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Using Tensor Analysis for EMC

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Using Tensor Analysis of Networks for EMC

Olivier MAURICE

March 17, 2026

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Introduction

The purpose of this MOOC is to submit a course for using tensor analysis of networks (TAN) for EMC. The idea is to construct a system and to show the technique through examples, presenting the theory as long as we construct the system, but with some theoretical preliminaries sometimes. Hoping this approach give you an easiest way for discovering TAN.

For readers who may want more information and deeper demonstrations please refer to *Maurice, O. (2022). Miscela de physique, systématique et électronique. AFSCET editor. BOOKELIS library.*

Olivier MAURICE

A simple circuit

To begin we consider a simple RC circuit. Figure 1 shows the setup and figure 2 the corresponding schematic.

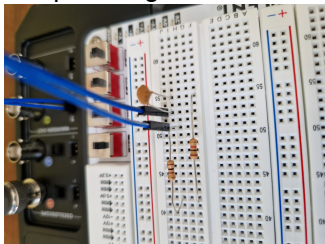


Figure: 1: Setup 1

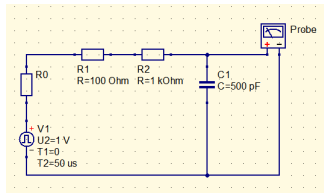


Figure: 2: Schematic
(QUCS@)

(for experiments, the platform used is *Digilent, Analog Discovery Studio*).

A simple circuit

To make this circuit we use a generator, 2 resistances, 1 capacitor and a probe. Each of these elements can be modeled by a Kirchhoff's edge including a source of potential energy and an impedance as self property. So we can create an edge space \mathbb{B} as $\mathbb{B} = \{b_1, b_2, \dots, b_5\}$. This set is the one you make when you place your components on the table, before to solder them. Each of these edge is made of two nodes. The border of an edge is a couple of nodes, for example: $\partial b_1 = (n_1, n_2)$. The set of nodes is \mathbb{Q} .

A simple circuit

Before to go further, we need to briefly recall the tensor algebra. First when two indices are repeated, it means that a summation is realized on this index omitting the sum symbol:

$$\sum_{i=1}^3 A_i B^i \Leftrightarrow A_i B^i = A_1 b^1 + A_2 B^2 + A_3 B^3 \quad (1)$$

The vectors, for example the edge currents vector \vec{I} has components with index up: $\vec{I} = I^a \vec{b}_a$. This vector has a dual, called a covector with index down. The dual of current is voltages: $\overleftarrow{V} = V_a \overleftarrow{b}^a$. The product of \mathbb{B} base and dual base is $\vec{b}_a \overleftarrow{b}^a = \delta_a^a$.

A simple circuit

The dual base makes a correspondance between a vector and a number (like a compass between its needle and the degrees). A scalar is a number. A vector (or covector) can be projected on a base. A tensor of order 2 can be projected on a tensorial product of bases (for example \vec{b}_u and \vec{c}_v which explores all the combinations $(\vec{b}_1, \vec{c}_1), (\vec{b}_1, \vec{c}_2), \dots, (\vec{b}_n, \vec{c}_n)$). When we inverse a tensor, index down goes up and reciprocally, and the order of the indices is reversed. A scalar is a tensor of order 0, a vector (or covector) of order 1, etc.

A simple circuit

The only way for transforming a vector in covector or a covector in vector is to use a metric: a space metric g , an impedance metric ζ or an admittance metric y .

$$x_u = g_{uv}x^v, \quad v_\alpha = \zeta_{\alpha\beta}\Omega^\beta \Leftrightarrow \Omega^\beta = y^{\beta\alpha}v_\alpha \quad (2)$$

The power P which is a scalar is an invariant: it does not depend of the graph used for a circuit. $\zeta_{\alpha\beta}$ is a tensor because it does not depend on the base $\vec{\Omega}_\sigma$ chosen for modeling the circuit.

A simple circuit

To construct our circuit we can either connect nodes or connect edges. The graph \mathcal{G} of the circuit is shown figure 3.

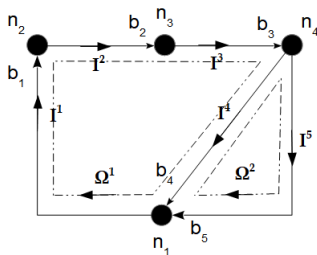


Figure: 3: Graph setup 1

A simple circuit

To connect edge we may use the connectivity between edges and nodes (often called adjacency) A . This leads to the "nodal method" easy to program (that's more or less the method used by SPICE-like softwares). But we prefer another connectivity, difficult to program but powerful for theoretical analysis: the edges to cycles connectivity Λ . We choose two cycles Ω^1 and Ω^2 in \mathbb{M} , the edges being oriented. Λ allows to write the connection $I^\alpha = \Lambda_\beta^\alpha \Omega^\beta$. When I^u and Ω^v have the same direction, $\Lambda_v^u = +1$. If they are no relations between them $\Lambda_v^u = 0$ and if the directions are opposed $\Lambda_v^u = -1$.

A simple circuit

For our example this leads to:

$$\Lambda = \begin{bmatrix} 1 & 0 \\ 1 & 0 \\ 1 & 0 \\ 1 & -1 \\ 0 & 1 \end{bmatrix} \quad (3)$$

A simple circuit

With the set \mathbb{B} and the graph \mathcal{G} we can define a vectorial space and its base \vec{b}_α . The scalar product between two base vector defines a **metric**:

$$g_{uv} = \left\langle \vec{b}_u, \vec{b}_v \right\rangle \quad (4)$$

With s : Laplace's operator, g and the tensor properties of matter ρ_{uv} , ϵ^{uv} and the edge volume ν , an edge property called an impedance can be defined by:

$$R_{uu} = \rho_{uu} \frac{g_{uu} |^u|^u}{\nu}, \quad \frac{1}{sC^{uu}} = \frac{g_{uu} |^u|^u}{s\epsilon^{uu}\nu} \quad (5)$$

A simple circuit

Let's consider Kirchhoff's equation for an edge (E electromotive force, z impedance, V potentials difference):

$$E_{\alpha} = z_{\alpha\beta} I^{\beta} + V_{\alpha} \quad (6)$$

Using the connection $I^{\alpha} = \Lambda_{\sigma}^{\alpha} \Omega^{\sigma}$ we obtain:

$$E_{\alpha} = z_{\alpha\beta} \Lambda_{\sigma}^{\beta} \Omega^{\sigma} + V_{\alpha} \quad (7)$$

Let's multiply all members on the left by $\Lambda_{\gamma}^{\alpha}$:

$$\Lambda_{\gamma}^{\alpha} E_{\alpha} = \Lambda_{\gamma}^{\alpha} z_{\alpha\beta} \Lambda_{\sigma}^{\beta} \Omega^{\sigma} + \Lambda_{\gamma}^{\alpha} V_{\alpha} \quad (8)$$

A simple circuit

but

$$\Lambda_\gamma^\alpha V_\alpha \equiv \oint_C d\mathbf{c} \cdot \mathbf{V} = 0$$

(cycles law and not the cycles space!) With $\Lambda_\gamma^\alpha z_{\alpha\beta} \Lambda_\sigma^\beta = \zeta_{\gamma\sigma}$ and $\Lambda_\gamma^\alpha E_\alpha = T_\gamma$ it remains:

$$T_\gamma = \zeta_{\gamma\sigma} \Omega^\sigma \quad (9)$$

In this equation, it lacks something: the electromagnetic inertia, or in other words, the inductances \mathcal{L} . Remember the three fundamental relations:

$$\int_t dt I^k(t) = C^{ku} V_u, \quad U_\sigma = R_{\sigma\sigma} I^\sigma, \quad \Phi_\alpha = \mathcal{L}_{\alpha\beta} \Omega^\beta \quad (10)$$

A simple circuit

The two first belong to \mathbb{B} while $\Omega \in \mathbb{M}$ and $\Phi \in \mathbb{F}$: the faces space. Nodal method and all equations written in \mathbb{B} CANNOT integrate inductances rigorously. $\partial\Phi = \Omega$ and $\Phi = \dot{e}$, $e \in \mathbb{M}$. In equation (9), $\overleftarrow{\zeta}$ is the impedance metric in \mathbb{M} , \overleftarrow{T} the sources in \mathbb{M} and $\overrightarrow{\Omega}$ the cycles currents. Finally, the circuit equation in \mathbb{M} becomes:

$$T_\gamma = \zeta_{\gamma\sigma}\Omega^\sigma + s\mathcal{L}_{\gamma\sigma}\Omega^\sigma \quad (11)$$

That's the relation we will use to solve all EMC problems. Spaces \mathbb{Q} , \mathbb{B} , \mathbb{M} can be complete with the circuit space (or connected graphs, referential space) \mathcal{R} and they obey Euler-Poincaré's relation:

$$\dim(\mathbb{B}) - \dim(\mathbb{Q}) + \dim(\mathcal{R}) = \dim(\mathbb{M})$$

A simple circuit

Let's solve our first circuit. The last relation gives $5 - 4 + 1 = 2$: 2 cycles, ok! We obtain for ζ :

$$\zeta = \begin{bmatrix} R_0 + R_1 + R_2 + \frac{1}{sC_1} & -\frac{1}{sC_1} \\ -\frac{1}{sC_1} & \frac{1}{sC_1} + Z_p \end{bmatrix} \quad (12)$$

R_0 is the generator self resistance and Z_p the probe self impedance. $T = [V_1, 0]$. Equations (11) are Lagrange's equations and we have to solve in Laplace formalism, neglecting $\mathcal{L}_{\mu\nu}$ (try to find the system before to read it):

A simple circuit

$$T_\gamma = \zeta_{\gamma\sigma} \Omega^\sigma \equiv \begin{cases} V_1(s) = (R_0 + R_1 + R_2 + \frac{1}{sC_1}) \Omega^1 - \frac{1}{sC_1} \Omega^2 \\ 0 = -\frac{1}{sC_1} \Omega^1 + (\frac{1}{sC_1} + Z_p) \Omega^2 \end{cases} \quad (13)$$

In ζ each diagonal component represents the own impedance of each cycle and the extradiagonal components represent the coupling between cycles. Note the minus sign link with the fact that Ω^1 and Ω^2 are of opposite direction in b_4 . It's very important to have a good understanding of how work these coupling components.

A simple circuit

1. If we suppress ζ_{21} and ζ_{12} , there are no couplings between the two cycles.
2. If we suppress ζ_{12} we neglect the influence of the probe on the RC circuit.
3. If we suppress ζ_{21} we neglect the influence of the circuit to the probe.

Let's consider the case 1. From (13) as $1/sC \neq 0$ and $Z_p \neq 0 \Rightarrow \Omega^2 = 0$. It remains with $R_0 + R_1 + R_2 = R$:

$$V_1(s) = \left(R + \frac{1}{sC_1} \right) \Omega^1 \quad (14)$$

A simple circuit

RLC components can be defined using their geometrical properties and the space properties, says:

$$R = \rho \left(\frac{X}{\Phi} \right), \quad C = \epsilon \left(\frac{\Phi}{Y} \right), \quad L = \mu \left(\frac{\Phi}{Z} \right), \quad M = \frac{\mu}{4\pi} \iint_{C_1, C_2} \frac{d\mathbf{c}_1 \cdot d\mathbf{c}_2}{d(1,2)} \quad (15)$$

Depending on the temperature T we have:

$$\rho = \rho(1 + \alpha_\rho T), \quad \epsilon = \epsilon(1 + \alpha_\epsilon T), \quad \mu = \mu(1 + \alpha_\mu T) \quad (16)$$

A simple circuit

All sources and circuits can be solved knowing the step response $V_1(s) = V_1/s$, as any signal can be developed like:

$$x(t) = \sum_{\tau} \frac{dx(t)}{dt} \Gamma(t) \Delta_{n\tau} \quad (17)$$

Here we take $V_1(s) = V_1/s$. If $u(s)$ is the voltage across $C = C_1$ we have:

$$u(s) = \frac{1}{sC} \Omega^1 = \frac{V_1}{s} \left[RC \left(\frac{1}{RC} + s \right) \right]^{-1} \quad (18)$$

A simple circuit

Going back to the time domain this gives:

$$u(t) = \int_t dt \frac{V_1}{RC} \exp\left(-\frac{t}{RC}\right) = -V_1 \left[\exp\left(-\frac{t}{RC}\right) \right]_0^t + K_0 \quad (19)$$

K_0 is an integration constant. As for $t = 0$, $u(t) = 0 \Rightarrow K_0 = 0$, and

$$u(t) = V_1 \left(1 - e^{-t/RC}\right) \quad (20)$$

Now to compute the response to the initial $V_1(t)$, we must remember the **initial conditions** in voltage and currents. Accross capacitors, the voltage remaining at the end of Γ is U_0/s . Accross an inductance, the emf stored at the end of Γ is $sLl_0/s = Ll_0$. For this time,

$$u(T) = V_1 \left(1 - e^{-T/RC}\right) \quad (21)$$

A simple circuit

For example, if we want to compute the response of our first circuit to a pulse $T_1(t) = V_1 \Pi_{[0, T]} \delta_0$, it means that the source is equal to 0 at time T . To model this source with step functions we write:

$$T_1(t) = V_1 (\Gamma(t = 0) - \Gamma(t = T)) \sqsupset \frac{V_1}{s} (1 - e^{-sT}) \quad (22)$$

In response to the first step we have $u(t) = V_1 (1 - e^{-t/RC})$. Then at $t = T$ we have the response to the second step with an initial condition $u(T)$, obtain by:

$$u(t) = \overset{t}{\mathcal{D}}_{[0, T]} V_1 (1 - e^{-t/RC}) + \overset{t}{\mathcal{D}}_{[T, \infty[} [u(T) - V_1 (1 - e^{-(t-T)/RC})] \quad (23)$$

$$\overset{t}{\mathcal{D}}_{[i, j]} = 1 \text{ if } t \in [i, j], 0 \text{ elsewhere}$$

A simple circuit

Comparison measurement versus computation. R_0 is unknown but we suppose $R_0 < R1$.



Figure: Measurement of $u(t)$

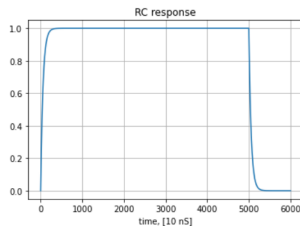


Figure: Computation of $u(t)$

Both curves follow the same time profile.

A simple circuit

The computation program is:

```
10 import numpy as np
11 import pylab as plt
12 #
13
14 dt=1E-8
15 T=50E-6
16 N=int(T/dt)+1000
17 V1=1.
18 R=1.1E3
19 C1=500E-12
20 # fonction échelon
21 u=np.zeros(N,dtype=complex)
22 D=0.
23 for n in range(1,N):
24     t=n*dt
25     if(t<=T):
26         u[n]=V1*(1-np.exp(-t/(R*C1)))
27     if(t>T):
28         u[n]=u[int(T/dt)]-V1*(1-np.exp(-(t-T)/(R*C1)))
29 plt.plot(u)
30 plt.grid(True)
31 plt.title('RC response')
32 plt.xlabel('time, [10 nS]')
33 plt.show()
34
```

Figure: Python listing

A simple circuit

To evaluate the uncertainties of our result depending on R_0 we can use a simple but efficient method: replacing R by $R \pm \delta R$ gives:

$$u(t) = V_1 \left(1 - e^{-t/(R \pm \delta R)C} \right) = V_1 \left(1 - e^{-t(R \mp \delta R)/(R^2 C)} \right), \quad \delta R^2 \approx 0 \quad (24)$$

with $e^\epsilon \approx 1 + \epsilon$ this leads to

$$u \mp \delta u = V_1 \left(1 - e^{-t/RC} \right) \mp V_1 \left(\frac{\delta R t}{R^2 C} \right) e^{-t/RC} \quad (25)$$

A simple circuit

Conclusion for the "simple circuit": what to remember.

1. The graph extract from the circuit schematic defines the circuit topology and the nodes \mathbb{Q} , edges \mathbb{B} and cycles \mathbb{M} sets.
2. each circuit has its own referential \mathcal{R} and
$$\dim(\mathbb{M}) = \dim(\mathbb{B}) - \dim(\mathbb{Q}) + \dim(\mathcal{R}).$$
3. ζ is an impedance metric, its diagonal components are the self impedance of each cycle and its extra-diagonal components are the coupling between them.
4. The tensor system equation in \mathbb{M} is $T_\alpha = \zeta_{\alpha\beta}\Omega^\beta + s\mathcal{L}_{\alpha\beta}\Omega^\beta.$
5. The power P is invariant whatever the circuit representation:
$$2P = \zeta_{uv}\Omega^u\Omega^{*v}.$$

Strings & Homologies

A **string** ζ_{ab} is a functor f between two referential $f : \mathcal{R} \rightarrow \mathcal{R}'$.
Simple strings are obtained as transfer function defined by:

$$\zeta_{ab} = \frac{e_a}{\Omega b} \quad (26)$$

Let's look this new graph:

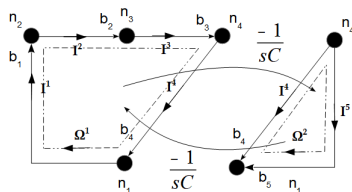


Figure: New graph

Strings & Homologies

The impedance metric of this new graph is:

$$\zeta = \begin{bmatrix} R_0 + R_1 + R_2 + \frac{1}{sC_1} & -\frac{1}{sC_1} \\ -\frac{1}{sC_1} & \frac{1}{sC_1} + Z_p \end{bmatrix} \quad (27)$$

It is completely identical to the one of the previous graph. Both graphs have the same impedance metric but not the same topology. The two graphs and the associated circuits are

homologous. In the second graph, the string is

$\zeta_{21} = \zeta_{12} = -(sC_1)^{-1}$. It creates an emf e_2 in the second cycle from the cycle current Ω^1 of the first cycle.

The marvellous fibered space \mathcal{W}

The time axis can be separated into intervals of constant duration, as we did previously with the domains $\mathcal{D}_{[0,T[}^t$ and $\mathcal{D}_{[T,\infty[}^t$. On each interval, the computation is conducted under Laplace's formalism. For one observable O^a or O_a , we can find the peak value of the time signal \hat{O}^a (or \hat{O}_a) over an interval by:

$$\hat{O}^a = \text{Sign} \cdot \sqrt{\langle O^a(s) | \mathcal{F} | O^a(s) \rangle} \equiv \text{Sign} \sqrt{g_{aa} O^{*a} \mathcal{F} O^a} \quad (28)$$

Where *Sign* is a sign function and \mathcal{F} is a functional depending on the interval duration and spectrum width.

The marvellous fibered space \mathcal{W}

For each time interval a value or function $t^\alpha \in \mathbb{R}$ comes from a spectrum calculated using Laplace $s^a \in \mathbb{C}$. The spaces product $\mathbb{C}(s^a) \times \mathbb{R}(t^\alpha) = \mathcal{W}$ creates the fibered space \mathcal{W} .

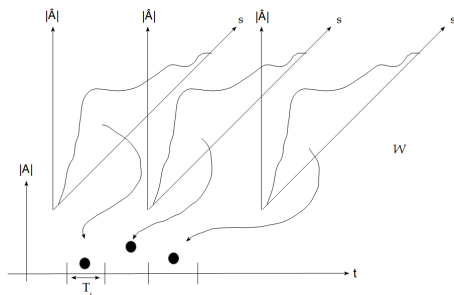


Figure: Fibered Space

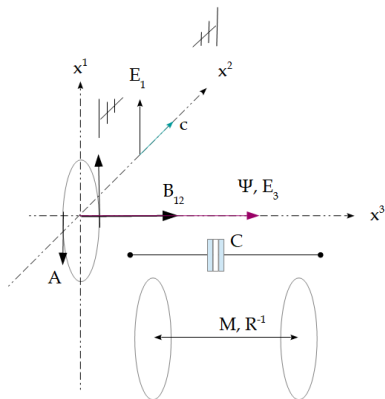
Waveguides

Electronics aside, modeling waveguides give access to all kind of delayed information exchange between circuits. It gives so the way for defining far field interactions.

As a preliminary, let's define the notion of near field: *To put it simply, we consider near interactions as those carried by longitudinal fields and far interactions as those carried by transverse fields, in the Coulomb gauge.*

Capacitive and inductive interactions are carried by longitudinal fields, while radiated field interactions in the sense of distant antennas are carried by transverse fields.

Waveguides

Figure: Various components of A^α

Waveguides

- ▶ **There are no direct relation between the field and the EMC disturbances.**
- ▶ We cannot measure or perceive the field. We can only perceive field-matter interactions.
- ▶ The transverse field is made up of photons that carry electromagnetic energy, while the near field stores electromagnetic energy.
- ▶ The best model for the electromagnetic field is the 4-vector potential $A^\alpha \leftarrow (\Psi, \mathbf{A})$.
- ▶ Three relationships explain it all: the electromagnetic energy W is $qg_{\alpha\sigma}v^\alpha A^\sigma$, the continuity equation $I = -dq/dt$ and the interaction field matter.

Waveguides

Interaction field matter.

Emission

$$d\Psi(r) = \frac{dQ}{4\pi\epsilon r} \quad dA^\alpha(r) = \mu_{\sigma\beta} \frac{I^\sigma(x^\beta, 0) dx^\beta}{4\pi r} \exp\left(-s \frac{r}{c}\right) u_t^\alpha \quad (29)$$

Reception

$$e_\sigma = - \oint_{l_\sigma} g_{\alpha\beta} dx^\alpha \frac{dA^\beta}{dt} \eta_\sigma \quad f^\sigma = \int_{l_\sigma} g_{\alpha\alpha} dx^\alpha \frac{\partial A^\alpha}{\partial x^\gamma} I^\gamma \eta^\sigma \quad (30)$$

$$J^\sigma(y) = -\epsilon \iint_{x,z} \frac{dx dz}{y} \cdot \frac{d\Psi}{dt}$$

Waveguides

How does field emission work?

A wave surface describing the transverse distribution of \mathbf{A} (or I and Ψ) is created and propagates in the direction of the line of sight (figure Transverse field mode).

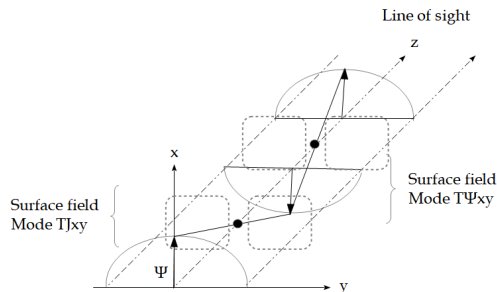


Figure: Transverse field mode

Waveguides

In the case of antennas, the wave surface is a portion of a sphere. TJ current modes are replaced by TA potential modes. However, potentials $F = F(\Psi, \mathbf{A}, \mathbf{J})$ follow a propagation law of the form

$$F(z) = \mathbf{F}_{xy} \alpha(z) \exp\left(-s \frac{z}{v}\right), \quad v = \frac{c}{\sqrt{1 - \left(\frac{\lambda}{\lambda_0}\right)^2}} \quad (31)$$

$\alpha(z)$ is a propagation dispersion function that can be written in general terms as

$$\alpha(z) = \sum_n \frac{a_n}{z^n} \exp(-\sigma z) \quad (32)$$

Waveguides

When should we use a propagation-free model (RLC) or a waveguide model?

We must compare the circuit dimension d with λ :

1. if $d < \lambda$, RLC models should be used.
2. if $d > \lambda$, waveguide models should be used.

Note the reader may rightly criticize the fact that I'm talking about guided waves for free radiation, but we'll see that the equations are similar and this reunion allows us to synthesize the classification of the models, which are already quite numerous!

Waveguides

λ criterion justification Let's consider a wave function $\Psi = C \exp(-ix/\lambda)$ and the hamiltonian $H = -\frac{\hbar}{i} \frac{\partial}{\partial x}$. We look at the probability to find a photon of energy $h\nu$ in a cavity of length X :

$$P[\Psi(x)] = \frac{1}{X} \int_x dx \Psi^* H \Psi = -\frac{1}{X} \int_x dx C^* e^{i\frac{x}{\lambda}} \frac{\hbar}{i} \left(-\frac{i}{\hbar}\right) C e^{-i\frac{x}{\lambda}} \quad (33)$$

So that, knowing that $h/\lambda = p$, the photon impulse:

$$P[\Psi(x)] = \frac{|C|^2}{X} (px) \quad (34)$$

But Heisenberg's uncertainty principle says that $px \geq \hbar/2$ which leads to $px \geq \hbar/2 \Rightarrow x \geq \lambda/2$. For simplifying memorisation we remember $x \geq \lambda$. In conclusion, being able to locate the photon in X , i.e. to see its propagation in X implies $x \geq \lambda$. Q.E.D (quod erat demonstrandum)

Waveguides

How does the waveguide model work? Let's take a look to Bergeron's diagram:

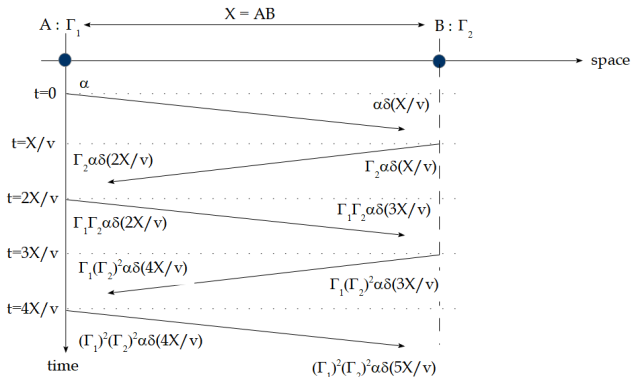


Figure: Bergeron's diagram

Waveguides

The diagram details the propagation of a wave from point A of one circuit to point B of another circuit at speed v . At B , part of the energy is absorbed at time X/v , while another part is reflected and backpropagates from B to A at speed v . Arriving in A at time $2X/v$, part of it is consumed again in A and the other part reflected back to B , and so on.

On each boundary, the reflected wave V_r is defined as a function of the incident wave V_i in Laplace's formalism by the relation $V_r = \Gamma V_i$, where Γ is the reflexion coefficient.

Waveguides

The model attached to the Bergeron diagram works simultaneously on two spaces: edge space \mathbb{B} and wave space \mathbb{O} . We know the first. The second is based on the incident and reflected potentials on the boundaries where the waves are measured.

Let's consider the simple case with two boundaries (i.e. two branches), which are the two boundary conditions between which the waves propagate and where the measurements are taken. The waves propagate in \mathbb{O} and the measurements and stimuli are carried out in \mathbb{B} .

Waveguides

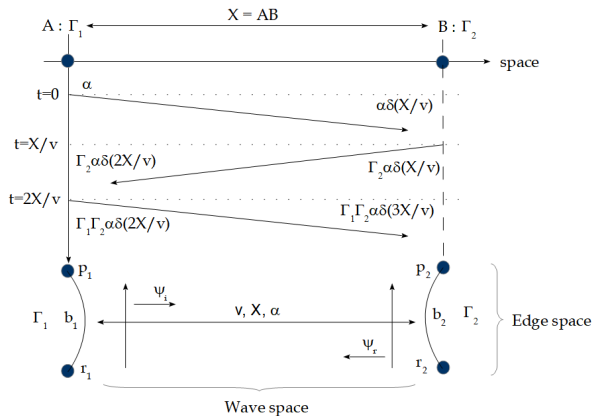


Figure: Wave space

Waveguides

To understand how these two spaces work in a concrete case, let's consider a line of propagation impedance η_c , speed v and length X . It is excited by a generator e_0 with its own impedance z_0 at the input (edge b_1 and ports p_1, r_1) and loaded at the output by the impedance z_L (edge b_2 and ports p_2, r_2). The ports p_i (incident waves) and r_j (reflected waves) belong to \mathbb{O} . In \mathbb{O} we define a wave covector \mathcal{V}_α which components are the potentials on the ports:

$$\mathcal{V}_\alpha = \left[\begin{array}{cccc} p_1 & r_1 & p_2 & r_2 \end{array} \right] \quad (35)$$

Next, we'll create a propagator γ (a matrix), the operation of which we'll understand later.

Waveguides

To fill in γ we create a table in which the rows and columns are the ports of the system under study. In this table we write the links between the ports. These are either reflection Γ_i or transmission t_i coefficients, or propagation functions G_{ij} .

	p1	r1	p2	r2
p1	0	Γ_1	0	0
r1	0	0	0	G_{12}
p2	G_{21}	0	0	0
r2	0	0	Γ_2	0

Figure: Gamma matrix tab

Waveguides

If we look at the box in row 1 column 2: the wave reflected at b_2 returning via r_1 is reflected at b_1 towards p_1 following a Γ_1 weighting. The reasoning goes from the column port to the row port, in this case from r_1 to p_1 .

Now if we look at row 2, column 4: G_{12} , the wave here propagates from port r_2 to port r_1 via the propagation function (Green's function) G_{12} .

The rest of the table is filled in according to the same principles. Once the table has been filled in, knowledge of the γ propagator is immediate.

Waveguides

$$\gamma = \begin{bmatrix} 0 & \Gamma_1 & 0 & 0 \\ 0 & 0 & 0 & G_{12} \\ G_{21} & 0 & 0 & 0 \\ 0 & 0 & \Gamma_2 & 0 \end{bmatrix} \quad (36)$$

Knowing γ , what is the wave vector at the initial time $t = 0$? The potential \mathcal{V}_1 transmitted to the η_c impedance line at time $t = 0$ with the e_0, z_0 generator is :

$$\mathcal{V}_1(s) = \left(\frac{\eta_c}{\eta_c + z_0} \right) e_0(s) \equiv e_0(s) \quad (37)$$

Waveguides

Let's apply the γ propagator to the \mathcal{V}_α wave vector. This operator modifies the wave vector, which becomes :

$$\gamma \overleftarrow{\mathcal{V}} = \begin{bmatrix} 0 & 0 & G_{21} e_0(s) & 0 \end{bmatrix} \quad (38)$$

Let's apply twice the product by γ :

$$\gamma \gamma \overleftarrow{\mathcal{V}} = \begin{bmatrix} 0 & 0 & 0 & \Gamma_2 G_{21} e_0(s) \end{bmatrix} \quad (39)$$

Then:

$$\gamma \gamma \gamma \overleftarrow{\mathcal{V}} = \begin{bmatrix} 0 & G_{12} \Gamma_2 G_{21} e_0(s) & 0 & 0 \end{bmatrix} \quad (40)$$

Let's continue and look at the evolution of the wave vector.

Waveguides

$$\gamma\gamma\gamma\overleftarrow{\mathcal{V}} = \left[\Gamma_1 G_{12} \Gamma_2 G_{21} e_0(s) \quad 0 \quad 0 \quad 0 \right]$$

$$\gamma\gamma\gamma\gamma\overleftarrow{\mathcal{V}} = \left[0 \quad 0 \quad G_{21} \Gamma_1 G_{12} \Gamma_2 G_{21} e_0(s) \quad 0 \right]$$

$$\gamma\gamma\gamma\gamma\gamma\overleftarrow{\mathcal{V}} = \left[0 \quad 0 \quad 0 \quad \Gamma_2 G_{21} \Gamma_1 G_{12} \Gamma_2 G_{21} e_0(s) \right]$$

We can go on and on. And if we look at the components obtained for \mathcal{V} , we find the expressions deduced on the boundaries in Bergeron's diagram, both the waves that leave the ports and those that reach them. **If** e_0 is a step function, the waves are cumulative at the ports. Furthermore, the induced emfs on the edges at the ends of the line have an equivalent pattern of 2 times the value of the wave arriving at the port, in series with the propagation impedance in the line.

Waveguides

The transition from \mathbb{O} to \mathbb{B} takes place via a Λ_i^j connectivity that returns the j component of $\overleftarrow{\mathcal{V}}$ to the i branch. Finally, we have at both line extremities (for a step function, that's why we add all the gamma products here):

$$\mathbf{e}_\beta = 2\Lambda_\beta^\sigma (\gamma + \gamma\gamma + \gamma\gamma\gamma + \dots) \mathcal{V}_\sigma \quad (41)$$

$\Lambda_\beta^\sigma = \pm 1$. Using $(1 + x)^{-1} = 1 + x + x^2 + \dots$, it becomes:

$$\mathbf{e}_\beta = 2\Lambda_\beta^\sigma [(U + \gamma)^{-1} - U] \mathcal{V}_\sigma \quad (42)$$

U being the unity matrix. $\mathbf{e}_\beta \in \mathbb{B}$, $\mathcal{V}_\alpha \in \mathbb{Q}$.

Waveguides

Let's apply the previous modeling to the case of our line.

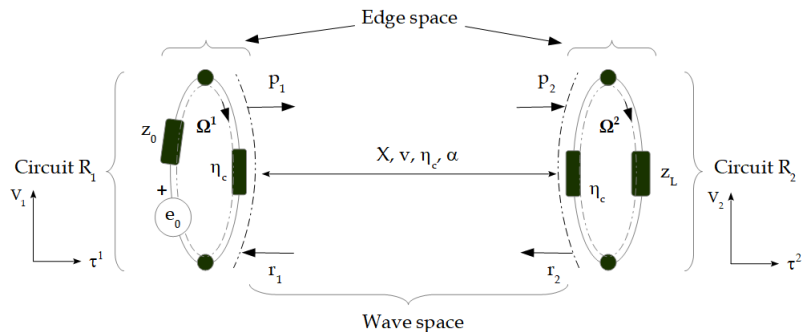


Figure: Line example

Waveguides

We compute the reflexion coefficients under Laplace's formalism:

$$\Gamma_1 = \frac{z_0 - \eta_c}{z_0 + \eta_c}, \quad \Gamma_2 = \frac{z_L - \eta_c}{z_L + \eta_c} \quad (43)$$

Note that for current waves, $\Gamma(J) = -\Gamma(V)$.

We compute the wave covector at $t = 0$:

$$\mathcal{V}_\alpha = \left[e_0 \left(\frac{\eta_c}{\eta_c + z_0} \right), 0, 0, 0 \right]$$

And under the assumption of using a lossless line in $T\Psi_{00}$ mode,

$$G_{12} = G_{21} = \exp\left(-s \frac{X}{v}\right), \quad v = \frac{c}{\sqrt{\epsilon_r}}$$

Waveguides

Each circuit (and associated referential) has their own time axis. On the left, the time domain values τ^1 are $0, 2X/v, 4X/v, \dots$. On the right, the time domain values τ^2 are $X/v, 3X/v, 5X/v, \dots$. For making some numerical application and tracing the voltage curves on \mathcal{R}_1 and \mathcal{R}_2 we choose $z_0 = 50[\Omega]$, $z_L = 30[\Omega]$, $\eta_c = 75[\Omega]$, $X = 100[m]$ and $v = 2.10^8[m][s]^{-1}$. Let's express the potential difference developed across z_L : $V_2 = z_L \Omega^2$.

At $t = 0$, $V_2 = 0$. At $t = X/v$, we have

$$e_2(s) = 2\Lambda_3^\sigma (\gamma \mathcal{V}_\sigma) = 2 \cdot \exp\left(-s \frac{X}{v}\right) \cdot \left(\frac{\eta_c}{\eta_c + z_0}\right) \frac{e_0}{s} \quad (44)$$

or

$$e_2(\tau^2) (\square e_2(s)) = \left(\frac{\eta_c}{\eta_c + z_0}\right) e_0 \delta_{X/v} = 1.2 e_0 \delta_{X/v} \quad (45)$$

If we take $e_0 = 2 [V]$, $e_2(\tau^2 = X/v) = 2.4 [V]$.

Waveguides

If we take $e_0 = 2 [V]$, $e_2(\tau^2 = X/v) = 2.4 [V]$. So that the equivalent circuit on the right (\mathcal{R}_2) for the traveling wave is:

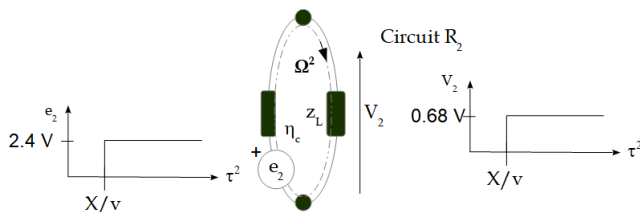


Figure: Circuit on the right

Waveguides

Part of the wave arriving at \mathcal{R}_2 is reflected and returns to \mathcal{R}_1 . Let's look at the equivalent circuit and the measurable level in \mathcal{R}_1 at the generator terminals. Note that each measurement location has a corresponding circuit. The temporal separation of the guide's inputs and outputs is fundamental, as is the impossibility of carrying out a measurement without disturbing propagation. At time $t = 2X/v$, the emf on cycle 1 issued from the wave coming on port r_1 is

$$e_1(s) = 2\Lambda_2^\sigma \gamma \gamma \gamma \mathcal{V}_\sigma = 2 \cdot (-1) \cdot G_{12} \Gamma_2 G_{21} \frac{e_0}{s}$$

or

$$e_1(s) \approx -2 \exp\left(-2\frac{X}{v}\right) \left(\frac{\eta_c}{\eta_c + z_0}\right) \frac{-0.43 e_0}{s}, \quad \Gamma_2 \approx -0.43$$

Waveguides

Given finally $e_1(s) \sqsubset -(-1.03)\delta_{2X/v} [V]$, and
 $\Omega^1(\tau^1 = 2X/v) = (2 + 1.03)/(75 + 50) = 24 [mA]$. So that
 $V_1(\tau^1 = 2X/v) = -1.03 + 75 \cdot 24 \cdot 10^{-3} = 0.8 [V]$.

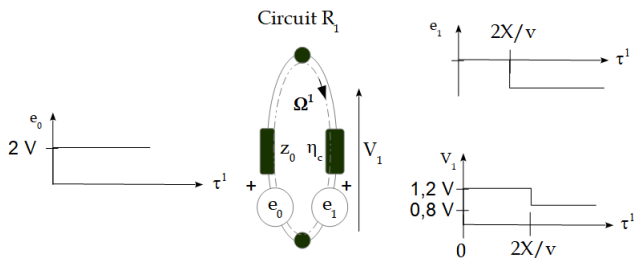


Figure: Circuit on the left

Waveguides

Where we need to be careful is that the induced emf on the input cycle is in the opposite direction to that chosen for the cycle current, in contrast to that on the output cycle. Orientations are important to calculate cycle currents correctly. In addition, the emfs induced by the waves arriving at the ports, multiplied by 2 and every $2nX/v$ for the input (for example), add up to the previous amplitude, since the generator e_0 is a step of infinite duration. From this point of view, the response to the step is unambiguous. This addition is translated by the summation over time

$$e_0 + e_1(2X/v) + e_1(4X/v) + \dots \Leftrightarrow e_0 + 2\Lambda_2^\sigma (\gamma + \gamma^2 + \dots) \mathcal{V}_\sigma \quad (46)$$

Please try computing the next values of V_1 and V_2 for this first example. You can verify by comparing with Bergeron's diagram.

Waveguides

At this point, we've seen a first example of waveguide modeling. If we wanted to work with TJ_{xy} modes, we'd take $\Gamma(J) = -\Gamma(\Psi)$ and the reflected current would be in the opposite direction to the incident current. So :

$$\Psi_t = \Psi_i + \Psi_r, \quad J_i = J_r + J_t \quad (47)$$

The index t being for transmit, i for incident and r for reflected. The waveguide model can be used to model waveguides and lines, as well as cavities, which are short-circuited guides at the input and output. Let's take a look at how to make a measurement in a waveguide.

Waveguides

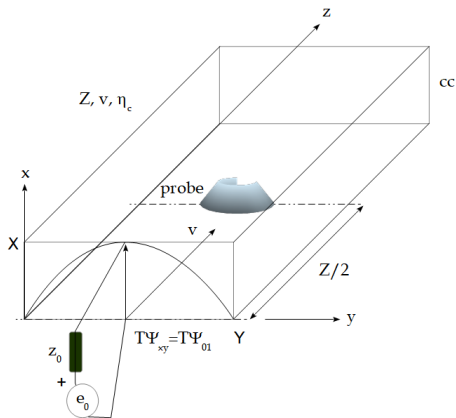


Figure: Measurement in a cavity

Waveguides

How can we calculate propagation impedance? The wave impedance is $\mu\nu$. Consider two parallel ribbons with the dimensions of the guide in our figure: the capacitance is ϵ_0ZY/X . The inductance is μ_0ZX/Y . The propagation impedance is $\eta_c = \sqrt{L/C} = \sqrt{\mu_0/\epsilon_0}X/Y$ or μ_0cX/Y . To evaluate the propagation impedance of a waveguide, we can always refer to parallel ribbons. Let's consider a sine-arched field distribution and compare it to a continuous field of width k . We want to equalize :

$$2 \int_{x=0}^{\pi/2} dx \sin(x) = 2 \int_{x=0}^{\pi/2} dx k \Leftrightarrow [-\cos(x)]_0^{\pi} = k [x]_0^{\pi} \quad (48)$$

hence: $k = 2/\pi$. For a waveguide where $X \approx Y$ with $\lambda_0 = 2Y$ and $\lambda = \lambda_0/2$ we find $\eta_c(T\Psi_{01}) = \sqrt{\mu_0/\epsilon_0}2X/(\pi Y) \approx 270[\Omega]$.

Waveguides

At low frequencies (below c/λ_0) the waveguide behaves like an inductance L_w . The waveguide cutoff frequency is fixed by $\lambda_0 \Rightarrow \omega_0 = \eta_c/L_w$, $\eta_c = \eta_c(\lambda \ll \lambda_0)$. This high-pass filter behavior makes the step response to become:

$$e^{-t/RC} \rightarrow e^{-\eta_c/L} \sqsupset [s + \eta_c/L]^{-1} \equiv \frac{L}{sL + \eta_c} \quad (49)$$

The time base for this function can be taken as $3/\omega_0 = 3L/\eta_c$. We can also use:

$$\delta\tau = \frac{3\lambda_0}{2\pi c}, \quad e_0(s) = [s + \lambda_0/(2\pi c)]^{-1}$$

Waveguides

Let's calculate the response of the previous half-closed cavity of length Z , empty of any object. The generator impedance is matched with the cavity impedance propagation, while its extremity is a wall acting as a short-circuit. Let's take a look at how the guide input behaves when excited by a step. We model this input by an inductance L in parallel with the propagation impedance. The incident voltage V_i , which is the first component of the wave vector, is defined by :

$$V_i = \left[\frac{e_0}{s} \right] \left(\frac{\eta_c s L}{\eta_c + s L} \right) \left[R_0 + \frac{\eta_c s L}{\eta_c + s L} \right]^{-1} \quad (50)$$

Waveguides

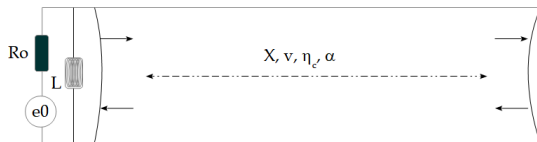


Figure: Semi-closed cavity

Waveguides

Noting

$$Z_e = \frac{\eta_c}{\eta_c + R_0}, \quad Z'_e = \frac{R_0 \eta_c}{L(R_0 + \eta_c)}$$

Hence

$$V_i = \frac{Z_e e_0}{s + Z'_e} \square Z_e \exp(-Z'_e t) \quad (51)$$

The guide acts as a high-pass filter, cutting out the low-frequency components of the step. The result is a decreasing exponential shape. This suggests not only that this propagative model is not suitable for low frequencies, but also that a different base shape might be more appropriate for high frequencies in this case. Let's find the expression for the reflection coefficient on the generator side (on the end, its value is trivial and equals -1).

Waveguides

$$\Gamma_1 = \frac{\frac{R_0 s L}{R_0 + s L} - \eta_c}{\frac{R_0 s L}{R_0 + s L} + \eta_c} \Rightarrow |\Gamma_1| = \sqrt{\frac{(\eta_c R_0)^2 + (R_0 - \eta_c)^2 \omega^2 L^2}{(\eta_c R_0)^2 + (R_0 + \eta_c)^2 \omega^2 L^2}} \quad (52)$$

To adapt the generator, we see that ω must be high enough and $R_0 = \eta_c$. An interesting timeform base for these high-frequency analyses is that of sines modulated by gate functions: $\sin(\omega_0 t) \Pi_T$.

$$\sin(\omega_0 t) \Pi_T \sqsupset \frac{\omega_0}{\omega_0^2 + s^2} (1 - e^{-sT}), \quad T = \frac{2k\pi}{\omega}, \quad k \in \mathbb{N} \quad (53)$$

If ω_0 is high enough and T large enough, this base behaves like a sinusoidal base, but with a limited time domain, defining the $n\tau$ samples of the time axis. Moreover, if we take $T < 2Z/v$, successive pulses are decorrelated at the boundaries. This could be an interesting possibility, depending on the studies to be carried out.

Waveguides

Taking $\omega_0 \gg Z'_e$ we obtain $\Gamma_1 \approx 0$ and $\Gamma_2 = -1$.

1. If $T < 2Z/v$, $e_1 = 2\Lambda_1^\alpha \gamma^n \mathcal{V}_\alpha$.
2. If $cf_0^{-1} < Z$ and $T > 2Z/v$ then $e_1 = 2\Lambda_1^\alpha (\gamma + \gamma^2 + \dots) \mathcal{V}_\alpha$.

If we send only one pulse under condition (1), then:

$$\begin{cases} |e_1(s)| = 2 \exp\left(-2s\frac{Z}{v}\right) \left[\frac{\omega_0}{\omega_0^2 + s^2} (1 - e^{-sT}) \right] \\ |e_2(s)| = 2 \exp\left(-s\frac{Z}{v}\right) \left[\frac{\omega_0}{\omega_0^2 + s^2} (1 - e^{-sT}) \right] \end{cases} \quad (54)$$

Equivalent diagrams can be drawn for each side.

Waveguides

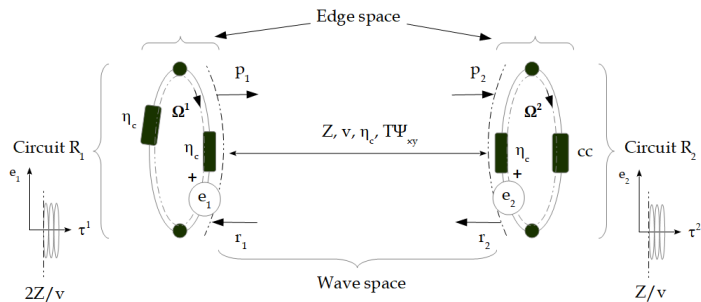


Figure: Waveguide response

You will have noticed that $e_0(2Z/v) = 0$.

Waveguides

We have three time bases to describe all EMC sources:

1. the step s^{-1} ,
2. decreasing exponential $(s + a)^{-1}$,
3. sine modulated by a gate function $\left[\frac{\omega_0}{\omega_0^2 + s^2} (1 - e^{-sT}) \right]$.

To work in \mathcal{W} , we need to determine the \mathcal{F} functional. Let's consider the last form.

$$V_i V_i^* = \frac{e_0^2}{4} \left(\frac{\omega_0}{\omega_0^2 - \omega^2} \right)^2 (1 - e^{-sT})(1 - e^{sT})$$

which leads to:

$$\sqrt{V_i V_i^*} = \left(e_0 \frac{\omega_0}{\omega_0^2 - \omega^2} \right) \text{Sin} \left(\frac{\omega T}{2} \right)$$

Waveguides

Noting the peak value \hat{V}_i we obtain:

$$\hat{V}_i = \sqrt{\langle V_i | \mathcal{F} | V_i \rangle} \Rightarrow \mathcal{F} = \alpha \left[\text{Sin} \left(\frac{\omega T}{2} \right) \right]^{-1} \quad (55)$$

α is a corrective coefficient coming from the sampling and energy. The main mode propagating in the cavity depends on the wavelength and the source. In our case:

$$T \Psi_{0n}, \quad n = \text{int} \left(2 \frac{Y}{\lambda} \right) \quad (56)$$

We now have all the information we need to code our problem. Note that in the next listing, we explore the whole frequency band, even if we may have only compute the time form pseudo-frequency, just to show an example of complete process.

Waveguides

```

7
8 import numpy as np
9 import pylab as plt
10 #
11 # Making the temporal form
12 N=1000 # number of points
13 dt=100E-12 # elementary time duration
14 sinmod=np.zeros(N,dtype=float) # temporal signal
15 fo=5E8 # signal central frequency
16 wo=2.*np.pi*fo # signal central pulsation
17 Tc=1./fo # signal central period
18 Tc=.5*Tc # signal duration when active
19 a=1E-6 # losses
20 for n in range(N):
21     if (n*dt>T):
22         sinmod[n]=np.sin(2.*np.pi*fo*n*dt)+np.exp(-a*n*dt)
23     if (n*dt<=T):
24         sinmod[n]=0.
25
26 plt.plot(sinmod) # plotting signal curve
27 plt.grid(True)
28 plt.xlabel('t/tau_e, [100 ps]')
29 plt.ylabel('V[V]')
30 plt.title('Temporal source [V]')
31 plt.show()
32 # we compute the signal peak value
33 # first defined Laplace's transform of sinmod.
34 sm=np.zeros(N,dtype=complex)
35 F=np.zeros(N,dtype=complex)
36 pvi=np.zeros(N,dtype=complex)
37 for u in range(1,N):
38     s=1+2.*np.pi*u/T+a
39     sm[u]=2.*wo/(wo**2.+s**2.)*(1.-np.exp(-s*T))
40     F[u]=2./np.pi*sin(2.*np.pi*u/2.)
41     pvi[u]=F[u]/sm[u]
42     pv=np.sqrt(np.dot(np.conj(sm),pvi))
43     print(np.real(pv))
44 plt.plot(abs(sm)) # plotting signal curve
45 plt.grid(True)
46 plt.xlabel('omega/T, [1/100 ns]')
47 plt.ylabel('|sm| [V]')
48 plt.title('Source spectrum [V]')
49 plt.xscale('log')
50 plt.show()

```

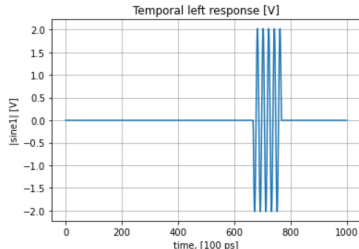
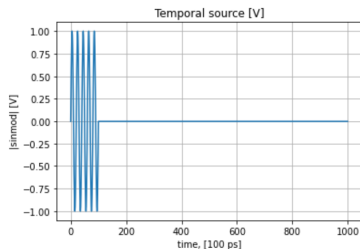
```

51 # waveguide response
52 uo=4.*np.pi*1E-7 # permeability
53 c=3E8 # light speed
54 e1=np.zeros(N,dtype=complex) # enf
55 e2=np.zeros(N,dtype=complex) # reflection coefficients
56 G2=-1.
57 G1=0.
58 G1=0.
59 Z=10. # waveguide length
60 Y=40E-2 # waveguide size
61 X=20E-2 # waveguide height
62 la=2.*Y
63 g=np.zeros((4,4), dtype=complex) # gamma matrix
64 Co=[[0.,-1.,0.,0],\
65     [0.,0.,1.,0.]] # connectivity matrix
66 for u in range(1,N):
67     V=np.zeros(4,dtype=complex) # wave covector
68     sm1=2.*np.pi*u/T+a
69     lae=(u/T)
70     if (la<lao):
71         v=c/np.sqrt(1.-|la/lao|**2.)
72         Gmp=np.exp(-s*T/vo) # green's function
73         if (la<=lao):
74             v=0. # simple model when lambda>=lambdao
75             Gm=
76             no=uo**X*Y
77             V[0]=wo/(wo**2.+s**2.)*(1.-np.exp(-s*T)) # generator matched, Vi=2./2.
78             g[0,1]=G1
79             g[1,1]=G2
80             g[2,0]=G2
81             g[3,2]=G2
82             Vmp=np.dot(g,np.dot(g,np.dot(g,V)))
83             e1[u]=2.*np.dot(Co,V[1])
84             e2[u]=2.*np.dot(Co,V[1])
85             pvi[u]=F[u]/e1[u]
86 plt.plot(abs(e1)) # plotting signal curve
87 plt.grid(True)
88 plt.xlabel('omega/T, [1/100 ns]')
89 plt.ylabel('|e1| [V]')
90 plt.title('Left enf spectrum [V]')
91 plt.xscale('log')
92 plt.show()
93 pv=np.sqrt(np.dot(np.conj(e1),pvi))
94 print(np.real(pv))
95 #
96 # temporal signal from e1
97 sine1=np.zeros(N,dtype=float)
98 for n in range(N):
99     if (n*dt<=2.*Z/c):
100         sine1[n]=e1
101     if (n*dt>=2.*Z/c):
102         sine1[n]=np.real(pv)*np.sin(2.*np.pi*fo*(n*dt-2.*Z/c))+np.exp(-a*n*dt)
103     if (n*dt<=+2.*Z/c):
104         sine1[n]=e1

```

Waveguides

Knowing the peak value and sign of the time waveform extracted from the Laplace's one, we can construct the time response of the measurements, only obtained in \mathbb{M} once the projection from \mathbb{O} realized. We continue working in \mathcal{W} .



Waveguides

Now we want to insert a probe to take a measurement in the cavity. If we introduce objects into a cavity, two cases occur:

1. If the objects are $< \lambda$ in size, the model is a localized pattern interposed between two cavities. If the objects are very small, we can approximate them without modifying the cavity's principal modes, but by incorporating these objects into the propagation losses.
2. If the objects are $> \lambda$, they modify the modes and characteristics of the cavity.

Waveguides

Different cavity assemblies:

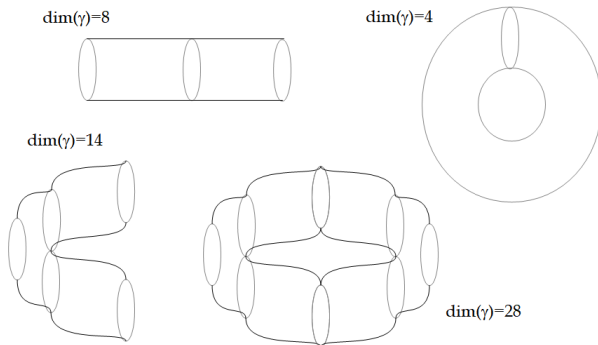
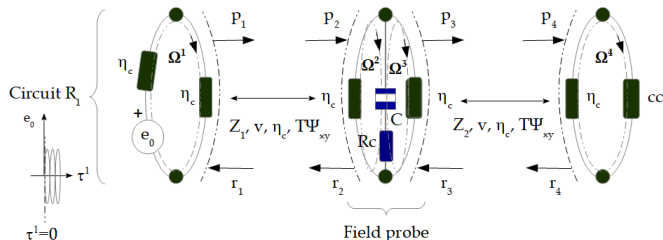


Figure: Waveguide assembling

Waveguides

Let's look at the first case by inserting an electric field probe centered at y in our guide. The equivalent diagram of such a probe is a capacitor C in series with the load R_C . Our complete schematic becomes:



Waveguides

The wave space now includes two waveguides in series $w_1, w_2 \in \mathbb{O}^2$, with the same characteristics and 8 ports. The wave covector therefore has 8 components. The two waveguides have a common boundary (R_c, C) . Let's calculate the transmission coefficients t_{32} and t_{23} . To do this, let's excite $c_2(\Omega^2)$ with an incident source e_2 . We measure the input of w_2 :

$$z_f = \frac{(R_c + \frac{1}{sC})\eta_c}{R_c + \frac{1}{sC} + \eta_c}, \quad V_i(w_2) = \left(\frac{z_f}{z_f + \eta_c} \right) 2e_2 \quad (57)$$

The reflection and transmission coefficients are equal:

$$\Gamma_2 = \Gamma_3 = \frac{z_f - \eta_c}{z_f + \eta_c}, \quad t_{32} = \frac{V_i(w_2)}{e_2} = 2 \left(\frac{z_f}{z_f + \eta_c} \right) = 1 + \Gamma_2 = t_{23} \quad (58)$$

Waveguides

$$\gamma = \begin{bmatrix} 0 & \Gamma_1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & G_1 & 0 & 0 & 0 & 0 \\ G_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \Gamma_2 & 0 & 0 & t_{23} & 0 & 0 \\ 0 & 0 & t_{32} & 0 & 0 & \Gamma_3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & G_2 \\ 0 & 0 & 0 & 0 & G_2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \Gamma_4 & 0 \end{bmatrix}, \quad \mathcal{V} = \begin{bmatrix} \frac{e_0}{2} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad (59)$$

The electric field measurement is obtained through:

$$\mathbf{E} = -\mathbf{grad} \left[\frac{1}{sC} (\Omega^2 - \Omega^3) \right], \quad \Omega^\alpha = y^{\alpha\sigma} \left\{ e_\sigma + 2N_\sigma^\beta (\gamma \dots \gamma) \mathcal{V}_\beta \right\} \quad (60)$$

$$G_1 = \exp(-sZ_1/v), \quad G_2 = \exp(-sZ_2/v)$$

Waveguides

An electric field probe is a special kind of electronic circuit, but one that in no way detracts from the mechanisms of field-matter coupling. So the level measured on the R_c charge could be a stress transferred to an electronic circuit by coupling with the ambient field. Let's calculate this stress using the narrow-band hypothesis. The question is "how many iterations should we perform on γ ?"

As the source is matched, and assuming the impact of the probe is negligible ($t_{32} \approx 1$), the Bergeron diagram shows us that an order of 5 (γ^5) should suffice to correctly evaluate the electric field. But we can explore various values to see how evolves the electric field intensity.

Waveguides

```

38 #
39 Ro=75.
40 C=100E-12
41 Rc=50.
42 etac=75.
43 s=1J*2.*np.pi*fo+a
44 zf=(Rc+1./(s*C))*etac/(Rc+1./(s*C)+etac)
45 G1=0.
46 G2=(zf-etac)/(zf+etac)
47 G3=G2
48 G4=(0.-etac)/(0.+etac)
49 t32=2.*(zf/(zf+etac))
50 t23=t32
51 Z1=4.
52 Z2=6.
53 v=c/np.sqrt(1.-(la/lao))
54 Ge1=np.exp(-s*Z1/v)
55 Ge2=np.exp(-s*Z2/v)
56 #
57 eo=2.
58 V=np.zeros(8,dtype=complex)
59 V[0]=eo/2.
60 gamma=[0.,G1,0.,0.,0.,0.,0.,0.],
61         [0.,0.,0.,Ge1,0.,0.,0.,0.],
62         [Ge1,0.,0.,0.,0.,0.,0.,0.],
63         [0.,0.,G2,0.,0.,0.,t23,0.],
64         [0.,0.,t32,0.,0.,0.,G3,0.],
65         [0.,0.,0.,0.,0.,0.,0.,Ge2],
66         [0.,0.,0.,0.,Ge2,0.,0.,0.],
67         [0.,0.,0.,0.,0.,0.,0.,G4,0.]

```

```

68 # impedance metric
69 zeta=[[etac+Ro,0.,0.,0.],
70       [0.,etac+1./(s*C),0.,0.],
71       [0.,0.,1./(s*C)+etac,0.],
72       [0.,0.,0.,etac]]
73 Lambda=[[0.,-1.,0.,0.,0.,0.,0.,0.],
74         [0.,0.,1.,0.,0.,0.,0.,0.],
75         [0.,0.,0.,0.,0.,-1.,0.,0.],
76         [0.,0.,0.,0.,0.,0.,1.,0.]]
77 # computing the wave covector
78 Q=30
79 mem0=np.zeros((0,2),dtype=float)
80 E=np.zeros(Q,dtype=float)
81 for q in range(Q):
82
83     V2=np.dot(gamma,np.transpose(V))
84     V=V2
85     T=2.*np.dot(Lambda,V)
86     print('q: '+str(q)+' e2: '+str(T[1])+' e3: '+str(T[2]))
87     0=np.zeros(4,dtype=complex) # cycle currents
88     0=np.dot(np.linalg.inv(zeta),T)
89     mem0[q]=[abs(0[1]),abs(0[2])]
90     E[q]=abs(-1./(s*C)*(mem0[q][0]-mem0[q][1])/X)
91     plt.plot(E, 'o')
92     plt.title(u'Electric field intensity [V/m]')
93     plt.xlabel(u'time sequence')
94     plt.ylabel(u'[V/m]')
95     plt.grid(True)
96     plt.show()

```

Waveguides

Sometimes the electric field amplitude is zero, because at these times, the wave covector component is null. When we search all values obtained for the various power of γ , we trace the curve shown below.

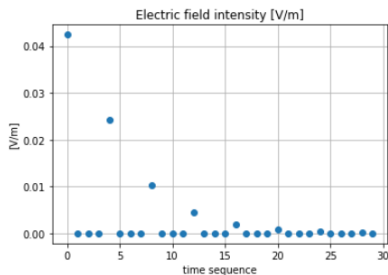


Figure: Electric field module

Waveguides

We could easily calculate the response for a probe in a different location, by varying the parameters Z_1 and Z_2 . The probe's equivalent diagram should reflect its spatial and temporal integration effect. If we had filled the cavity with a printed circuit board, we could have created two parallel cavities, separated by the printed circuit board, or one cavity with a symmetrical $T\Psi$ mode around the printed circuit board acting as a “central septum”. Try treating this second case as an exercise.

Crosstalk

Crosstalk concerns the coupling between two lines. There are two kind of couplings: the capacitive coupling which refers to the scalar potential and the magnetic coupling which refers to the vector potential coupling. As always, we study the models in the two cases: inferior or superior to λ . In the first case, the graph corresponding of two lines coupled is given figure 4.

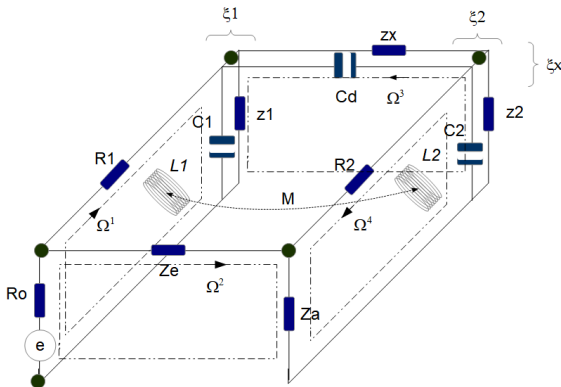


Figure: Fig.4: Crosstalk

Crosstalk

The impedance metric of this structure is:

$$\zeta = \begin{bmatrix} R_0 + z_1 + \xi_1 + sL_1 & R_0 & \xi_1 & -sM \\ R_0 & R_0 + z_e + z_a & 0 & z_a \\ \xi_1 & 0 & \xi_1 + \xi_2 + \xi_x & \xi_2 \\ -sM & z_a & \xi_2 & z_a + R_2 + \xi_2 + sL_2 \end{bmatrix} \quad (61)$$

Crosstalk

The near-end crosstalk (on the source side) transfer function is given by (knowing that $T_1 = e$ is the only source component different from zero and with $y = \zeta^{-1}$):

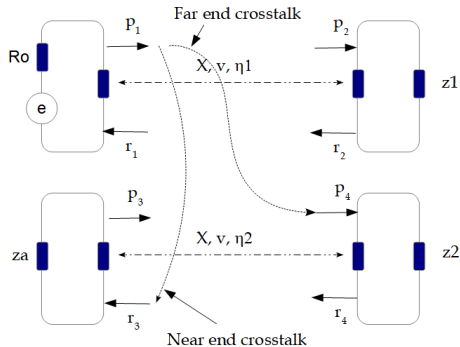
$$H_n = \frac{z_a(y^{21} + y^{41})T_1}{T_1 - R_0(y^{11} + y^{21})T_1} \Leftrightarrow H_n = \frac{(y^{21} + y^{41})z_a}{1 - (y^{11} + y^{21})R_0} \quad (62)$$

The far-end crosstalk (opposite to the source side) is:

$$H_f = \frac{V(\xi_2)}{e - R_0(\Omega^1 + \Omega^2)} \Leftrightarrow H_f = -\frac{(y^{31} + y^{41})\xi_2}{1 - (y^{11} + y^{21})R_0} \quad (63)$$

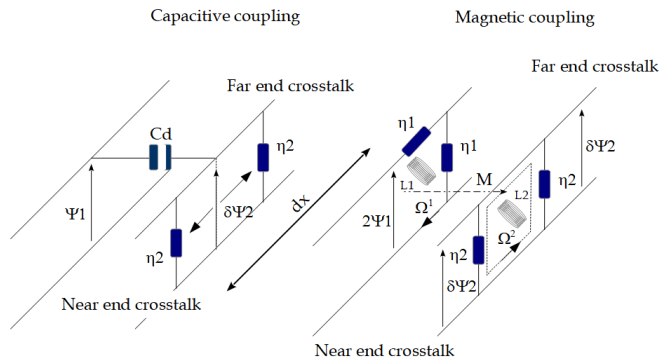
Crosstalk

For long lines ($x > \lambda$), our purpose is to determine the propagator γ . We work in a 8-dimension wave space as shown below.



Crosstalk

For computing γ components, we use both schematics shown below, making some approximations.



Crosstalk

τ being the wave propagation delay, the elementary voltage induced from the capacitive coupling is:

$$d\Psi_2 = \frac{1}{2} \left(\frac{\eta_2}{\eta_2 + (sC_d dx)^{-1}} \right) \Psi_1 \approx \frac{1}{2} \eta_2 s C_d dx \Psi_1 e^{-s\tau} \quad (64)$$

while from the magnetic coupling we obtain (with $sL_i dx \ll \eta_i$):

$$d\Psi_2 = \pm \frac{1}{2} s M dx \frac{\Psi_1}{\eta_1} e^{-s\tau} \quad (65)$$

Crosstalk

It remains for us to integrate $d\Psi_2$ over the whole line length. At this level, the two processes are fundamentally different. Let's take a look to the figure 6.

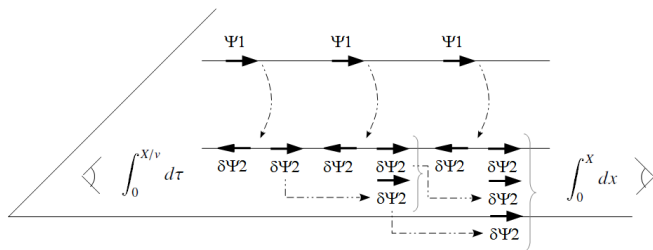


Figure: 6: Crosstalk process

Crosstalk

Seeing to this figure we understand that the integration is in the time domain on the near-end side while it is in the space domain for the far-end side. The time duration is fixed by the propagation delay $\tau = x/v$ that we can integrate from 0 to X/v . So, for the capacitive coupling we obtain on the near-end side (\mathcal{V} is our wave covector, $\Psi_1 = \mathcal{V}_1$):

$$\mathcal{V}(r_3) = \int_0^{X/v} d\tau \frac{1}{2} \eta_2 s C_d v \mathcal{V}(p_1) e^{-2s\tau} = \frac{1}{4} \eta_2 C_d v (1 - e^{-2sX/v}) \mathcal{V}(p_1) \quad (66)$$

On the far-end side it gives:

$$\mathcal{V}(p_4) = \int_0^X dx \frac{1}{2} \eta_2 s C_d \mathcal{V}(p_1) e^{-s\tau} = \frac{1}{2} \eta_2 s C_d X \mathcal{V}(p_1) e^{-sX/v} \quad (67)$$

Crosstalk

Making the same exercise for the magnetic coupling we find for the near-end crosstalk:

$$\mathcal{V}(r_3) = - \int_0^{X/v} \frac{1}{2} sMv d\tau \frac{\mathcal{V}(p_1)}{\eta_1} e^{-2s\tau} = M \frac{v}{4} \frac{\mathcal{V}(p_1)}{\eta_1} (1 - e^{-2sX/v}) \quad (68)$$

and for the far-end crosstalk:

$$\mathcal{V}(p_4) = - \int_0^X \left(\frac{1}{2} sM \right) dx \frac{\mathcal{V}(p_1)}{\eta_1} e^{-s\tau} = - \left(\frac{1}{2} sM \right) X \frac{\mathcal{V}(p_1)}{\eta_1} e^{-sX/v} \quad (69)$$

Crosstalk

Finally, the whole coupling function including both capacitive and magnetic couplings leads to the next near-end formula:

$$\mathcal{V}(r_3) = \frac{1}{4}v(1 - e^{-2sX/v})\mathcal{V}(p_1)\left[\eta_2 C_d + \frac{M}{\eta_1}\right] \quad (70)$$

and the far-end formula:

$$\mathcal{V}(p_4) = \frac{1}{2}sX\mathcal{V}(p_1)e^{-sX/v}\left[\eta_2 C_d - \frac{M}{\eta_1}\right] \quad (71)$$

In the case of harnesses, the previous local schematics of couplings give the way for establishing the crosstalks inside harnesses. In final the voltages and currents come from $2\Lambda_\alpha^\sigma(\gamma \dots \gamma)\mathcal{V}_\sigma$ as usual.

Crosstalk

The lines are 12 cm long. The distance between the lines is 5 mm and they are 4 mm height. The wire diameter is 1 mm. All the loads are 100Ω . We consider the resistivity as $10^7 [\Omega][m]$. Using sinusoidal source at various frequencies, we obtain the measurements given in the next tab.

frequency	Near-end crosstalk	Far-end crosstalk	V_i
100k	2,5 mV	2 mV	1,8 V
200k	4 mV	4 mV	1,8 V
500k	4,5 mV	4,5 mV	1,8 V
1M	7,5 mV	7,5 mV	1,8 V
2M	11 mV	10 mV	1,8 V
5M	15 mV	17 mV	1,2 V
10M	15 mV	18 mV	0,5 V

Crosstalk

The python listing for computing this case is given below.

```

8 import numpy as np
9 import pylab as plt
10
11 # data
12 X=12E-2 # line length
13 d=5E-3 # distance between wires
14 h=4E-3 # wires height
15 phi=1E-3 # wire diameter
16 rho=1E-7 # wire resistivity
17 uo=4.*np.pi*1E-7
18 #
19 # parameters computation
20 # propagation impedances
21 n1=20.*np.log(4.*h/phi)
22 n2=20.*np.log(4.*h/phi)
23 v=2E8
24 C1=1./(v*n1)
25 C2=1./(v*n2)
26 Cd=1./(v*120.*np.log(d/phi))
27 L1=n1/v
28 L2=n2/v
29 M=3.9*np.sqrt(L1*L2)*X
30 R1=rho*X/(np.pi*phi**2./4.)
31 R2=rho*X/(np.pi*phi**2./4.)
32
33 # load impedances
34 Ro=98.
35 za=98.
36 z1=98.
37 z2=98.
38
39 # frequency tab
40 freq=[100E3,200E3,500E3,1E6,2E6,5E6, 10E6]
41
42 # tabs
43 N=7
44 Omega=np.zeros(N,3,dtype=complex)
45 source=[1,8,1,8,1,8,1,8,1,2,0,5]
46 mespara=[2,5,4,4,5,7,5,11,15,15] # near end C. measurement in mV
47 mesdia=[2,4,4,5,7,10,17,18] # idem FE
48 para=np.zeros(N,dtype=float)
49 dia=np.zeros(N,dtype=float)
50

```

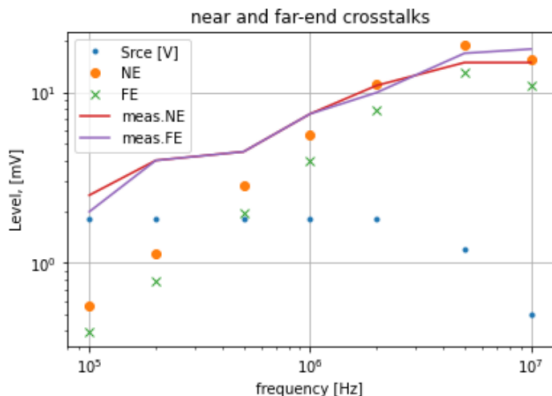
```

51 # computation
52 ind=0
53 for f in freq:
54     s=1*2.*np.pi*f
55     se=1./np.sqrt(np.pi*f*uo/rho) # skin depth
56     R1f=R1+rho*X/(np.pi*phi*se) # resistance in frequency
57     R2f=R2+rho*X/(np.pi*phi*se)
58     x1=z1/(1.+s**2*C1*X)
59     x2=z2/(1.+s**2*C2*X)
60     #
61     zeta=[(Ro+R1f+x1+s**2*L1*X,x1.-s*M],
62           [x1,x1+1./(s*Cd*X)+x2,x2],
63           [-s*M,x2,za+R2f+x2+s**2*L2*X]]
64     y=np.linalg.inv(zeta)
65     T=[source[ind],0,0,0]
66     Omega[ind]=np.dot(y, T)
67     para[ind]=IE3*abs(Ce*Omega[ind,2])
68     dia[ind]=IE3*abs(x2*(Omega[ind,1]+Omega[ind,2]))
69     ind=ind+1
70
71 plt.plot(freq,source,'.',label='Srce (V)')
72 plt.plot(freq,para,'o',label='NE')
73 plt.plot(freq,dia,'x',label='FE')
74 plt.plot(freq,mespara,'-',label='meas.NE')
75 plt.plot(freq,mesdia,'-',label='meas.FE')
76 plt.grid(True)
77 plt.title(u'near and far-end crosstalks')
78 plt.xlabel(u'frequency [Hz]')
79 plt.ylabel(u'Level, [mV]')
80 plt.xscale('log')
81 plt.yscale('log')
82 plt.legend()
83 plt.show()

```

Crosstalk

This computation leads to the next results, where measurements and calculation are in good accordance, except in low frequencies. The difference comes from the signal to noise ratio that becomes very bad as the frequency decreases.



Crosstalk

Some remarks on the previous example: the results was obtained with roughly estimated parameter values. In general we don't know the resistivity exact value, the distances are not strictly constant, etc. What we want are simple and robust models that give the right orders of magnitude.. This does not mean oversimplifying the physical relationships involved. Besides in this example, we take into account the skin effect for the resistances, which is not so trivial and is not considered by SPICE-like tools for example. Once the orders of magnitude are known, we must add a margin which depends on the electronic function failure criticality.

Margin

The EMC risk R for an electronic to remain safe can be defined as:

$$R = \left(\frac{\mathcal{E} \cdot H}{p} \right) \quad (72)$$

\mathcal{E} is the noise source and H the transfer function between the external source \mathcal{E} and the power p transmitted across some electronics. The fact that an electronic function may be disturbed is a probabilistic function. If n is the level induced across the electronic, $C(n) = H \cdot \mathcal{E}$ is the probability that the constraint is higher than a medium level $C_x(n)$ and $t(n)$ is the probability function saying the chance there is for disturbing the electronic. This probability is equal to 1/2 if $P(R) = t(n) > t_m$. $C(n)$ translates the probability that the source is higher than a given amplitude n . Under the value C_m , the probability that $P(H \cdot \mathcal{E}) > n = 1$.

Margin

t is linked with p^{-1} : up to a given value t_x , the probability to disturb the electronic is 1, it means that the threshold power is very small. So, in correspondence with R we have the probability function to disturb the electronic: $C(n) \cdot t(n)$.

What kind of function may be take for C and t ?

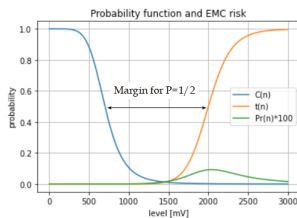
As input data we must recall that almost always **we don't know $t(n)$ accurately**, all the more when we work at high frequencies. At least, what we can estimate is the threshold t_x for which we are sure to disturb the electronic. After what we can estimate the speed with which the function t decreases to a probability of 1/2 to disturb the electronic.

Margin

Many function can be used to model C and t . We choose quite easy controlled functions:

$$C = \left[1 + \left(\frac{n}{C_x} \right)^a \right]^{-1}, \quad t = \left[1 + \left(\frac{t_m}{n} \right)^b \right]^{-1} \quad (73)$$

We obtain for example:



Margin

On this curve we read that the maximum probability $P(R)$ for disturbing the electronic is 10^{-3} .

$$P(R) = \max (C(n) \cdot t(n)) \quad (74)$$

It is clear that we have a big uncertainty on $P(R)$. But it's not a blocking point. In fact if your safety target is for example 10^{-6} , you can choose the parameters in order to be sure for obtaining $P(R)$ maximum. If the result is around or higher than the target $P(R) \approx 10^{-6}$, you must increase the system robustness (prevention: decrease \mathcal{E} or H or protection: decrease H or increase p). If $P(R) < 10^{-6}$ then you have reached the requirement compliance.

Margin

Added to the previous probability calculation, it's often interesting to take into account the source occurrence $P(\mathcal{E})$ during the system mission. Then:

$$P(R) = \max(C(n) \cdot t(n)) \cdot P(\mathcal{E}) \quad (75)$$

The safety target depends on the function criticality and depends on the kind of failure. We can have:

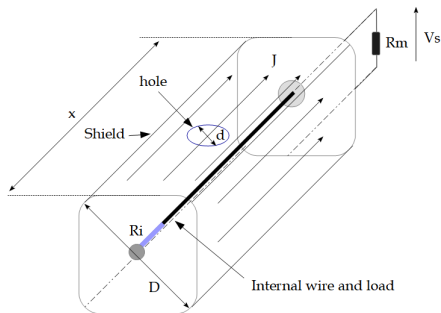
1. a disturbance existing during a limited time duration (smaller margin);
2. a disturbance that implies a reset of the electronics for starting next (medium margin);
3. a failure implying an electronic repair (higher margin).

Margin

- ▶ The EMC tests verify on some functional points and for various margins/constraints level that the system under test is robust in front of these environments.
- ▶ **Compliance with an EMC test does not constitute a demonstration of robustness.**
- ▶ **Only the design work in EMC supported by equations and related justifications can, with the verification that is the tests, constitute a start of proof and demonstration.**

Shield transfer impedance

Shield transfer impedance is defined by the ratio between the voltage measured across an external load, over the current running on the shield.



Shield transfer impedance

The electromotive force e induced on the internal line is given by:

$$e(s) = \int_x dx z_T(s) J(s) \quad (76)$$

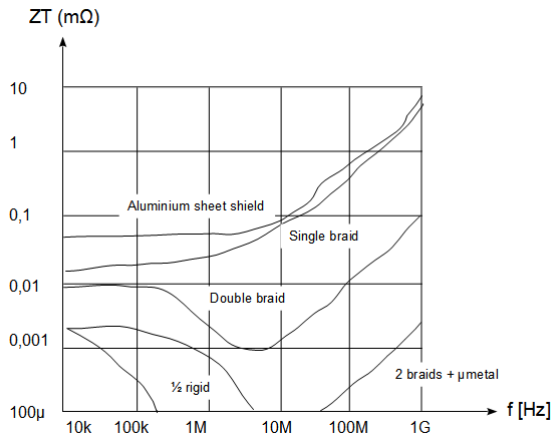
The transfer impedance z_T depends on the shield nature and thickness, and depends on the shield size and number of holes. We approximate:

$$z_T = R_T + sL_T, \quad R_T = \rho \frac{x}{S} \left(\frac{(1+i)t/k}{sh[(1+i)t/k]} \right), \quad L_T \approx n \frac{\mu_0 d}{6\pi^2 D^2} \quad (77)$$

S is the shield section, t the shield thickness, k the skin depth ($\sqrt{\pi f \mu \sigma^{-1}}$). n is the number of hole per meter. R_T and L_T are lineique parameters. For braids we just take $R_T = \rho x / S$.

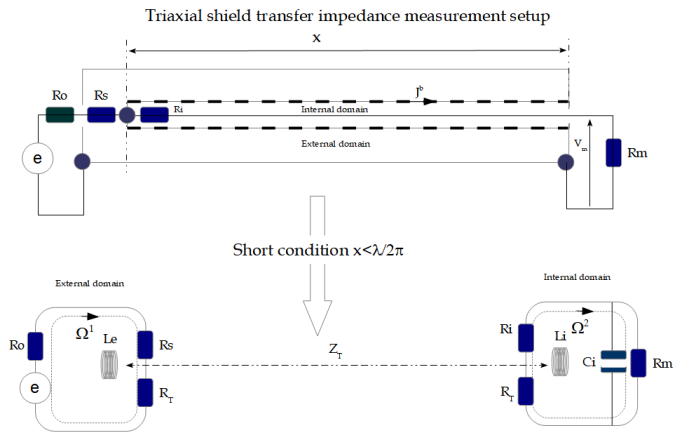
Shield transfer impedance

Some typical transfer impedance functions¹.



¹Extract from Mardiguian's EMC book

Shield transfer impedance

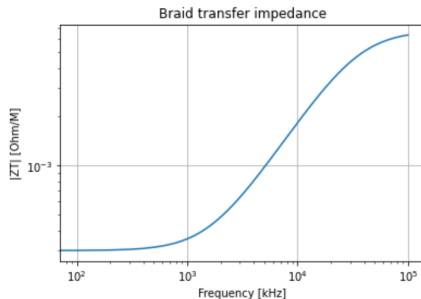


Shield transfer impedance

```

11 # components
12 De=10E-2 # external domain diameter
13 Di=4E-2 # internal domain diameter
14 Da=1E-3 # awm wire diameter
15 x=30E-2 # sample length
16 c=3E8
17 rho=1E-7
18 #
19 ne=60.*np.log(De/Di) # external domain propagation impedance
20 ni=60.*np.log(Di/Da) # internal domain propagation impedance
21 Ro=50.
22 Rs=50.
23 RT=1E-3*x # transfer resistance
24 LT=1E-10*x # transfer inductance
25 Le=nc/c*x
26 Li=ni/c*x
27 Ci=1./(ni*c)*x
28 Ri=rho*x/(np.pi*Di**2./4.)
29 Rm=50.
30 #
31 # frequency band
32 fmin=1E3
33 fmax=100E6
34 N=1000
35 df=(fmax-fmin)/N
36 #
37 # tabs
38 e=np.zeros(N,dtype=complex)
39 Omega=np.zeros(N,2,dtype=complex)
40 ZT=np.zeros(N,dtype=float)
41 absx=np.zeros(N,dtype=float)
42 #
43 for f in range(1, N):
44     s=1j*2.*np.pi*f*df
45     absx[f]=f*df*1E-3
46     ZT=RT+s*LT
47     zeta=[(Ro+Rs+RT+s*Le, -ZT)\
48           [-ZT, Ri+RT+Rm+s*Li]]
49     T=[[1, [0]]
50        [ymp, linalg.inv(zeta)]]
51     Omega[f,:]=np.transpose(np.dot(y,T))
52     ZTx[f]=abs((Ri+Rm)*Omega[f,1]/Omega[f,0])
53     ZTx[0]=ZTx[1]
54 plt.plot(absx,ZTx)
55 plt.title('Braid transfer impedance')
56 plt.xlabel('Frequency [kHz]')
57 plt.ylabel('|ZT| [Ohm/M]')
58 plt.grid(True)
59 plt.xscale('log')
60 plt.yscale('log')
61 plt.show()

```



Shield transfer impedance

Case $x > \lambda$. The process is similar to the one for crosstalk. We have for near and far-end crosstalks through the shield transfer impedance:

$$\Psi_{NE} = \frac{1}{2} \int_0^{X/v} d\tau v Z_T(\tau) J^1 e^{-s\tau}, \quad \Psi_{FE} = -\frac{1}{2} \int_0^X dx Z_T(x) J^1 e^{-sX/v} \quad (78)$$

if $\dot{z}_T(x) = 0$, these relations can be simplified:

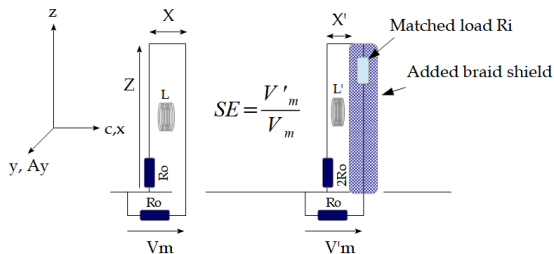
$$\Psi_{NE} = \frac{1}{2s} v Z_T J^1 (1 - e^{-sX/v}), \quad \Psi_{FE} = -\frac{1}{2} Z_T J^1 X e^{-sX/v} \quad (79)$$

Using the propagating current J^1 on the shield line as wave vector, we can include the shield transfer impedance coupling as components of γ in \mathbb{O} .

Shield transfer impedance

Shielding Effectiveness

The shielding effectiveness is another dimension for characterizing braid shields effectiveness, but only in far field condition. It is less well defined than the shield transfer impedance. The definition I suggest here, is for me the most accurate one, but not the most usual one. The classical method presents a change in the current modes induced on the measured line in the two conditions, with or without shielding braid.



Shield transfer impedance

Shielding Effectiveness

Let's compute the relation between SE and z_T in the short case $Z < \lambda$.
The electromotive force induced by the external field in both case is:

$$e = -sZX \frac{\partial A_y}{\partial x}, \quad e' = -sZX' \frac{\partial A_y}{\partial x} \quad (80)$$

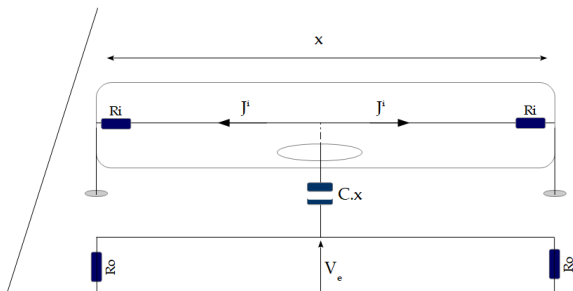
R_w being the wire resistance with $R_w \ll R_0$ and R_T the braid resistance with $R_T \ll R_0$, we have:

$$V_m = R_0 \left(\frac{e}{sL + 2R_0} \right), \quad V'_m = z_T \frac{R_0}{R_0 + R_i} \left(\frac{e'}{sL' + 2R_0} \right) \quad (81)$$

if $X' \approx X \Rightarrow e' \approx e$, $L' \approx L$, then $z_T \approx (R_0 + R_i) SE$.

Shield transfer impedance

Another parameter characterizes the coupling between a wire inside a braided shield and an environment: the transfer admittance: $Y^T \cdot x = \frac{J^i}{V_e}$. It's a near field interaction.



Field to line interaction

Equivalence principle: In Coulomb's gauge, the transverse far electric field is defined by

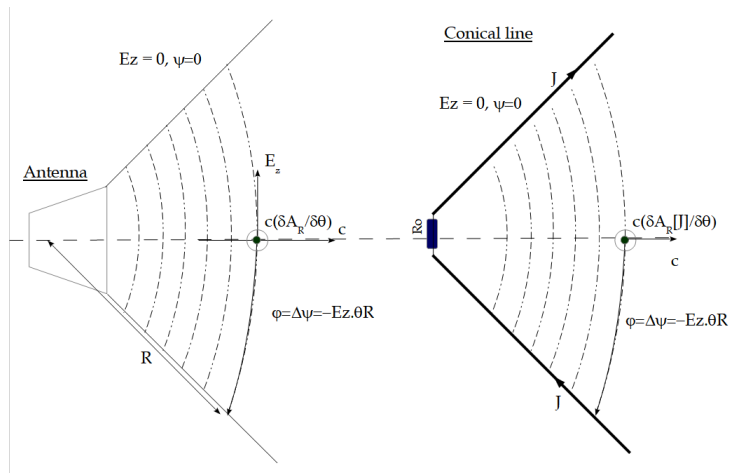
$$\mathbf{E}_z = -\frac{d}{dt}\mathbf{A}_z \quad (82)$$

but we have also: $\exists \varphi_z / \mathbf{E}_z = -\text{grad}_z(\varphi_z)$ so that $\text{grad}_z(\varphi_z) = d\mathbf{A}_z/dt$. Seen from the receiver, a scalar potential makes the same effect as a potential vector if:

$$\varphi_z = \int_z dz \frac{d\mathbf{A}_z}{dt} \quad (83)$$

We will be able to analyze the field to object interactions, including transmission lines, using this equivalence and all previous relations on crosstalk.

Field to line interaction



Field to line interaction

It's always possible to create a kind of strip-line in order to have the same fields and potentials as the ones radiated by an antenna. By increasing the width of its plate proportionally to the radius, it keeps $\eta_0 = \mu\nu G$ constant. The only difference comes from the possible near field interaction between the line plates and the illuminated object. The field amplitude decreases with the plates distance:

$$\vec{E}_\theta = -\frac{\vec{\varphi}_\theta}{\theta R} \propto \frac{1}{R} \quad (84)$$

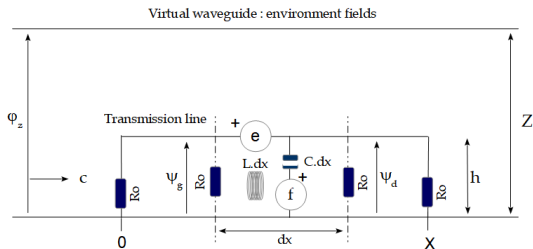
It keeps the same distance evolving as any antenna electric far field. The second transverse field comes from the vector potential rotational or obtained by

$$\vec{B}_y = -\frac{2}{c} \frac{\vec{\varphi}_\theta}{\Theta R} \quad (85)$$

Θ : maximum height antenne opening.

Field to line interaction

A field to line interaction becomes a waveguide to line coupling (this equivalence is also used in some EMC tests).



$$f = h \frac{\varphi_Z}{Z}, \quad e = dxh \frac{S}{c} \frac{\varphi_Z}{Z} \quad (86)$$

Note that f is a 1-form coming from \dot{A} , and e a 2-form coming from $\epsilon^{\gamma\alpha\sigma} \nabla_\alpha A_\sigma$.

Field to line interaction

First we consider the short case, $dx = X < \lambda$. We can use the overlay theorem. Looking to f , the voltage reported on R_0 is on both sides ($1/(sCX) \gg R_0$):

$$\Psi_g = \left(\frac{R_0}{2(R_0/2 + 1/(sCdx))} \right) hE \approx \frac{1}{2} sCX R_0 \frac{h}{Z} \varphi_z = \Psi_d \quad (87)$$

From e ($sLdx \rightarrow 0$):

$$\Psi_g = R_0 \left(\frac{e}{2R_0 + sLX} \right) \approx hX \frac{s\varphi_z}{2cZ} = -\Psi_d \quad (88)$$

Finally:

$$\Psi_g = \frac{1}{2} sCX R_0 \frac{h}{Z} \varphi_z + hX \frac{s\varphi_z}{2cZ}, \quad \Psi_d = \frac{1}{2} sCX R_0 \frac{h}{Z} \varphi_z - hX \frac{s\varphi_z}{2cZ} \quad (89)$$

Field to line interaction

Long case $x > \lambda$. Using the same technique that for the crosstalk and starting from our previous results with $X \rightarrow dx$ and integrating over time or space we find:

$$\Psi_g = \frac{1}{4} R_0 C \frac{h}{Z} \varphi_{z0} v (1 - e^{-2sX/v}) + \frac{1}{4} \frac{vh}{scZ} \varphi_{z0} (1 - e^{-2sX/v}) \quad (90)$$

$$\Psi_d = \frac{1}{2} \frac{h}{cZ} \varphi_{z0} X e^{-sX/v} - \frac{R_0 Ch}{2Z} c \varphi_{z0} X e^{-sX/v} \quad (91)$$

The emf induced on each cycles at the transmission line extremities are obtained as usually computing

$$e_{\sigma} = 2\Lambda_{\sigma}^{\alpha} (\gamma \dots \gamma) \mathcal{V}_{\alpha}.$$

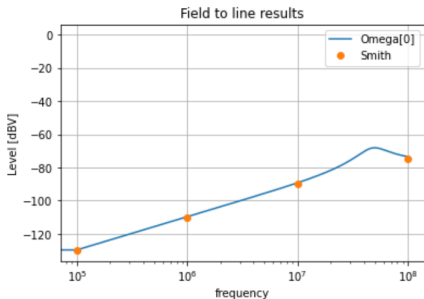
Field to line interaction

As an example we take a line of 1 mm diameter, 1 m long and $h = 10$ cm above the ground plane. The field coming on its left, \mathbf{E} parallel to h with \vec{c} colinear with the line wire. We compute both short and long cases. Results are compared with Smith Coupling of external electromagnetic fields to transmission line, page 25.

```

8 import numpy as np
9 import pylab as plt
10 #
11 X=1. # line length
12 h=10E-2 # line height over the ground plane
13 phi=1E-3 # line wire diameter
14 c=3EB
15 #
16 no=60.*np.log(4.*h/phi)
17 # short case
18 C=1./no.*X
19 L=no/c*X
20 Z1=1. # left load
21 Z2=635. # right load
22 #
23 Zc=Z1*Z2/(Z1+Z2)
24 #
25 fmin=10E3
26 fmax=1E8
27 N=1000
28 df=(fmax-fmin)/N
29 Eo=1.
30 #
31 Omega=np.zeros((N,2),dtype=complex)
32 V1=np.zeros(N,dtype=float)
33 V2=np.zeros(N,dtype=float)
34 freq=np.zeros(N,dtype=float)
35 I=np.zeros(N,dtype=float)
36 freqSmith=[1E5, 1E6, 1E7, 1E8]
37 Smith=[-130, -110, -90, -75.]
38 for q in range(1,N):
39     f=q*df
40     freq[q]=f
41     sc=1+2.*np.pi*f
42     #
43     zeta=[[Z1+1./sc, 1./sc], [1./sc, Z2+1./sc]]
44     q=x*h*c+Eo/c
45     f=h*Eo
46     T=[sc,f,[]]
47     y=np.linalg.inv(zeta)
48     Omega[q,:]=np.transpose(np.dot(y,T))
49     V1[q]=abs(Z1*Omega[q,0])
50     V2[q]=abs(Z2*Omega[q,1])
51     I[q]=20.*np.log10(abs(Omega[q,1]))
52 #
53 # plt.plot(freq,V1,label='V1')
54 # plt.plot(freq,V2,label='V2')
55 #
56 plt.plot(freq,I,label='Omega[0]')
57 plt.plot(freqSmith,Smith,'o',label='Smith')

```

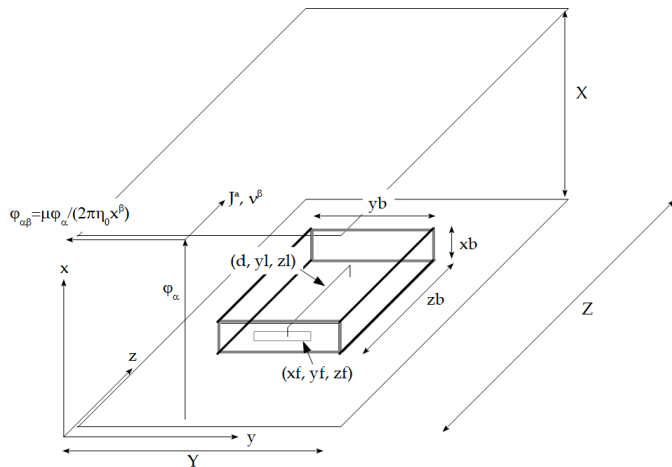


Field to cavity interaction

We can use the same artefact as for lines, studying the interaction between a large strip-line and a cavity, neglecting near-field interactions. This approach has many advantages:

- ▶ it gives the way for comparing easier experiments and computations,
- ▶ it simplifies the understanding and use of potentials rather than fields,
- ▶ it fixes clearly the field limit conditions, distribution and modes.

Field to cavity interaction



Field to cavity interaction

For this case we introduce the potential $\varphi_{\alpha\beta}$, associated with $\partial A_\alpha / \partial x^\beta$ (as said Feynman, *there's always more than one way to do it*).

How may we easily evaluate the constraint induced on the internal wire by the external field, in the case $z_b < \lambda$?

If we can evaluate the current J^a induced by the field on the cavity walls, we can compute the internal constraint using the cavity transfer impedance z_T . In our experiment,

$$J^a = \frac{\varphi_\alpha}{\eta_{a\alpha}} \Rightarrow \varphi_{\alpha b} \leftrightarrow \langle B_{\alpha b} \rangle = \mu_{\alpha a} \frac{J^a}{2\pi X^b / 2} = \mu_{\alpha a} \frac{\varphi_\sigma}{\pi X^b \eta_{\sigma a}} \quad (92)$$

so, knowing $\langle B_{\alpha b} \rangle$ defines

$$J^a = \frac{\pi X^b}{\mu_{\alpha a}} \langle B_{\alpha b} \rangle \quad (93)$$

Field to cavity interaction

Knowing J^a , the electromotive force e_l induced on any link inside the cavity is given by:

$$e_l = (z_T)_{la} J^a \quad (94)$$

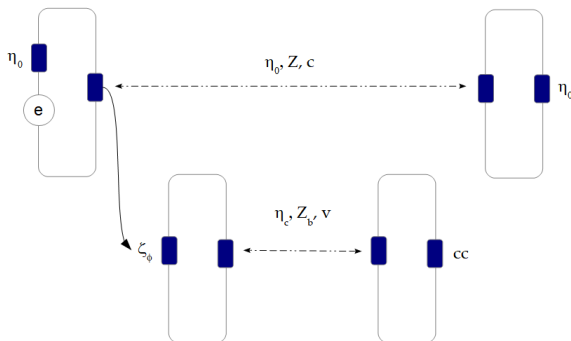
with (ϵ being the cavity walls depth)

$$R_T(DC) \approx \rho \frac{z_b}{(2y_b + 2x_b)\epsilon}, \quad L_T \approx \mu_0 \frac{\sqrt{x_f^2 + y_f^2}}{6\pi^2 \sqrt{x_b^2 + y_b^2}} \quad (95)$$

$$z_T \approx R_T + sL_T$$

Field to cavity interaction

What about the long case $z_b > \lambda$? The couplings between the field modes inside and outside the cavity only come from the openers. The openers appear as an intermediate circuit ζ_ϕ between the environment (the external line) and the cavity inside. We see that on the problem graph.



Field to cavity interaction

The opener impedance ζ is in our example
 $\zeta^{-1} = 1/R_f + sC_f + 1/(sL_f)$, with

$$R_f \approx \rho \frac{x_b}{(y_b - y_f)\epsilon}, \quad C_f = \epsilon_0 \left(\frac{y_f \epsilon}{x_f} \right), \quad L_f = \frac{\mu_0}{4\pi} \left(\frac{y_f x_f}{\epsilon} \right) \quad (96)$$

The potential φ_α running on the external line creates a mmf J^0 (current source or magnetomotive force) on the opener capacitor $sC_f x_f \varphi_\alpha / X$. Then, if the wavelength is short enough to respect $\lambda \leq 2y_b$, the potential developed across the opener will be transmitted to the first potential mode of the cavity. This potential is maximum at the opener resonance.

Field to cavity interaction

The potential distribution inside the cavity depends on the transverse mode number n given by

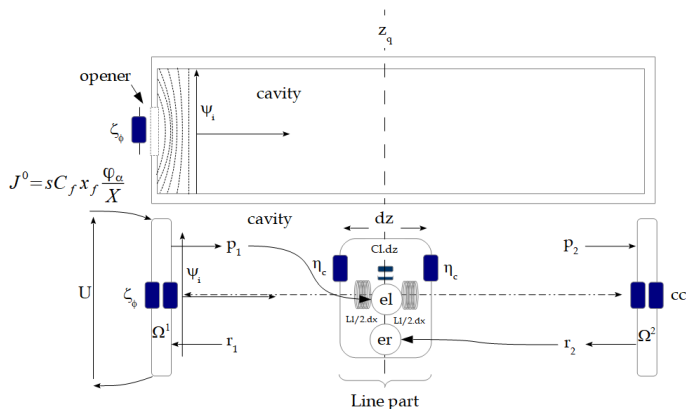
$$n = \text{int} \left(2 \frac{y_b}{\lambda} \right) \quad (97)$$

and depends on the limit conditions and γ . The potential anywhere on Z is obtained using the wave coming from ports p_1 and r_2 delayed. With the initial condition

$$\mathcal{V}_1(\tau = 0) = \mathcal{J}^0 \frac{\zeta_\phi}{\zeta_\phi + \eta_c} \quad (98)$$

Field to cavity interaction

A portion dz of the line inside the cavity capture the field like if it was a sensor which equivalent schematic is a portion dz of T telegraph's model, where each load on the edge has the emf coming from p_1 on the left and r_2 on the right.



Field to cavity interaction

Once we have computed the environment wave $\mathcal{V}(N\tau)$ (the cavity potential wave), at any abscissa z_q we have:

$$e_l = 2\Lambda_1^\sigma \frac{x_l}{x_b} \mathcal{V}_\sigma(N\tau) \exp\left(-s \frac{z_q}{v}\right), \quad e_r = 2\Lambda_4^\sigma \frac{x_l}{x_b} \mathcal{V}_\sigma(N\tau) \exp\left(-s \frac{Z - z_q}{v}\right) \quad (99)$$

so that the potential transmitted on both sides is:

$$\Psi_l = sC_l dz \frac{\eta_c}{2} (e_l - e_r) \quad (100)$$

Each contributor can be operated separately. The wave propagating in the same direction as the source wave being integrated in the space domain z , and the opposite in the time domain τ . Same mechanisms must be realized with $\varphi_{\alpha\beta}$.

Field to cavity interaction

Conclusion for H at high frequencies When many cavities (i.e. equipments, structures, ...) are imbricated at high frequencies ($x > \lambda$), the previous mechanisms seen give the way for solving any situation. It is important, as the structures behavior are in most cases linear, to write the system transfer function H , then by convolution, to apply it to various excitations.

In all these cases, the establishment of \mathbb{O} becomes a major step for obtaining the equations in \mathbb{M} . The power of our method comes from the fact that you only have to know the initial wave vector expression, then to be able to solve the whole problem, with all the waves round trips.

Radiated emissions

In near field interaction, emission or radiation is just a choice of source location in the system graph, the source coming from the environment or from internal noisy components. But the graph remains the same (in other words, the impedance metric ζ remains unchanged while just the source T changes).

So our purpose concerns only the radiated emission in far field, i.e. $x > \lambda$. The basic relation for this radiation is the vector potential emitted value in the Coulomb's gauge:

$$d\vec{A}_\sigma(r) = \mu_{\sigma\alpha} \left(\frac{I^\alpha dx^\alpha}{4\pi r} \right) \exp\left(-s\frac{r}{c}\right) \vec{b}_\alpha \quad (101)$$

\vec{b}_α is the base vector in the local direction of the current, perpendicular to the line of sight.

Radiated emissions

It's interesting to work in \mathbb{M} rather than \mathbb{B} and to evaluate the radiated emission of a cycle. By equating a cycle with a loop and with the cycle radius $a \ll r$, we find in antenna's book for an azimuthal referential (r , θ , height, ϕ , azimuth):

$$A_{\sigma} \vec{u}_{\phi}(r) = \frac{\mu_{\sigma\alpha} a^2 \Omega^{\alpha}}{4} \frac{s}{rc} e^{-s \frac{r}{c}} \text{Sin}(\theta) \vec{u}_{\phi} \quad (102)$$

r is the distance from the observation point to the loop center. We have also:

$$A_{\sigma} \vec{u}_r(r) = \vec{0}, \quad A_{\sigma} \vec{u}_{\theta}(r) = \vec{0} \quad (103)$$

The radiation resistance of such a loop is²

$$R_l = 320\pi^2 \frac{a^2}{\lambda^2} \quad (104)$$

²see Balanis 2005, page 238

Radiated emissions

Once the cycles defined for a circuit, its radiated emissions are implicitly defined also, using relation 102.

For the radiation coming from an unmatched open transmission line of length X , we already know the emission of the two cycles at each extremity of the line. In $\mathcal{O}(J)$ we know the current at any point of the line. We can approximate the line as a succession of cycles of length $2X/\lambda$. It remains just to add vectorially the emissions of these cycles to the one of the limit conditions.

Ground plane

Perhaps the only EMC rule that must be memorized is:

In EMC conception, I only do what I can modeling.

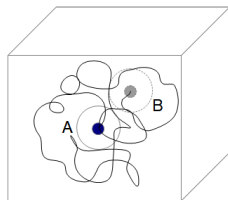
Knowing this rule, the ground plane role becomes clear. We use ground planes for being able to model various electromagnetic situations. The ground plane is an electrical referential giving access to all the models in relation with waveguides, and there are many waveguides in any electronics for linking all the functions.

An electronic may be compliant with all the requirements without any EMC conception study. Let's looking to that through an illustrative example.

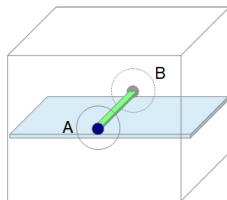
And remember what said Sherlock Holmes if you do experiments:
correlation does not imply causation. An experiment is not a proof.

Ground plane

An engineer has to realize an equipment having one input and one output. Its mission is to transmit a signal as perfectly as possible in a wide frequency range. A first engineer pulls a wire anyhow: option 1. A second engineer uses a high frequency printed circuit board with a matched strip-line: option 2.



Option 1



Option 2

Ground plane

In option 1, the conception time was 10 minutes. But:

- ▶ the probability for it respects the requirements is near to zero;
- ▶ more than all, if it doesn't reach the requirements target, it's impossible to understand how it works, and so to repair the system.

In option 2, the conception time was 1 hour, but:

- ▶ the probability for it respects the requirements is near to one;
- ▶ if it doesn't reach completely the requirements target, we have the model for improving the equipment and to correct its defaults.

Shields and ground should not be designed intuitively, but on the base of modeling.

Components EMC models

The electronic components models are separated in three categories:

- ▶ the models "in-band" when the frequency band is in the self frequency band of the component;
- ▶ the emission models, giving the noise emitted by the component itself;
- ▶ the models "out-band" when the frequency band is out off the self frequency band of the component.

Informations (like the susceptibility threshold) for the first and second category can be extracted from IBIS and ICEM³ models, or datasheets. In the second case, the information is often unknown.

³Levant, J. L., Ramdani, M., Perdriau, R., & Drissi, M. H. (2005, February). ICEM model. In 16th International Zurich Symposium and Technical Exposition on Electromagnetic Compatibility (pp. 631-658). IEEE.

Components EMC models

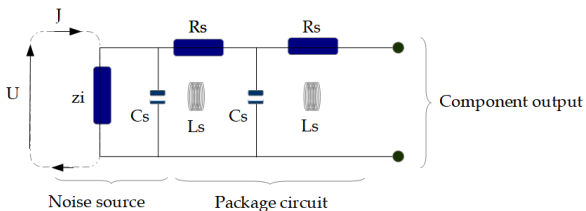
The concept of in and out-band is true even for passive components. Following this principle:

- ▶ a resistance becomes a resistance in parallel with a capacitance (the inductance depends on the whole circuit);
- ▶ an inductance becomes an inductance in parallel with a capacitance and in serie with a resistance;
- ▶ a capacitor becomes a capacitance in parallel with a resistance and in serie with another resistance.

these "high frequency models" must be taken into account for obtaining correct modeling and comparisons with measurements.

Components EMC models

In emissions, the global approach consider the waveforms spectra generated by the components. They can be seen as voltage or current generators. In parallel to these sources, an impedance represents the component circuit self impedance, then a network model the component package circuit including inductances and capacitors. Such a source of noise is associated with each component output.

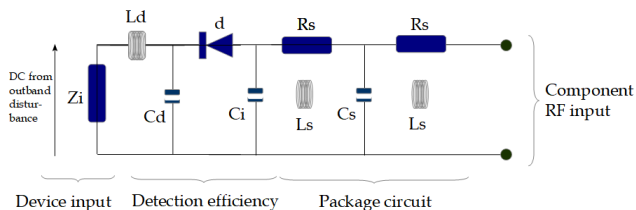


Example of ICEM model

Components EMC models

Out-band EMC model.

The detection effect transforms the out-band energy of the constraint into in-band energy that can disturb the component. For modeling this kind of mechanism, the simplest way consists in adding a non linear layer in front of the functional model. This added layer wears the energy transformation and filtering.



Components EMC models

The detection efficiency De is defined by:

$$De = \left| \frac{V_{DC}}{V_{HF}} \right| \quad (105)$$

Is typically around $[10^{-1}, \dots, 10^{-2}]$. For digital doors, some volts transmitted on a device input can create a change of state. A lower level can delay the time of change of state front (see⁴).

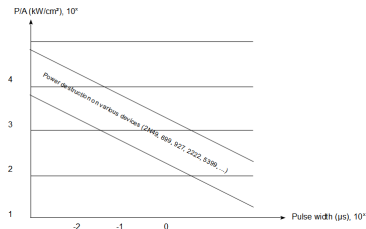
⁴Maurice, O., & Pigneret, J. (1997, September). Digital circuit susceptibility characterization to RF and microwave disturbances. In RADECS 97. Fourth European Conference on Radiation and its Effects on Components and Systems (Cat. No. 97TH8294) (pp. 178-181). IEEE.

Components EMC models

Components destruction can come from two effects:

- ▶ Joule effect that burn out the junctions;
- ▶ overvoltage which creates a spark in the junction and destroy them.

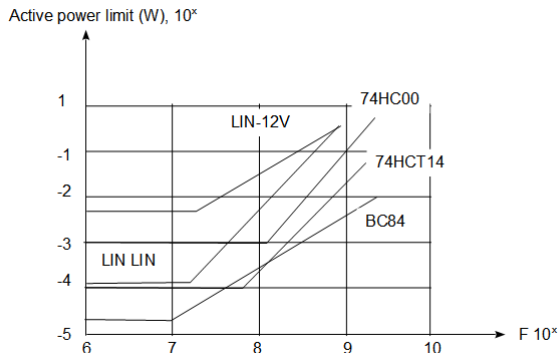
Next curve extract from⁵ shows the power per junction section that destroy the junction, compared with experiments with standard functions.



⁵Orvis, W. J., Khanaka, G. H., & Yee, J. H. (1984). A Review of the Physics and Response Models for Burnout of Semiconductor Devices.

Components EMC models

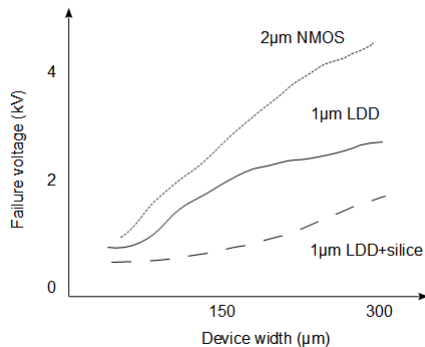
Other measurements was done with more modern devices and in frequency⁶



⁶Lafon, F., Ramdani, M., Perdriau, R., Drissi, M. H., & de Daran, F. (2009, July). An industry-compliant immunity modeling technique for integrated circuits.

Components EMC models

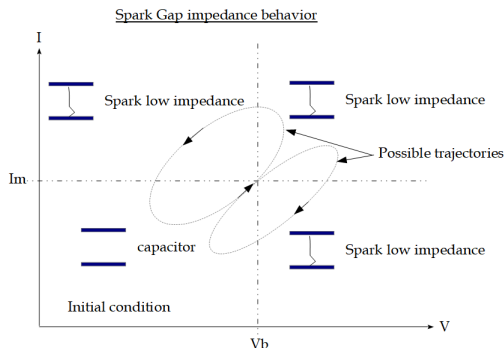
The maximum overvoltage supports by a component can be evaluated reading its breakdown voltage in ESD on the datasheet.



Components EMC models

Spark Gap Modeling

There are various working modes for spark gap. One we are looking for flows next description in impedance depending on the open voltage and short circuit current across them:



Components EMC models

Each of the fourth parts of impedance behavior can be attached with a combination of domain function for these parameters. We write (is R_{cc} is the impedance of the spark gap in active mode, and C its capacitance in non active mode):

$$Z_{sp} = \mathcal{D}_{<I_m}^I \mathcal{D}_{<V_b}^V \left(\frac{1}{sC} \right) + \mathcal{D}_{>I_m}^I \mathcal{D}_{<V_b}^V (R_{cc}) + \mathcal{D}_{<I_m}^I \mathcal{D}_{>V_b}^V (R_{cc}) + \mathcal{D}_{>I_m}^I \mathcal{D}_{>V_b}^V (R_{cc}) \quad (106)$$

V and I are the open circuit voltage and short-circuit current peak values in the time domain acquired across the spark gap but without its presence.

Diodes and others non linear components can be modeled using the same principles with parametrized domain function for their impedances.

Components EMC models

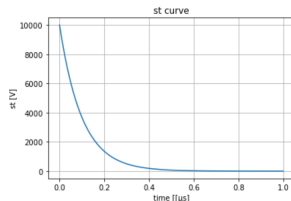
We can program a spark gap response inside a simple circuit including the spark gap and the signal generator.

1. Source signal creation in the time domain.

```

8 import numpy as np
9 import pylab as plt
10 #
11 # create source signal in time
12 N=1000
13 st=np.zeros(N,dtype=float)
14 at=np.zeros(N,dtype=float)
15 T=1E-3
16 dt=T/N
17 #
18 for n in range(N):
19     t=n*dt
20     at[n]=t*1E3
21     st[n]=10E3*np.exp(-1E4*t)
22 plt.plot(at,st)
23 plt.grid(True)
24 plt.xlabel('time [[us]')
25 plt.ylabel('st [V]')
26 plt.title('st curve')
27 plt.show()

```



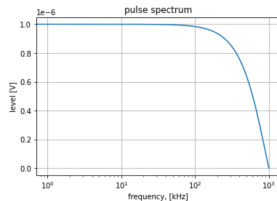
Components EMC models

2. Signal development as a pulse sequence.

```

28 # st development in pulse sequence
29 pu=np.zeros(N,dtype=complex)
30 stc=np.zeros(N,dtype=float)
31 ax=np.zeros(N,dtype=float)
32 for f in range(1,N):
33     w=2.*np.pi*f/T
34     s=1j*w
35     ax[f]=w/(2.*np.pi)/1E3
36     #
37     pu[f]=1./s*(1.-np.exp(-s*dt))
38 pu[0]=pu[1]
39 plt.plot(ax,abs(pu))
40 plt.title(u'pulse spectrum')
41 plt.grid(True)
42 plt.xscale('log')
43 plt.xlabel(u'frequency, [kHz]')
44 plt.ylabel(u'level [V]')
45 plt.show()

```



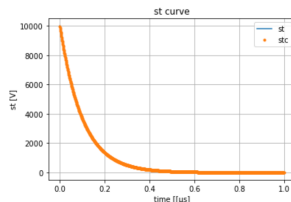
Components EMC models

3. Signal development as a pulse sequence II, verifying correspondence and working in \mathcal{W} .

```

46 pui=np.zeros(N,dtype=complex)
47 F=2.2E3/(dt)
48 for n in range(N):
49     t=n*dt
50     for f in range(N):
51         pui[f]=st[n]*pui[f]
52     stc[n]=abs(np.sqrt(np.dot(np.conj(pui),F*pui)))
53 plt.plot(at,st,label='st')
54 plt.plot(at,stc,'.',label='stc')
55 plt.grid(True)
56 plt.xlabel('time [[μs]]')
57 plt.ylabel('st [V]')
58 plt.title('st curve')
59 plt.legend()
60 plt.show()

```



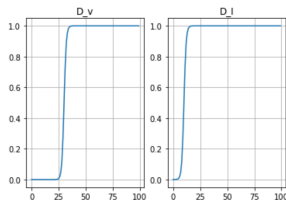
Components EMC models

4. Spark Gap modeling: time domain functions (sigmoides) for the spark gap non linear impedance.

```

62 # spark gap modeling
63 Rcc=1E-3 # SG closed impedance
64 C=1E-12 # SG open capacitance
65 Ro=10. # generator self impedance
66 Vb=3E3 # SG breakdown voltage
67 Im=100. # SG minimum current
68 # SG impedance construction
69 zsg=np.zeros((100,100),dtype=complex)
70 D_v=np.zeros(100,dtype=float)
71 D_I=np.zeros(100,dtype=float)
72 for dv in range(100):
73     v=dv*1E2
74     D_v[dv]=(1.+np.exp(-1E-2*(v-Vb)))*(-1.)
75     for di in range(100):
76         I=di*10.
77         D_I[di]=(1.+np.exp(-1E-1*(I-Im)))*(-1.)
78     #
79 plt.subplot(1,2,1)
80 plt.plot(D_v)
81 plt.title('D_v')
82 plt.grid(True)
83 plt.subplot(1,2,2)
84 plt.plot(D_I)
85 plt.grid(True)
86 plt.title('D_I')
87 plt.show()

```



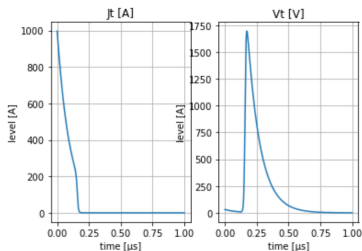
Components EMC models

5. Spark Gap modeling

```

89 J=np.zeros(N,dtype=complex)
90 Jt=np.zeros(N,dtype=float)
91 Vt=np.zeros(N,dtype=float)
92 dw=np.zeros(N,dtype=complex)
93 diw=np.zeros(N,dtype=complex)
94 z=np.zeros(N,dtype=float)
95 iz=np.zeros(N,dtype=float)
96 #
97 Rinfy=1E6 # open voltage measurement
98 Rzero=1E-3 # short circuit measurement
99 for n in range(N):
100     t=n*dt
101     puist[n]*pu
102     for f in range(N):
103         w=2.*np.pi*f/T
104         s=1j*w
105         #
106         zeta=Ro+zsg
107         # 1. open voltage and short circuit acquisition
108         zetao=Ro+Rinfy
109         diw[f]=1E6*puif[f]/zetao
110         zetacc=Ro+Rzero
111         diw[f]=puif[f]/zetacc
112         iv[n]=abs(np.sqrt(np.dot(np.conj(dw),F*dw)))
113         ii[n]=abs(np.sqrt(np.dot(np.conj(diw),F*diw)))
114         for f in range(1,N):
115             w=2.*np.pi*f/T
116             s=1j*w
117             #
118             D_vs=(1.+np.exp(-1E-2*(iv[n]-Vb)))*(-1.)
119             O_Is=(1.+np.exp(-1E-1*(ii[n]-Im)))*(-1.)
120             zsg=(D_vs+O_Is)*Rcc+(1.-D_vs)*(1.-O_Is)*1./(s*c)
121             J[f]=puif[f]/(Ro+zsg)
122             J[0]=J[1]
123             Ji[n]=abs(np.sqrt(np.dot(np.conj(J),F*J)))
124             Vt[n]=st[n]-Ro*Jt[n]
125 plt.subplot(1,2,1)
126 plt.plot(at,Jt)
127 plt.title(u'Jt [A]')
128 plt.grid(True)
129 plt.xlabel(u'time [us]')
130 plt.ylabel(u'level [A]')
131 plt.subplot(1,2,2)
132 plt.plot(at,Vt)
133 plt.title(u'Vt [V]')
134 plt.grid(True)
135 plt.xlabel(u'time [us]')
136 plt.ylabel(u'level [A]')
137 plt.show()

```



Components EMC models

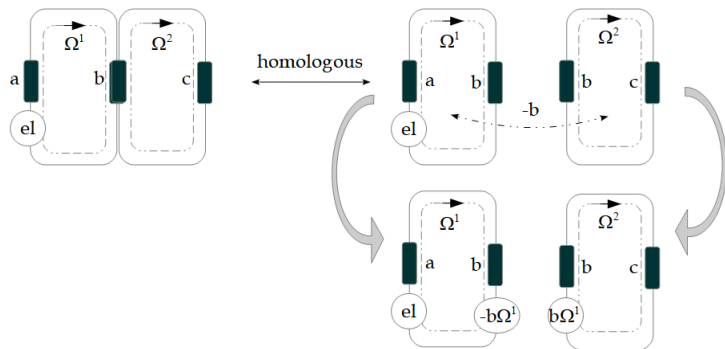
In the previous example we neglect the capacitor voltage memory, knowing its weak value (1 pF).

Usually, from one time t_1 to the next one t_2 , final voltage across capacitances and currents in inductances at end of $t_1: t_1 + dt$, are initial conditions for t_2 with:

$$U(t_2, C) = \frac{U_C(t_1 + dt)}{s} = \frac{1}{s} \left(\frac{1}{sC} \Omega \right) \quad (107)$$

$$U(t_2, L) = \frac{U_L(t_1 + dt)}{s} = \frac{1}{s} (sL\Omega) = L\Omega \quad (108)$$

Reduction & Diakoptic



Reduction & Diakoptic

We imagine a network where only the first cycle has its own source. This can be written:

$$\begin{cases} \mathbf{e}_1 = \zeta_{11}\Omega^1 + \zeta_{1\alpha}\Omega^\alpha \\ 0 = \epsilon_{\sigma 1}\Omega^1 + \beta_{\sigma\alpha}\Omega^\alpha \end{cases} \quad (109)$$

So, $-\epsilon_{\sigma 1}\Omega^1 = \beta_{\sigma\alpha}\Omega^\alpha$ which leads to $\Omega^\alpha = -\epsilon_{\sigma 1}(\beta_{\sigma\alpha})^{-1}\Omega^1$ and:

$$\mathbf{e}_1 = \left(\zeta_{11} - \zeta_{1\alpha} \frac{\epsilon_{\sigma 1}}{\beta_{\sigma\alpha}} \right) \Omega^1 \quad (110)$$

Then the second equation of system (105) can be solved with $-\epsilon_{\sigma 1}\Omega^1$ as source covector.

System to Graph

When we go from a real system to its representation as a graph, we consider their similar shapes, in a sense that is much broader than homeomorphism.

An everyday example is a thick letter, let's take **X** and its corresponding thin one *X*.

We can shrink **X** to *X* by sliding each point of **X** – *X* into *X* along the line segment that contains it. Points that are already in *X* do not move.

System to Graph

A deformation retraction $f_\alpha : \mathbf{X} \rightarrow X$, $X < \lambda$ is a homotopy. If X can be cutted out using a parameter α : $X = \{x_\alpha\}$. Then

$$\forall x_\alpha, \exists \Phi \in \mathbb{R}^2 / \mathbf{X} = \{\Phi(x_\alpha)\}$$

So that $\forall \Phi(x_\alpha)$, $df_\alpha : \Phi(x_\alpha) \rightarrow x_\alpha$, $\forall \alpha$, and $f_\alpha = \cup_\alpha df_\alpha$.

Using this homotopy, we can shrink all volumes V representing a real system made of a succession of faces F along a curve $L \in$ a graph G : $V = F \times L$ associated with an edge $\varphi : L \rightarrow b \in \mathbb{B}$ through the map φ . As a consequence, all the other elements of the cellular topology (chain space) $\mathcal{T}(\varphi)$ are defined, in correspondance with the graph G deduced from the system.

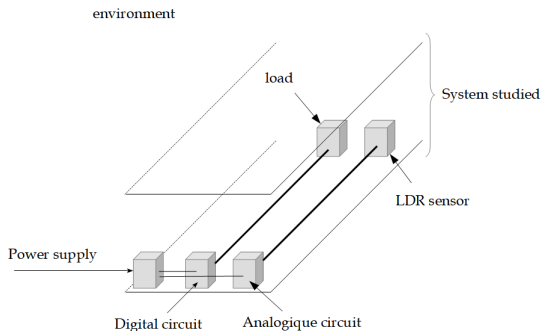
Complete study of a realistic case

What are we going to do?

1. We will define and test an electronic system as a prototype.
2. Having done this first conception, we are able to define its environment and mission and requirements.
3. Then we can classify the signals by families., and do the same for the environment signals.
4. We will construct a major tool; the interaction matrix.
5. And compute the interactions with risks, for deciding of the final design for a known safety target.

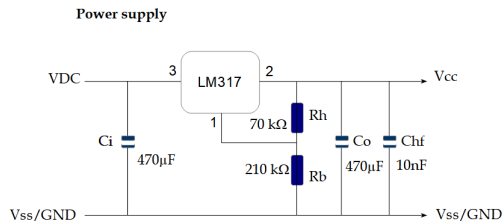
Complete study of a realistic case

The system we consider includes two electronic functions and a power supply. It may be embedded in a vehicle, and for this reason is not shielded. Next figure shows an illustration of this system.



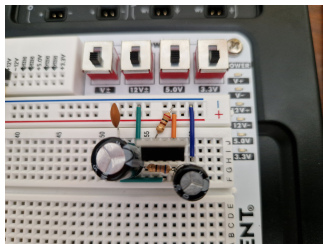
Complete study of a realistic case

We will make each function step by step, in order to verify it works. Then we will begin the EMC study itself. It's clear that the good working of the electronic is a preliminary to any EMC analysis. Let's begin by the power supply. Next figure gives its schematic.



Complete study of a realistic case

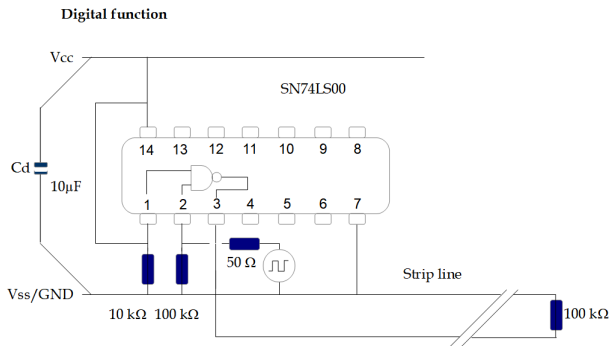
Next photography shows a top view of the setup of this circuit on a breadboard.



The measured input voltage is 11,96 V while the output is 4,91 V.

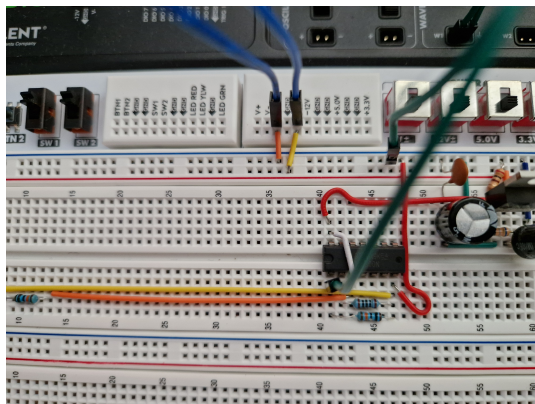
Complete study of a realistic case

This power supply is here for giving energy to the digital circuit:



Complete study of a realistic case

Once made, our set-up has this appearance:



Complete study of a realistic case

The input and output signals measured are:

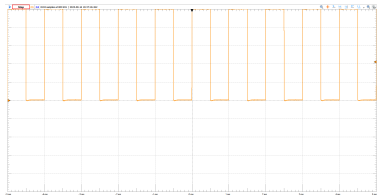


Figure: Input

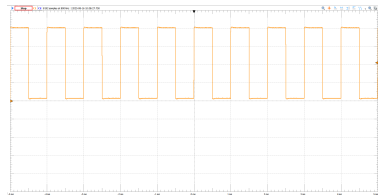


Figure: Output

Complete study of a realistic case

The simple digital function we realize creates the complementary of the input digital signal. If the input signal is:

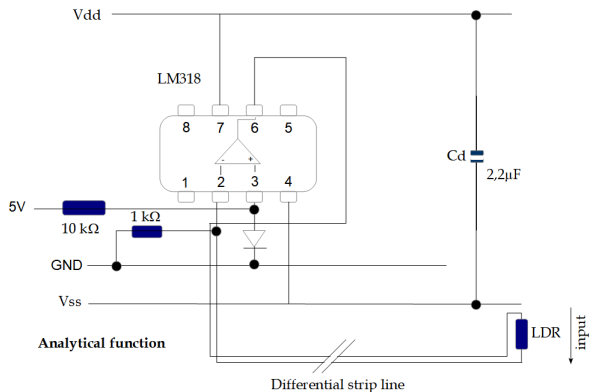
$$i(s) = \frac{\alpha}{s} \left(\frac{1 - \exp(-s\tau)}{1 - \exp(-sT)} \right), \quad \tau \in [\epsilon, T] \quad (111)$$

As the signal is periodic, the NAND door appears as a simply delaying door with some losses β and noise \tilde{n} :

$$o(s) = \beta \frac{\alpha}{s} \left(\frac{1 - \exp(-s\tau)}{1 - \exp(-sT)} \right) \exp\left(-s \frac{T}{2}\right) + \tilde{n}, \quad \tau \in [\epsilon, T] \quad (112)$$

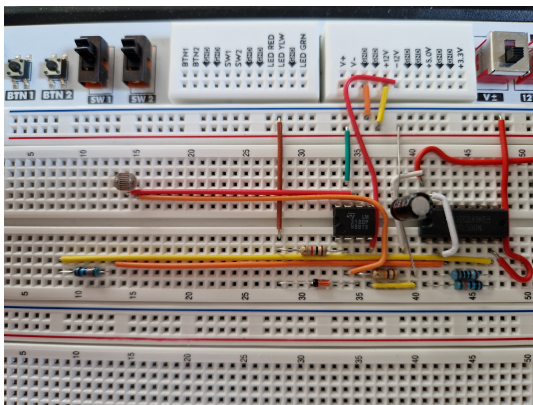
Complete study of a realistic case

The analytical function graph is given here below:



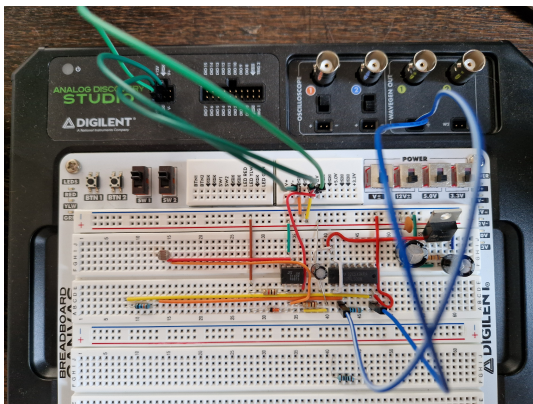
Complete study of a realistic case

Set-up picture:



Complete study of a realistic case

After some minor corrections (inversion of LS74 inputs), the final circuit is:



Complete study of a realistic case

The measured analogic function output follows next curve

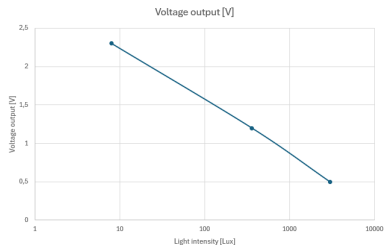
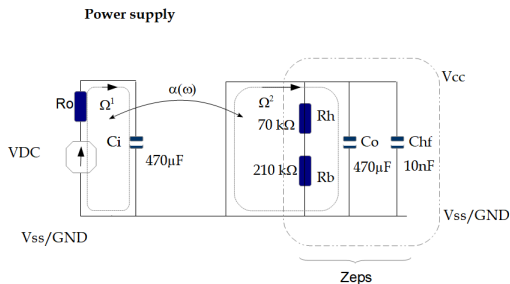


Figure: Tab 1

Next step for us consists in drawing the graph of each function, for being able to write the whole problem impedance metric.

Complete study of a realistic case

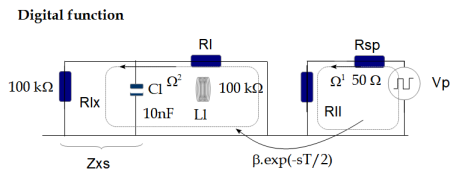
For the power supply, using parallel equivalent impedance Z_{eps} for the regulator output, we obtain a two cycles graph shown below:



The coupling α is bidirectional, including the back capacitive like coupling, important for conducted emissions.

Complete study of a realistic case

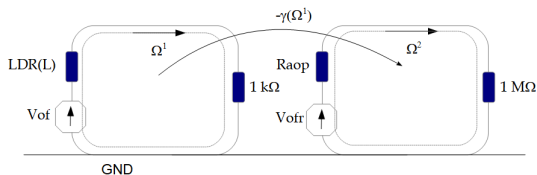
For the digital function, the delay function transmits the signal into a short line model loaded by the $100\text{ k}\Omega$ resistor.



The coupling *beta* is here for translating the first cycle current into the adequate second cycle emf.

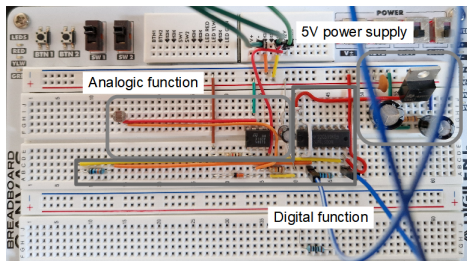
Complete study of a realistic case

The low level signal coming from the LDR component is amplified by the operational amplifier by what we write symbolically $-\gamma(\Omega^1)$. In fact the emf transmitted is a function like $(a - \gamma.\Omega^1)$. The simplest way consists in reporting the offset voltage V_{of} on the receiving cycle V_{ofr} .



Complete study of a realistic case

Having the functions graphs, we can take a view to our system in order to evaluate the coupling inside it (note that we don't take care of our layout showing a bad example, not to be followed!).



Having all this material, next step consists in defining the environment.

Complete study of a realistic case

This environment definition should be a customer requirement. For our example we will consider only one external source: lightning. One target of the EMC test is to transform the environment constraint into possible experiments to realize. If our electronic is submitted to an external lightning field, we should be able to simulate this by applying a similar field coming from a strip line located up to the circuit. Using the same electrical reference as the circuit itself, we just have to apply a voltage V so that for a distance h to the components, the electric field will be V/h , or in other words if J is the line current, the magnetic field seen by the components is $\mu_0 J / (2\pi h)$.

Complete study of a realistic case

Let's imagine a lightning spark at 1000 [m] with a 200 kA intensity, it leads to a magnetic field of amplitude 40 [μT]. For example, if we create a loop of $h = 10$ cm height on the circuit board, for reaching $|B|_0 = 40$ [μT] we need a current of:

$$J \approx 2\pi h |B|_0 = 0,025 [mA] \quad (113)$$

It's clear that it's easier to create such a source than the original one of 200 [kA]!

We can now begin the **interaction matrix construction**.

Complete study of a realistic case

The interaction matrix gives the engineer the best way for managing their EMC conception. It consists of lines wearing the sources of noises, including the ones of the circuit itself, and rows having the same function completed by other sensitive receivers. In our case we can list three functions :

- ▶ the power supply;
- ▶ the digital function;
- ▶ the analogic function.

As emitter we have in more the lightning external source. And as receiver, we consider one standard radiofrequency receiver. Finally, the interaction matrix has 4 lines and 4 rows: let's draw it.

Complete study of a realistic case

At the beginning, the matrix is empty and has next appearance:

	A	B	C	D	E
1		Power supply	digital F	analogic F	RF receiver
2	Power supply				
3	digital F				
4	analogic F				
5	Lightning (through added loop)				

Our first work consists in coloring in grey the self interactions for which we make the assumption that the circuit cannot disturb itself, or it won't work. Then we gray each interaction that we consider presenting no risk. This leads to the new interaction matrix:

Complete study of a realistic case

	A	B	C	D	E
1		Power supply	digital F	analogic F	RF receiver
2	Power supply				
3	digital F				
4	analogic F				
5	Lightning (through added loop)				

The lightning cannot disturb the RF receiver as they both belong to the environment. As the power supply is a continuous signal, it cannot disturb the other signals. The analogic signal is a low level, low frequency signal, for the same reason. So it remains the interaction between lightning and all the circuit signals, and the interaction between the digital signal and the power supply (conducted noise emission risk), the analogic signal (it can induce noise) and the RF receiver (radiated emissions).

Complete study of a realistic case

Finally, our EMC conception work consists in studying 6 interactions: 3 interactions concern the circuit susceptibility. and three others concern the circuit emissions. We can number the interactions to organize the work:

	A	B	C	D	E
1		Power supply	digital F	analogic F	RF receiver
2	Power supply				
3	digital F	4		5	6
4	analogic F				
5	Lightning (through added loop)	1	2	3	

Complete study of a realistic case

The objective of the EMC conception engineer is to reach the requirements defined through the EMC tests and their specifications include in the customer needs description. By the fact, we must be able to say how the EMC tests will be conducted. That's why the first EMC conception task is to write the **EMC test plan** (ETP). As previously, we write here a very simplified ETP.

The ETP must include:

1. how the test must be applied;
2. the levels ot be applied in immunity, and the limits in emissions;
3. the conditions of work of the electronics, and the electronics failures definitions and measurements.

Complete study of a realistic case

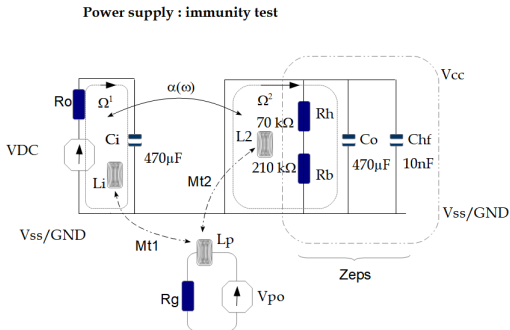
In our case, we use a loop, able to create a $400 \mu T$ magnetic field, and through a radio receptor, we will appreciate the noise emitted by the electronic. It's clear that real facilities are many more complex, complete and expensive! The failure criteria consists in loosing the capacity to measure the LDR state correctly (we may specify a limit value), or to loose the digital function working. In emission, the electronics working should not disturb the radio receptor receiving sound. This last material gives us all what we need to begin the EMC conception tasks.

Complete study of a realistic case: case 1 study

As we said we are only concerned by immunity on this case. Having the power supply equivalent graph, we just have to value the parameter depending on the layout, and add the emf coming from the lightning magnetic field. For this exercise, it is important to remember the fundamental relations on capacitance, inductance and resistance given previously on slide 123. The approach remains always the same: by computation we determine the EMC conception of our electronics including margins, then the EMC tests should confirm that the EMC requirements are reached. **Making a test is never a proof.** When people says *I've made a measurement which proof that ...* it's completely false! A measurement has meaning only if it comes as a verification of a result obtained with a theory and a numerical application.

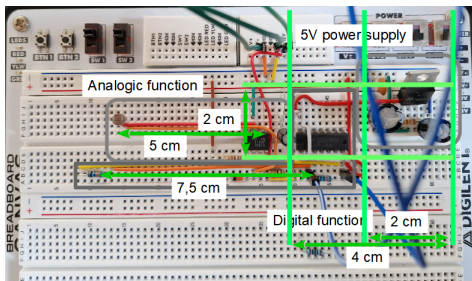
Complete study of a realistic case: case 1 study

The final objective of the EMC conception is to solve the equation $T_{\alpha} = [\zeta_{\alpha\beta} + s\mathcal{L}_{\alpha\beta}] \Omega^{\beta}$. After some evaluation and adding inductances, we complete the power supply graph for immunity:



Complete study of a realistic case: case 1 study

Inductances L_i and L_2 belong to the circuit, coming from the loops created with the wires on the board. Inductance L_p is the loop made for creating a $400 \mu T$ magnetic field applied to the circuit, and V_{po} , R_g are the generator power supplying this loop. L_{t1} and M_{t2} are the mutual inductances between the injection loop and the circuit loops. Note the difference between L_p which is a component with L_i or L_2 which are undesired loops associated with the circuit layout. For evaluating the circuit inductances, we need approximate dimensions:



Complete study of a realistic case: case 1 study

For determining the circuit inductances, remember that for a loop of typical dimension $2r$, $L \approx (\mu\pi r)/2$. On another side we have to make a probe for generating the field. At a height h from a loop plan (r, θ) , the vertical magnetic field in (r, θ, z) is (Biot & Savart law):

$$B_z = \int_{\theta} d\theta \mu_0 \frac{I r}{4\pi (\sqrt{r^2 + h^2})^2} \text{SinArct}\left(\frac{r}{h}\right) \quad (114)$$

So for N loops:

$$B_z = N\mu_0 \frac{I r}{2 (\sqrt{r^2 + h^2})^2} \text{SinArct}\left(\frac{r}{h}\right) \quad (115)$$

We verify that with 20 loops, a radius of 2 cm with 100 mA at a distance of 1 cm from the circuit, the magnetic field reaches is $44 \mu T$, which is a little more than the requirement. The associate inductance is $L_p \approx 0,70 [\mu H]$, while $L_i = 39 [nH]$ and $L_2 = 59 [nH]$. We evaluate (by the reluctance method) that $Mt1 \approx 5 [nH]$ and $Mt2 \approx 11 [nH]$.

Complete study of a realistic case: case 1 study

Circuit immunity computation. As we said, for the equipment conception, only the EMC test results has meanings. For immunity, we have to compute the graph presented slide 178, with $V_{po} \approx 5 [V]$ for $R_g = 50 [\Omega]$. Taking Ω^3 for the generator we obtain the impedance metric (considering that the component schematics are wide band ones, i.e. covering the lightning frequency band):

$$\zeta_{\alpha\beta} + s\mathcal{L}_{\alpha\beta} = \begin{bmatrix} R_0 + \frac{1}{sC_i} + sL_i & \alpha(\omega) & -sM_{t1} \\ \alpha(\omega) & sL_2 + Z_{eps} & -sM_{t2} \\ -sM_{t1} & -sM_{t2} & R_g + sL_p \end{bmatrix} \quad (116)$$

The CW source covector including the reports of final to initial conditions is given by:

$$T = \begin{bmatrix} -\frac{Q_i}{s} & -\frac{Q_{ohf}}{s} & V_{po} \end{bmatrix} \quad (117)$$

Our purpose is to solve $\Omega^\beta = (\zeta_{\alpha\beta} + s\mathcal{L}_{\alpha\beta})^{-1} T_\alpha$, then to look if the power supply $Z_{eps}\Omega^2$ is significantly disturbed or not. We take in dynamics, $\alpha(\omega) = -1/(sC_\alpha)$, $C_\alpha \approx 1 [pF]$.

Complete study of a realistic case: case 1 study

```

1 # -*- coding: utf-8 -*-
2 ---
3 Created on Mon Sep 1 15:14:22 2025
4
5 @author: olivi
6 ---
7
8 import numpy as np
9 import pylab as plt
10 #
11 # components
12 Ro=10.
13 Ci=100E-6
14 Ca=10E-12
15 Co=470E-6
16 Chf=10E-9
17 Rh=70E3
18 Rb=210E3
19 Li=39E-9
20 L2=59E-9
21 Lp=0.7E-6
22 Mt1=5E-9
23 Mt2=11E-9
24 Rg=50.
25 #
26 # lightning source temporal form
27 N=100
28 LG=np.zeros(N,dtype=float)
29 abst=np.zeros(N,dtype=float)
30 Tlg=1E-3
31 for n in range(N):
32     dt=Tlg/N
33     abst[n]=n*dt
34     LG[n]=5.*(np.exp(-n*dt/150E-6)-np.exp(-n*dt/10E-6))
35 plt.plot(abst,LG,'x',label='origin')
36 plt.grid(True)
37 plt.xlabel(u'time')
38 plt.ylabel(u'Voltage')
39 plt.title(u'Source form')
40 plt.show()
41
42 # simulation in W
43 M=1000
44 LG=np.zeros(M,dtype=float) # LG reconstruct par rMSE
45 pS=np.zeros(M,dtype-complex) # pulse spectrum
46 for q in range(1,M):
47     df=1./Tlg
48     s=1j*2.*np.pi*q*df
49     pS[q]=1./s*(1.-np.exp(-s*dt))
50 pS[0]=pS[1]
51 F=50E6/abs(np.dot(pS,np.conj(pS)))
52 print(u' F : '+str(F))
53 for n in range(N):
54     for q in range(M):
55         LGr[n]=abs(np.sqrt(np.dot(LG[n]*np.conj(pS[q]),np.dot(F,LG[n]*pS[q])))
56 plt.plot(abst,LGr,label='from W')
57 plt.grid(True)
58 # plt.xlabel(u'time')
59 # plt.ylabel(u'Voltage')
60 # plt.title(u'Source form')
61 plt.legend()
62 plt.show()
63 # computation of the noise induced by the lightning magnetic field
64 Obs=np.zeros(N,dtype=float) # noise induced in temporal space
65 Omega=np.zeros((M,3),dtype-complex)
66 ResF=np.zeros(M,dtype-complex)
67 QfI=np.zeros(M,dtype-complex)
68 #
69 ## For each n we compute the Laplace's response of the circuit then the special
70 # rms value for going back to the time space, following W process
71 # Initial conditions for capacitive interactions
72 Qohf=0.
73 Ql=0.
74 for n in range(N):
75     for q in range(1,M):
76         df=1./Tlg
77         s=1j*2.*np.pi*q*df
78         T=[0.,-Ql/s,-Qohf/s,LG[n]*pS[q]]
79         Zeps=(Rh+Rb)/(1.+s*(Rh+Rb))*(Co+Chf)
80         zeta=[(Ro+1./s*Ci),-1./s*Ca),-1./s*Ca),-s*Mt1],\
81              [-1./s*Ca),-1./s*Ca),-s*Mt1],\

```

Complete study of a realistic case: case 1 study

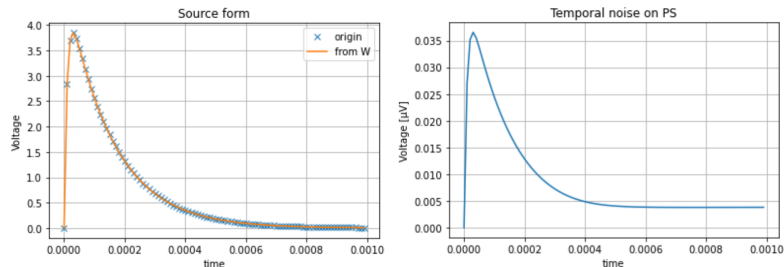
```

73 for n in range(N):
74     for q in range(1,M):
75         zeta=[[Ro+1./(s*Ci), -1./(s*Ca), -s*Mt1],\
80              [-1./(s*Ca), s*Lz+zeps, -s*mtz],\
81              [-s*Mt1, -s*Mt2, Rg+s*Lp]]
82         y=np.linalg.inv(zeta)
83         Omega[q,:]=np.transpose(np.dot(y,T))
84         ResF[q]=Zeps*Omega[q,1]
85         Qfi[q]=1./(s*Ci)*Omega[q,0]
86         #
87         ResF[0]=ResF[1]
88         Qfi[0]=Qfi[1]
89         # final conditions computation for reporting as initial conditions on T
90         Obs[n]=abs(np.sqrt(np.dot(np.conj(ResF), np.dot(F, ResF))))
91         Qohf=Qohf+Obs[n]
92         Qi=Qi+abs(np.sqrt(np.dot(np.conj(Qfi), np.dot(F, Qfi))))
93         #
94     plt.plot(abst, Obs*1E6)
95     plt.grid(True)
96     plt.xlabel(u'time')
97     plt.ylabel(u'Voltage [V]')
98     plt.title(u'Temporal noise on PS')
99     plt.show()

```

Complete study of a realistic case

The source form and output signals computed are:



The induced noise being far less than 1% of the functional power supply, remains acceptable (the conclusion is evident and we don't need any safety calculation here for confirming the safety risk level). Note that we consider the order of amplitude, not the value strictly by itself (i.e. 0,035 μV is similar to 0,1 μV !).

Complete study of a realistic case: case 1 study

Corresponding EMC test plan. The EMC test plan must describe accurately:

1. what is the test principle, see standard reference;
2. how the equipment is located and how it works;
3. how the constraint is applied;
4. how the electronics is spied for verifying that it is not disturbed.

So, in our example the test was inspired by NF EN 61000-4-9. The electronics is located on a table, as for functional tests.

Complete study of a realistic case: case 1 study

The power supply is connected with the wire leaving the electronic board rightly, at 5 cm from the common ground plane. The electronics are power supplied at their nominal voltage (± 12 [V]).

The constraint was defined previously by a declension of the lightning environment. A current of 100 mA lightning biexponential form is applied in a loop of 2 cm diameter and 20 turns, at 1 cm height from the circuit. The loop axe is located up upon the circuit loops made by its wires. An oscilloscope is used for measuring the signal on the power supply output (5 [V]). The added noise should be $< 0,1$ V.

This is an illustration: in "real life" the EMC test plan must be many more detailed, with pictures for describing accurately the setups and **not only copy-paste of the standard drawings**. Both validated by the customer and the supplier, the EMC test plan is the reference for all computations and discussions around the equipment or system EMC conception.

Complete study of a realistic case: case 1 study

We have conducted the immunity test following the EMC test plan requirements. We power supply one loop made as required for generating the desired B field.

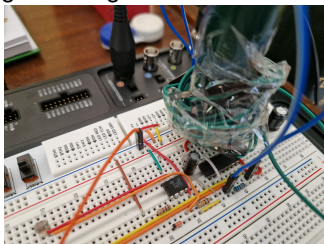


Figure: Test view

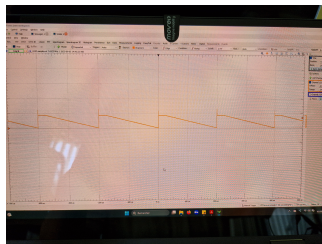


Figure: Source signal

The level of noise was less than 1 [mV] as waited by the conception.

Complete study of a realistic case: case 6 study

Now we take a look to the emission case 6.

The digital signal has its own temporal form with a time rise of 100 ns. The line output of this function is 7,5 cm long with 1 mm between each wire. The far field radiated emission coming from such a waveguide can be computed using the potential vector:

$$d\mathbf{A}^\alpha = \mu_0 \frac{d\Omega^\alpha [\mathbf{u}_r \times d\mathbf{x} \times \mathbf{u}_r]}{4\pi r} \exp\left(-s \frac{r}{c}\right) \quad (118)$$

From $d\mathbf{A}^\alpha$ we can obtain $d\mathbf{e}^\alpha$ which is a normalized (for 1 meter):

$$d\mathbf{e}^\alpha = -sd\mathbf{A}^\alpha = \mu_0 s \frac{d\Omega^\alpha [\mathbf{u}_r \times d\mathbf{x} \times \mathbf{u}_r]}{4\pi r} \exp\left(-s \frac{r}{c}\right) \quad (119)$$

From $d\mathbf{e}^\alpha$ we can compute the emf induced on any receptor, including the antennas used for the EMC radiated emission test. That's the configuration we will consider for analyzing the radiated emission of the digital function.

Complete study of a realistic case: case 6 study

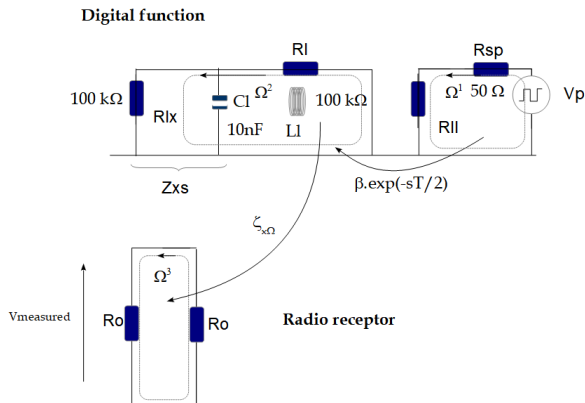
EMC test plan for emissions

Knowing the signal rise time, we are looking for radiofrequencies between 100 kHz and 26 MHz. At these frequency we remain with an antenne at 1 meter far, in near field. The receiving antenne should be parallele to the digital signal emitting line. At the end of the computation, we will determine some frequencies to be tested in the band. The 7,5 cm line is short compare to the wavelength. Let's compute the emf induced on the receiving antenna. As $y = 7,5 [cm] \ll r = 1 [m]$ we have at the antenna feet level:

$$e_x \approx -sy\mu_0\Omega \left[\frac{\exp(-s\frac{r}{c})}{r} - \frac{\exp\left(-s\frac{\sqrt{r^2+\epsilon^2}}{c}\right)}{\sqrt{r^2+\epsilon^2}} \right] \Rightarrow \zeta_{x\Omega} \approx -sy\mu_0h \left[\frac{\exp(-s\frac{r}{c})}{r} - \frac{\exp\left(-s\frac{\sqrt{r^2+\epsilon^2}}{c}\right)}{\sqrt{r^2+\epsilon^2}} \right] \quad (120)$$

ϵ being the distance between wires. The emf created by e_x , h being the effective height of the antenna, is simply he_x . At this level we have all the knowledge for establishing the graph of our emission EMC test.

Complete study of a realistic case: case 6 study



As the EMC emissions are measured entirely in frequencies, we can calculate the circuit also entirely in the Laplace's domain.

Complete study of a realistic case: case 6 study

The impedance metric for this circuit is:

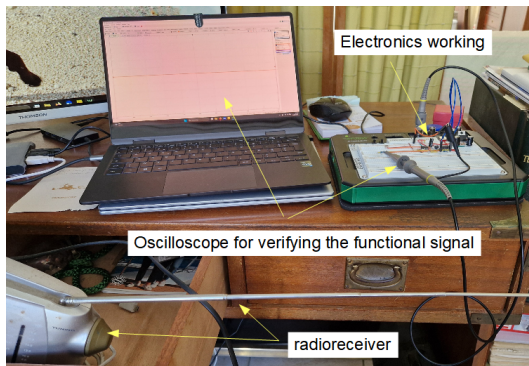
$$\zeta = \begin{bmatrix} R_{sp} + R_{ll} & 0 & 0 \\ \beta \exp\left(-s\frac{T}{2}\right) & R_l + Z_{xs} + sL_l & 0 \\ 0 & \zeta_{x\Omega} & 2R_0 \end{bmatrix} \quad (121)$$

and the source covector:

$$T = \begin{bmatrix} \frac{5}{s} \left[1 - \exp\left(-s\frac{T}{2}\right)\right] & 0 & 0 \end{bmatrix} \quad (122)$$

Complete study of a realistic case

We made the experience, locating a radioreceiver near the electronics working. No added noise was heard in all the AM and SW bands.



Complete study of a realistic case

Each time an interaction is studied, its case is painted of grey, showing that the task is made.

	A	B	C	D	E
1		Power supply	digital F	analogic F	RF receiver
2	Power supply				
3	digital F	4		5	
4	analogic F				
5	Lightning (through added loop)		2	3	

Once the matrix is completely covered, the EMC work is done!

EMC conception global approach

There are some simple rules that help to solve any EMC problem:

1. we always start with the simplest model as possible. This implies for $x < \lambda$, then if necessary (depending on the sources), to work with models for $x > \lambda$.
2. Working under the interaction matrix user's guide, once a result is validated with a first level model we stop the study. This means that the result is far less or far over the limits accepted, with a factor 2 or 3. The unique condition is to demonstrate that the first order model gives results worse than a more accurate one.
3. If observables and limits are close, then we must work with more accurate models, and so on.
4. What must never be forgotten in EMC is that we are seeking to demonstrate the absence of risk, not to accurately reproduce measurements. Measurements should only confirm the prediction, and therefore necessarily fall below the calculated levels.

EMC conception global approach

When EMC design work begins, a lot of information is missing. Nevertheless, the **EMC test plan** must be written, even if this means leaving certain details pending and making assumptions. However, the initial fundamental elements, such as the validation test configurations, can be expressed. At the same time, the document detailing the analyses conducted (the **EMC design report**), with its interaction matrix and related calculations, will gradually feed into the test plan. The two documents are enriched and converge after a few weeks or months into their final versions, reflecting a well-executed EMC design.

EMC conception global approach

I hope that this course has provided you with a solid foundation for your EMC design and inspired you to further explore tensor analysis of networks for EMC.

Olivier Maurice