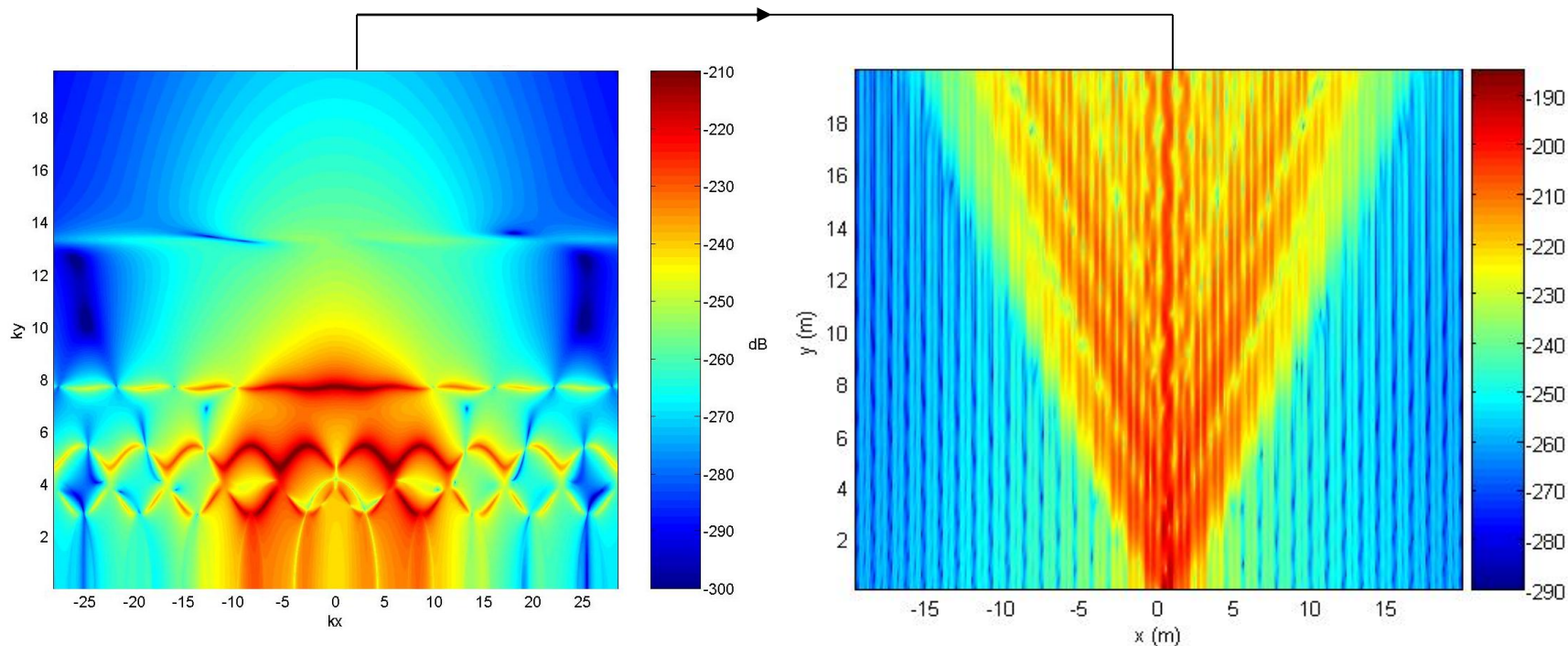


Input mobility versus frequency

Solid, results of Mace, dash, present approach.

EXAMPLES OF RESULTS

FFT⁻¹ (~30 seconds)



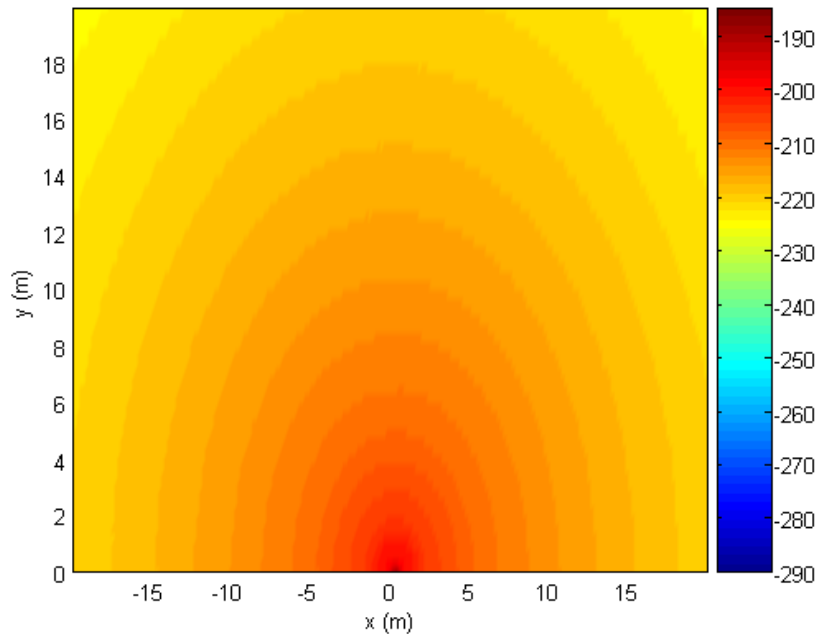
(a) in the wavenumber space (k_x, k_y)

(b) in the physical space (x, y)

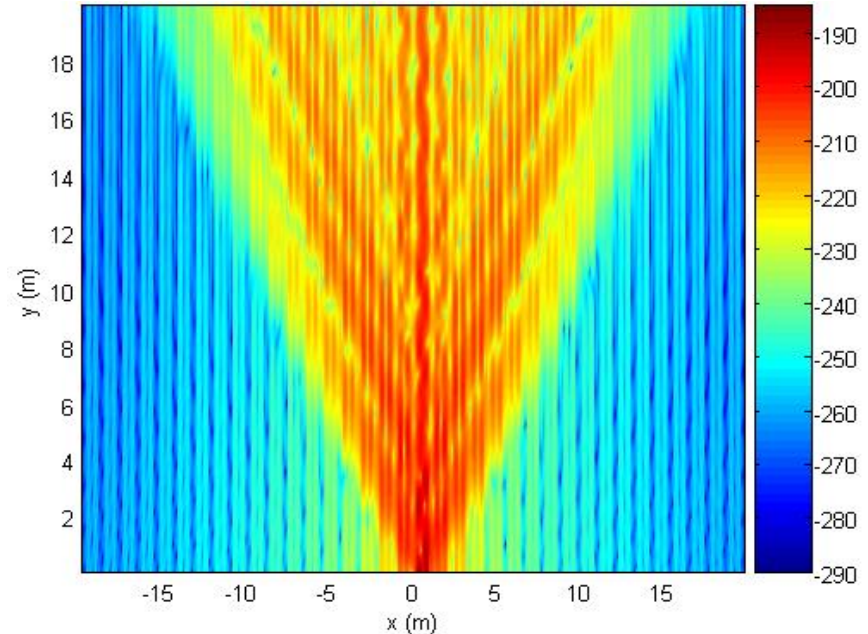
Level of the displacement at 1000 Hz – $x_0=0.3$ m
Stiffened plate (naval test case)

EXAMPLES OF RESULTS

Comparison with and without stiffeners



(a) Un-stiffened plate



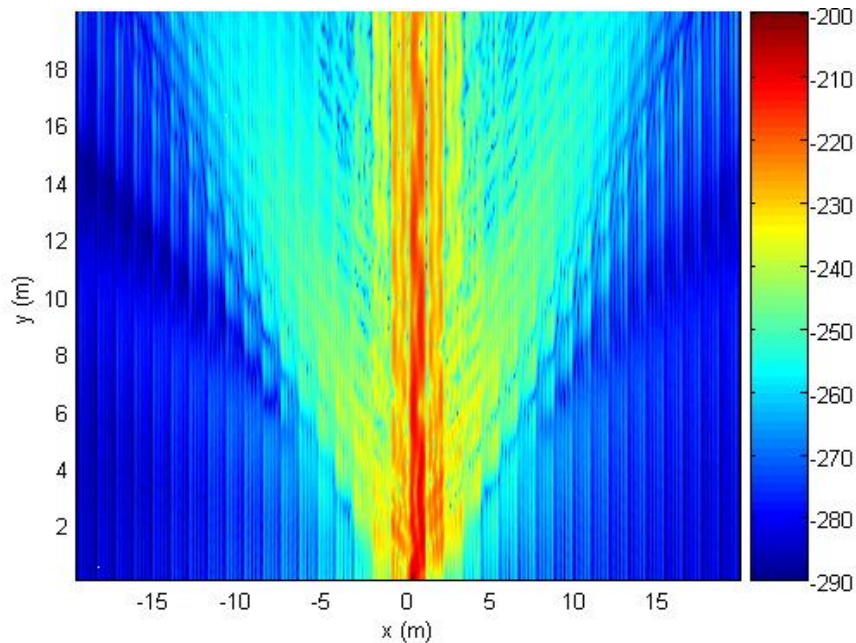
(b) Stiffened plate

Level of the displacement at 1000 Hz – $x_0=0.3\text{m}$

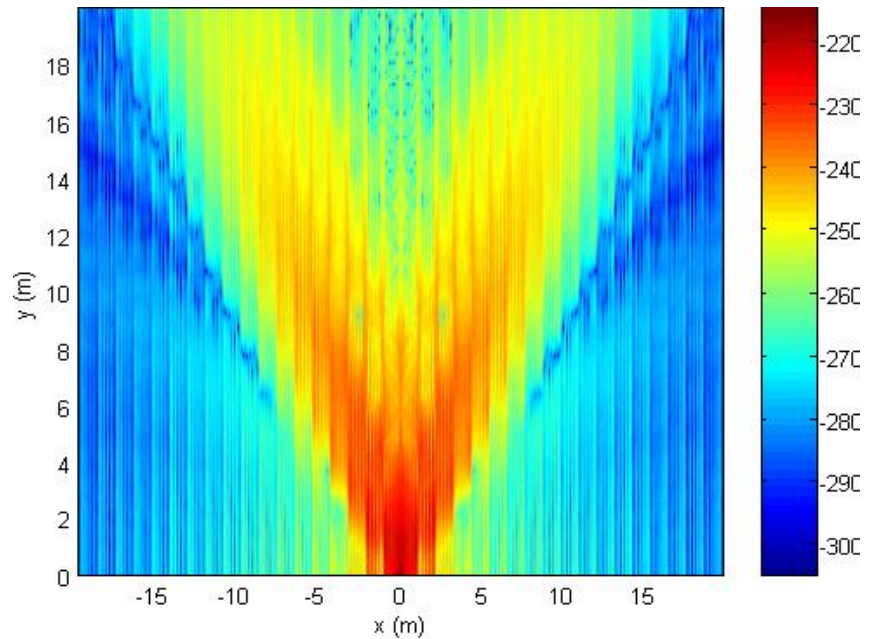
Results in the physical space (x,y)

EXAMPLES OF RESULTS

Comparison with different excitation points



(a) $x_0=0.3\text{m}$ (excitation between 2 ribs)



(b) $x_0=0\text{m}$ (excitation on a rib)

Level of the displacement at 3000 Hz – $x_0=0.3\text{m}$

Results in the physical space (x,y)

EXAMPLES OF RESULTS

Plate accelerations in the wavenumber space (k_x, k_y) -400 Hz - dB ref. $1 \mu\text{m/s}^2$

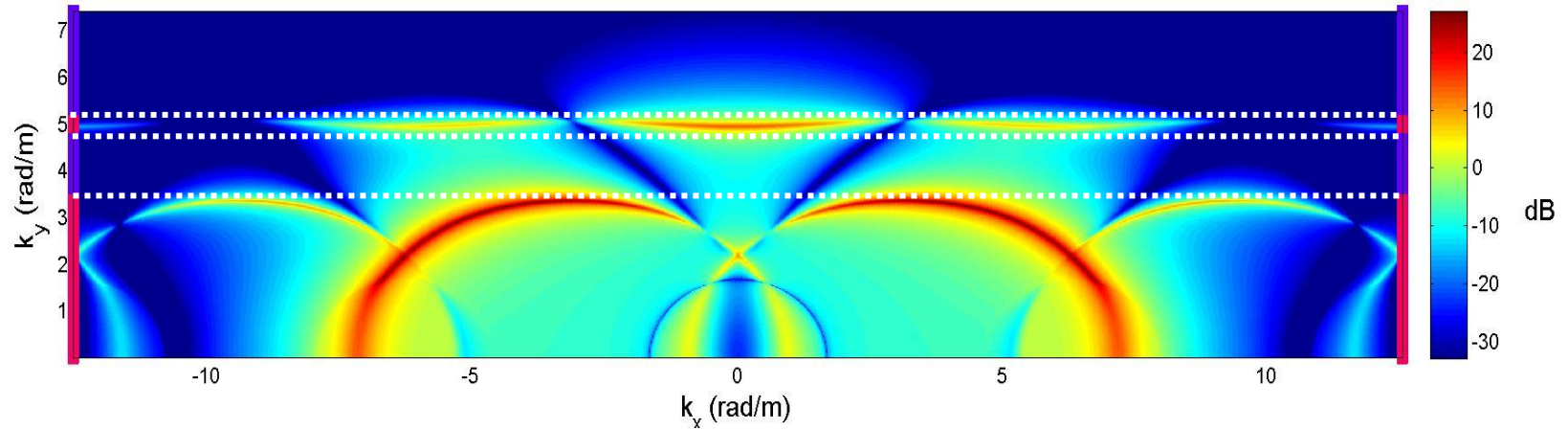


Plate accelerations in the wavenumber space (k_x, k_y) -400 Hz - dB ref. $1 \mu\text{m/s}^2$

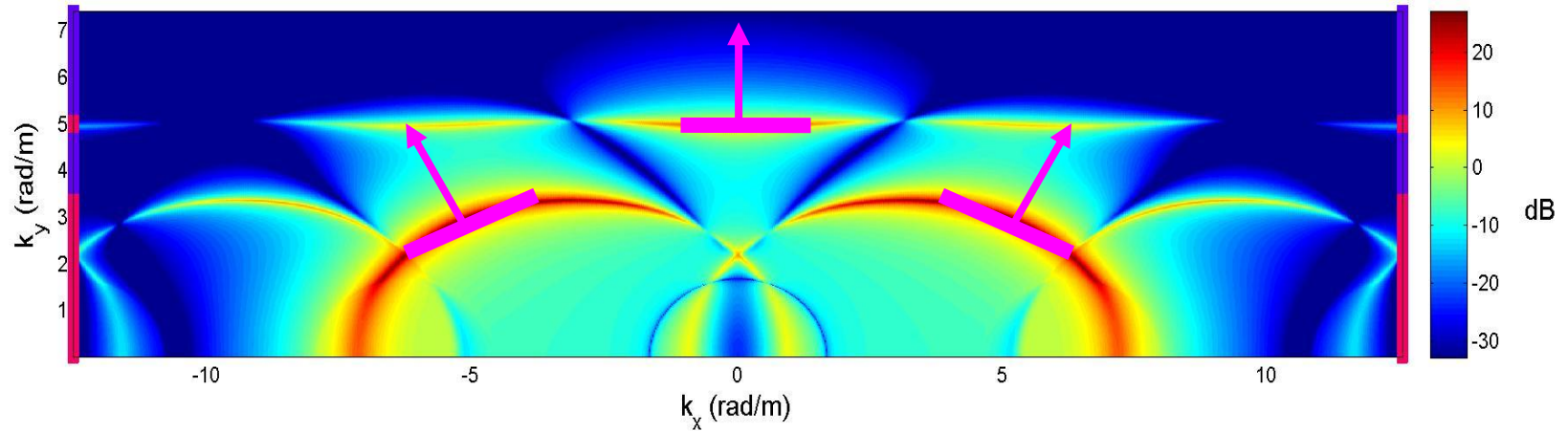


Plate accelerations in the physical space (x,y) -400 Hz - dB ref. $1 \mu\text{m/s}^2$

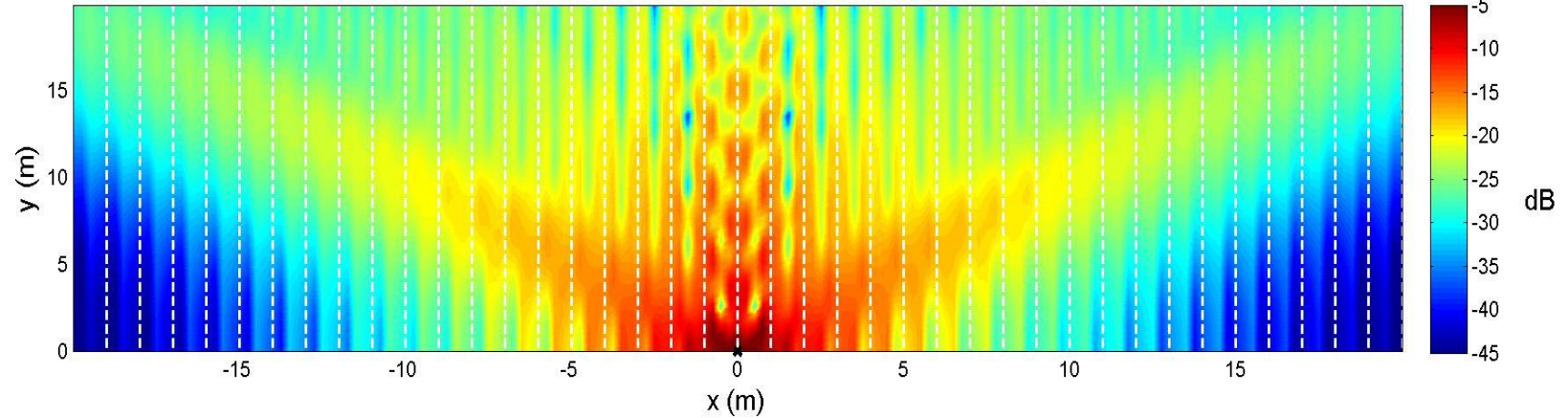
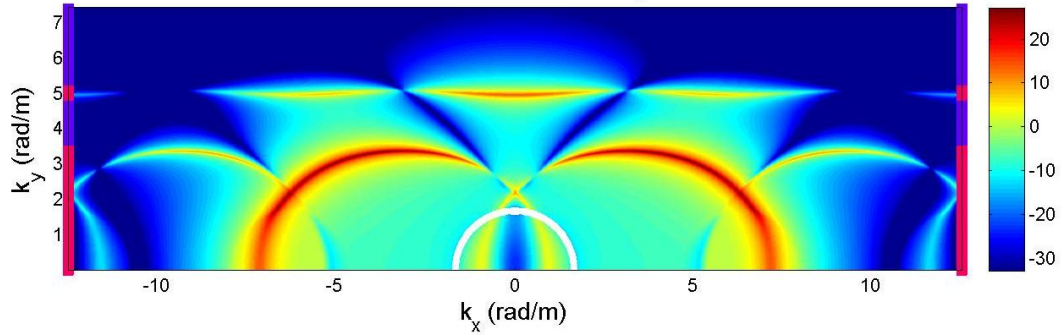
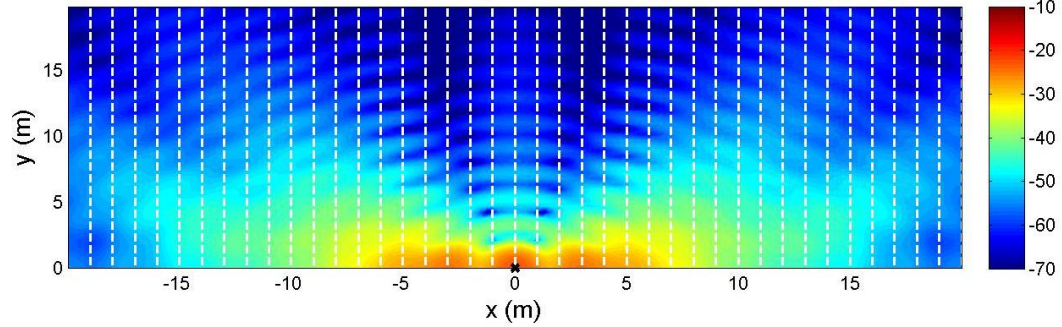


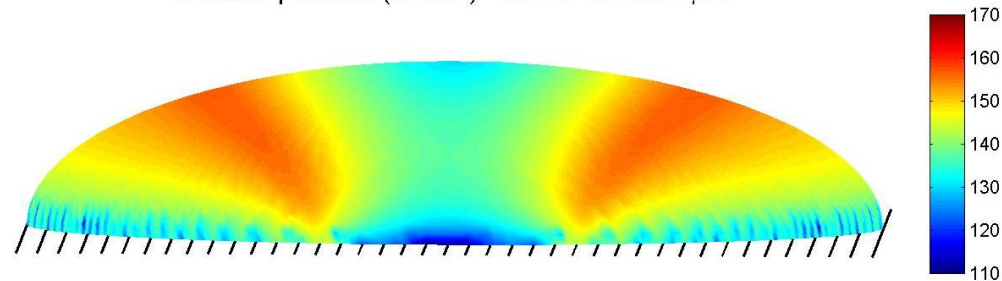
Plate accelerations in the wavenumber space (k_x, k_y) -400 Hz - dB ref. $1 \mu\text{m/s}^{-2}$



Radiated plate accelerations in the physical space (x,y) -400 Hz - dB ref. $1 \mu\text{m/s}^{-2}$

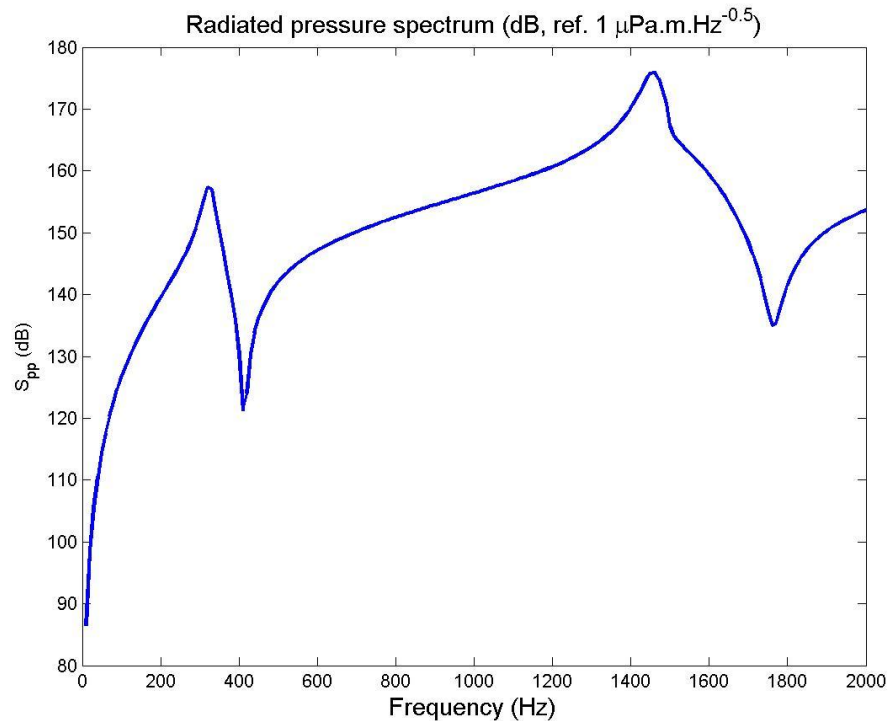


Radiated pressure (R=20m) -400 Hz - dB ref. $1 \mu\text{Pa}$

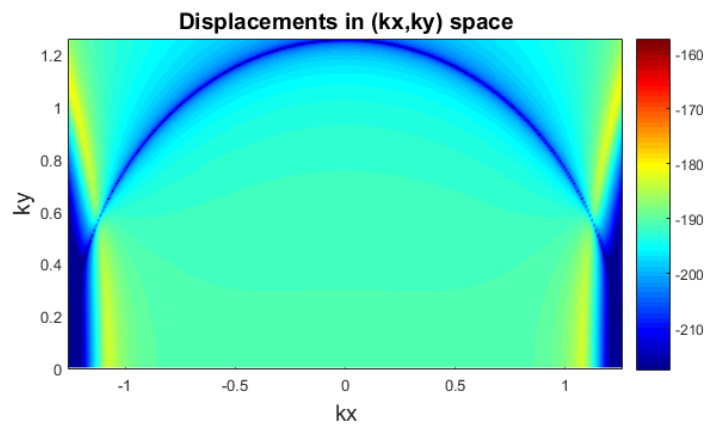


EXAMPLES OF RESULTS

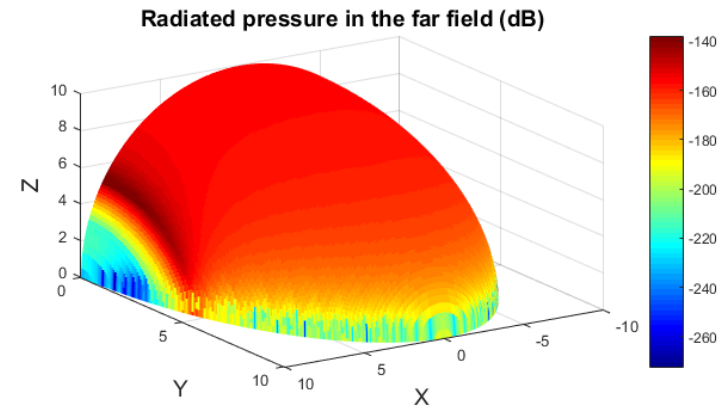
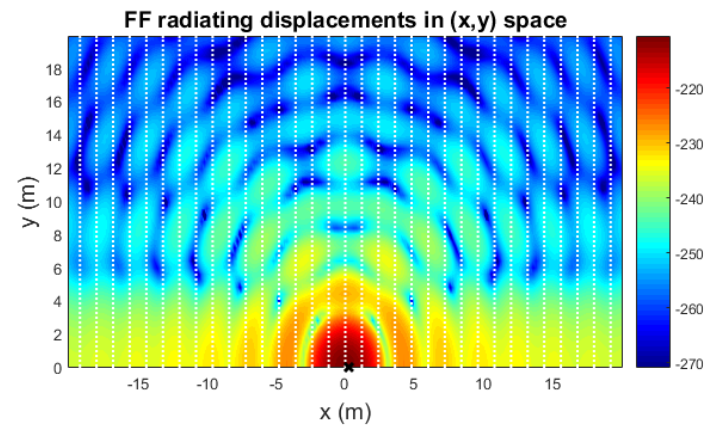
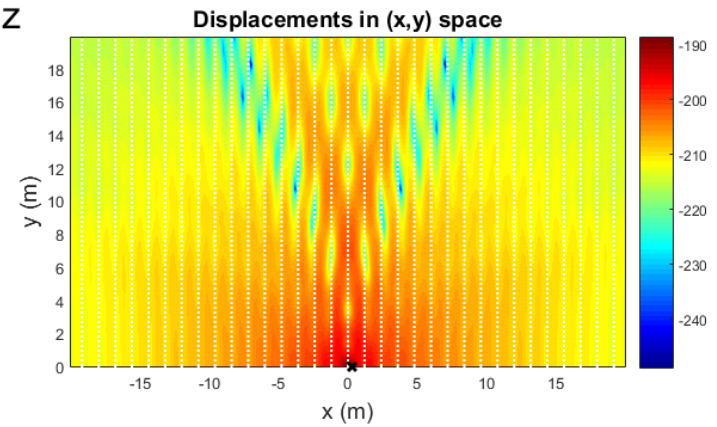
Sound radiated by a stiffened plate

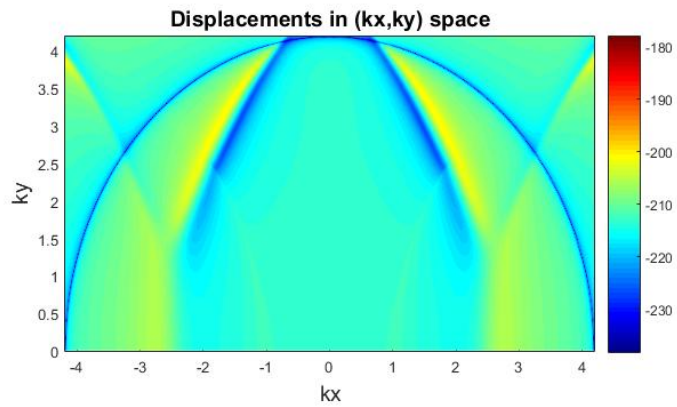


Spectrum of the radiated pressure in the normal direction to the plate ($R=20\text{m}$)



300 Hz





1000 Hz

